Field-space Surprises in Multi-field Preheating

Evangelos Sfakianakis

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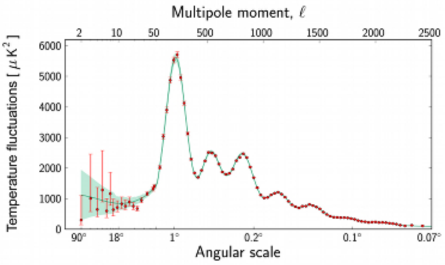
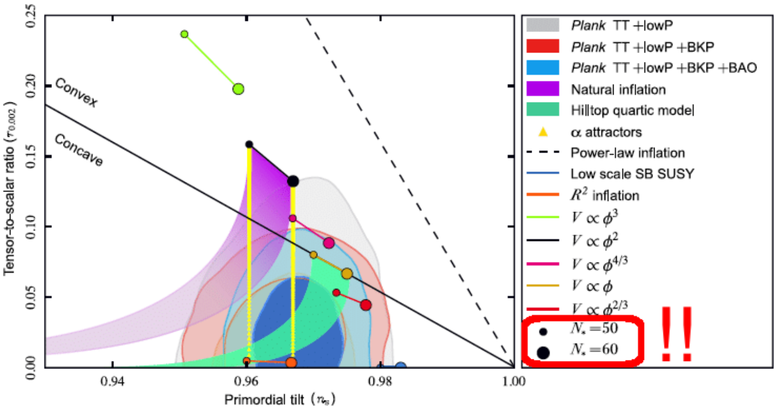
(Simple) Single field inflation:

- **Solves** horizon, flatness, monopole **problems**
- **Explains fluctuations** as stretched quantum mechanical perturbations
- Predicts a **nearly scale invariant** spectrum (of tunable amplitude)
- Predicts **Gaussian** perturbations



- Spectral index not flat by 5σ
- Spectral index running is small
- $|f_{NL}| \lesssim \mathcal{O}(1)$

Hints from the sky



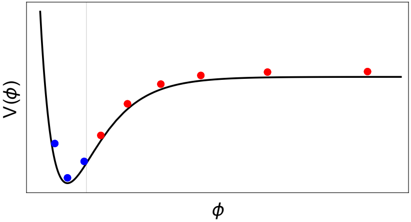
Many models with **different motivation**.

$\Downarrow$

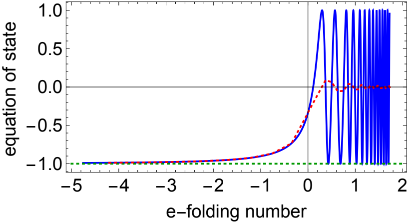
They all share the **same uncertainty**.

Inflation must end

- The inflaton rolls on a flat potential.
- The inflaton oscillates.



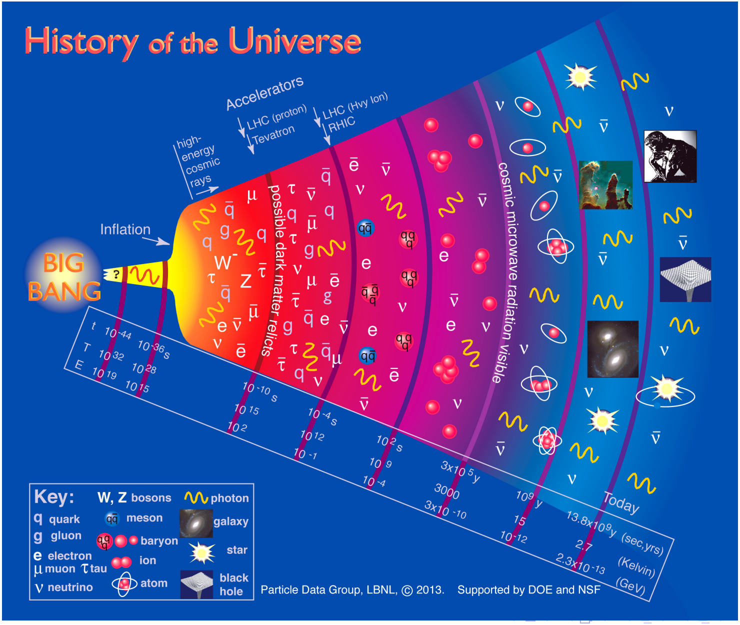
- During inflation: $p \simeq -\rho$
- After inflation: $V(\phi) \approx \frac{1}{2} m^2 \phi^2$ and $p \to 0$



