

# Phenomenology of heavy-ion collisions

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# The goal of studies with heavy-ion collisions

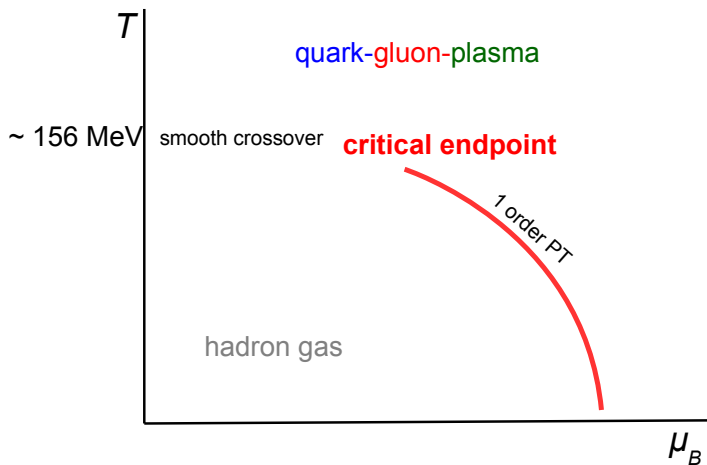
To study the properties of strongly interacting matter in extreme conditions.

- Equation of State  
(complementary to neutron stars and their mergers)
- phase transition(s), critical point
- transport coefficients:
  - shear and bulk viscosity
  - momentum transfer from hard parton to medium ( $\hat{q}$ )
  - ...

To learn how the created matter looks like

- temperature, pressure, volume, evolution, ...
- vorticity (most vortical fluid)
- emergent hydrodynamics (why hydro works, when it should not)
- ...

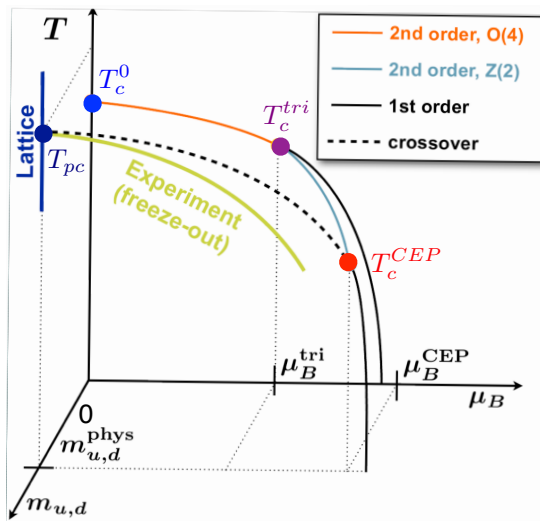
# The roadmap for these studies: the phase diagram



$\mu_B$ : baryochemical potential

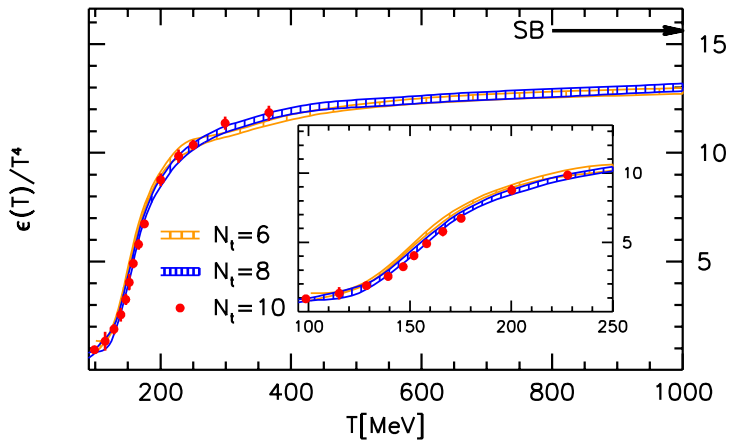
ratio of the numbers of baryons to antibaryons:  $B/\bar{B} \propto \exp(2\mu_B/T)$

# The phase diagram: points and transitions



plot: Heng-Tong Ding @ Quark Matter 2019

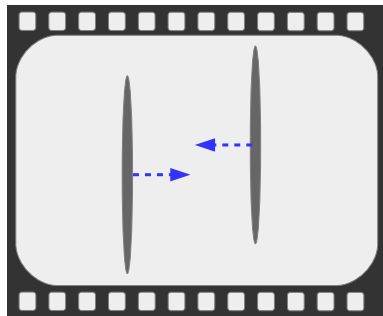
# Smooth crossover observed the lattice



[S. Borsanyi, et al., JHEP 11 (2010) 077]

