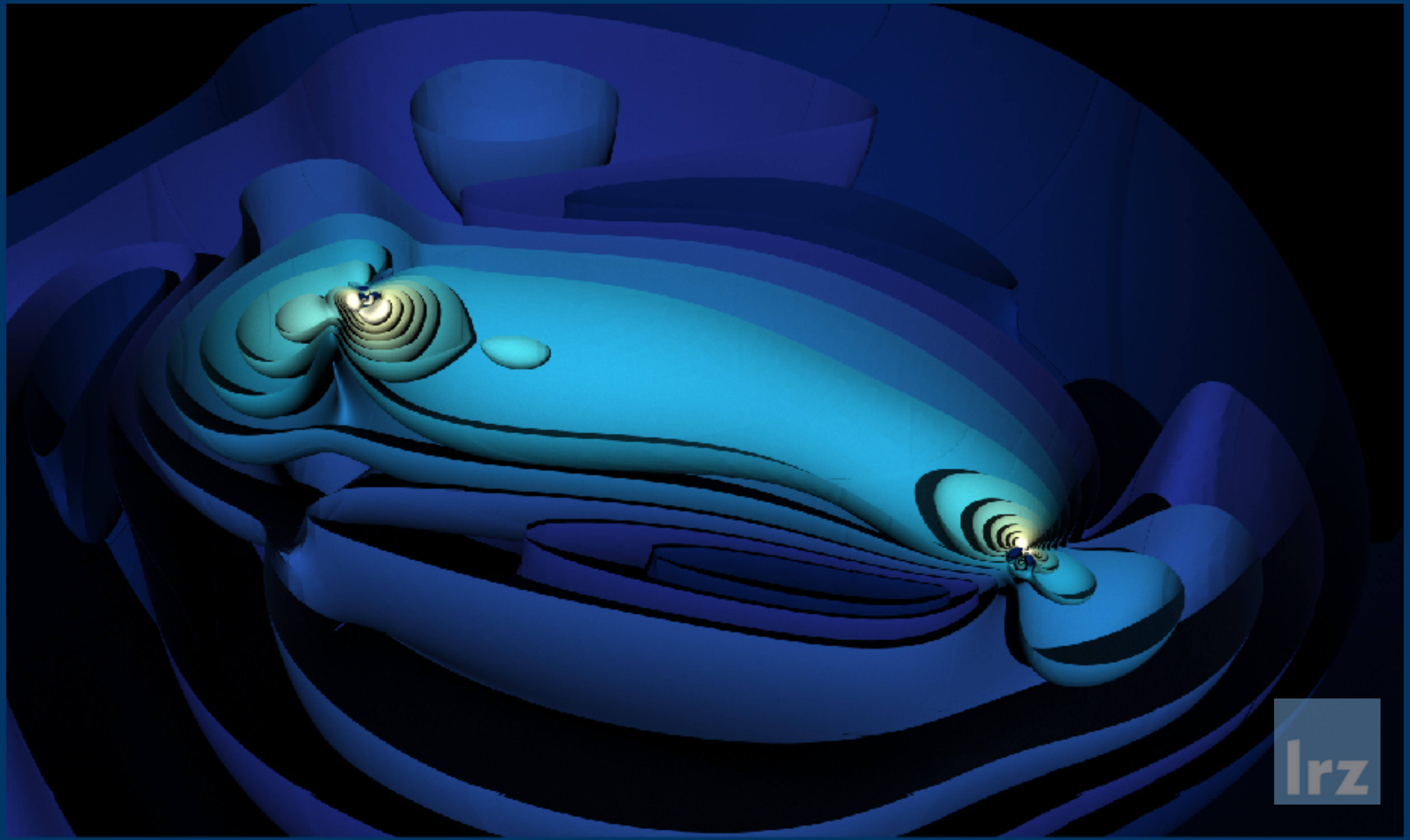
What can numerical relativity (NR) do for dark matter particle physics?

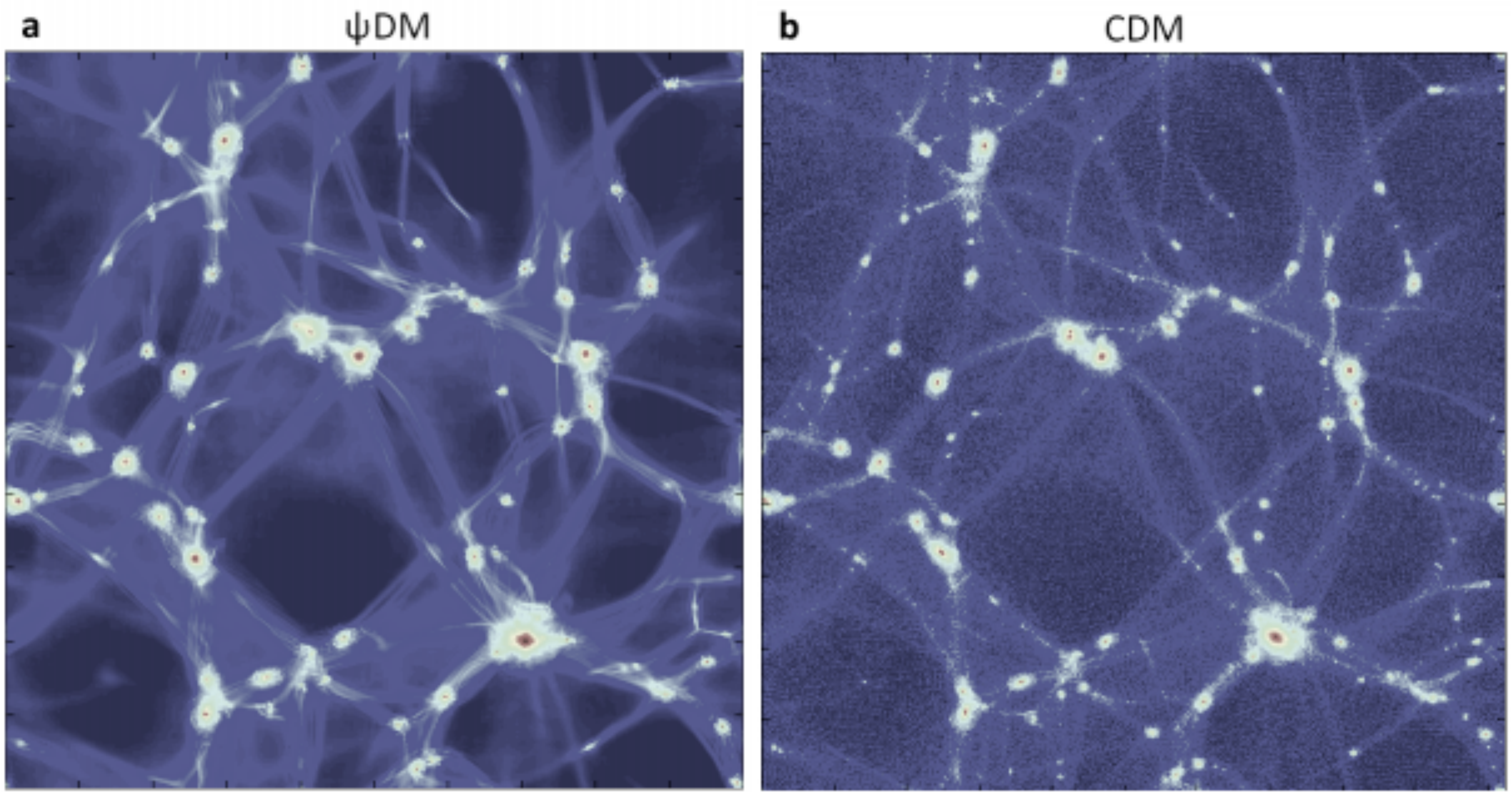
A collaborator's guide to numerical relativity



Katy Clough

UNIVERSITY OF OXFORD

Why study light DM in strong gravity?



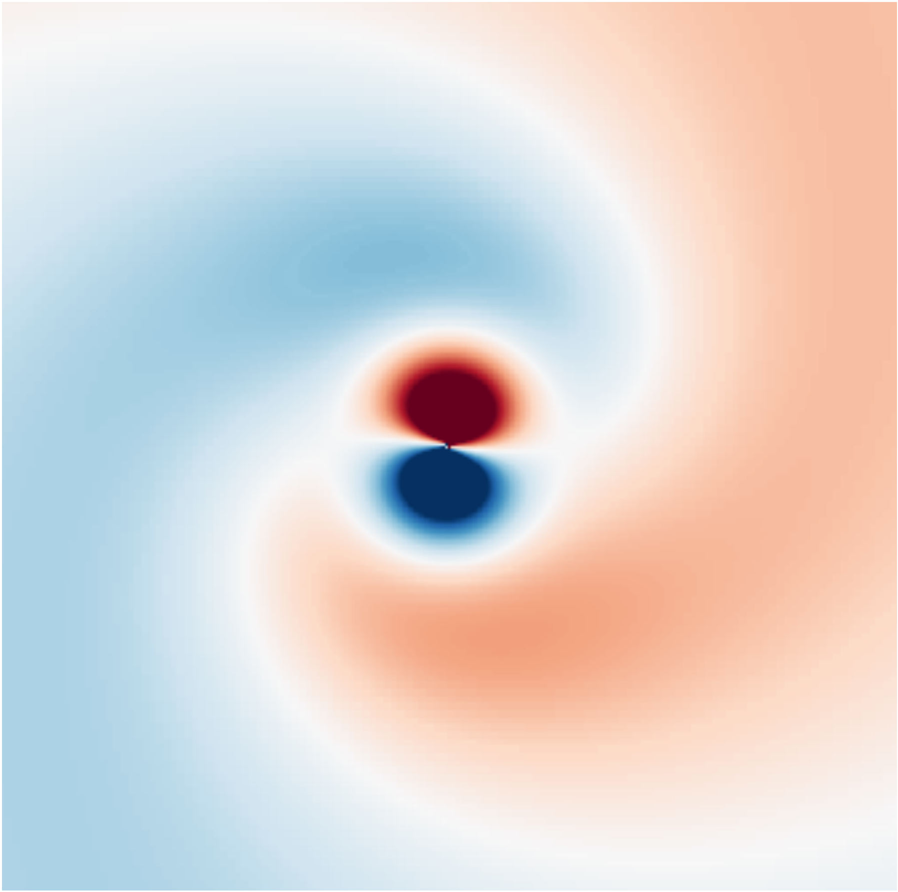
Nature Phys. 10, 496-499
Schive et al. 2014

Plan for the talk

What can numerical relativity do for dark matter particle physics?

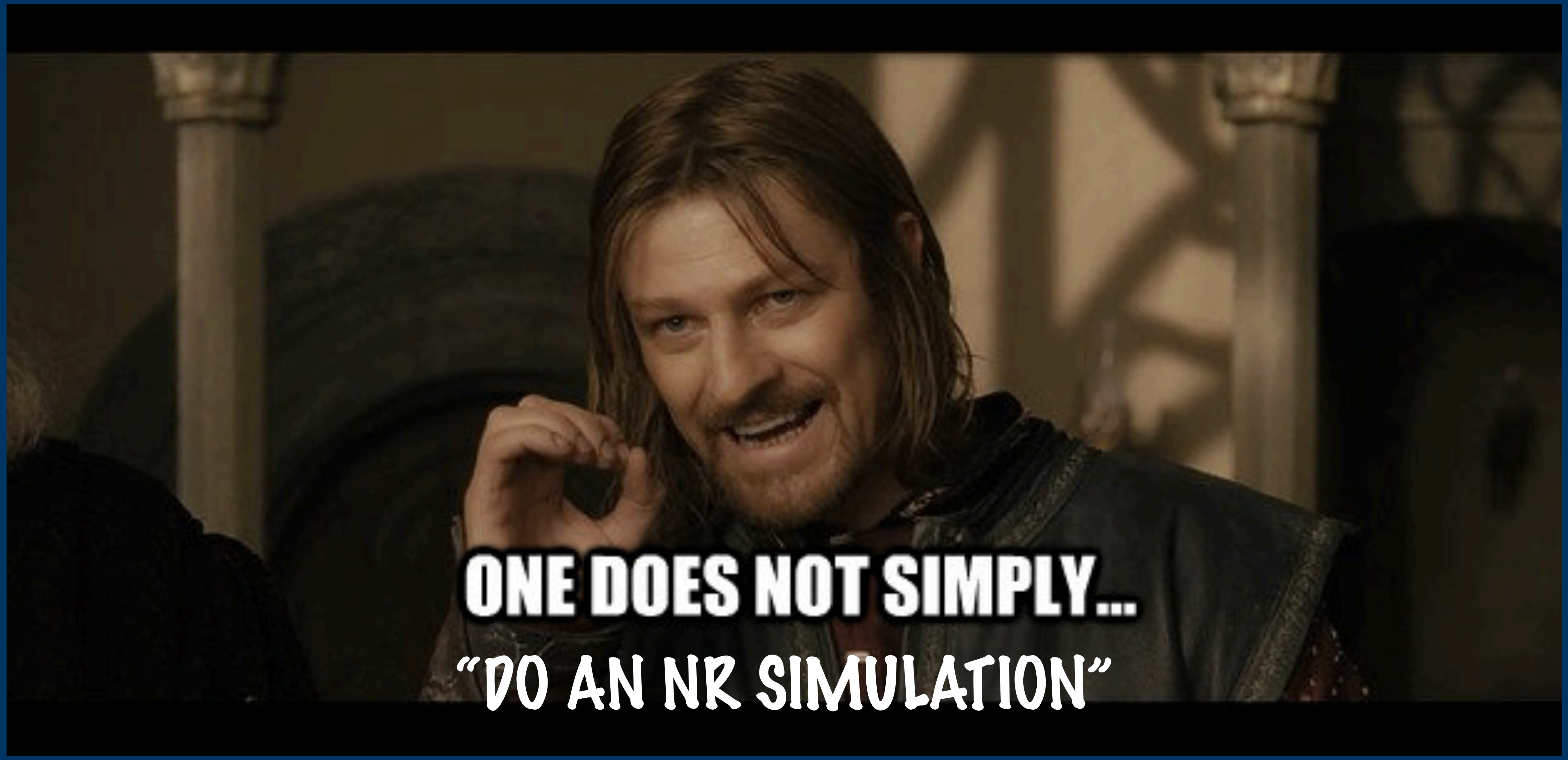
- A collaborators guide to numerical relativity
- Application to dark matter (DM)

