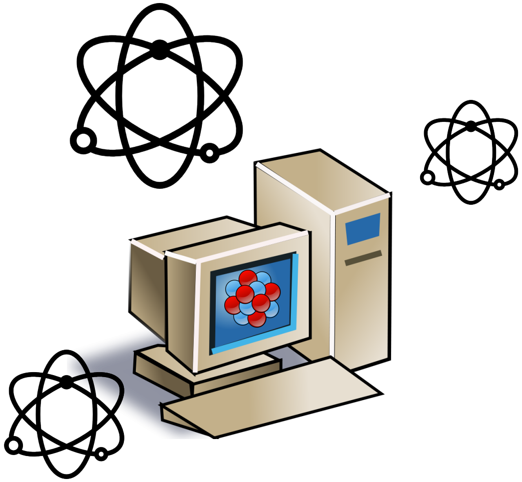
SU(2) gauge theory on digital quantum computers



Jesse Stryker
Institute for Nuclear Theory

work done w/ Natalie Klco (INT)
& Martin Savage (INT)

arXiv:1908.06935

Lawrence Berkeley National Lab
Quantum Computing Mini-Workshop
2019/10/30

BERKELEY LAB

INSTITUTE for NUCLEAR THEORY

Big picture

Conjectured phase diagram credit: G. Endrödi J.Phys.Conf.Ser. 503 (2014) 012009

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Physics targets:

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    - Hadronization
    - Microscopic understanding of nuclear interactions

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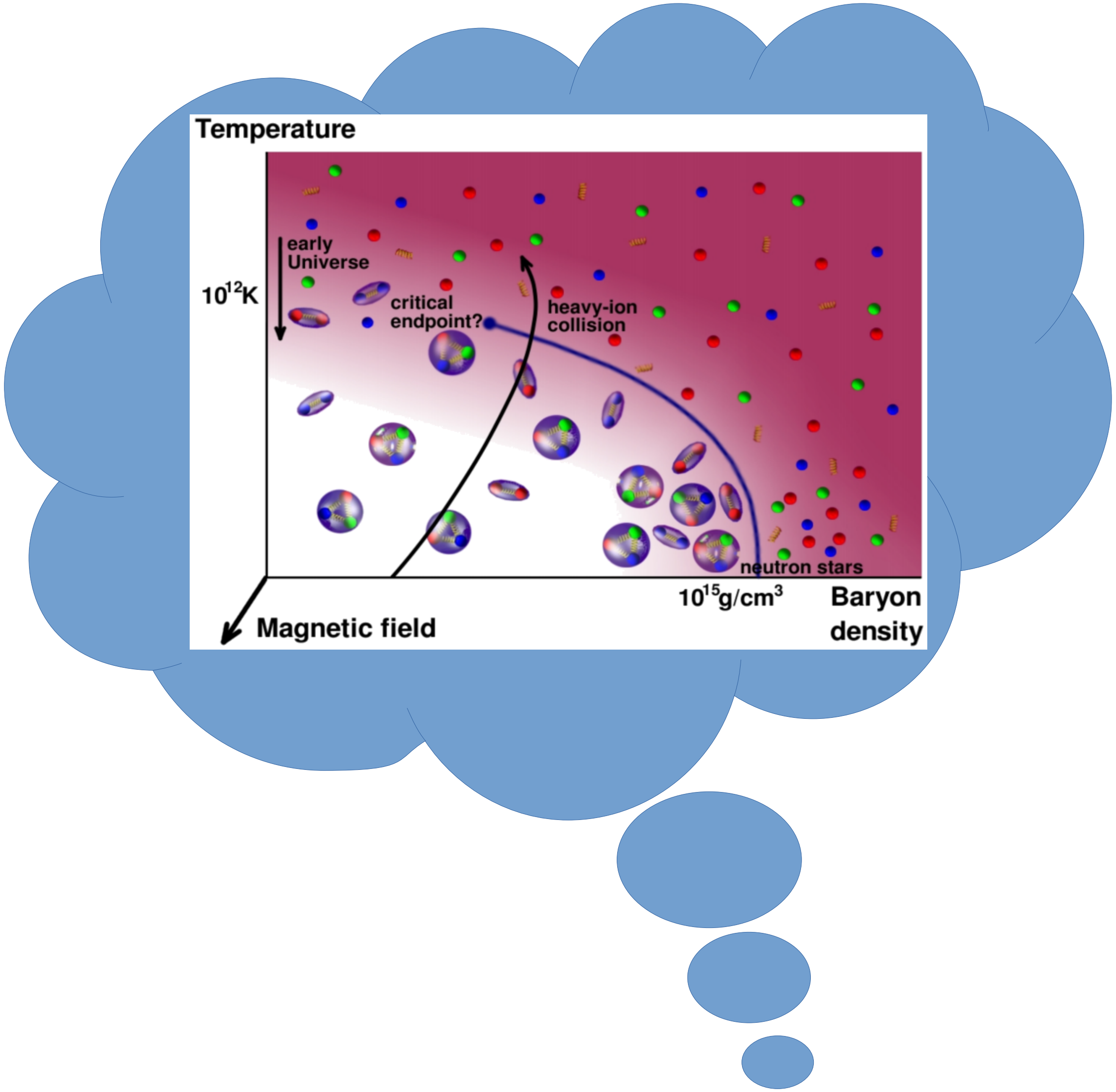