# The evolution of a heavy-ion collision 1

## Before the collision



- (highly) Lorentz contracted nuclei approach  
 $\gamma$  between 1 and 2750
- geometry important for later evolution

# The evolution of a heavy-ion collision 2

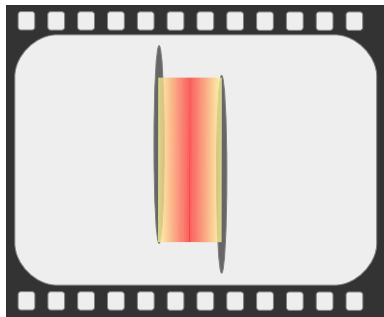
## Production of quanta, “energy transformation”



- low  $\sqrt{s_{NN}}$ 
  - low  $\sqrt{s_{NN}}$  hadronic mechanisms
  - gradual as nuclei pass through each other
- high  $\sqrt{s_{NN}}$ 
  - partonic, depends on parton distributions
  - nuclear effects on parton distributions
  - quickly emerging hydrodynamics behaviour
- setting up initial conditions for collective behaviour
- crucial for (some of) later correlations

# The evolution of a heavy-ion collision 3

## Flowing of hot (deconfined) matter

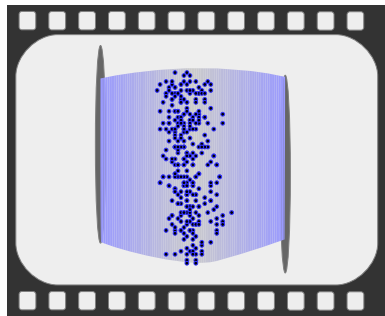


- expansion!
- (s)QGP for  $\sqrt{s_{NN}}$  above ...
- (surprisingly) well described by hydrodynamic models (particularly if long in the QGP phase)
- properties of matter clearly enter modelling
  - Equation of State
  - viscosities



# The evolution of a heavy-ion collision 4

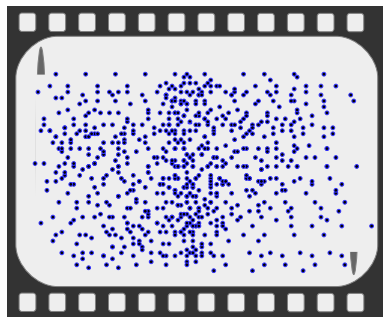
## Hadronisation and hadron gas



- chemical freeze-out happens here
- gas of interacting hadrons and resonances (HRG)
- suitably described by transport models
- continues the strong expansion
- seems to start in chemical equilibrium and get out of this equilibrium
- acts like “firewall” for the deconfined fireball

# The evolution of a heavy-ion collision 5

## Freeze-out

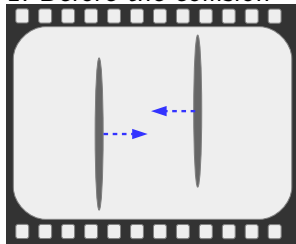


- Hadrons produced with their final state momenta and correlations
- Happens gradually
- sometimes modelled as sudden process (Cooper-Frye formalism)

# The evolution of heavy-ion collision: summary

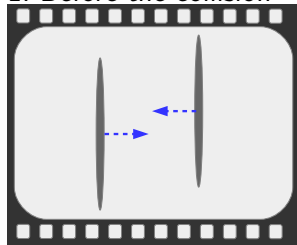
# The evolution of heavy-ion collision: summary

## 1. Before the collision

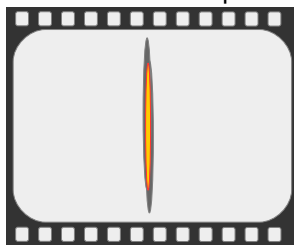


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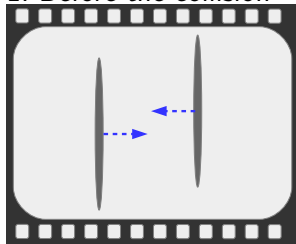


