Phenomenology of heavy-ion collisions
Part 2

Boris Tomášik

FNSPE, České vysoké učení technické, Praha, Czech Republic
and Univerzita Mateja Bela, Banská Bystrica, Slovakia

boris.tomasik@cern.ch

THOR School
Jahorina, Bosnia and Hercegovina

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Observables: hadrons from the bulk fireball

Statistical production of hadrons

Enrico Fermi, 1950s

Suppose:

- we have collision of two hadrons in which energy E is released
- there is only one kind of hadrons (pions) which can be produced
- the matrix elements for all channels are constant, i.e. no microscopic mechanisms are at work

Then:
The probability to produce n hadrons is proportional to the total phase space occupied by states with n hadrons.

In this way, n grows with energy.

Statistical model of hadron gas

- gas of hadrons in chemical equilibrium
- global variables: T, V, chemical potentials for conserved quantum numbers
- for each hadron species: m_i, degeneracy g_i, Bose of Fermi
- chemical potential given by $\mu_i = B_i\mu_B + S_i\mu_S + I_{3,i}\mu_I$
- partition function for the species

$$\ln Z_i = \pm \frac{Vg_i}{2\pi^2} \int d^3\vec{p} \ln\left(1 \pm \exp(-(E_i(m_i,p) - \mu_i)/T)\right)$$

- particle densities

$$n_i = \frac{N_i}{V} = -\frac{T}{V}\frac{d\ln Z_i}{d\mu_i} = \frac{g_i}{(2\pi)^3}\int d^3\vec{p}\,\frac{1}{\exp((E_i - \mu_i)/T) \pm 1}$$

- for free hadrons this can be integrated

$$n_i = \frac{g_i}{2\pi^2} m_i^2 T \sum_{n=1}^{\infty} \frac{(\pm 1)^{n-1}}{n} e^{n\mu_i/T} K_2\left(\frac{nm_i}{T}\right)$$

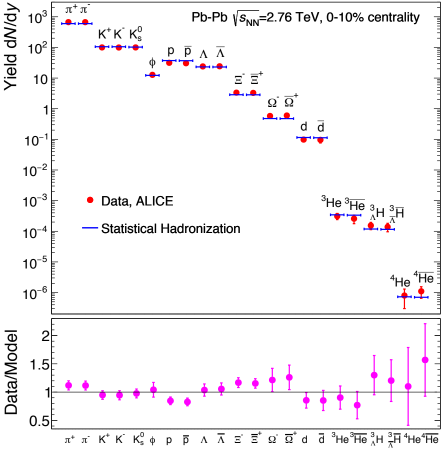
Statistical model: a few more details

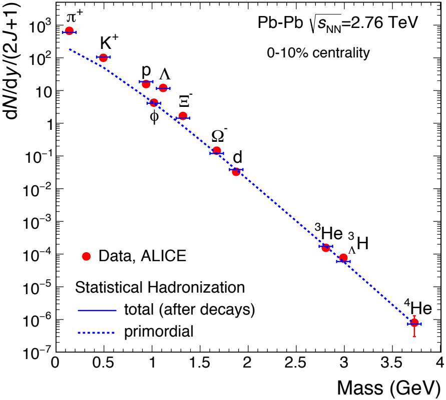
- interacting gas: interactions included via inclusion of resonances
  [R. Dashen, S.K. Ma, J. Bernstein, Phys. Rev. 187 (1969) 345-370]
- to obtain final state hadron abundances after freeze-out, production from the decays of resonances must be included
- μ_S determined from strangeness neutrality,
  (positive μ_B leads to more Λs than $\bar{\Lambda}$s and thus to negative strangeness, which must be balanced by kaons)
- μ_I determined from the initial state
- sometimes fit to ratios, to get rid of the volume
- other details of the model (depending on implementation)
  - list of included resonances
  - fugacity factors for strangeness (suppress both $S = \pm 1$ hadrons in same way)
  - fugacity factors for non-strange hadrons (quarks)
  - excluded volume corrections
  - ...