  - e.g. Light DM structures in strong gravity regimes around black holes



  - Simulating heavy (particle) DM in numerical relativity

A collaborator’s guide to numerical relativity

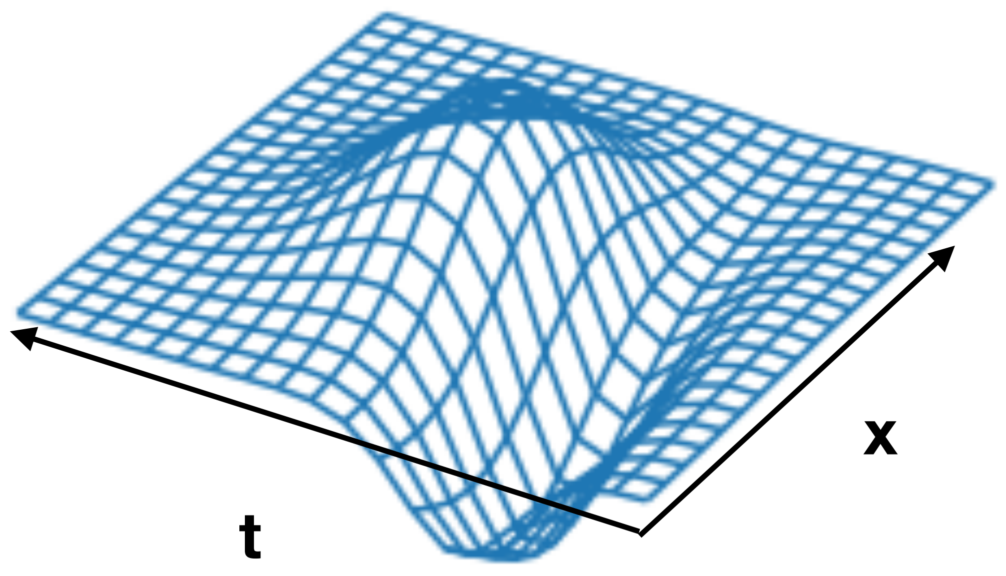


A collaborators guide to numerical relativity

What is NR?

The metric

$$ds^2 = \begin{pmatrix} dt & dx & dy & dz \end{pmatrix} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$



"The spacetime metric"

$$g_{ab}(t, \vec{x})$$

The evolution equation

$$R_{ab} - R/2\ g_{ab} = 8\pi\ T_{ab}$$

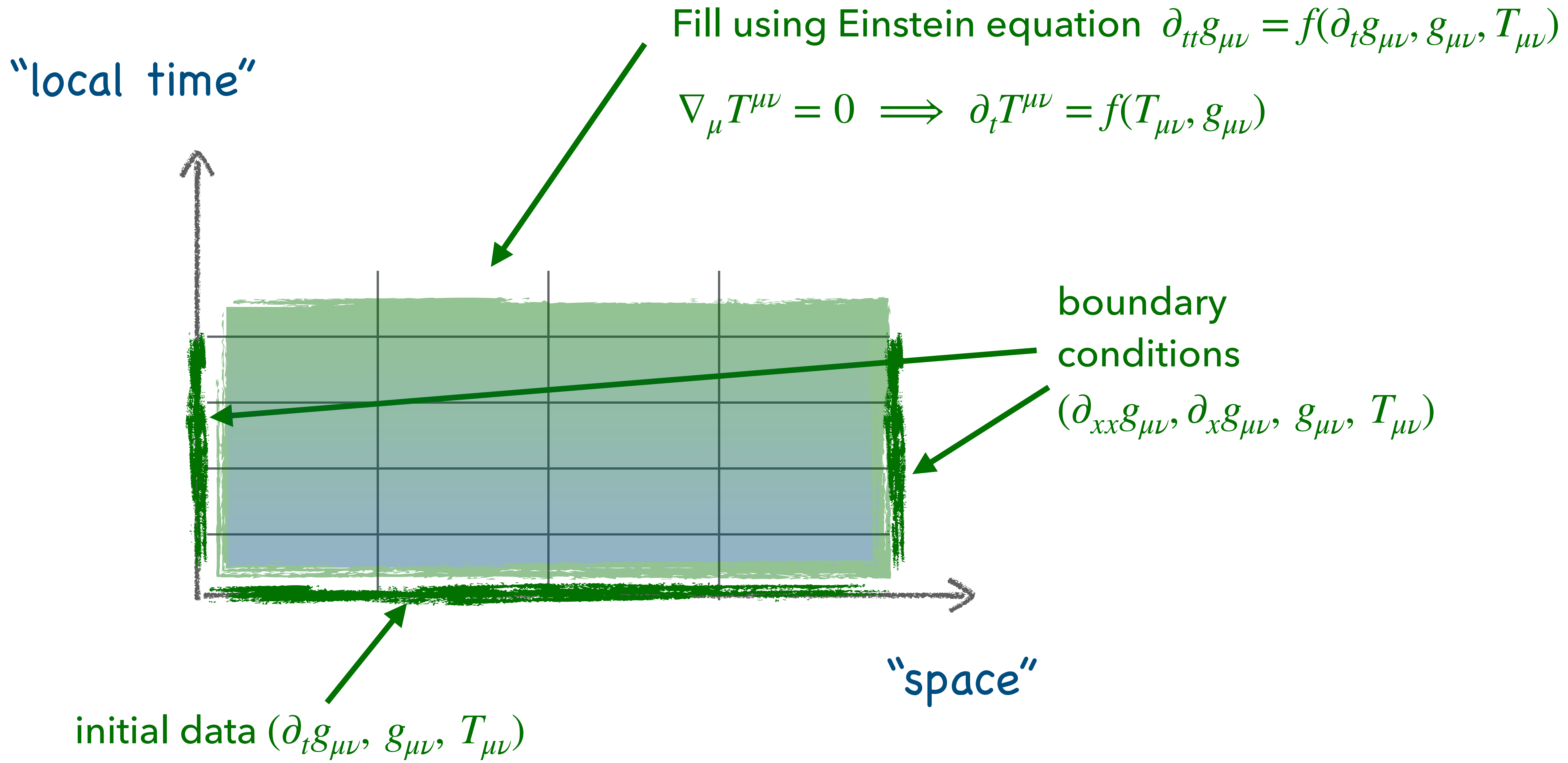
$f(\partial^2 g_{ab}, \partial g_{ab}, g_{ab})$

"Curvature"

"Energy-Momentum"

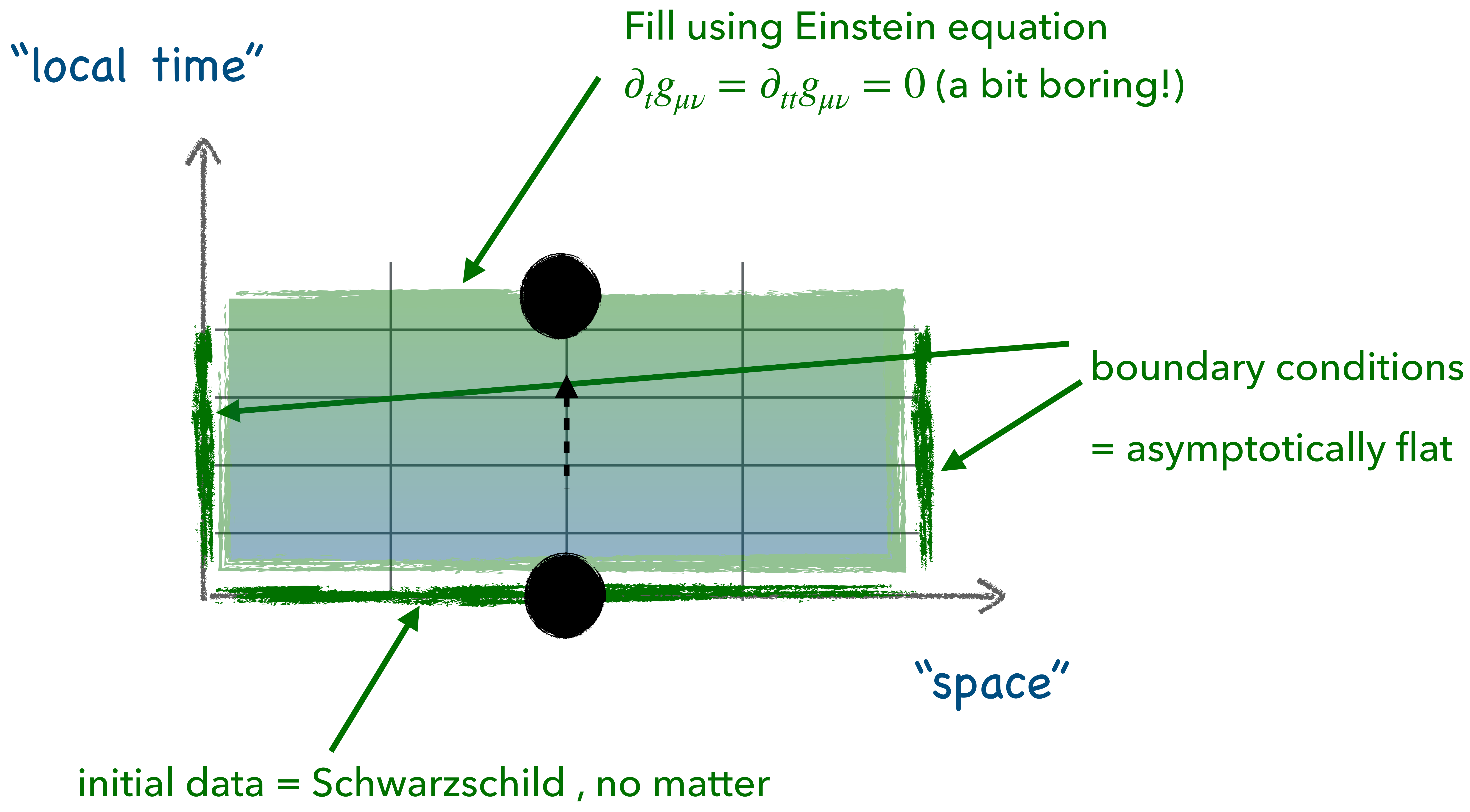
A collaborators guide to numerical relativity

What is NR?



A collaborators guide to numerical relativity

What is NR?



A collaborators guide to numerical relativity

Practical issues in NR

I will now gloss over a large number of practical issues:

- Initial data
- Gauge choice
- Stability of formulation
- Constraints
- Interpretation of results
- …

A collaborators guide to numerical relativity

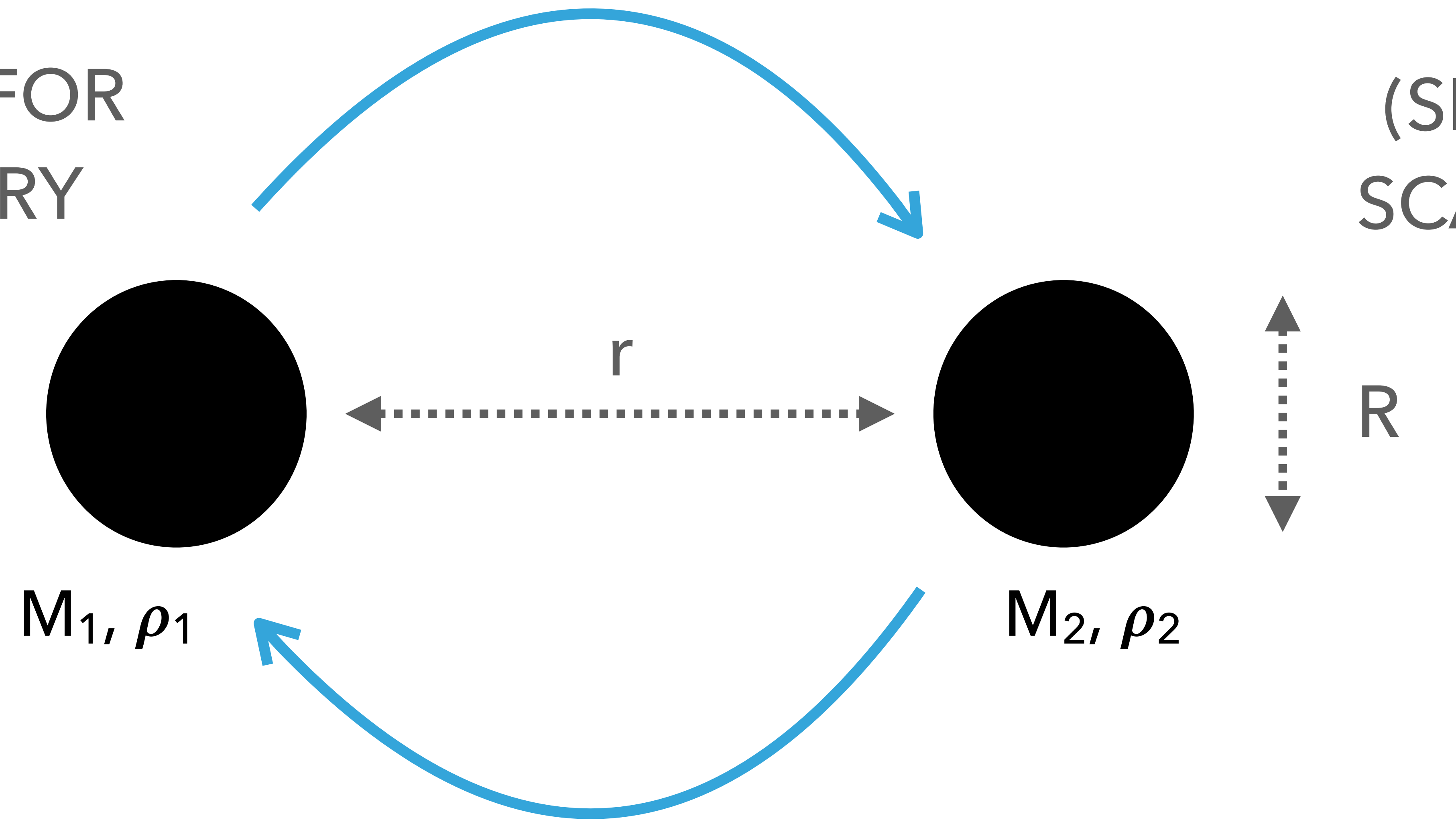
When should I use NR?