2. Production of quanta

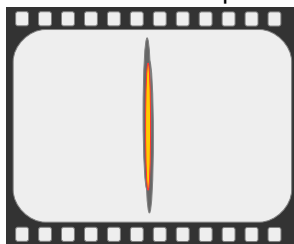


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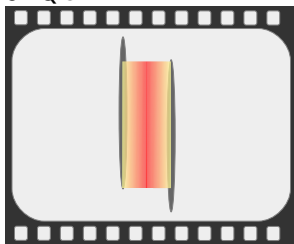
1. Before the collision



2. Production of quanta

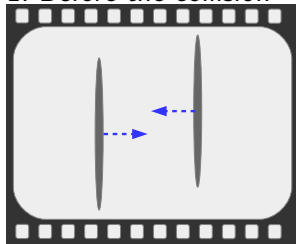


3. QGP

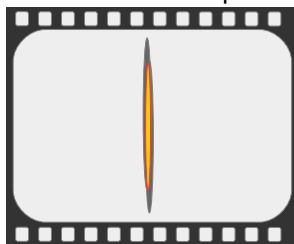


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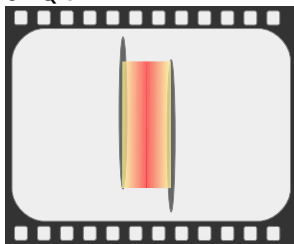
1. Before the collision



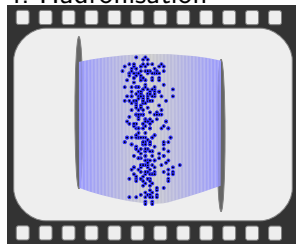
2. Production of quanta



3. QGP

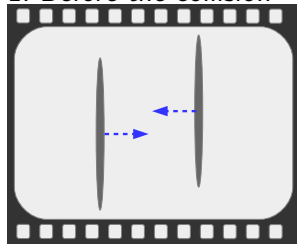


4. Hadronisation

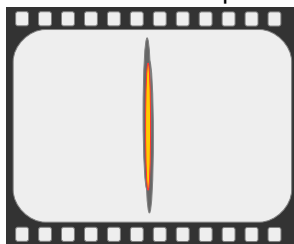


# The evolution of heavy-ion collision: summary

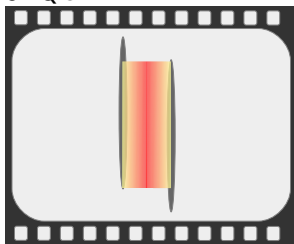
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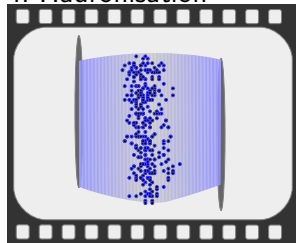
2. Production of quanta



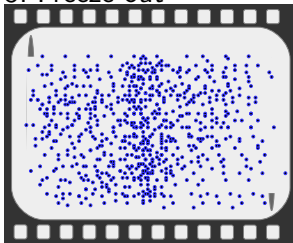
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4. Hadronisation

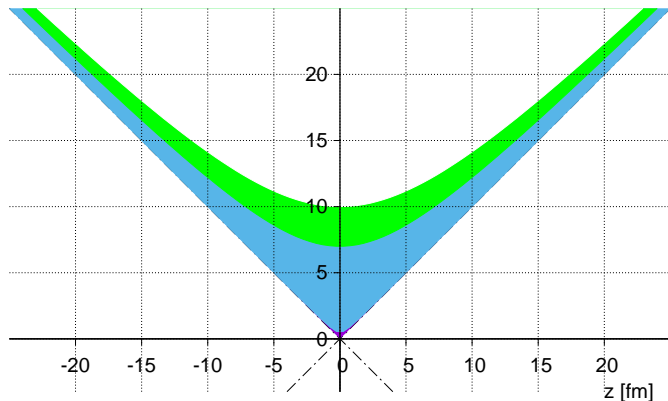


5. Freeze-out





# The space-time diagram of the evolution



# Space-time parametrisation: the Milne coordinates

More suitable for longitudinally rapidly expanding systems:

longitudinal proper time

$$\tau = \sqrt{t^2 - z^2}$$

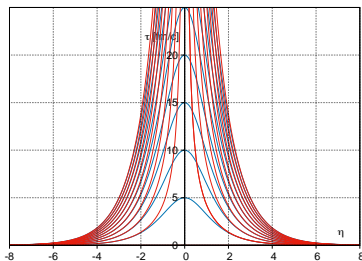
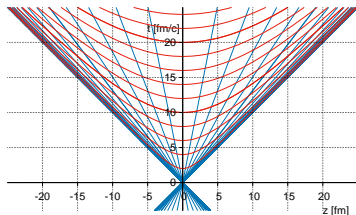
space-time rapidity

$$\eta = \frac{1}{2} \ln \frac{t+z}{t-z}$$

Inverse transformation

$$t = \tau \cosh \eta$$

$$z = \tau \sinh \eta$$



# Momentum parametrisation

4-momentum of a particle with the mass  $m$ :