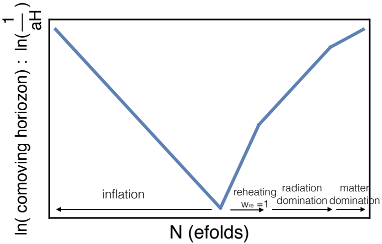
The inflaton **must** transfer its energy to radiative degrees of freedom, setting the stage for BBN.

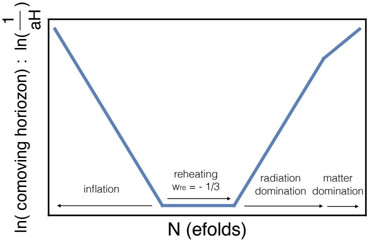
This process is called **reheating**.

The Big picture

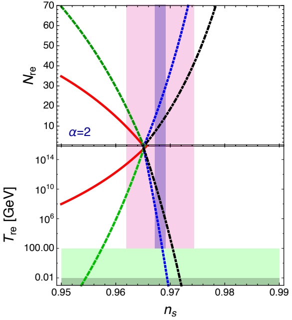


Reheating effects





Cook et al. 2015



The **reheating history** connects the times of horizon exit & re-entry of perturbations $\Rightarrow$ **shifts CMB observables**

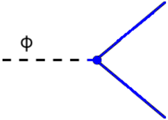
"The value of $\mathcal{N}_*$ is not well constrained and depends on unknown details of reheating"

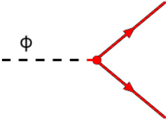
CMB-S4 Science Book, 2016

Perturbative reheating

Introduce couplings $g\phi\chi^2$ or $h\phi\bar{\psi}\psi$ and assume $m_\phi \gg m_\chi, m_\psi$

$$\Gamma_{\phi\to\chi\chi} = \frac{g^2}{8\pi m_\phi}$$





$$\Gamma_{\phi\to\bar{\psi}\psi} = \frac{h^2 m_\phi}{8\pi}$$

We can describe the decays as an extra friction term

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + m^2\phi = 0$$

Reheating occurs for $H \leq \Gamma$. In thermal equilibrium

$$\rho \simeq \frac{\pi^2}{30} g_* T^4 = 3M_{\rm Pl}^2 H^2$$

where the reheat temperature is

$$T_{\rm reh} \simeq \left(\frac{90}{g_* \pi^2}\right)^{1/4} \sqrt{\Gamma M_{\rm Pl}}$$

Parametric resonance: preheating

Bose enhancement changes the game

$$\ddot{\chi} + 3H\dot{\chi} + \frac{k^2}{a^2}\chi + g\phi\chi = 0$$

Neglect the expansion ($H = 0$) and take $\phi(t) = \Phi_0 sin(mt)$

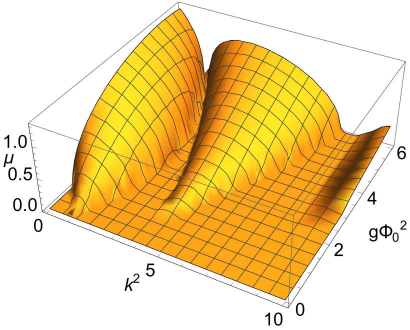
$$\boxed{\ddot{\chi} + \left[k^2 + g\Phi_0 \cos(mt)\right]\chi = 0}$$

An equation of the form $\dot{x} = A(t)x$, where $A(t)$ is periodic, $A(t+T) = A(t)$, has solutions of the form

$$x(t) = c_1 P(t) e^{\mu t} + c_2 P(t) e^{-\mu t}$$

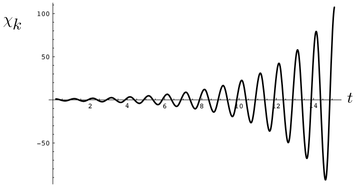
where μ is called the **Floquet exponent**.