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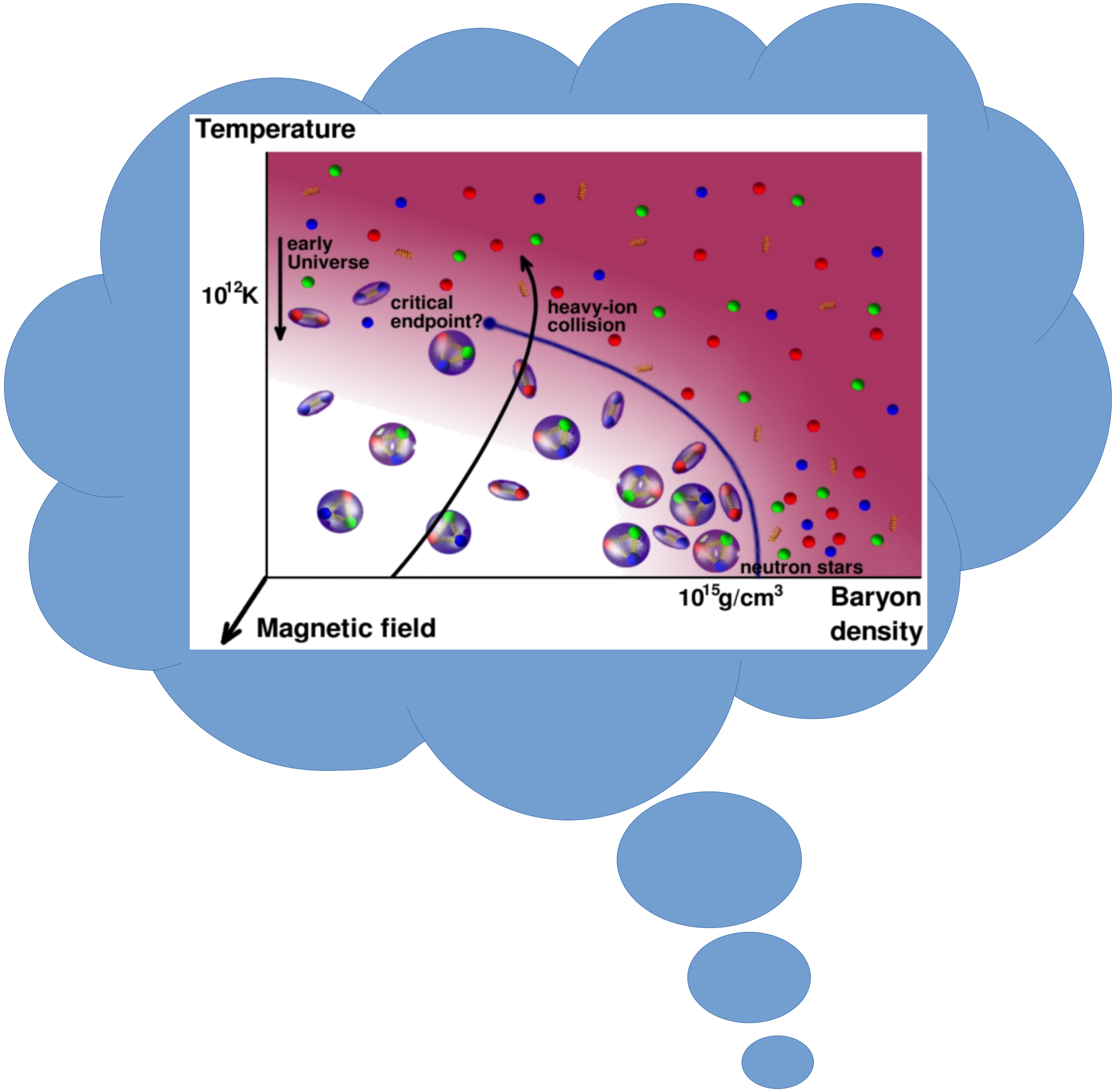

