Phenomenology of heavy-ion collisions
Hands-on problems

1. Derive how the velocity of a particle depends on its rapidity (in natural units $c = 1$).
   (a) Show that for small rapidities $v \approx y$.
   (b) Plot the function $v(y)$ for y up to 5.
   (c) Make such a plot that the velocity can be read off the plot even for large y. Plot this for y up to 12.
   (d) Determine the rapidity (and the velocity) for the ion beam at the LHC (5.5 TeV per colliding nucleon pair), RHIC (200 GeV), SPS (take the centre of mass system: 17.6 GeV per colliding nucleon pair), SIS ($\sqrt{s_{NN}} = 2.42$ GeV), and also for the planned FCC ($\sqrt{s_{NN}} = 39$ TeV).

2. Pseudorapidity
   (a) From $\eta = \frac{1}{2}\ln\frac{|p|+p_z}{|p|-p_z}$ show that $\eta = -\ln\tan\frac{\theta}{2}$. (Where θ is the angle between $\vec{p}$ and the beam axis.)
   (b) Prove that $|p| = p_t \cosh\eta$ and $p_z = p_t \sinh\eta$.
   (c) Provided that pion spectrum is flat in rapidity, plot it in pseudorapidity for $p_t = 200$ MeV, 500 MeV, and 1000 MeV? Do the same for proton spectrum. (This exercise should show that the pseudorapidity is good approximation to rapidity for $m \ll p_t$.)

3. Transverse energy and transverse mass.
   (Thanks to Dušan for the problem :-))
   These quantities are defined as

$$
\begin{aligned}
m_t &= \sqrt{m^2 + p_t^2} \\
E_t &= E\sin\theta
\end{aligned}
$$

   where θ is the angle between $\vec{p}$ and the beam axis. Derive that

$$
E_t = m_t \frac{\cosh y}{\cosh\eta}\,.
$$

4. Glauber model. Realise that in a collision of nuclei A and B, $T_{AB}(\vec{s})\sigma_{inel}^{NN}$ is the probability for one inelastic nucleon-nucleon interaction to occur. The maximum number of binary NN collision is then AB. Show that the mean number of binary collisions is $ABT_{AB}(\vec{s})\sigma_{inel}^{NN}$.

5. Resonance decays.
   This problem takes us to the lecture on group theory by Ilja Doršner, and to a part of my lecture which I had no time to deliver (but will do so on Thursday).
   In calculating the number of final state hadrons, which are observed in an experiment, it is important to account for those hadrons, which resulted from decays of resonances.
   (a) The Δ resonance appears in four isospin states: Δ^-, Δ^0, Δ^+, Δ^{++}. It decays exclusively into one nucleon and one pion. What are the possible decay channels of Δ^+ and what are their branching ratios? (Branching ratio is the probability of a given decay channel.)
   (b) The resonance ρ^0, which is a part of an isospin triplet decays exclusively into two pions. What are the possible decay channels and what are the branching ratios?