Models: PBM& Stachel& comp., THERMUS, THERMINATOR, SHARE

Example of a fit to data

[A. Andronic, P. Braun Munzinger, K. Redlich, J. Stachel: Nature 561 (2018) 321-330]





Parameters:

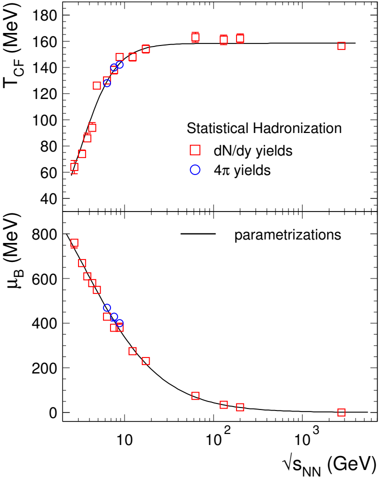
$T_{CF} = 156.5 \pm 1.5$ MeV, $\mu_B = 0.7 \pm 3.8$ MeV, $V = 5280 \pm 410$ fm^3

Collision energy dependence of the chemical freeze-out

[A. Andronic, P. Braun Munzinger, K. Redlich, J. Stachel: Nature 561 (2018) 321-330]

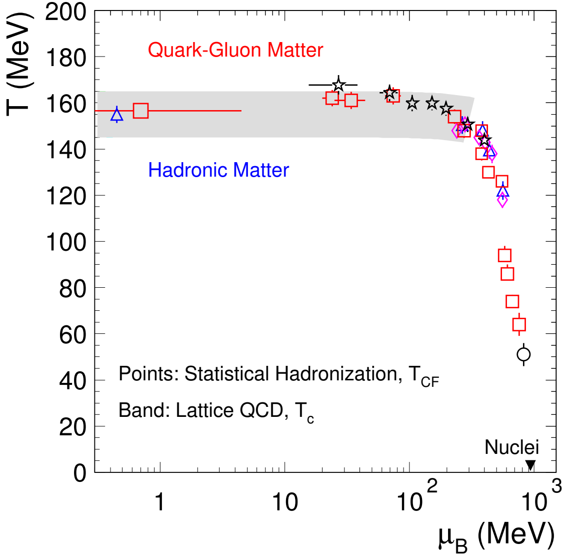
$$
\begin{aligned}
T_{CP}(\sqrt{s_{NN}}) &= \frac{T_{CF}^{lim}}{1 + 13.46(\sqrt{s_{NN}})^{20/9}} \\
T_{CF}^{lim} &= 158.4 \pm 1.4\,\mathrm{MeV} \\
\mu_B(\sqrt{s_{NN}}) &= \frac{a}{1 + 0.288\sqrt{s_{NN}}} \\
a &= 1307.5\,\mathrm{MeV}
\end{aligned}
$$

$\sqrt{s_{NN}}$ in GeV



Chemical freeze-out in the phase diagram

[A. Andronic, P. Braun Munzinger, K. Redlich, J. Stachel: Nature 561 (2018) 321-330]



I will not talk about Strangeness

Strangeness production
must be enhanced in QGP.

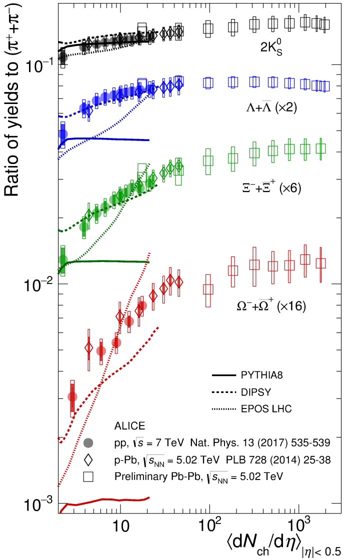
[J. Rafelski, B. Müller, PRL 48 (1982) 1066]

Thresholds

QGP: $gg \to s\bar{s}$, $q\bar{q} \to s\bar{s}$
$E_{th} = 300$ MeV

HG: $\pi + N \to K + \Lambda$
$E_{th} = 531$ MeV

It is also suppressed in small systems.



[ALICE collab. Nature Physics 13 (2017) 535]

I will not talk about Deuterons and clusters

- abundances of clusters are well described by a statistical model at $T = 156.5$ MeV
- binding energy of a deuteron is 2.2 MeV
- the more appropriate mechanism for cluster production is coalescence (and it also works)

Questions:

- Can we distinguish which mechanism is at work here?
- What does it tell us when the statistical model also works?