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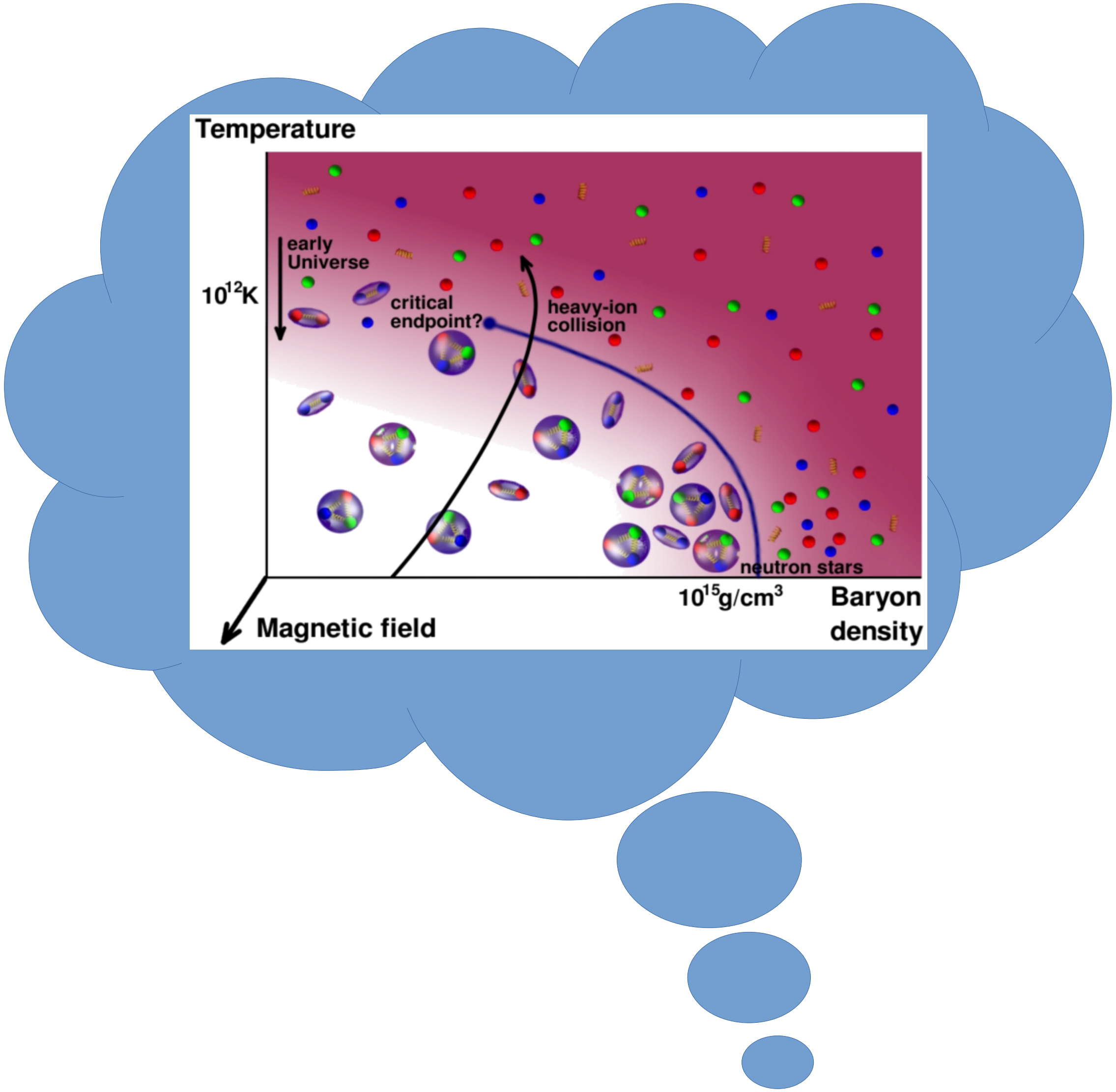

How to make these predictions?