DYNAMICAL SPACETIME

(ALTHOUGH CAN BE USEFUL FOR FINDING MATTER STATIONARY STATES DYNAMICALLY)

NO PERTURBATIVE EXPANSION

(SIMILARITY OF LENGTH SCALES BOTH A FEATURE AND A BUG)



GRAVITATIONAL BACKREACTION (STRONG GRAVITY)

(ALTHOUGH CAN BE USEFUL FOR FINDING EVOLUTION OF MATTER IN DECOUPLING LIMIT)

Application to light dark matter (sub eV)

Light dark matter is wave-like

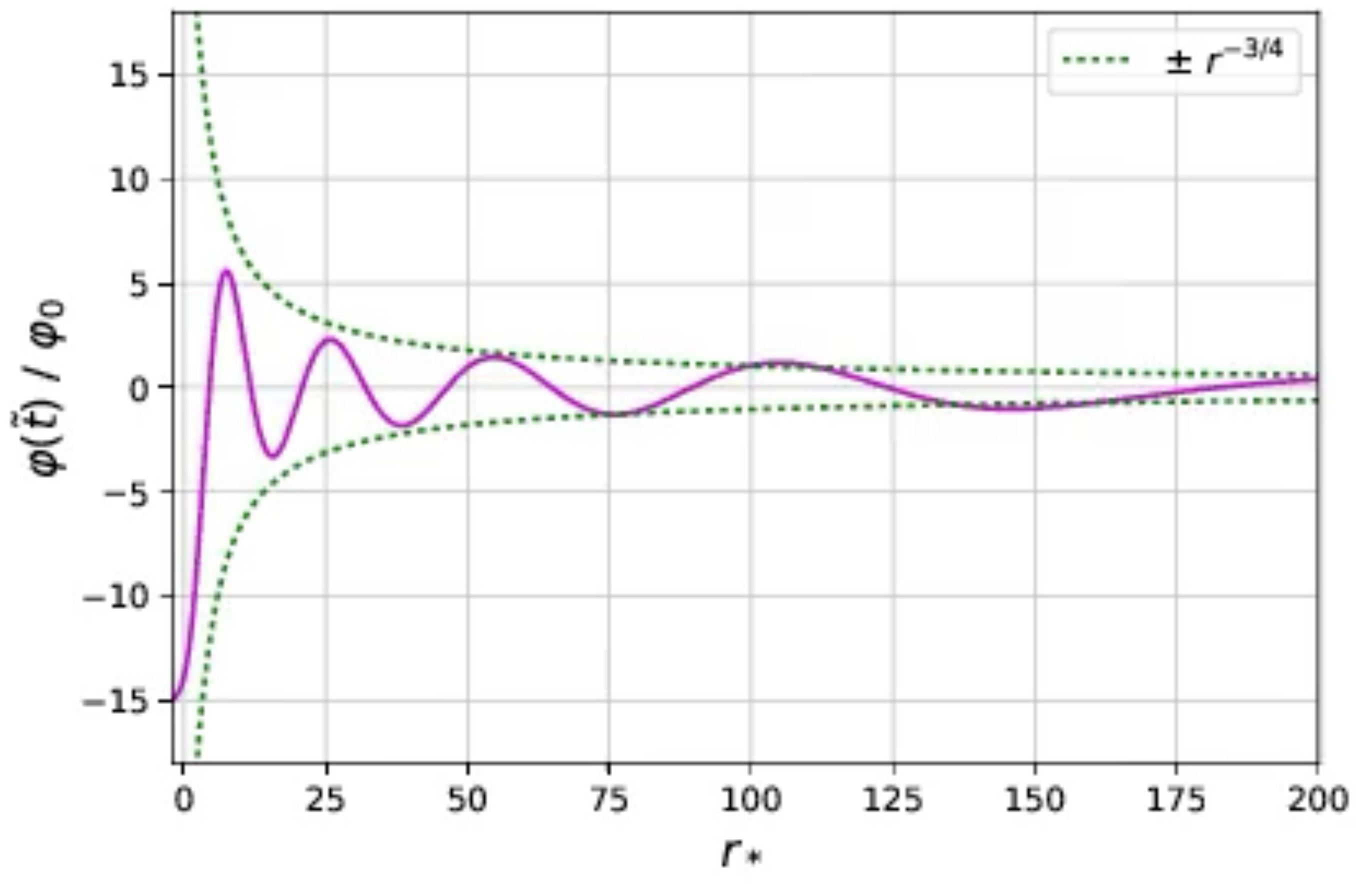
- In the limit of small particle mass m (sub eV), where number density is high, dark matter can be described as a classical field, obeying (for a spin 0 particle) the relativistic Klein Gordon equation:

$$\nabla^\mu \nabla_\mu \phi = -\frac{dV(\phi)}{d\phi}$$

- It has a **wave like behaviour**, and **pressure support** on scales of order $\lambda \sim 1/m$
e.g. 10^{-10} eV ~ 1 km

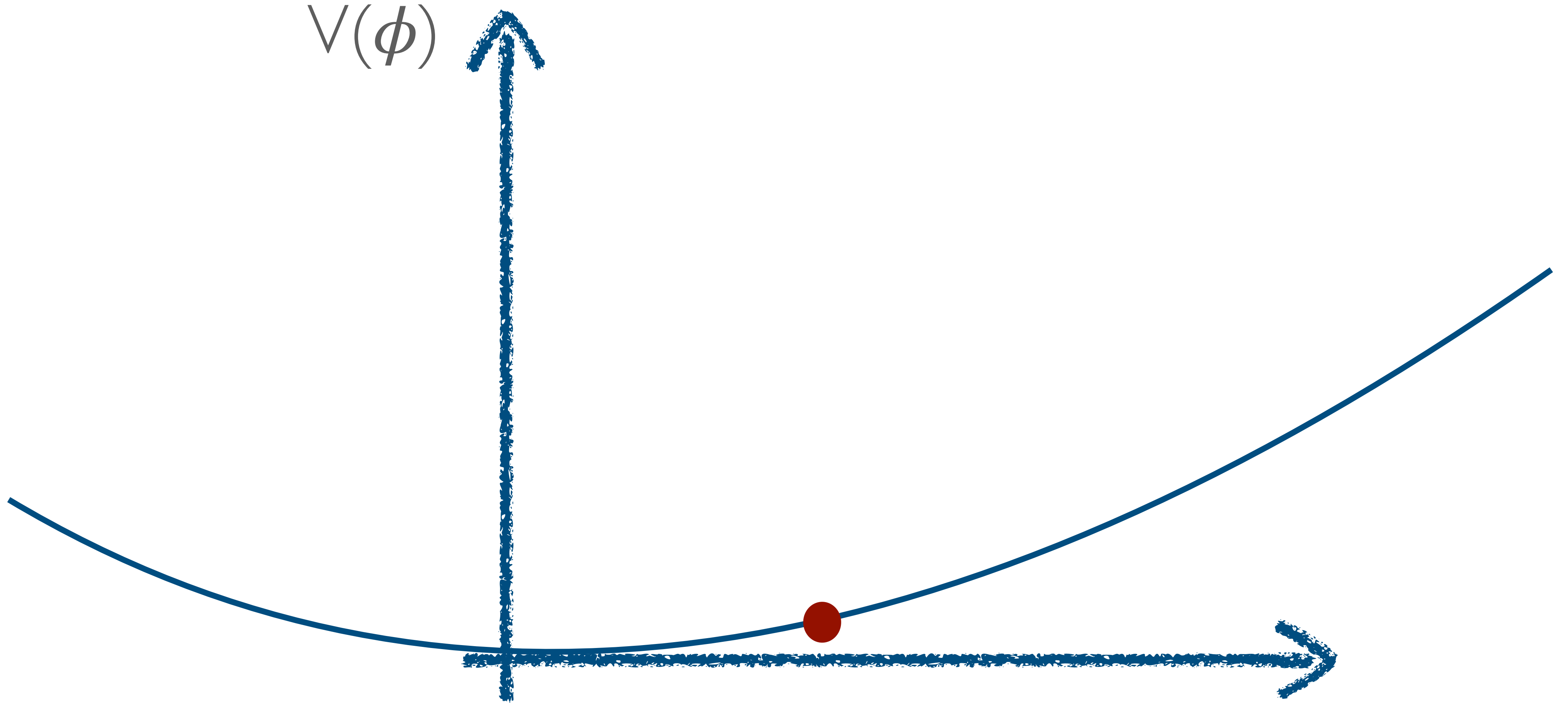
- The underlying properties of the particles (number density, velocity) at each point can be inferred by considering properties of the field, for example:

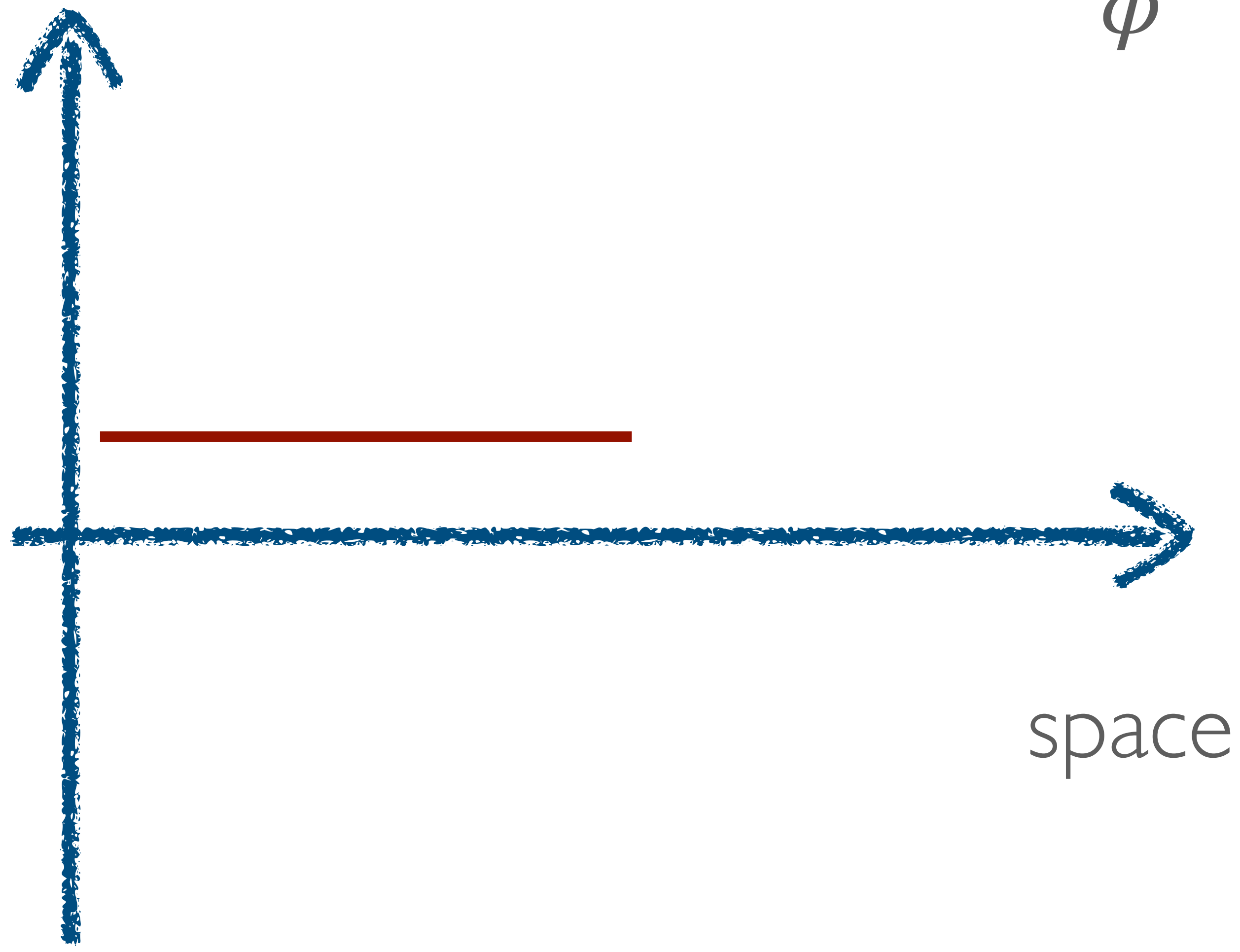
$$k_i \sim \dot{\phi}\ \partial_i \phi \qquad \rho \sim \dot{\phi}^2 + m^2\phi^2 + (\partial_i \phi)^2$$