- transverse momentum  $p_t$
- azimuthal angle  $\phi$
- transverse mass

$$m_t = \sqrt{m^2 + p_t^2}$$

- rapidity

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

Momentum parametrisation:

$$p = (E, \vec{p}) = (m_t \cosh y, p_t \cos \phi, p_t \sin \phi, m_t \sinh y)$$

Experimental proxy for the rapidity: pseudorapidity (same letter  $\eta$  used :- ( )

$$\eta = \frac{1}{2} \ln \frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} = -\ln \tan \frac{\theta}{2}$$

easy to measure, since  $\theta$  is the angle between  $\vec{p}$  and the beam  
good approximation for  $|\vec{p}| \gg m$

# Typical energies, sizes, times

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- energy of central collision of Pb+Pb at the LHC (CERN):  
 $208 \times 5.5 = 1144 \text{ TeV} \approx 0,2 \text{ mJ}$  (energy of a flying hornet)

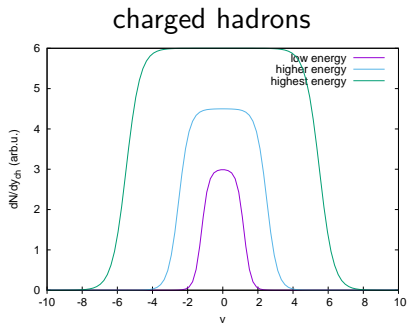
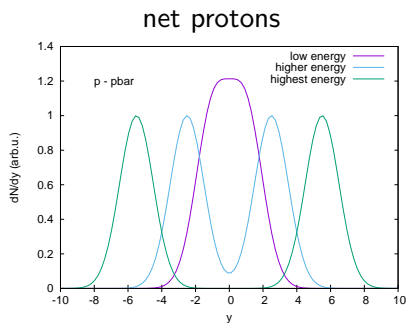
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- typical size of the fireball:  $10^{-14} \text{ m}$  (10 fm)  
(If QGP were as big as poppy seed, poppy seed would be as big as the Earth.)

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- typical size of the fireball:  $10^{-14} \text{ m}$  (10 fm)  
(If QGP were as big as poppy seed, poppy seed would be as big as the Earth.)
- typical lifetime of the fireball:  $10^{-22} \text{ s}$  (10 fm/c)  
(The time it takes for the light to pass through a nucleus.)

# Collisions at different energies



- Incoming baryon number is somewhat stopped with respect to beam rapidity.
- At sufficient energies, produced hadrons exhibit plateau in rapidity.

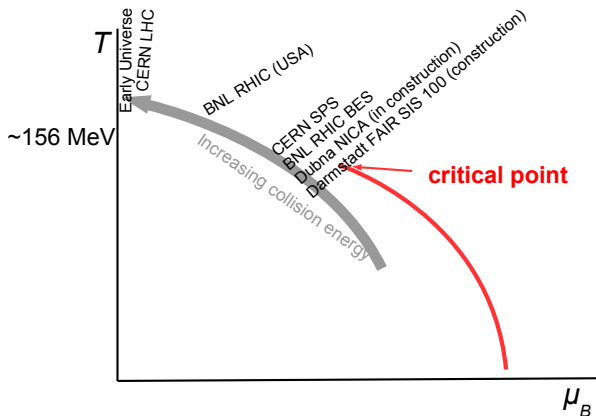


# Mapping the phase diagram

conserved quantity:  $B - \bar{B}$

higher collision energy  $\Rightarrow$  higher pair production  $B\bar{B}$

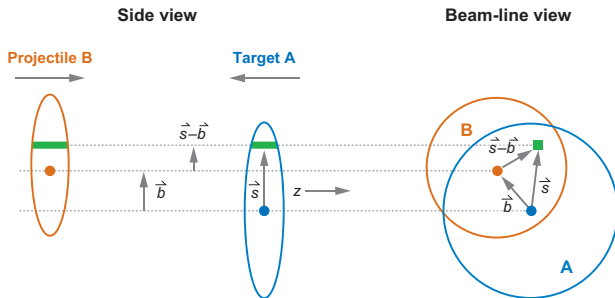
$\Rightarrow$  smaller  $B/\bar{B} \Rightarrow$  smaller  $\mu_B$