Fluctuations of conserved charges . . . (WHAT!?)

$$\langle N \rangle = \sum_i N_i P_i = \frac{\sum_i N_i w_i}{\sum_i w_i} = \frac{\sum_i N_i \exp\left(-\frac{E_i - \mu N_i}{T}\right)}{\sum_i \exp\left(-\frac{E_i - \mu N_i}{T}\right)} = \frac{\frac{\partial Z}{\partial \frac{\mu}{T}}}{Z} = \frac{\partial \ln Z}{\partial \frac{\mu}{T}}$$

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Relativistic system:

- creation and annihilation of particle-antiparticle pairs
- study charges which are **conserved in microscopic interactions**
- fluctuations by exchange with the heatbath

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mean baryon number

$$\langle B \rangle = \frac{\partial \ln Z}{\partial \frac{\mu_B}{T}}$$

Statistical physics: Fluctuations of a conserved charge

Higher moments of the (net) baryon number distribution obtained via derivatives of $\ln Z$:

$$
\begin{aligned}
\frac{\partial^2 \ln Z}{\partial \left(\frac{\mu}{T}\right)^2} &= \langle N^2\rangle - \langle N\rangle^2 = \mu_2 = \kappa_2 = \sigma^2 = VT^3\chi_2 \\
\frac{\partial^3 \ln Z}{\partial \left(\frac{\mu}{T}\right)^3} &= \langle N^3\rangle - 3\langle N^2\rangle\langle N\rangle + 2\langle N\rangle^3 = \mu_3 = \kappa_3 = VT^3\chi_3 \\
\frac{\partial^4 \ln Z}{\partial \left(\frac{\mu}{T}\right)^4} &= \langle N^4\rangle - 4\langle N^3\rangle\langle N\rangle - 3\langle N^2\rangle^2 + 12\langle N^2\rangle\langle N\rangle^2 - 6\langle N\rangle^4 \\
&= \mu_4 - 3\mu_2^2 = \kappa_4 = VT^3\chi_4
\end{aligned}
$$

Here:

μ_i : central moments

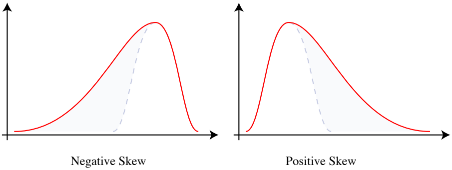
κ_i : central cumulants

χ_i : susceptibilities

Other coefficients that characterise statistical distribution

Skewness:

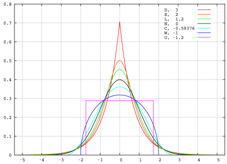
$$S = \frac{\kappa_3}{\kappa_2^{3/2}} = \frac{\mu_3}{\sigma^3}$$



[Rodolfo Hermans on Wikipedia, and Wikipedia]

Kurtosis:

$$\kappa = \frac{\kappa_4}{\kappa_2^2} = \frac{\mu_4}{\mu_2^2} - 3$$



Volume-independent ratios

$$S\sigma = \frac{\kappa_3}{\kappa_2} = \frac{\mu_3}{\sigma^2} = \frac{\chi_3}{\chi_2}$$

$$\kappa\sigma^2 = \frac{\kappa_4}{\kappa_2} = \frac{\mu_4}{\sigma^2} - 3\sigma^2 = \frac{\chi_4}{\chi_2}$$

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Because we look for the state of matter where $\ln Z$ changes dramatically (phase transition, crossover).

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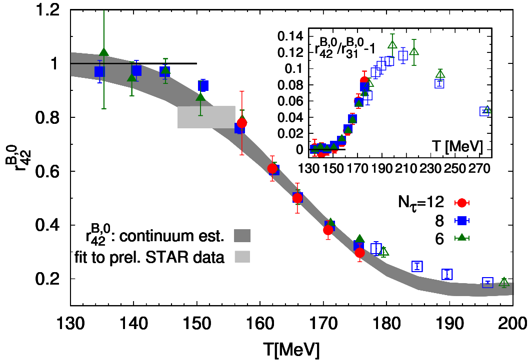
Example: $r_{42}^{B,0} = \chi_4^B / \chi_2^B = \kappa\sigma^2$ at $\mu_B = 0$



[A. Bazavov *et al.*, Phys. Rev. D **96** (2017) 074510]

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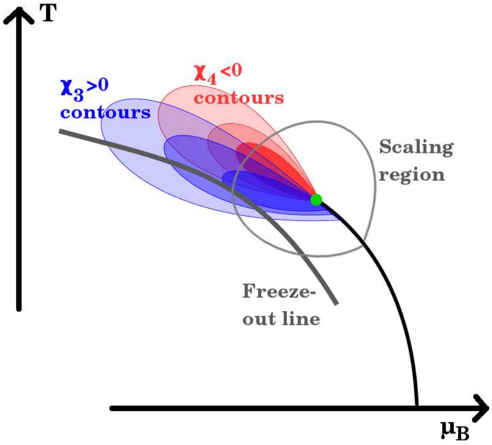
Example: susceptibilities in the Ising model (same universality class)