Reviews:
Wave Dark Matter by Lam Hui - arXiv 2101.11735
Ultra-Light Dark Matter by Elisa G. M. Ferreira - arXiv 2005.03254

Light dark matter is wave-like





Cold!

$$k_i \sim \dot{\phi}\ \partial_i \phi \sim 0$$

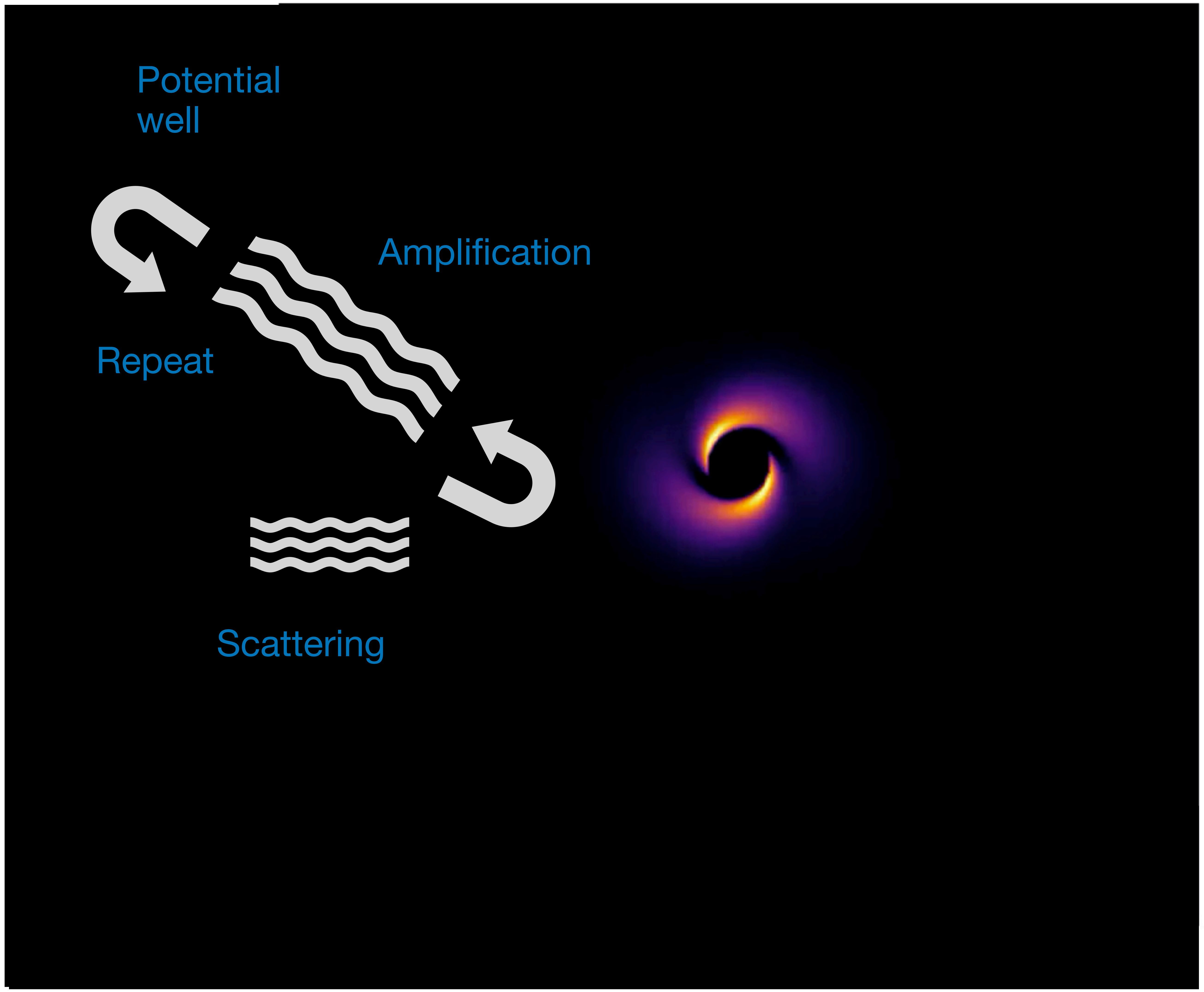
Energy density set by local amplitude

$$\rho \sim \dot{\phi}^2 + m^2\phi^2 \implies \langle \rho \rangle \sim m^2\phi^2$$

Pressureless (on average)

$$p \sim \dot{\phi}^2 - m^2\phi^2 \implies \langle p \rangle \sim 0$$

Superradiance



Simulation credit: KC / Thomas Helfer

Just a few of the key NR works:

H Witek et al
Phys.Rev.D 87 (2013) 4, 043513

M Zilihao et al
Class.Quant.Grav. 32 (2015) 234003

W East
Phys.Rev.D 96 (2017) 2, 024004

W East and F. Pretorius
Phys.Rev.Lett. 119 (2017) 4, 041101

N Sanchis-Gual, et al
Phys.Rev.D 102 (2020) 10

T Ikeda et al
Phys.Rev.D 102 (2020) 10, 101504

“Pros and cons” of superradiance

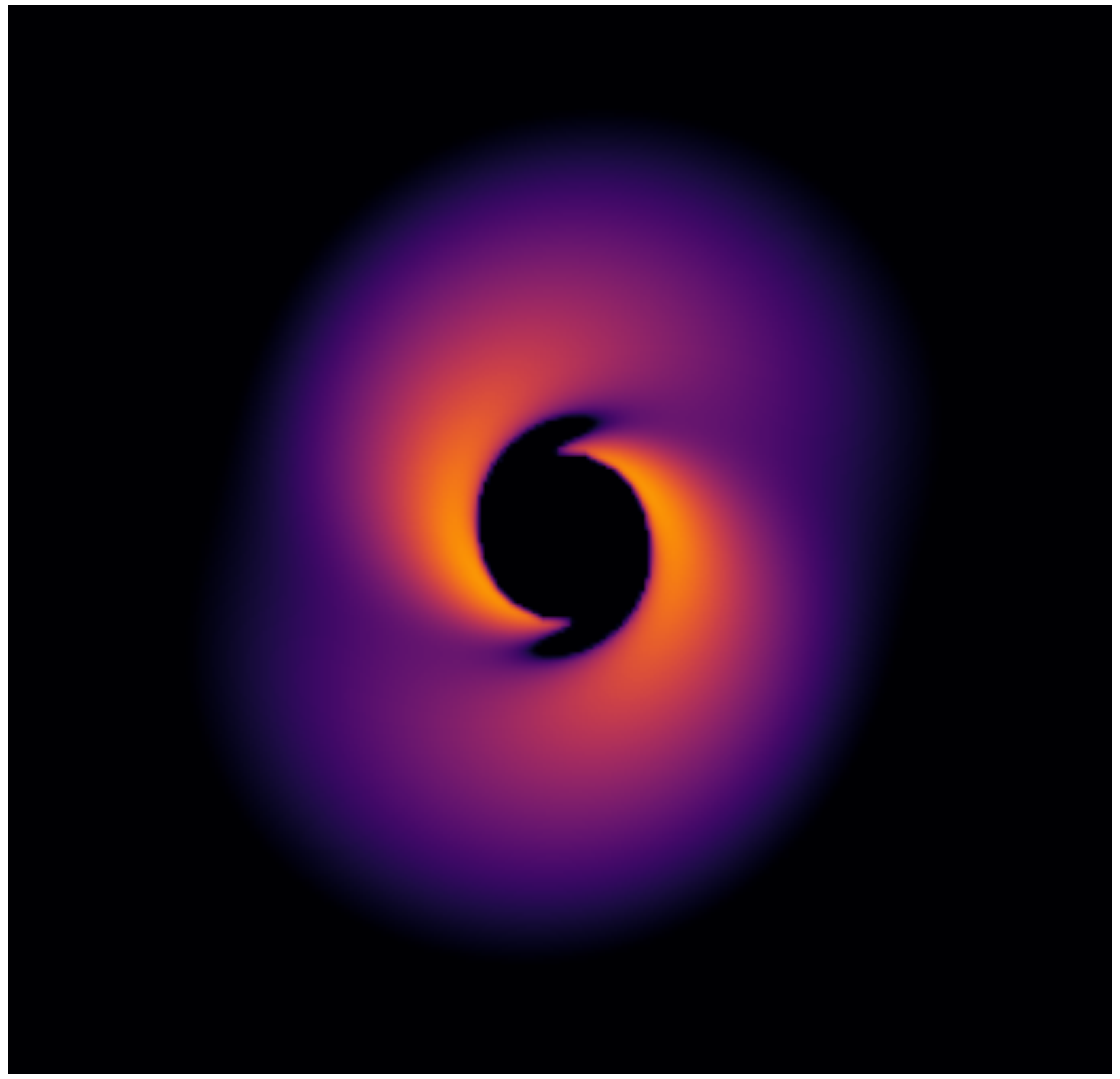
- Requires a resonance in length scales $1/m \sim r_s$

- Requires high spins ($a/M > 0.7$ but ideally higher)

+ Superradiant scalar does not have to be all of the DM (but can be)

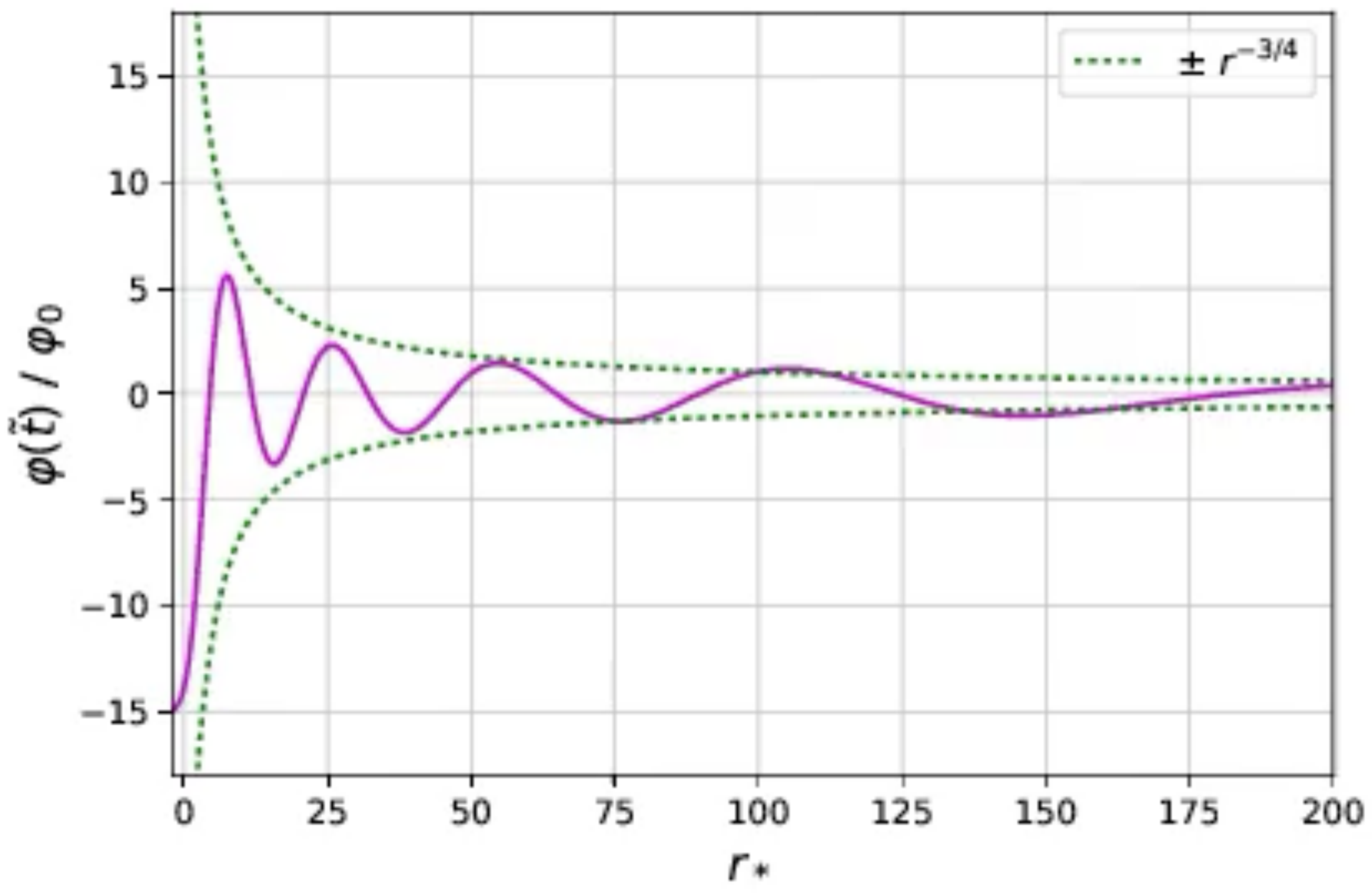
+ Superradiant bound states may achieve high densities