$\mu_B$  also depends on rapidity

# Centrality

- Each collision happens at different impact parameter  $b$ .
- Impact parameter  $b$  is not measurable.
- Nevertheless,  $b$  influences the geometry and future evolution



**spectators:** nucleons that did *not* interact

**participants:** nucleons that did interact

**wounded nucleons:** nucleons that did interact *inelastically*

figure: M.L. Miller, K. Reygers, S.J. Sanders, P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57 (2007) 205

# Optical Glauber model

Projected nuclear density

$$\hat{T}_A(\vec{s}) = \int \hat{\rho}(\vec{s}, z_A) dz_A$$

Overlap function

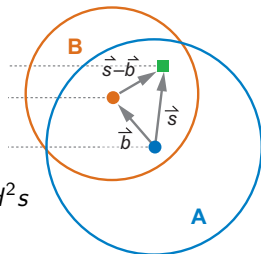
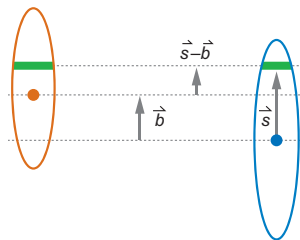
$$\hat{T}_{AB}(\vec{s}) = \int \hat{T}_A(\vec{s}) \vec{T}_B(\vec{s} - \vec{b}) d^2s$$

Number of binary NN collisions

$$N_{coll}(\vec{s}) = AB T_{AB}(\vec{s}) \sigma_{inel}^{NN}$$

Number of wounded nucleons

$$N_w(\vec{s}) = A \int \hat{T}_A(\vec{s}) \left\{ 1 - \left[ 1 - \hat{T}_B(\vec{s} - \vec{b}) \sigma_{inel}^{NN} \right]^B \right\} d^2s \\ + B \int \hat{T}_B(\vec{s} - \vec{b}) \left\{ 1 - \left[ 1 - \hat{T}_A(\vec{s}) \sigma_{inel}^{NN} \right]^A \right\} d^2s$$



# Monte-Carlo Glauber model

Even at the same  $\vec{s}$  the numbers of  $NN$  collisions will fluctuate.

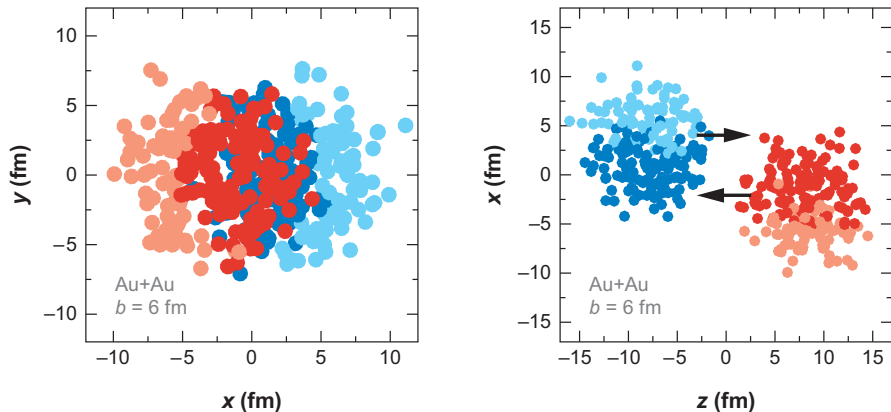


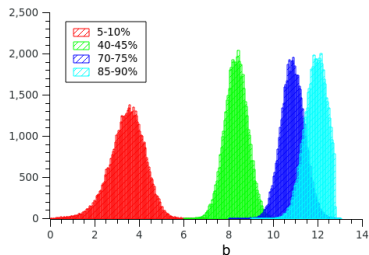
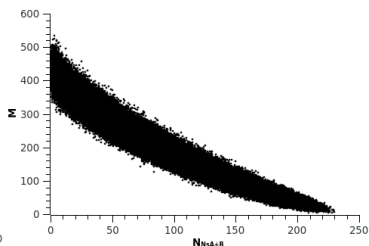
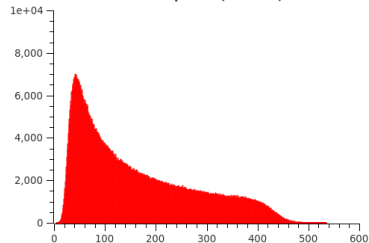
figure: M.L. Miller, K. Reyers, S.J. Sanders, P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57 (2007) 205