Floquet charts

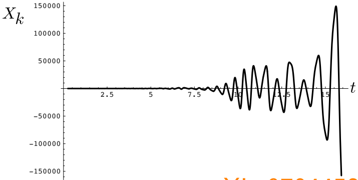


We can read off the regions where the Floquet chart leads to large amplification $\chi_k(t) \sim e^{\mu_k t}$.

In a static universe we see exponential enhancement $e^{\mu t}$



In an expanding universe, the qualitative behaviour remains



arXiv:9704452

Non-adiabaticity

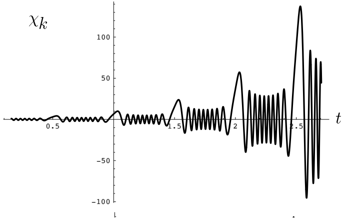
$$\ddot{\chi} + \omega^2(t)\chi = 0$$

For $\omega^2 \gg 1/T$ and $\frac{\dot{\omega}}{\omega^2} \ll 1$

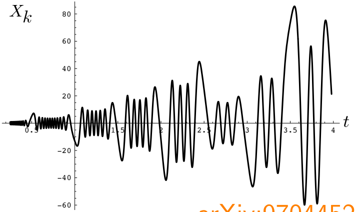
$$\chi \simeq \frac{1}{\sqrt{\omega}} \exp\left[\pm i \int \omega dt\right]$$

When the **adiabaticity condition** is violated, we get a sudden **burst of particle production**.

In a static universe particle production is clear.



In an expanding universe, we see **stochastic resonance**.



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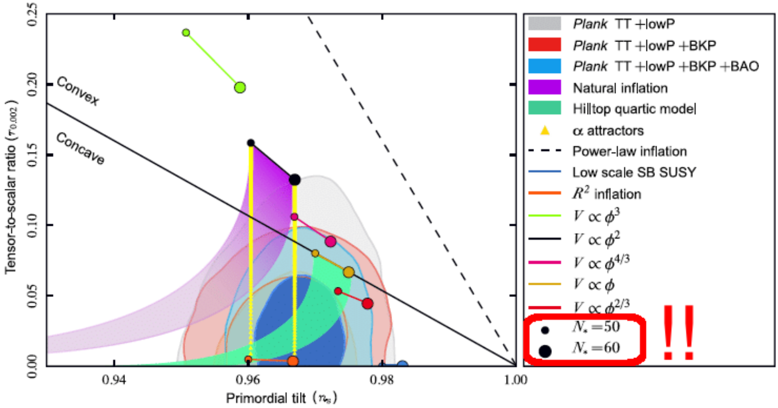
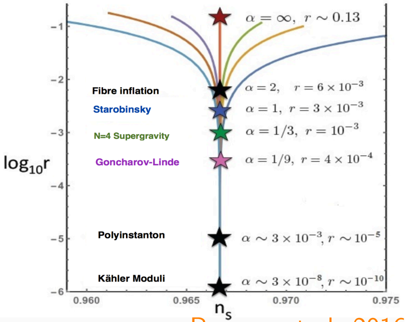
Anticipating upcoming data



The time of horizon-exit is being constrained, begging for a better understanding of reheating.