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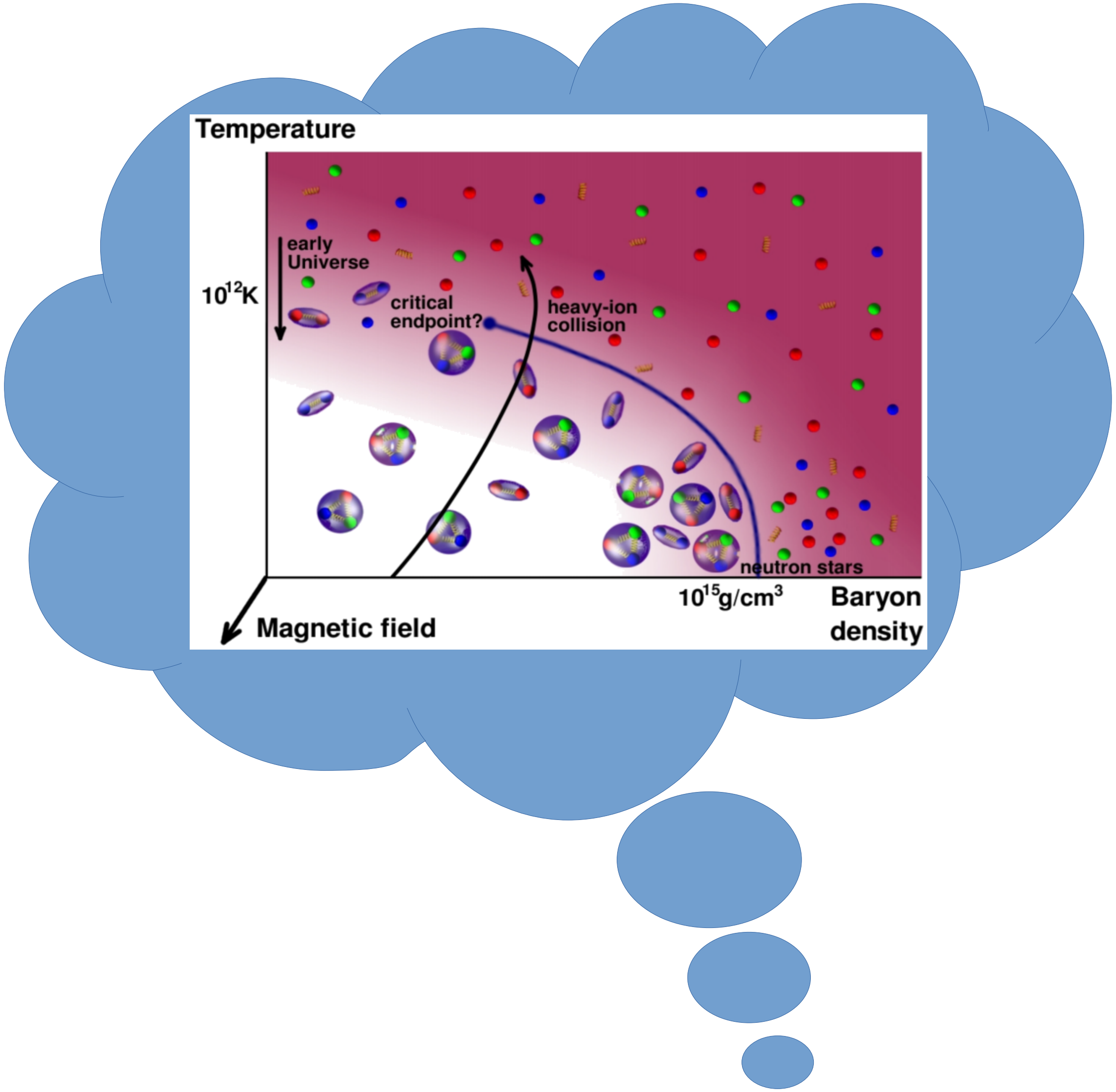

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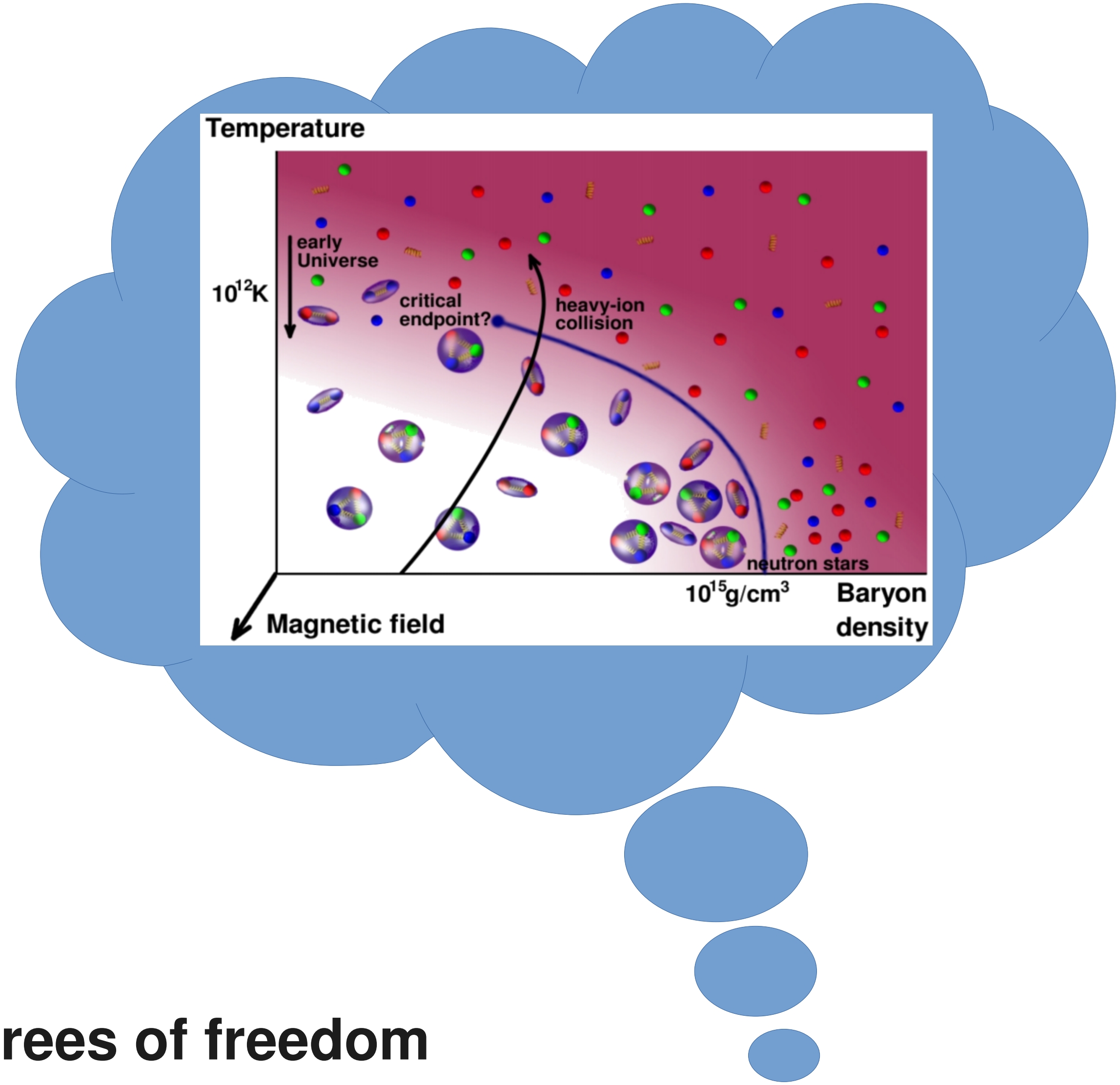