# Experimental determination of centrality

Multiplicity:  $M \propto \left(\frac{1-\alpha}{2} N_w + \alpha N_{coll}\right)$

( $\alpha$  determined to fit centrality dependence of multiplicity)

Use measurable quantities:  $M$ ,  $N_{spect}$ , number of spectator neutrons



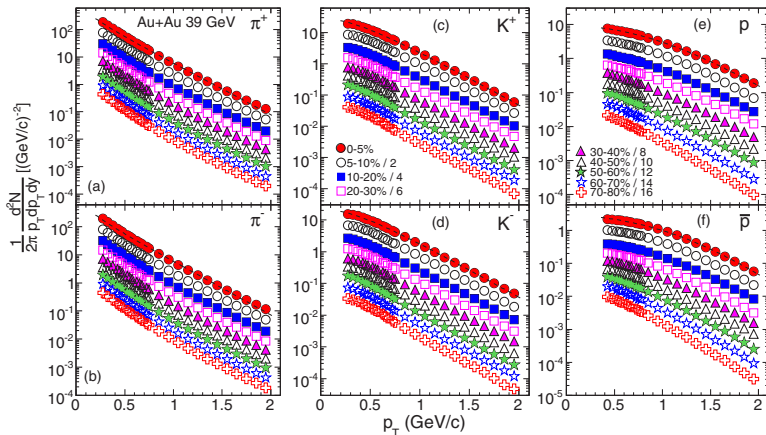
[simulations Matěj Gajdoš, Grammar School Ustí nad Labem]

## Observables: hadrons from the bulk fireball

# Distribution of hadrons

We can obtain temperature at the kinetic freeze-out and the expansion velocity from transverse momentum spectra.

Example:



[STAR collaboration, Phys. Rev. C 96 (2017) 044904]

# Thermal equilibrium in longitudinally expanding fireball

Emission function (pre-blast-wave model)

- longitudinal boost invariance  $u^\mu = \gamma(1, 0, 0, z/t) = (\cosh \eta, 0, 0, \sinh \eta)$
- surface of the cross-cut  $S$
- integrate over all coordinates  $\eta$
- energy of hadron in the fluid rest frame

$$E^* = p^\mu u_\mu = m_t \cosh y \cosh \eta - m_t \sinh y \sinh \eta = m_t \cosh(\eta - y)$$

$$\begin{aligned} \frac{dN}{m_t dm_t} &\propto S \int_{-\infty}^{\infty} \tau d\eta m_t \cosh(\eta - y) \exp\left(-\frac{p^\mu u_\mu}{T}\right) \\ &= S \int_{-\infty}^{\infty} \tau d\eta m_t \cosh(\eta - y) \exp\left(-\frac{m_t \cosh(\eta - y)}{T}\right) \\ &= S \tau m_t K_1\left(\frac{m_t}{T}\right) \end{aligned}$$

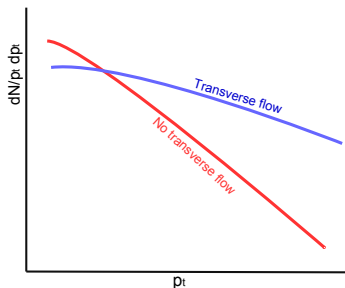
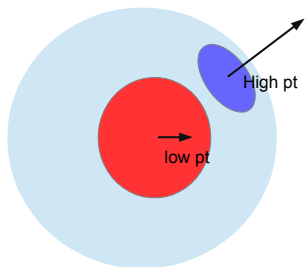
Scaling in  $m_t$ !



# Spectra from transversely expanding fireball

Hadrons for a given  $p_t$  are produced by corresponding *region of homogeneity*.