[J.W. Chen et al.: Phys. Rev. D 95 (2017) 014038]

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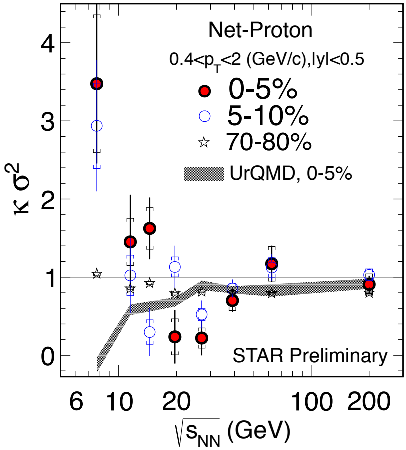


Net proton number fluctuations.

[STAR, PRL 112 (2014) 032302, CPOD2014, QM2015]

Huge increase of $\kappa\sigma^2 = \chi_4/\chi_2$ at $\sqrt{s_{NN}} = 7.7$ GeV.

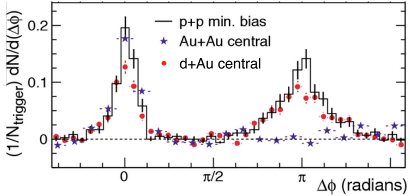
No theoretical understanding, but look at A. Bzdak et al.

Measure the net proton number fluctuations

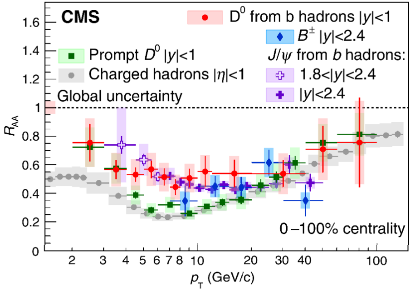
- baryon number susceptibilities χ_i^B calculated on the lattice
- enhancement of susceptibilities near the critical point
- susceptibilities are measurable as cumulants of baryon number distribution
- B-number not measurable, since no neutrons are measured
- Conflict!
  - susceptibilities are calculated in grand-canonical ensemble
  - cumulants are measured in real collisions which conserve B, have limited acceptance, and measure (almost) only protons

But it is very important!

[J. Adams *et al.* (STAR collab.), PRL **91** (2003) 072304]



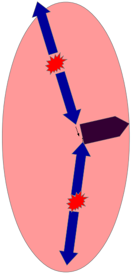
[A.M. Sirunyan *et al.* (CMS collab.), PRL **123** (2019) 022001]



- suppression at high p_t due to quenching of hard partons by deconfined medium
- originally expected less quenching for heavy quarks (dead-cone effect, less gluon bremsstrahlung)
- needs the size and evolution of the flowing bulk in which partons are quenched
- (Experts in the room! Discussion)

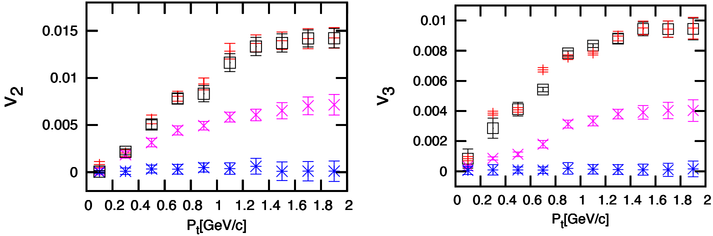
I will not talk about back-reaction on the medium

But I can't help showing our results. . . :-)



Flows induced by hard partons merge
and create azimuthally anisotropic distribution

Anisotropic flow in $b = 0$ collisions: (no jets, hot spots, jets)



[M. Schulc, B. Tomášik, Phys.Rev. C 90 (2014) 064910]

Summary

- we can map the QCD phase diagram with colliding nuclei at different energies
- gross features of particle production are statistical
- expansion, including its anisotropies, can be mapped via hadron distributions
- interaction of hard partons with the medium provides an important probe
- this brings us to study the properties of QCD matter

A list of questions/topics. . .

- How do we identify the critical point?
  - experimentally
  - theoretically
- Find a measurable smoking gun signature of chiral symmetry restoration.
- Link the knowledge about QCD matter with that coming from compact stars and their mergers.
- Arrive at unified description of both high p_t and low p_t production.