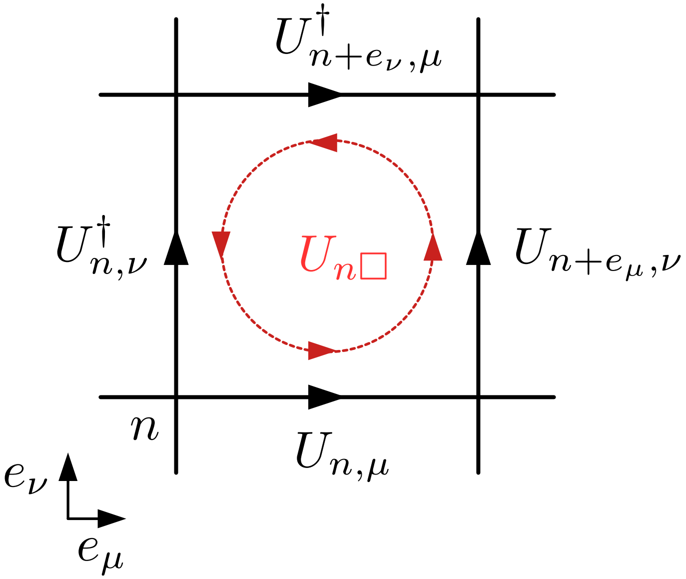
How to make these predictions?

- **Non-perturbative problems**
- **Numerically simulate QCD degrees of freedom**

Conjectured phase diagram credit: G. Endrödi J.Phys.Conf.Ser. 503 (2014) 012009

Traditional lattice ingredients

$x^\mu \to a n^\mu$



Wilson gauge action

$$S_W = -\beta \sum_{n,\mu} \mathrm{tr}(\underbrace{U_{n,\mu} U_{n+e_\mu,\nu} U^\dagger_{n+e_\nu,\mu} U^\dagger_{n,\nu}}_{U_\Box} + U^\dagger_\Box)$$

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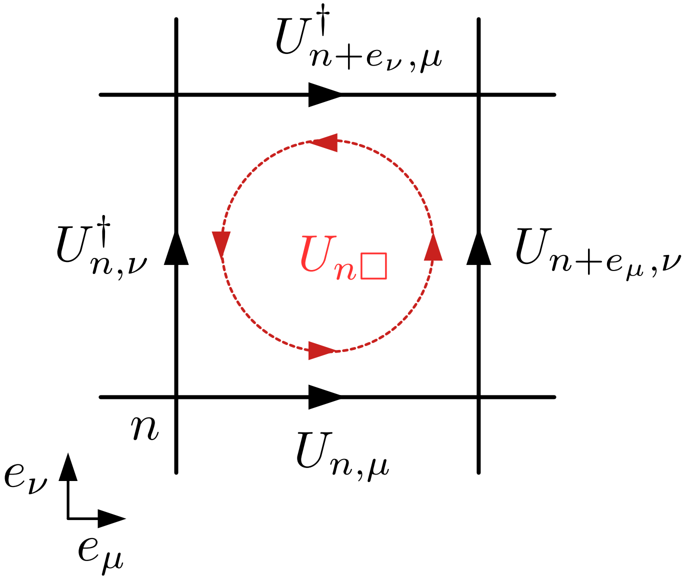

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"link operators" $U_{n,\mu}$ in gauge group G