α-attractors



Burgess et al. 2016

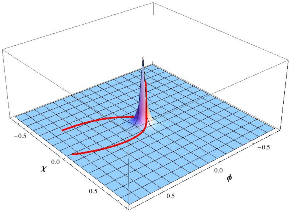
- String theory compactification: Fibre inflation
- Supergravity, e.g. T-model

all lead to a **hyperbolic field-space**.
Canonically normalizing the inflaton leads to a potential

$$V \sim V_0 \left(1 - 2e^{-\sqrt{2}\phi/\sqrt{3\alpha}} + \ldots\right)$$

and the "universal" predictions $\boxed{n_s \simeq 1 - \frac{2}{N}\,, \qquad r \simeq \frac{12\alpha}{N^2}}$

Non-minimal couplings



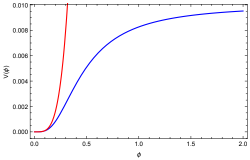
Non-minimal coupling to gravity:

$$\mathcal{L} \subset \frac{1}{2} M_{\rm Pl}^2 R \to \frac{1}{2} M_{\rm Pl}^2 R + \xi \phi^2 R$$

Example: **Higgs inflation**

The conformal transformation from the Jordan to the Einstein frame leads to

$$\tilde{V} \sim \lambda \phi^4 \to V \sim \frac{\lambda}{\xi^2} \left(1 - \frac{2}{\xi \phi^2} \right)$$



The predictions are those of the Starobinsky model $\mathcal{L} = R + \xi R^2$

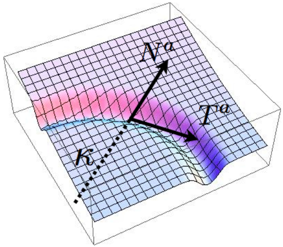
$$\boxed{n_s \simeq 1 - \frac{2}{N}\,, \quad r \simeq \frac{12}{N^2}}$$

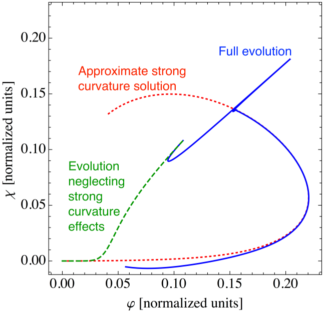
General Model-building

At high energies, we expect

- multiple fields and
- more complicated couplings, e.g. $\mathcal{L} \subset f(\phi)(\partial\chi)^2 + \tilde{f}(\chi)(\partial\phi)^2$

leading to interesting inflationary dynamics.





During inflation, field-space features received significant attention (van Tent et al 2003, Achucarro et al 2010, ...).

Recent **novel trajectories** supported by field-space curvature reveal interesting connections to the Swampland program (a whole other talk!!)

“Family tree” of this work

inflation (80’s)

$\Downarrow$

preheating (late 90’s)

$\Downarrow$

field-space effects (2000’s - ...)

$\Downarrow$

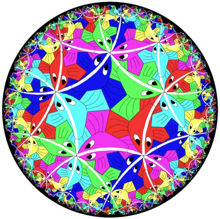
Higgs inflation (2008) + α-attractors (2010’s)

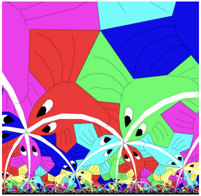
$\Downarrow$

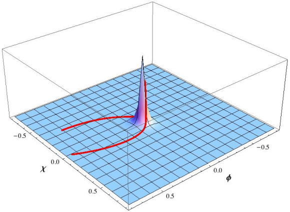
Field-space effects in multi-field preheating,
focusing on Higgs-like inflation & α attractors

Both families of models (and many more) **necessarily** include multiple fields **AND** non-canonical kinetic terms.

The field-space manifolds have either global or local features







General Model set-up

Action

$$\mathcal{S} = \int d^3x dt \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} \mathcal{G}_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi^I) \right]$$

Background fields:

$$\boxed{\mathcal{D}_t \dot{\phi}^I + 3H\dot{\phi}^I + \mathcal{G}^{IK} V_{,K} = 0}$$

where $\mathcal{D}_t \dot{\phi}^I \equiv \ddot{\phi}^I + \Gamma^I_{JK} \dot{\phi}^J \dot{\phi}^K$

Fluctuations:

$$\boxed{\ddot{Q}^I_k + 3H\dot{Q}^I_k + \left[\frac{k^2}{a^2} \delta^I_J + \mathcal{M}^I_J \right] Q^I_k = 0}$$

where

$$\mathcal{M}^I_J = \mathcal{G}^{IK} \mathcal{D}_J \mathcal{D}_K V - \mathcal{R}^I_{LMJ} \dot{\phi}^L \dot{\phi}^M - \frac{1}{M_{Pl}^2 a^3} \mathcal{D}_t \left(\frac{a^3}{H} \dot{\phi}^I \dot{\phi}_J \right)$$