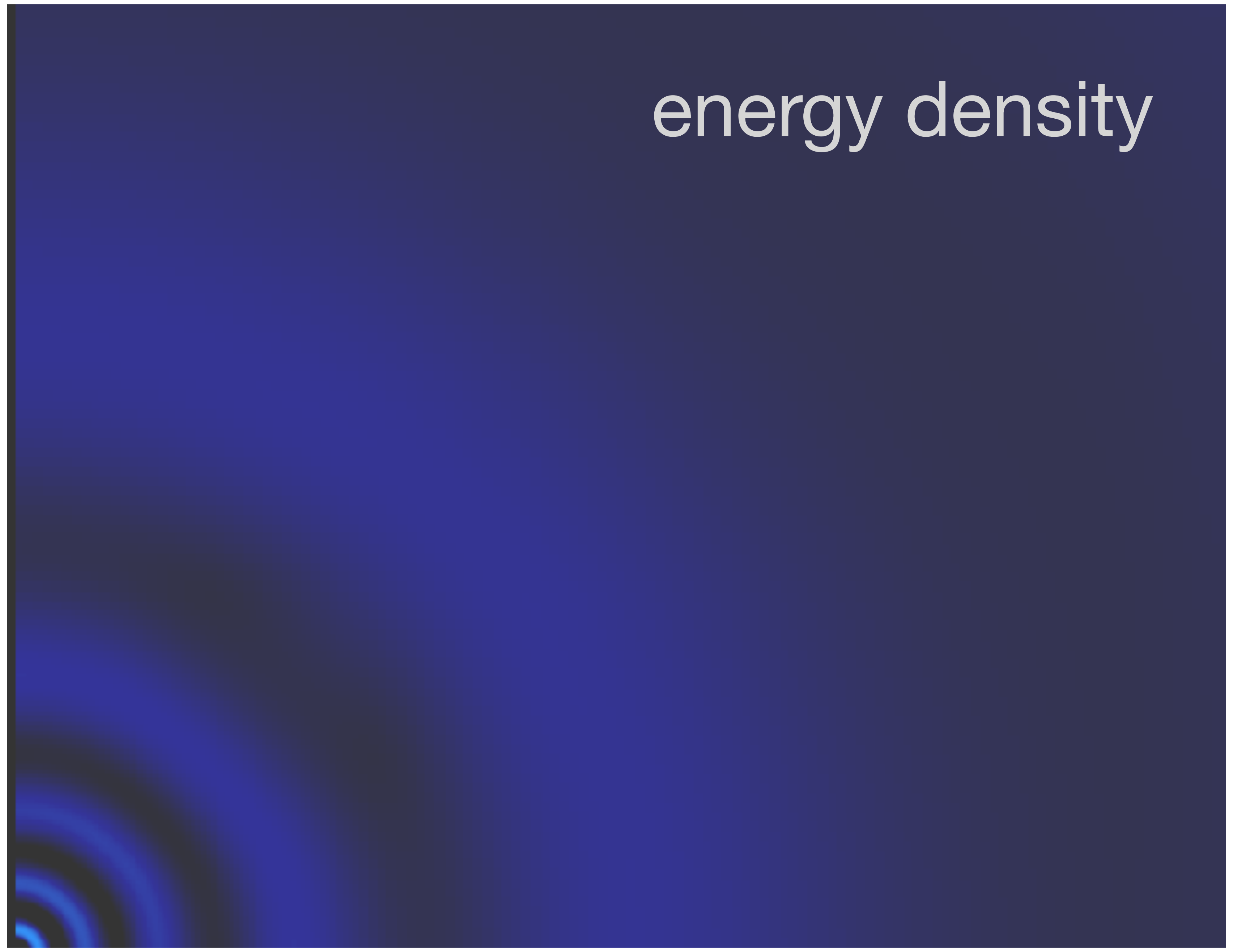
$$\rho \sim \frac{0.1 M_{BH}}{\frac{4\pi}{3}(10 r_s)^3} \sim 10^{-5}\ [r_s^{-2}]$$



What does accretion look like for scalar DM?

radial
field profile



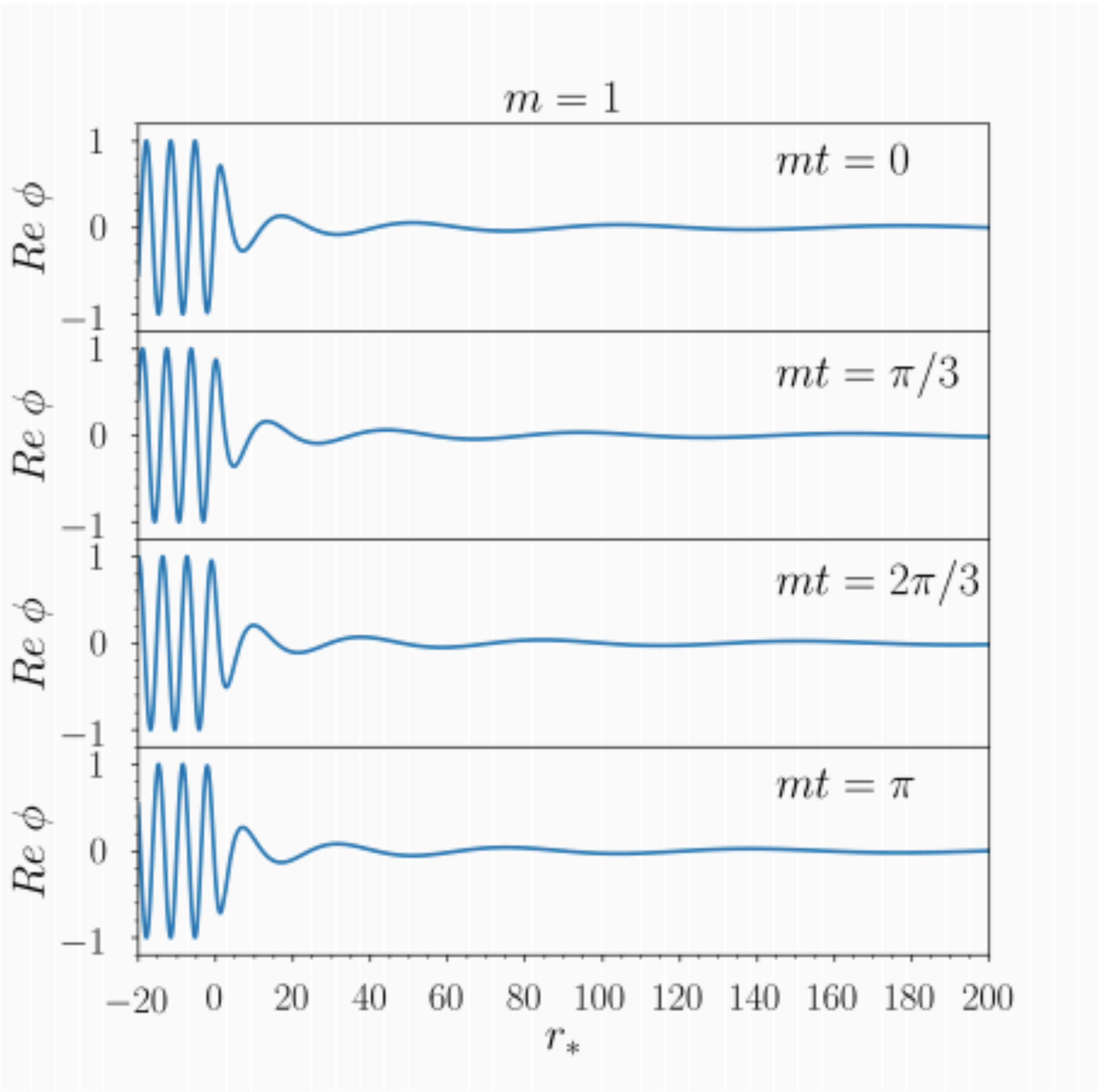


Growth of massive scalar hair around a Schwarzschild black hole
Phys.Rev. D100 (2019) 063014, KC, P Ferreira and M Lagos

Confluent Heun functions

Plamen P Fiziev (2006) Class. Quantum Grav. 23 2447
V B Bezerra et al (2014) Class. Quantum Grav. 31 045003

How big is the impact?



- Enhancement in energy density near the horizon

$$\rho \sim 10^9\,\rho_0 \left(\frac{r_0/r_s}{10^6}\right)^{3/2}$$

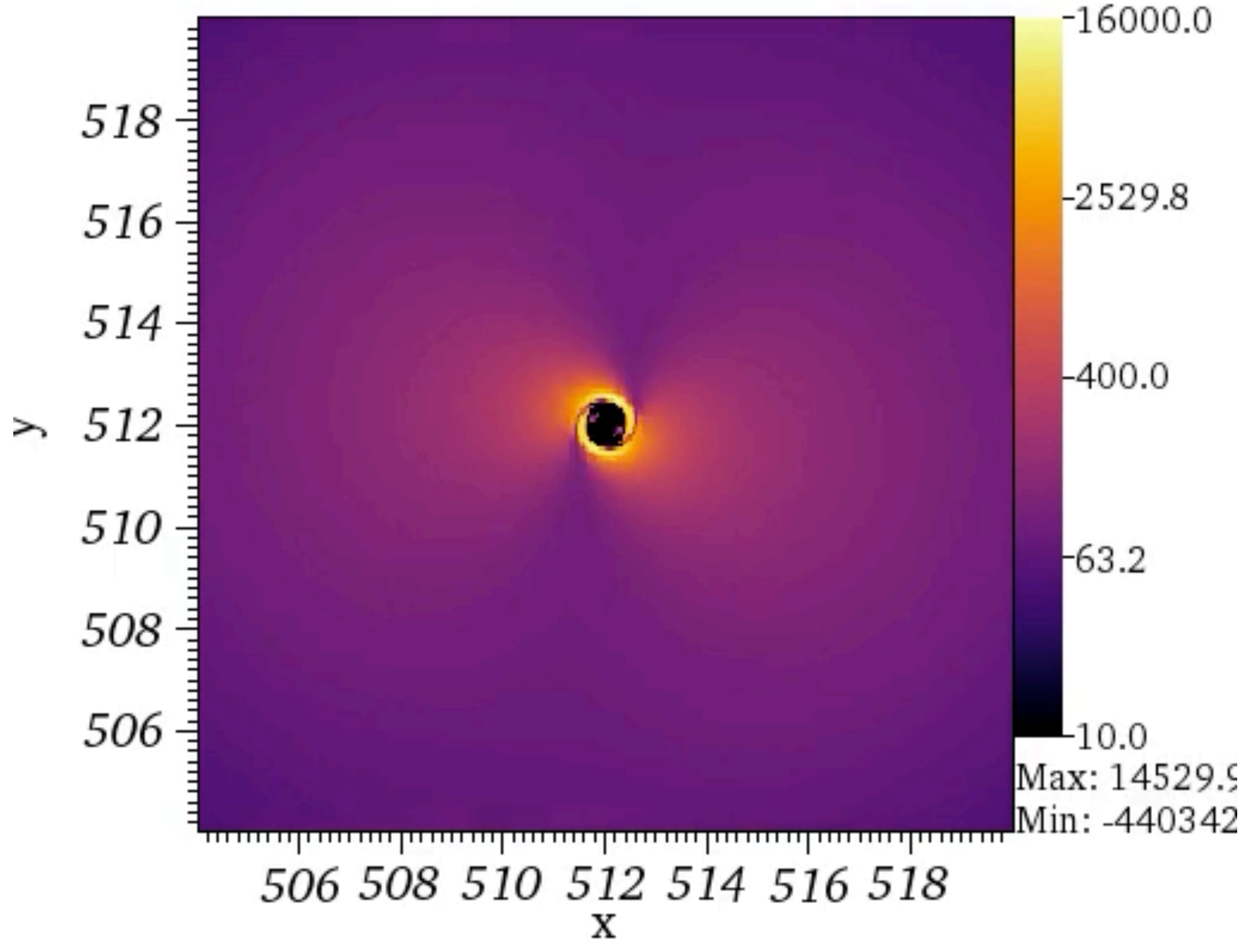
- Comparison to curvature scale of BH

$$\frac{16\pi G\rho}{[r_s^{-2}]} \sim 10^{-12}\left(\frac{M_{BH}}{10^9 M_\odot}\right)^2\left(\frac{\rho_0}{\mathrm{GeV\ cm^{-3}}}\right)\left(\frac{r_0/r_s}{10^6}\right)^{3/2}$$