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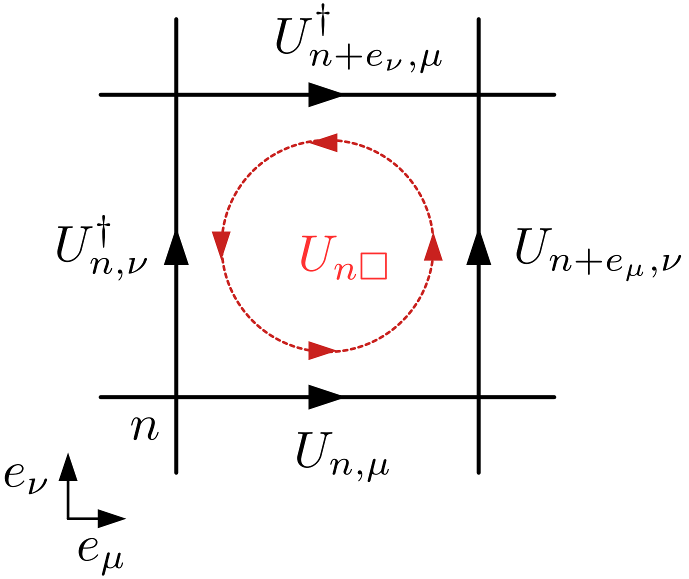

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"plaquette" operator

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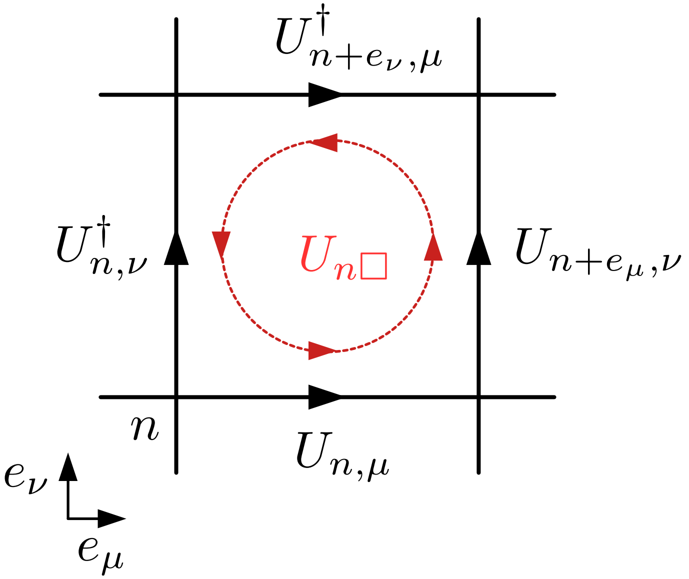

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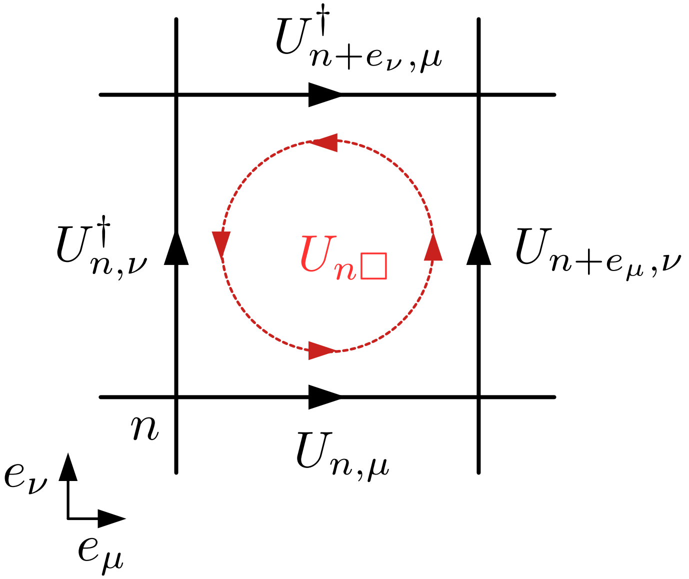

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$$Z = \int \prod_{n,\mu} [\mathcal{D} U_{n,\mu}] e^{-S[U]}$$

Monte Carlo on this

Traditional lattice ingredients

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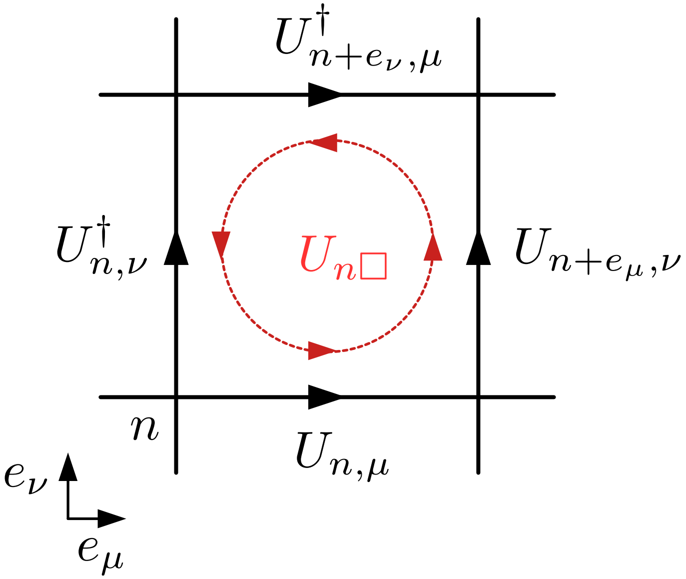