Quantizing the fluctuations

$$\hat{X}^{\phi}(x^{\mu}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\left(v_k e_1^{\phi} \hat{b}_{\mathbf{k}} + w_k e_2^{\phi} \hat{c}_{\mathbf{k}} \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \right.$$
$$\left. + \left(v_k^* e_1^{\phi} \hat{b}_{\mathbf{k}}^{\dagger} + w_k^* e_2^{\phi} \hat{c}_{\mathbf{k}}^{\dagger} \right) e^{-i\mathbf{k}\cdot\mathbf{x}} \right],$$

$$\hat{X}^{\chi}(x^{\mu}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\left(y_k e_1^{\chi} \hat{b}_{\mathbf{k}} + z_k e_2^{\chi} \hat{c}_{\mathbf{k}} \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \right.$$
$$\left. + \left(y_k^* e_1^{\chi} \hat{b}_{\mathbf{k}}^{\dagger} + z_k^* e_2^{\chi} \hat{c}_{\mathbf{k}}^{\dagger} \right) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

$$\left(v_k'' + \Omega_{(\phi)}^2 v_k \right) e_1^{\phi} = -a^2 \mathcal{M}^{\phi}{}_{\chi} y_k e_1^{\chi},$$
$$\left(w_k'' + \Omega_{(\phi)}^2 w_k \right) e_2^{\phi} = -a^2 \mathcal{M}^{\phi}{}_{\chi} z_k e_2^{\chi},$$
$$\left(y_k'' + \Omega_{(\chi)}^2 y_k \right) e_1^{\chi} = -a^2 \mathcal{M}^{\chi}{}_{\phi} v_k e_1^{\phi},$$
$$\left(z_k'' + \Omega_{(\chi)}^2 z_k \right) e_2^{\chi} = -a^2 \mathcal{M}^{\chi}{}_{\phi} w_k e_2^{\phi},$$

Quantizing the fluctuations

For motion **along a single-field attractor**, quantization is simple

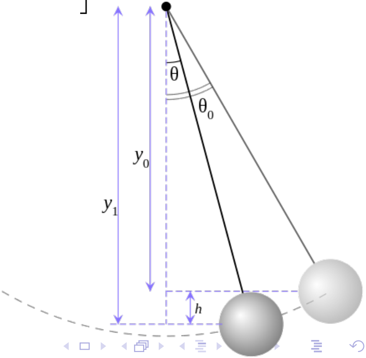
$$\hat{Q}^{\phi}(x^{\mu}) = \sqrt{\mathcal{G}^{\phi\phi}}\, a(t) \int \frac{d^3k}{(2\pi)^{3/2}} \left[v_k \hat{b}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + c.c. \right]$$

$$\hat{Q}^{\chi}(x^{\mu}) = \sqrt{\mathcal{G}^{\chi\chi}}\, a(t) \int \frac{d^3k}{(2\pi)^{3/2}} \left[z_k \hat{c}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + c.c. \right]$$

Re-write as a "harmonic" oscillator

$$\delta\tilde{\phi}_k{}'' + \Omega^2_{(\phi)}(k,\tau)\, \delta\tilde{\phi}_k \simeq 0$$

$$\delta\tilde{\chi}_k{}'' + \Omega^2_{(\chi)}(k,\tau)\, \delta\tilde{\chi}_k \simeq 0$$