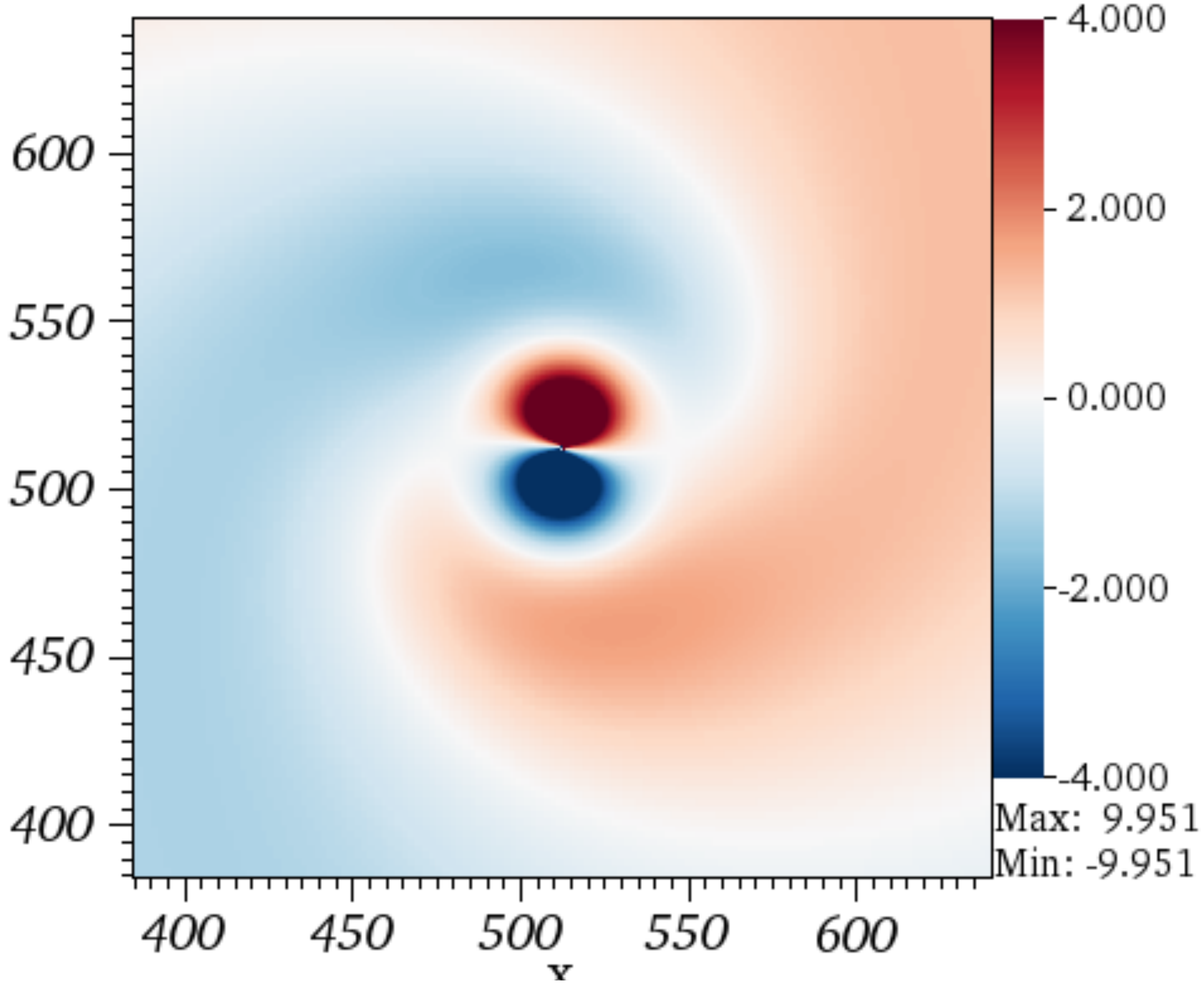
Black Hole Hair from Scalar Dark Matter
Lam Hui, Daniel Kabat, Xinyu Li, Luca Santoni, Sam S. C. Wong
JCAP 1906 (2019) no.06, 038

Angular momentum reduces accretion and changes the structure of the clouds (potential for coherent GW signal?)

energy density

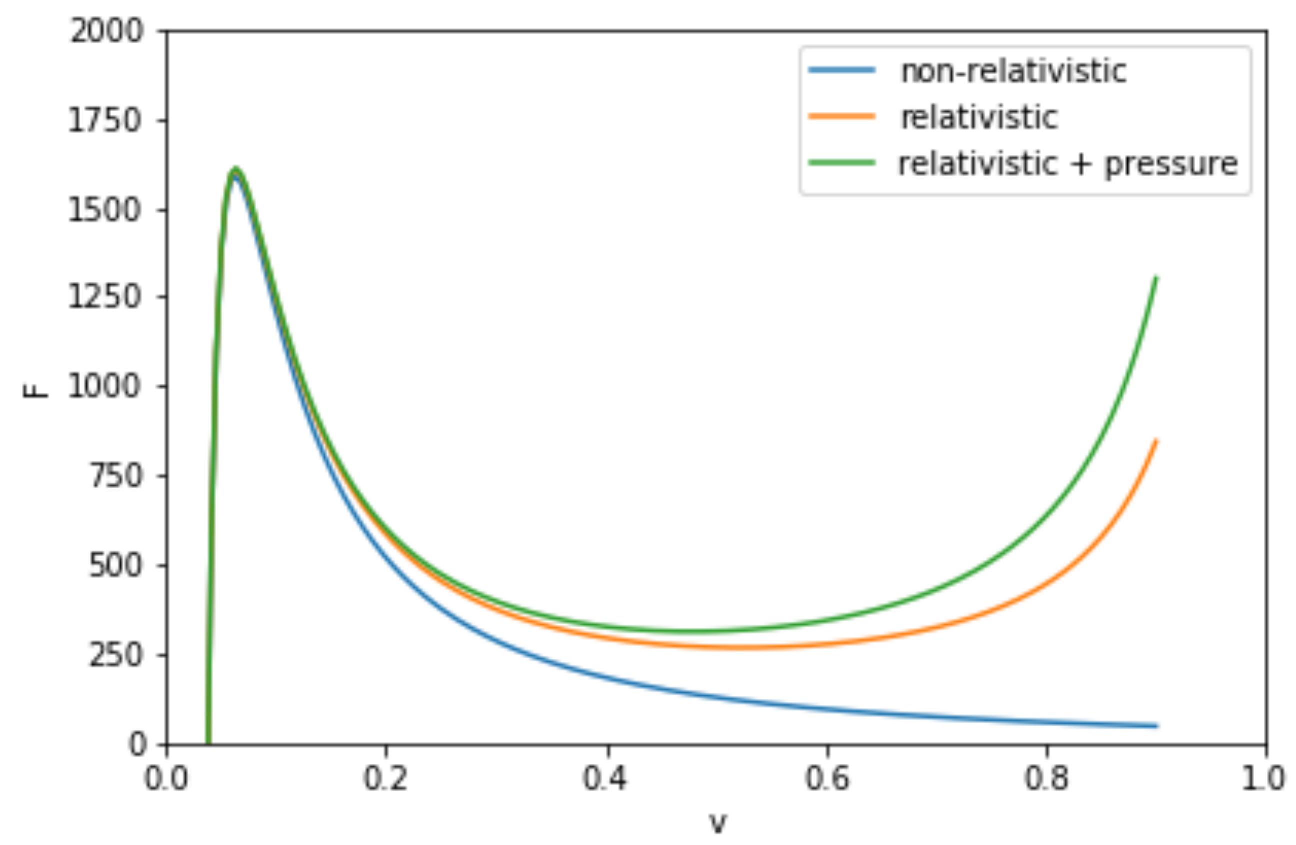


field profile

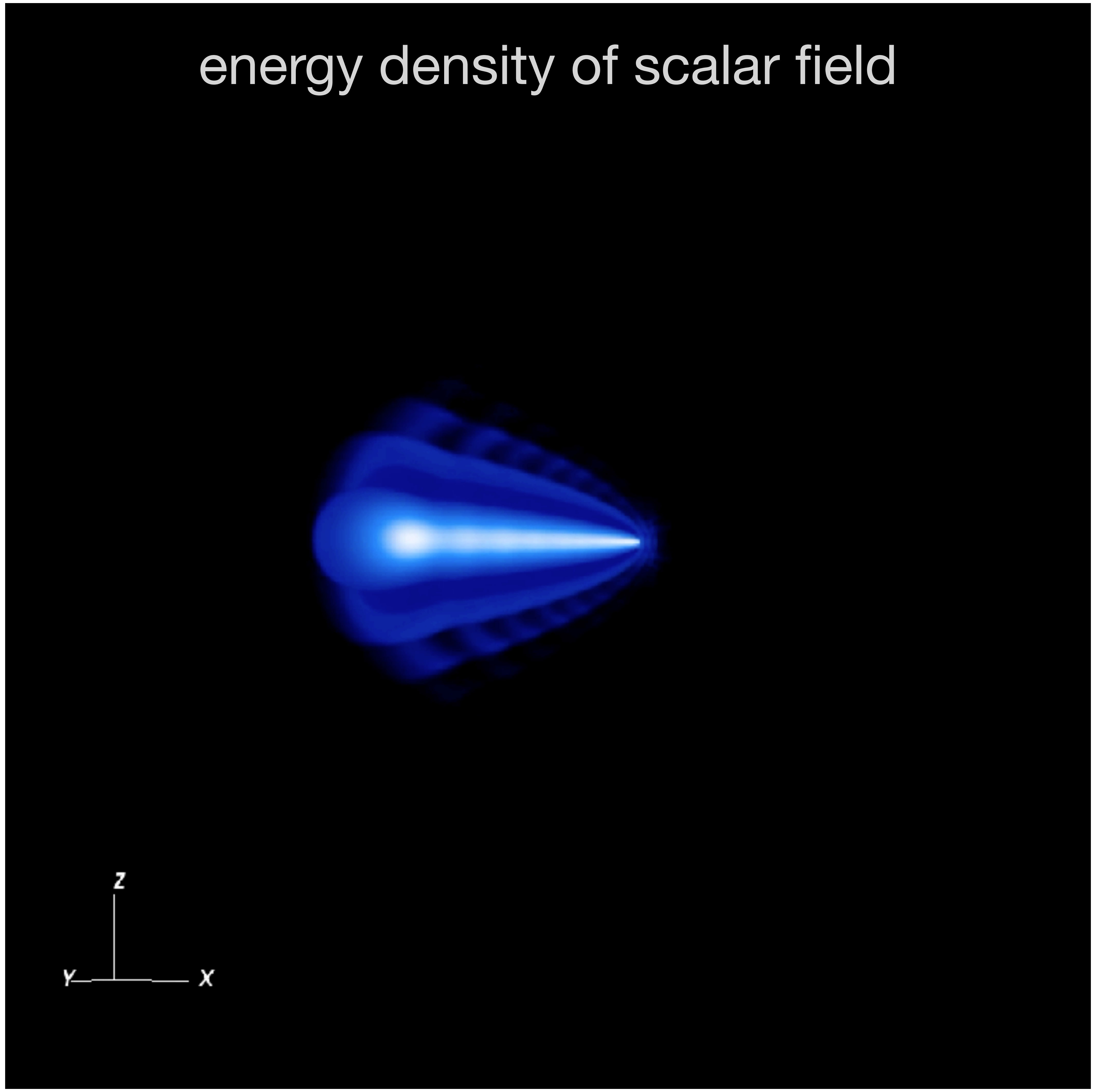


Growth of accretion driven scalar hair around Kerr black holes
Phys.Rev.D 103 (2021) 4, 044059
Jamie Bamber, KC, P Ferreira, L Hui, M Lagos

Linear momentum results in a "heavy tail" which slows the BH (dynamical friction on EMRIs?)



Dynamical friction from scalar dark matter in the relativistic regime (in prep)
Dina Traykova, KC, E Berti, P Ferreira, T Helfer



Simulation credit: Josu Aurrekoetxea / Dina Traykova

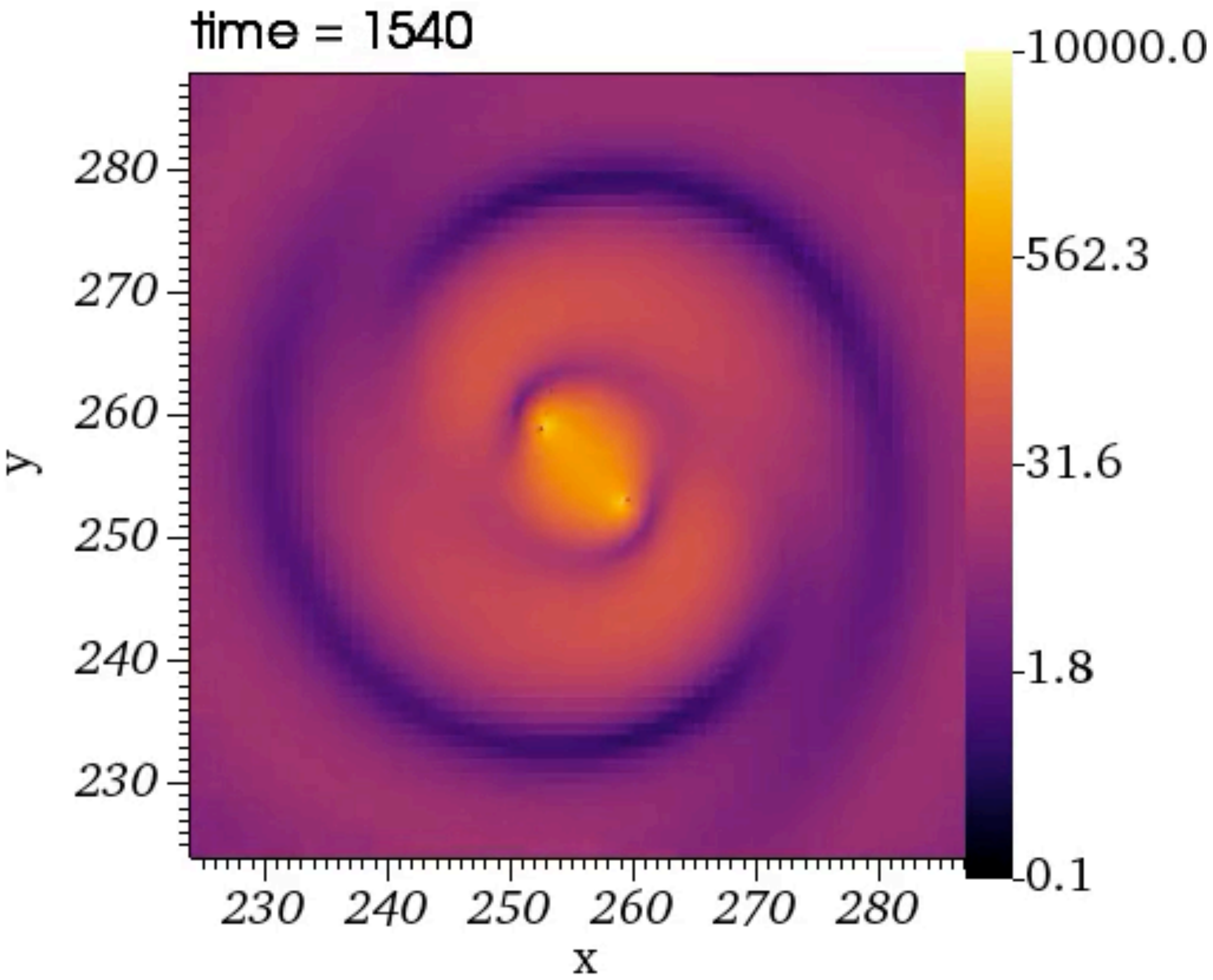
Probing the existence of ultralight bosons with a single gravitational-wave measurement
OA Hannuksela , K Wong , R Brito , E Berti , TGF Li
Nature Astronomy volume 3, pages 447–451(2019)

Stirred and shaken: dynamical behavior of boson stars and dark matter cores
L. Annulli, V. Cardoso, R. Vicente, Phys.Lett.B 811 (2020) 135944

Ultralight scalars as cosmological dark matter
L. Hui, J. Ostriker, S. Tremaine, E. Witten, Phys. Rev. D 95, 043541 (2017)

What about the binary case?