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Relation to continuum gauge fields:

$$U_{n,\mu} = e^{iaA_\mu(x)} \qquad S_W = a^4 \sum_n F^\alpha_{\mu\nu} F^{\mu\nu}_\alpha + \cdots$$

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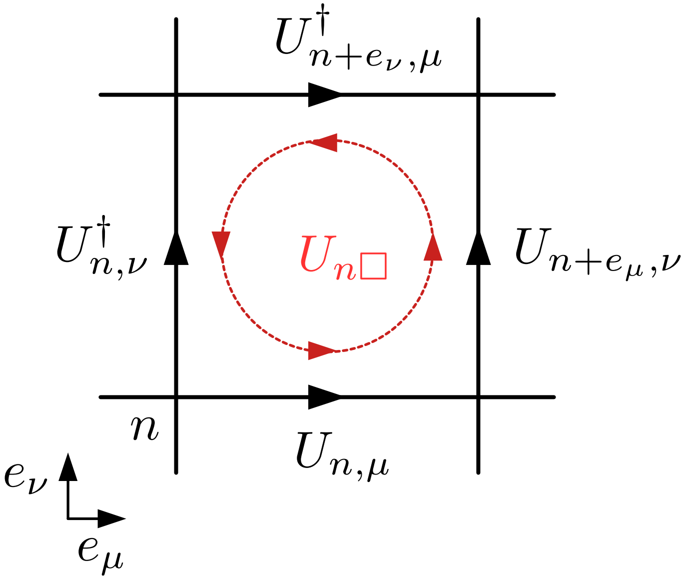

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$U_\square$ "plaquette" operator

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Fermionic matter:

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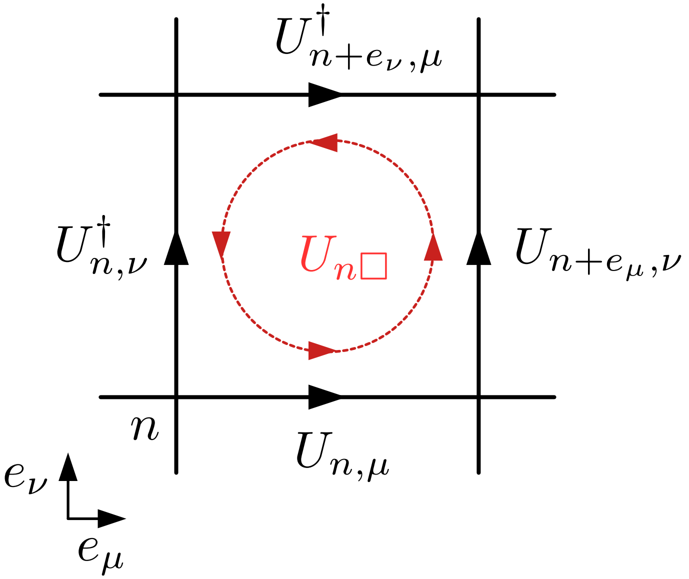

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Fermionic matter:

- Grassmann integrals done analytically → “Fermion determinant”

Traditional lattice

Euclidean path integral Monte Carlo

- **Great for static, equilibrium properties**
- **Real-time dynamics? Nonzero density? Topological term?**

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$-S[U]$ generically complex-valued

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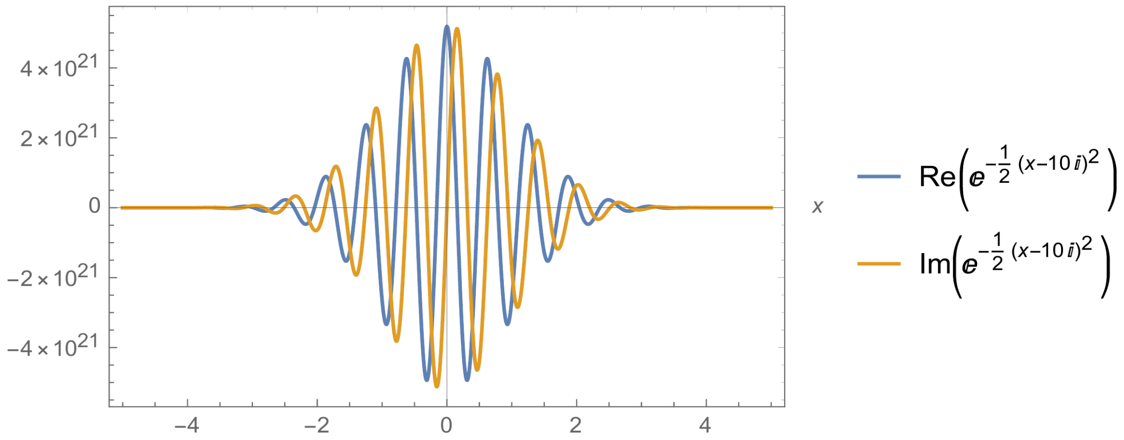
Oscillations of a Gaussian: $S=\frac{1}{2}(x-10\,i)^2$