Effective Mass-squared: Ingredients

$$\partial_\tau^2 \delta\tilde{\chi}_k + (k^2 + a^2 m_{\text{eff},\chi}^2)\delta\tilde{\chi} = 0$$

$$\boxed{m_{\text{eff},\chi}^2 = m_{1,\chi}^2 + m_{2,\chi}^2 + m_{4,\chi}^2}$$

$$m_{1,\chi}^2 \equiv \; \mathcal{G}^{\chi K}\left(\mathcal{D}_\chi \mathcal{D}_K V\right) \;\longleftrightarrow\; \text{potential gradient}$$

$$\boxed{m_{2,\chi}^2 \equiv \; -\mathcal{R}^{\chi}{}_{LM\chi}\dot{\varphi}^L\dot{\varphi}^M = \frac{1}{2}\mathcal{R}\dot{\phi}^2 \;\longleftrightarrow\; \text{non-trivial field-space manifold}}$$

$$m_{4,\chi}^2 \equiv \; -\frac{1}{6}R \;\longleftrightarrow\; \text{changes in the background spacetime}$$

Complex fields in supergravity lead to the **2-field Lagrangian**

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu \chi \partial^\mu \chi + e^{2b(\chi)} \partial_\mu \phi \partial^\mu \phi\right) - V(\phi, \chi)$$

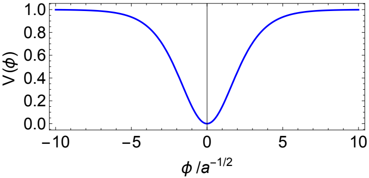
For single-field motion $\chi = 0$

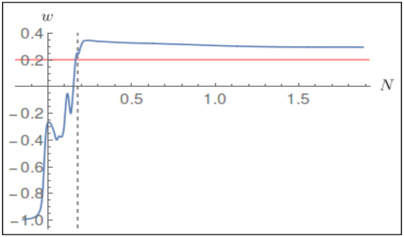
$$V(\phi, \chi = 0) = \mu^2 \alpha \left| \tanh(\phi/\sqrt{6\alpha}) \right|^2$$

The **field-space Ricci scalar** is

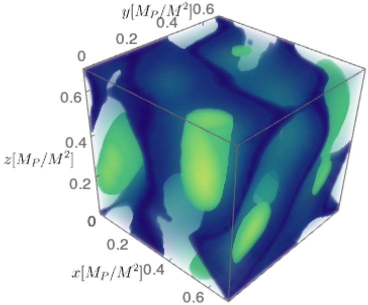
$$\mathcal{R} = -\frac{4}{3\alpha}$$

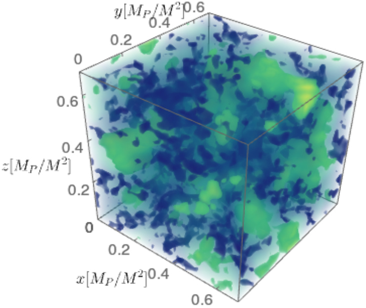
Smaller $\alpha \Rightarrow$ highly curved manifold



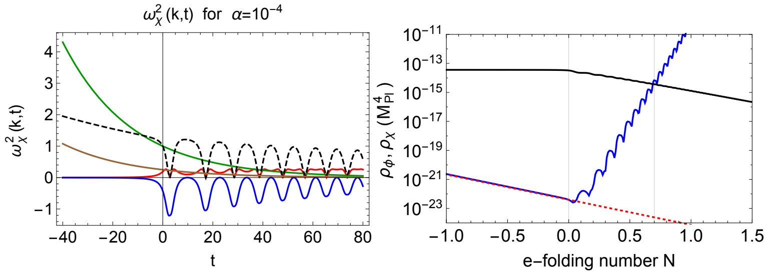


Lattice simulations
(Krajewski et al, 2018)
showed
very efficient preheating
for $\alpha \ll 1$





Effective frequency



$$m_{2,\phi}^2 = \frac{1}{2} R \left(\frac{d\phi}{dt}\right)^2 \propto -\frac{1}{\alpha} \times \left(\frac{\sqrt{\alpha}}{\mathcal{O}(1)}\right)^2 = -\mathcal{O}(1)$$

During each background oscillation
the χ field undergoes **tachyonic amplification**.

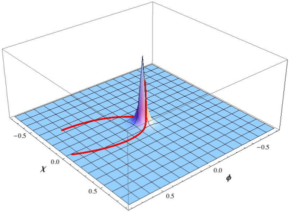
$\Rightarrow$ **Preheating is faster for larger curvature**.