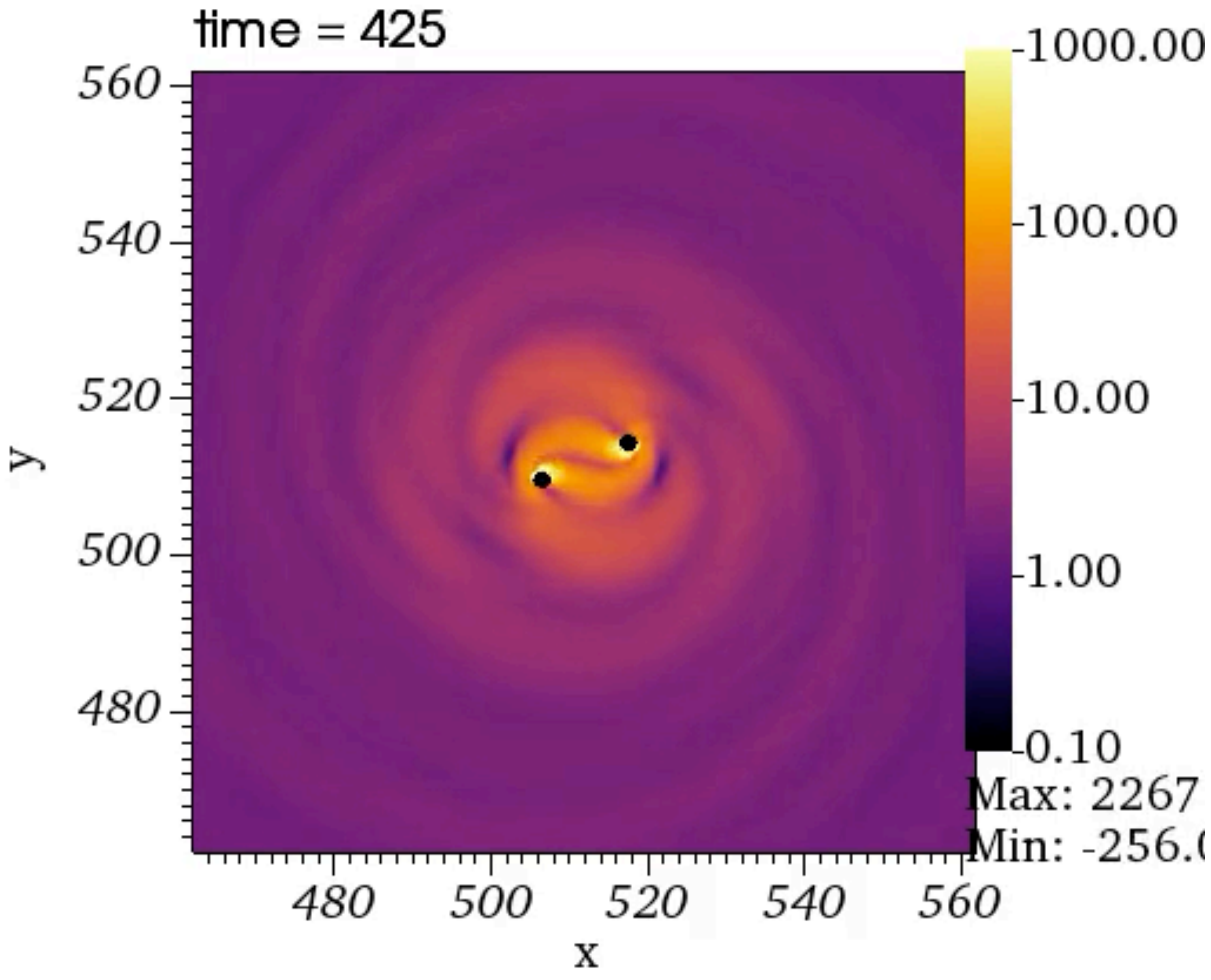
energy density
of the scalar DM



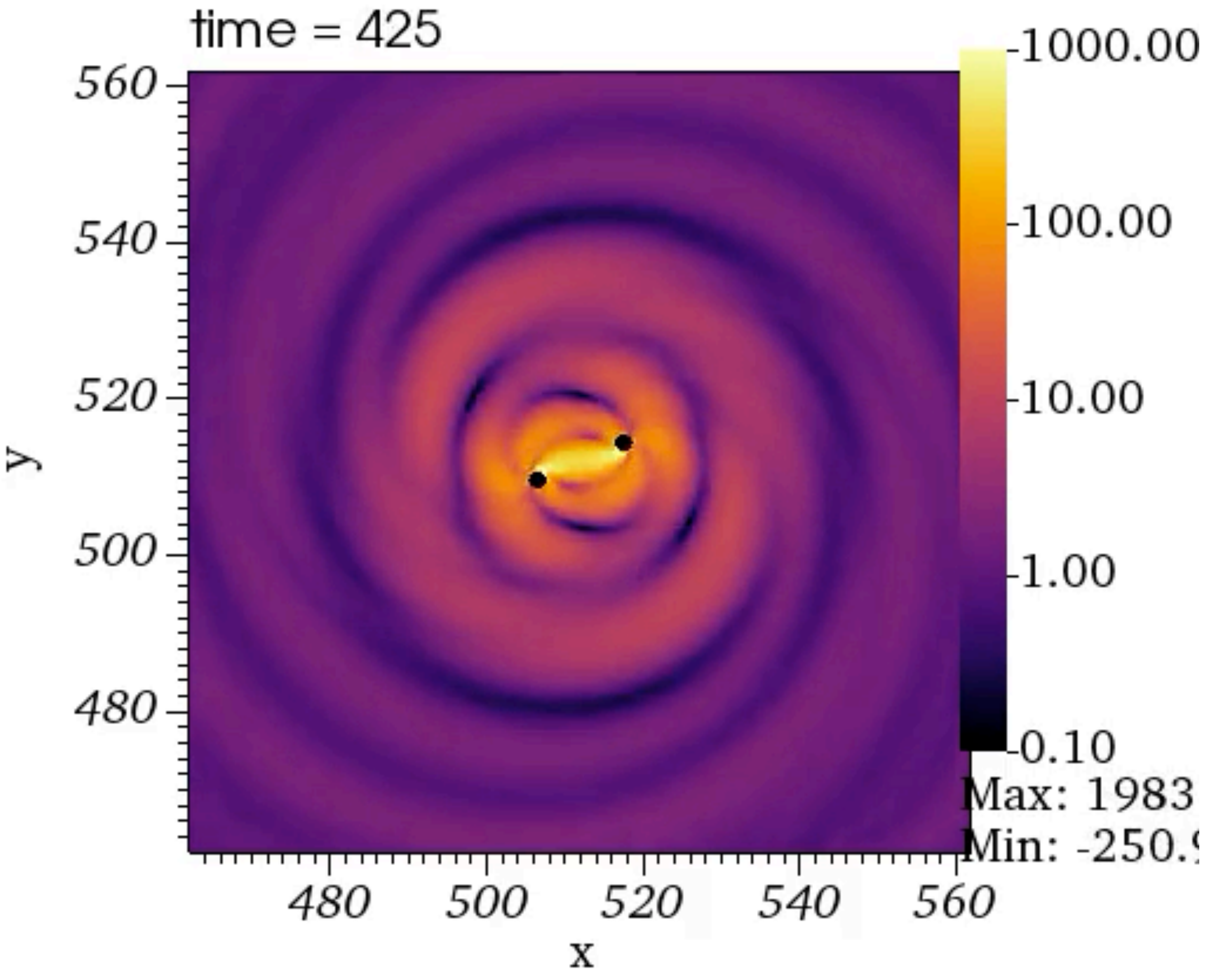
Scalar waves extract energy and angular momentum from the binary and cause it to inspiral faster

Simulation credit: KC / J Bamber

Use Newtonian simulations to create better motivated initial conditions



energy density l=m=1



energy density l=m=0

Future challenges and opportunities

- Simulations with backreaction
  - Initial data
  - Timescales
  - Numerical precision
- Treat more complex scenarios:
  - Different spin - e.g. “dark photons”
  - Impact of self interactions
  - Interactions with standard model matter
- Degeneracies with other effects