This enhances production of higher  $p_t$ , i.e. shorter wavelength = blue shift



$T^* \approx T + m\langle v_t \rangle \Rightarrow$  obtain  $T$  and  $\langle v_t \rangle$  from spectra of different sorts of identified particles

## A simple formula for $p_t$ spectra—blast-wave model

Transverse flow velocity  $v_t(r) = \tanh \eta_t(r)$

$$u^\mu = (\cosh \eta \cosh \eta_t(r), \cos \psi \sinh \eta_t(r), \sin \psi \sinh \eta_t(r), \sinh \eta \cosh \eta_t(r))$$

Energy in the fluid rest frame

$$E^* = p^\mu u_\mu = m_t \cosh(\eta - y) \cosh \eta_t(r) - p_t \sinh \eta_t(r) \cos(\phi - \psi)$$

Transverse momentum spectrum

$$\begin{aligned} \frac{dN}{m_t dm_t} &\propto \int_{-\infty}^{\infty} \tau d\eta \int_0^R r dr \int_0^{2\pi} d\psi m_t \cosh(\eta - y) \Theta(R - r) \exp\left(-\frac{p^\mu u_\mu}{T}\right) \\ &= 2\pi\tau m_t \int_0^R r dr K_1\left(\frac{m_t \cosh \eta_t(r)}{T}\right) I_0\left(\frac{p_t \sinh \eta_t(r)}{T}\right) \end{aligned}$$

Resonance contributions are missing here!

# Analysis of the kinetic freeze-out

[I. Melo, B. Tomášik, J. Phys. G to appear, arXiv:1908.03023 [nucl-th]]

- a fit with the blast-wave model

$$S(x, p) d^4x = g_i \frac{m_t \cosh(\eta - y)}{(2\pi)^3} \left( \exp\left(\frac{p_\mu u^\mu - \mu_i}{T_k}\right) + s_i \right)^{-1} \\ \theta\left(1 - \frac{r}{R}\right) \times r dr d\varphi \delta(\tau - \tau_0) \tau d\tau d\eta$$

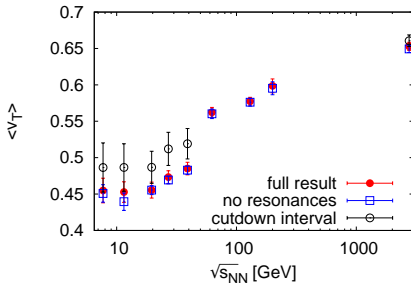
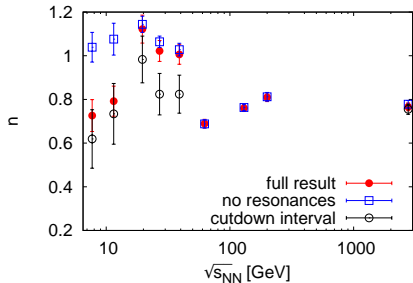
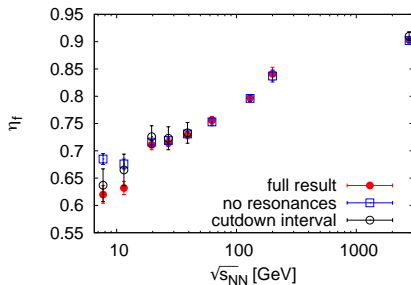
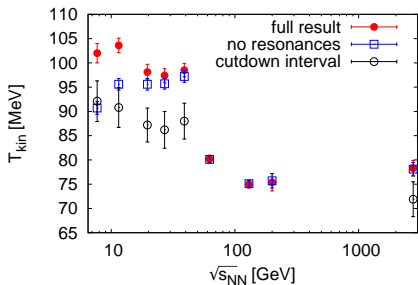
$$E \frac{d^3N}{dp^3} = \int_{\Sigma} S(x, p) d^4x$$

- transverse velocity

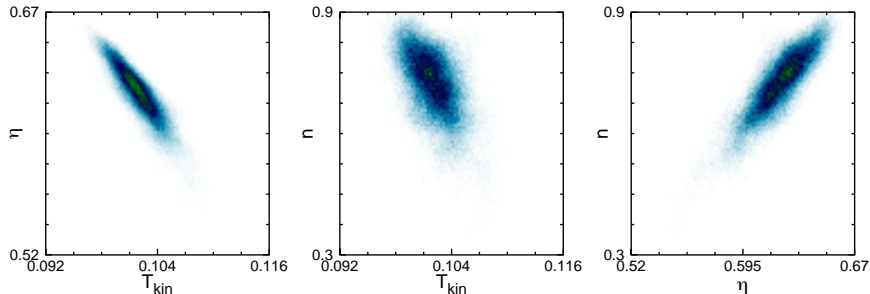
$$v_t = \tanh \eta_t = \eta_f \left(\frac{r}{R}\right)^n$$