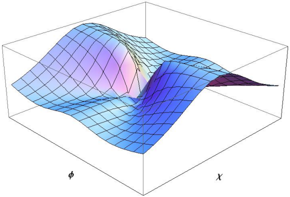
Higgs-like inflation

$$S_{\mathrm{Jordan}} = \int d^4x\sqrt{-\tilde{g}}\left[f(\phi^I)\tilde{R} - \frac{1}{2}\delta_{IJ}\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \tilde{V}(\phi^I)\right]$$

$$\Downarrow \boxed{g_{\mu\nu}(x) \propto f(\phi^I(x))\,\tilde{g}_{\mu\nu}(x)} \Downarrow f(\phi^I) \subset \xi\phi^2$$

$$S_{\mathrm{Einstein}} = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}\mathcal{G}_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$





- $\frac{1}{2}G_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J$ leads to a **locally curved manifold**.
- The potential exhibits plateaus, where inflation proceeds.

Effective Mass-squared reminder

$$\partial_\tau^2 \delta\tilde{\chi}_k + (k^2 + a^2 m^2_{\text{eff},\chi})\delta\tilde{\chi} = 0$$

$$\boxed{m^2_{\text{eff},\chi} = m^2_{1,\chi} + m^2_{2,\chi} + m^2_{4,\chi}}$$

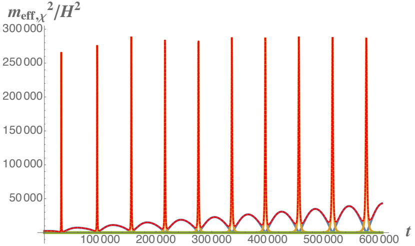
$$m^2_{1,\chi} \equiv \ \mathcal{G}^{\chi K} \left(\mathcal{D}_\chi \mathcal{D}_K V\right) \ \longleftrightarrow \text{potential gradient}$$

$$\boxed{m^2_{2,\chi} \equiv \ -\mathcal{R}^{\chi}{}_{LM\chi}\dot{\varphi}^L \dot{\varphi}^M = \frac{1}{2}\mathcal{R}\dot{\phi}^2 \ \longleftrightarrow \text{non-trivial field-space manifold}}$$

$$m^2_{4,\chi} \equiv \ -\frac{1}{6}R \ \longleftrightarrow \text{changes in the background spacetime}$$

Effective Mass-squared: $\xi \gg 1$

An "unusual" way for **adiabaticity violation** for $m^2_{2,\chi} \propto \mathcal{R}\dot{\phi}^2$



We define

$$\mathcal{A} \equiv \frac{\Omega'}{\Omega^2}$$

where

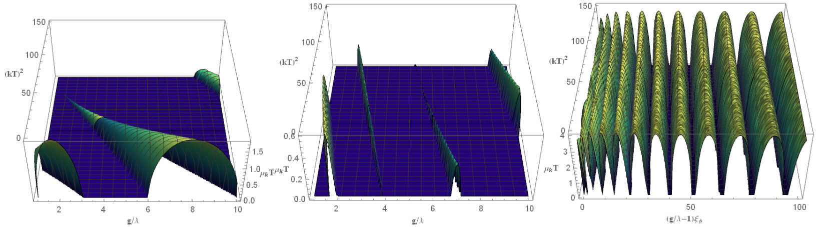
$$\Omega^2 = k^2 + a^2 m^2_{\mathrm{eff},\chi}$$

Adiabaticity is violated for $\Omega' \gg \Omega^2$, rather than $\Omega \approx 0$.