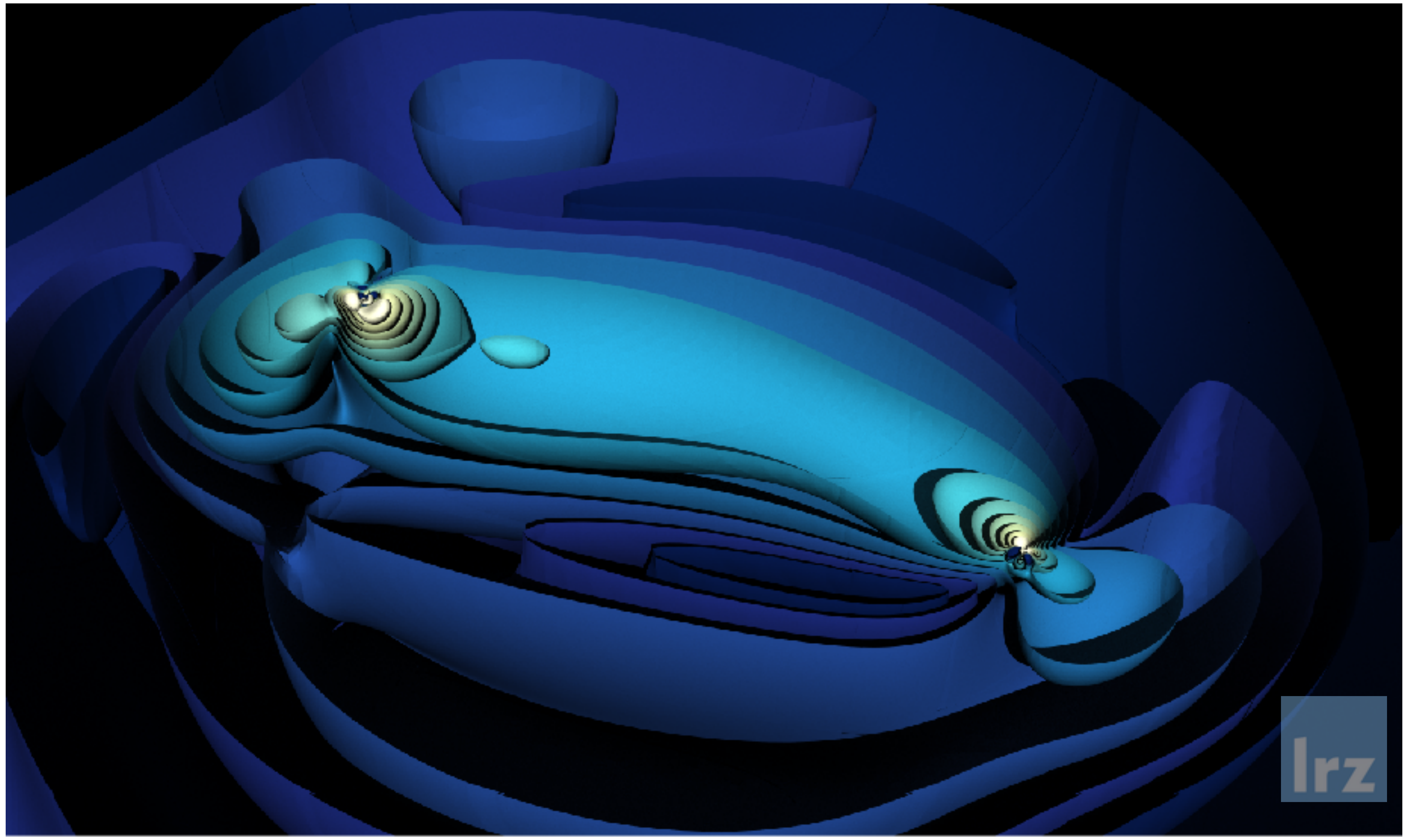
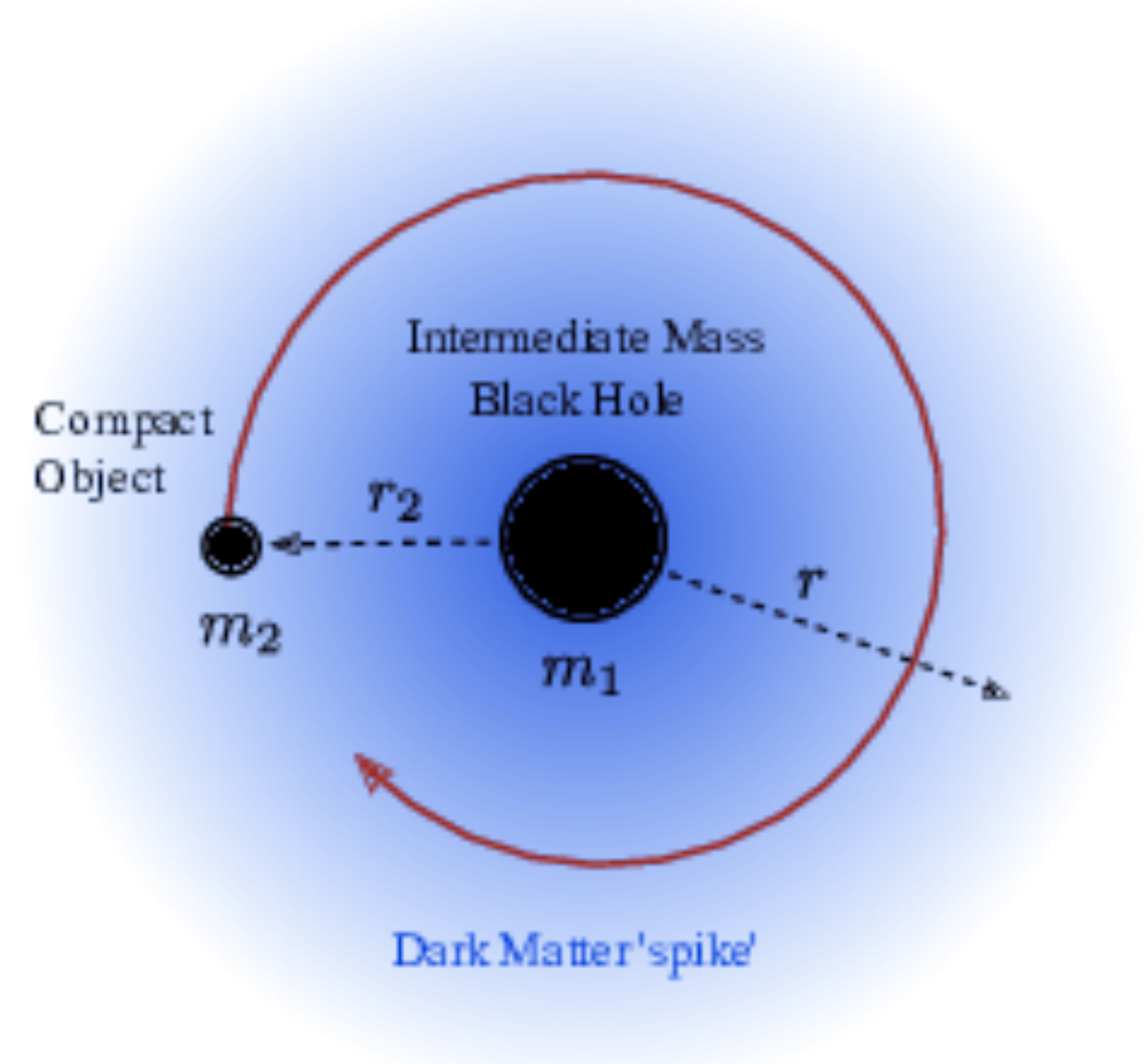


Image credit: Bamber/KC/Cielo

What about WIMPs?

Surely, to learn anything specifically about light dark matter, we need to compare to the heavy DM case?

- See this morning's talk by Bradley Kavanagh
- Why has this not been tackled more in NR simulations?

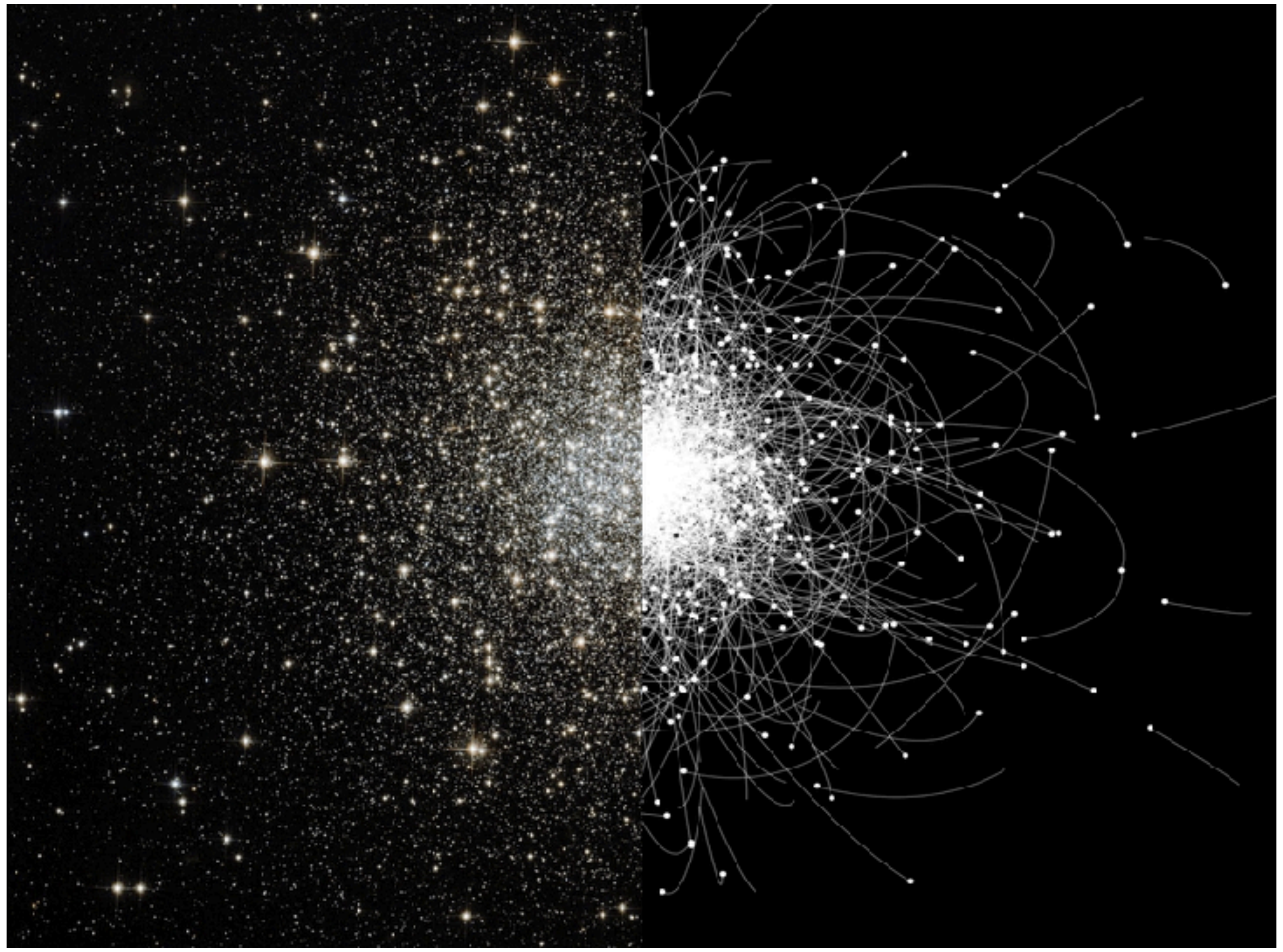


Detecting dark matter around black holes with gravitational waves: Effects of dark-matter dynamics on the gravitational waveform - B Kavanagh et al,
Phys.Rev.D 102 (2020) 8, 083006

Heavy dark matter is particle-like and virialised on small scales

- High DM masses inaccessible with scalar field description because small wavelength cannot be resolved (also no longer valid for low number densities)
- Pressureless perfect fluid not an appropriate description on small scales because of “shell crossing”
- Require N body simulations - but with fully “GR” treatment - backreaction onto the metric, geodesic motion, singularity excision etc
- (to my knowledge) 3 NR codes with particles

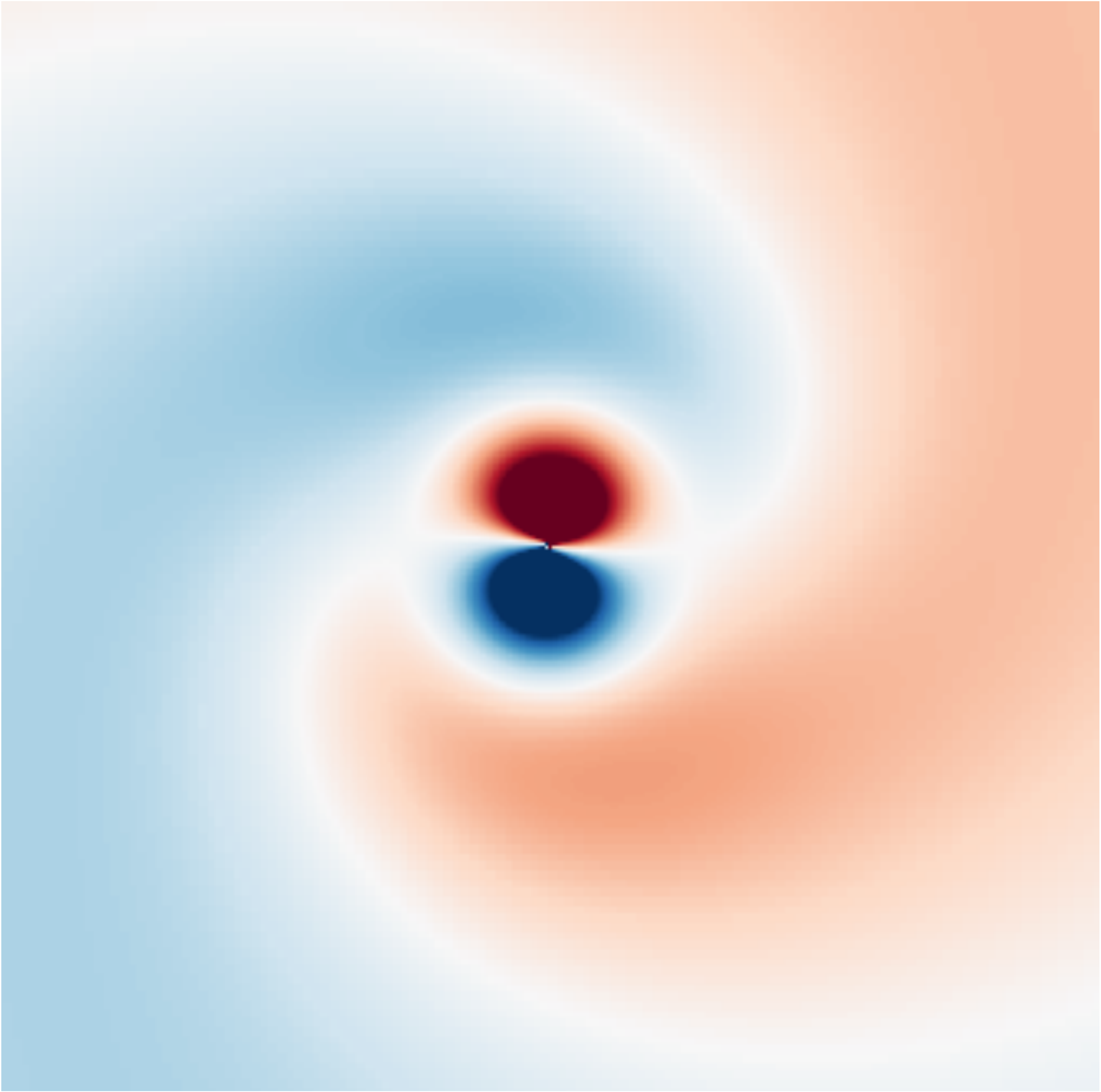
  - PAMR/HAD (Will East, Frans Pretorious)
  - CosmoGRaPH (Jim Mertens, Chi Tian)
  - GRAMSES (Baojiu Li, Cristian Barrera-Hinojosa)
- I am currently developing this capability in the open source code GRChombo - very happy to discuss this further with anyone who is interested!



Summary

What can NR do for dark matter particle physics?

- Dark matter environments create distinctive structures around black holes
- These potentially contain information about the mass of the particle, as well as its spin and (self) interactions
- Many (but not all) cases require numerics of some kind (which may or may not mean full NR)
- Binary merger signals (EMRI and equal mass) and coherent GWs from the decay of DM clouds give us access to the information, although signals probably weak
- NR simulations are challenging and not a black box, so we have to be tactical about how we use them!



Questions?

Please follow the NR code used in this work on Twitter!
@GRChombo