No free diffusion during Inflation
without overproducing Primordial Black Holes
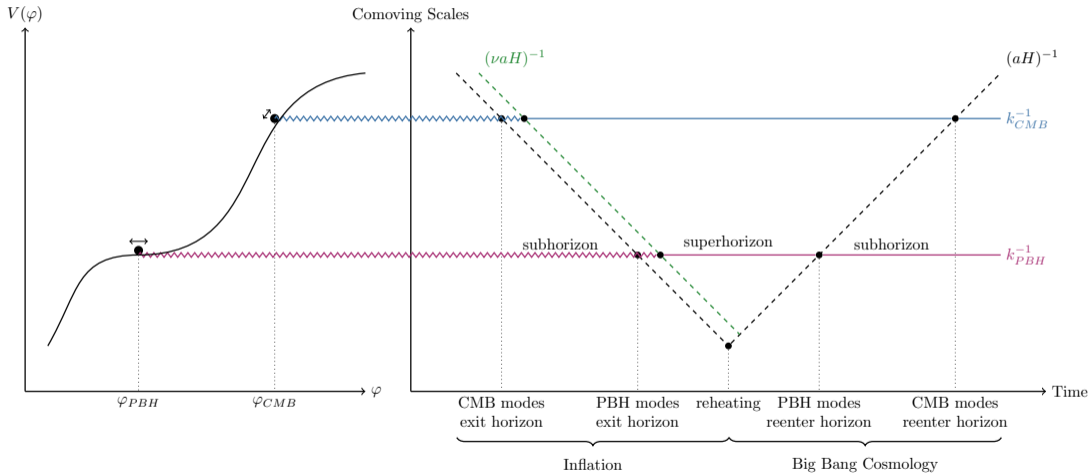
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Inflationary Perturbations

The separate universe approach indicates that, for long wavelengths, each point in space follows its own K-G and Friedmann equation i.e.

$$\ddot{\phi}(t, \mathbf{x}) + 3H(t, \mathbf{x})\dot{\phi}(t, \mathbf{x}) + V_{,\phi} = 0 \tag{1}$$

$$H^2(t, \mathbf{x}) = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2(t, \mathbf{x}) + V(\phi(t, \mathbf{x}))\right) \tag{2}$$

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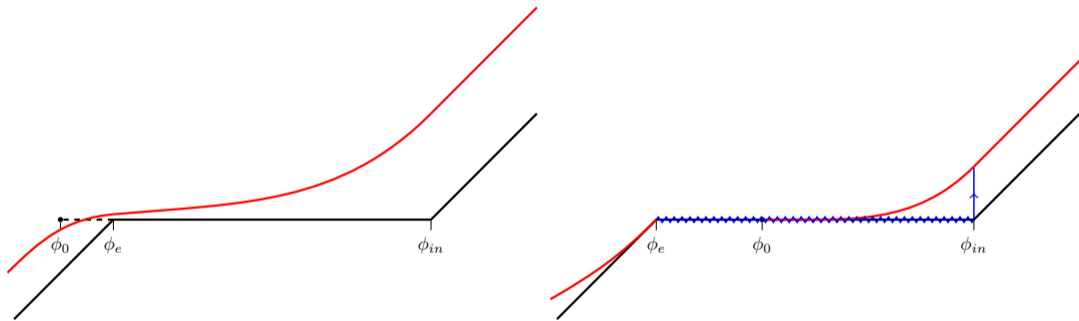However incorporating the GR momentum constraint reduces the problem to first order:

$$\ddot{\phi}(t, \mathbf{x}) + 3H(t, \mathbf{x})\dot{\phi}(t, \mathbf{x}) + V_{,\phi} = 0 \quad \rightarrow \quad \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{2}{H(\phi)}\frac{\mathrm{d}H(\phi)}{\mathrm{d}\phi} \tag{1}$$

$$H^2(t, \mathbf{x}) = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2(t, \mathbf{x}) + V(\phi(t, \mathbf{x}))\right) \quad \rightarrow \quad \left(\frac{\mathrm{d}H(\phi)}{\mathrm{d}\phi}\right)^2 = \frac{3}{2}H^2(\phi) - \frac{1}{2}V(\phi) \tag{2}$$
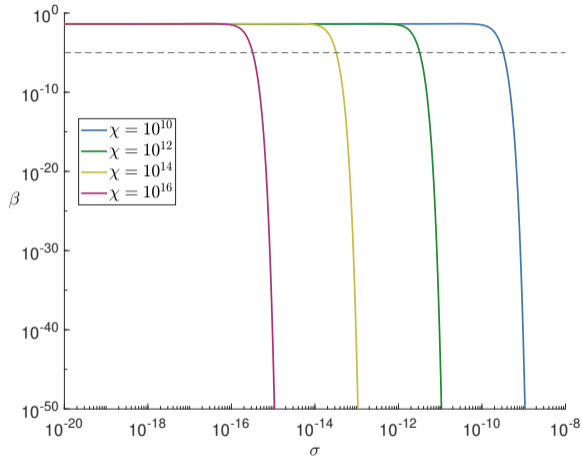
Define the distance travelled by classical drift, $\Delta\phi_{cl}$, and the length of the plateau, $\Delta\phi_{pl}$, as:

$$\Delta\phi_{cl} \equiv \phi_{in} - \phi_0 = \sqrt{\epsilon_{in}/3}, \quad \Delta\phi_{pl} \equiv \phi_{in} - \phi_e \tag{3}$$

Figure: Scenario A (left) corresponding to $\Delta\phi_{pl} \leq \Delta\phi_{cl}$ and Scenario B (right) corresponding to $\Delta\phi_{pl} > \Delta\phi_{cl}$. The Plateau is at a height $V_0 = v_0 \cdot 24\pi^2 M_{Pl}^4$

Figure: How the mass fraction, β, depends on $\sigma \equiv \dfrac{\Delta\phi_{cl} - \Delta\phi_{pl}}{\Delta\phi_{cl}}$ for four values of $\chi \equiv \sqrt{\dfrac{3}{2v_0}}\Delta\phi_{cl}$

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Classically dominated evolution

PBHs will be overproduced before the inflaton's evolution is dominated by quantum diffusion