- contributions from resonance decays included
- partial chemical equilibrium:  
chemical potentials for each species

# Excitation functions of the freeze-out parameters



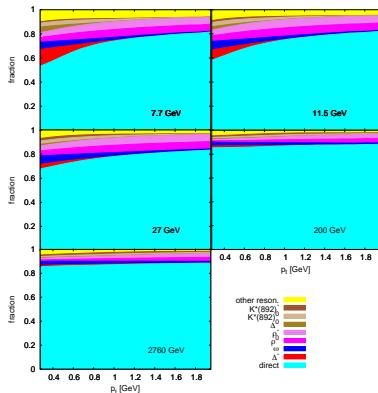
# The parameters are correlated



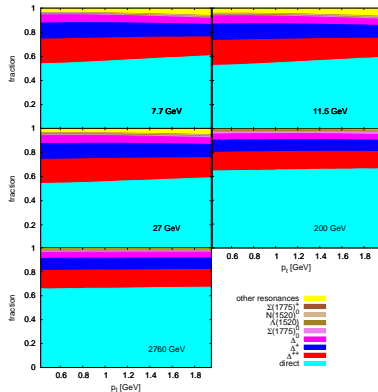
$$\sqrt{s_{NN}} = 7.7 \text{ GeV}$$

# Contributions from the resonances

Relative contributions to  $p_t$  spectra  
pions



protons



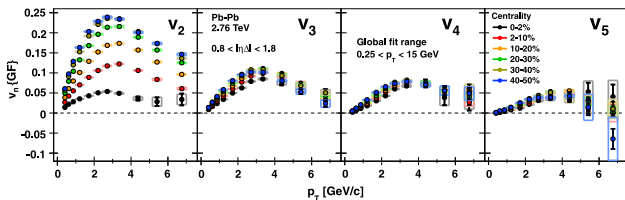
# Azimuthal anisotropy of hadronic momentum distributions

- parametrized by Fourier expansion

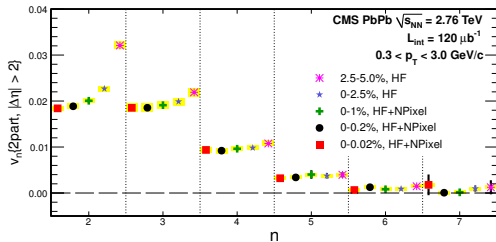
$$\frac{dN}{p_t dp_t dy d\phi} = \frac{1}{2\pi} \frac{dN}{p_t dp_t dy} \left( 1 + 2 \sum_{n=1}^{\infty} v_n(p_t, y) \cos(n(\phi - \phi_n)) \right)$$

- summation over many events in symmetric collisions at midrapidity  
 $\Rightarrow$  symmetry constraints:  $\phi_n = 0, n = 2, 4, 6, \dots$
- all  $v_n$ 's non-vanishing in individual events
- may be a result of stronger blueshift in some directions

# Examples of data



[ALICE collab: Phys. Lett. B **708** (2012) 249]

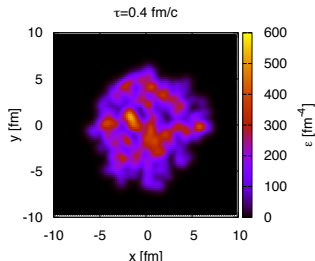
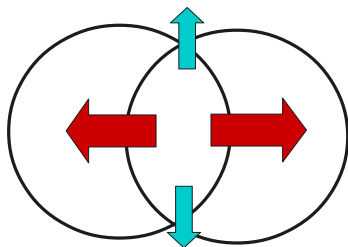


[CMS collab: JHEP 02 (2014) 088]



# From azimuthal anisotropy to momentum anisotropy

- expansion accelerates due to pressure gradients
- higher  $\nabla p \Rightarrow$  stronger expansion
- response to pressure: depends on EoS and transport coefficients
- inhomogeneities in real collisions due to event-by-event fluctuations



[B. Schenke, S. Jeon, C. Gale, PRL106 (2011) 042301]

# Summary

- we can map the QCD phase diagram with colliding nuclei at different energies
- gross features of particle production are statistical
- expansion, including its anisotropies, can be mapped via hadron distributions
- this brings us to study the properties of QCD matter