A broad range of wavenumbers is excited $k \lesssim \xi H_{\mathrm{end}}$

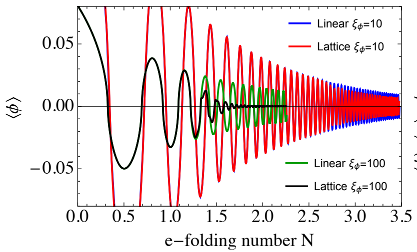
Linear analysis (VERY briefly)

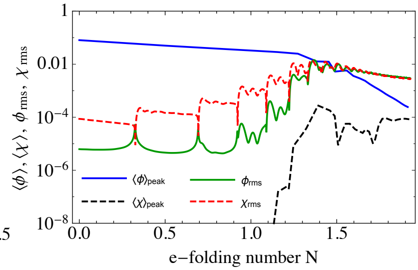
$\xi = 0$ $\xi = 10$ $\xi = 100$

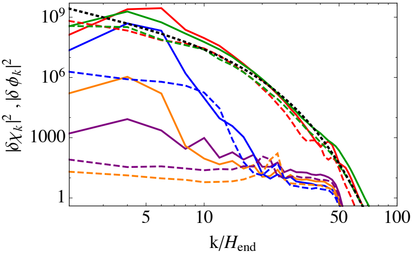


Dense instability bands hint at
efficient particle production

Need for
lattice simulations



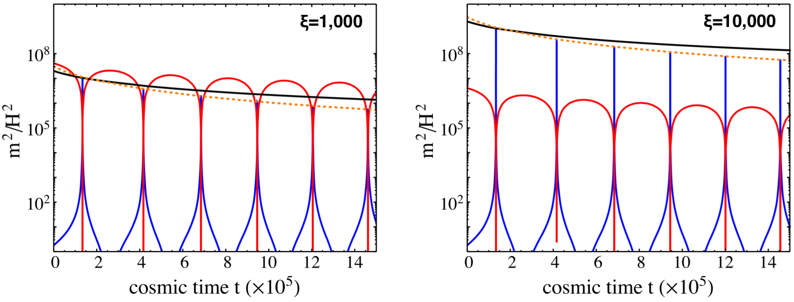




Non-minimal couplings quickly lead to a thermal radiation bath while preserving CMB predictions

Actual Higgs inflation

Higgs inflation is a multi-field non-minimally coupled model with known SM couplings $\Rightarrow$ the inflaton decays into W, Z bosons.

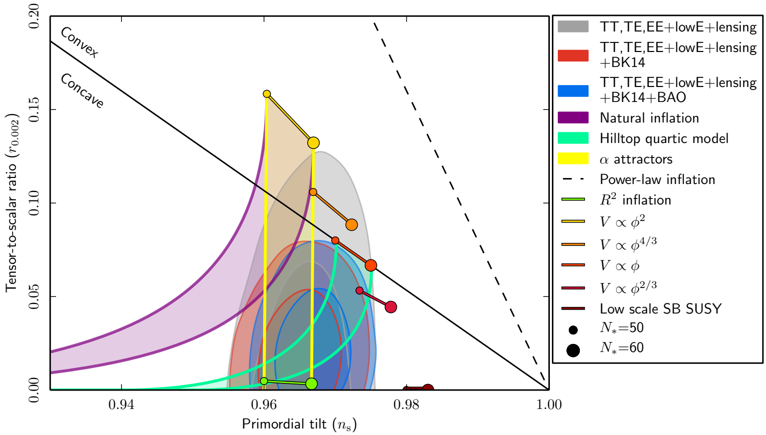


$$m_{\rm spike} \sim \xi H_{\rm end}$$

$$m_B \sim \frac{10^5}{\sqrt{\xi}} H_{\rm end}$$

For $\xi \gtrsim 10^3$ preheating completes within **ONE** oscillation

Thank you . . .



Understanding **preheating** in major plateau models
reduces theoretical **error-bars** of the $n_s - r$ plot
& allows for **comparison of Higgs inflation models**

References

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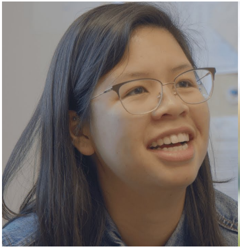
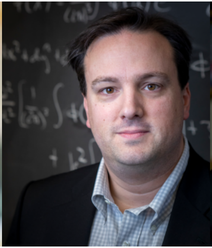
Acknowledgments

My fantastic preheating collaborators!






Questions?