$$\int dx\, e^{-\frac{1}{2}(x-10i)^2} = \sqrt{2\pi} \ll 10^{21}$$

$$\langle e^{i\theta}\rangle = e^{-50}$$

Exponentially hard to sample oscillating path integral

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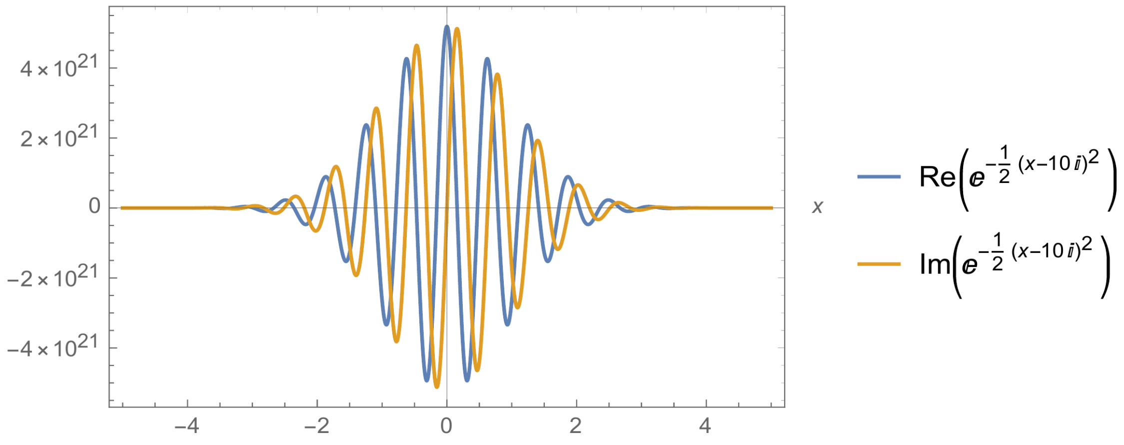
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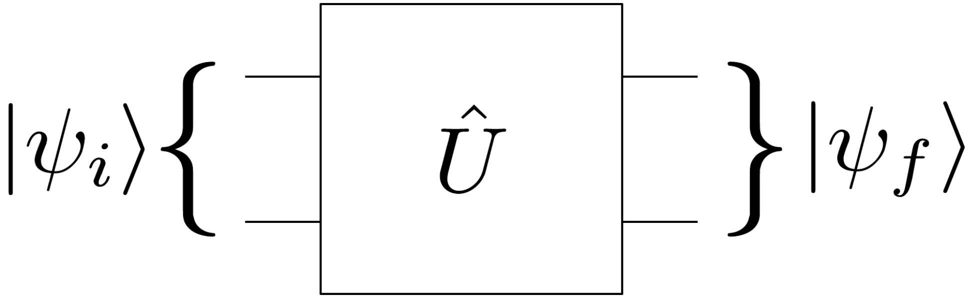

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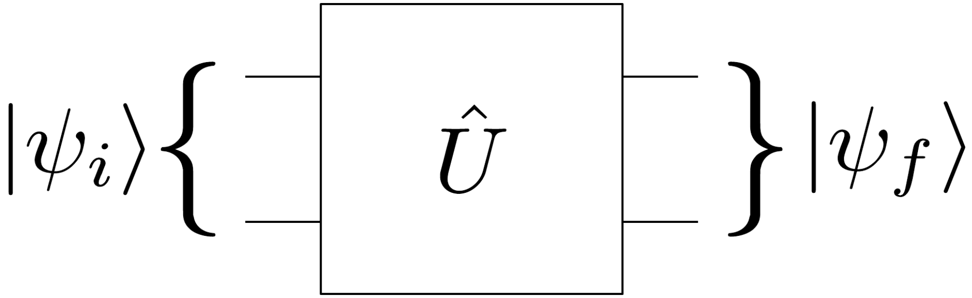

Unitary gates: $e^{-it\hat{H}}$ with your favorite Hamiltonian

- Want to simulate non-perturbative gauge theory
  - Gauge theory on the lattice
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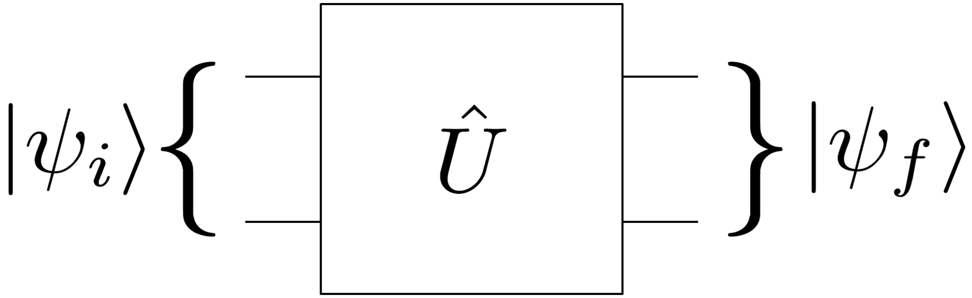

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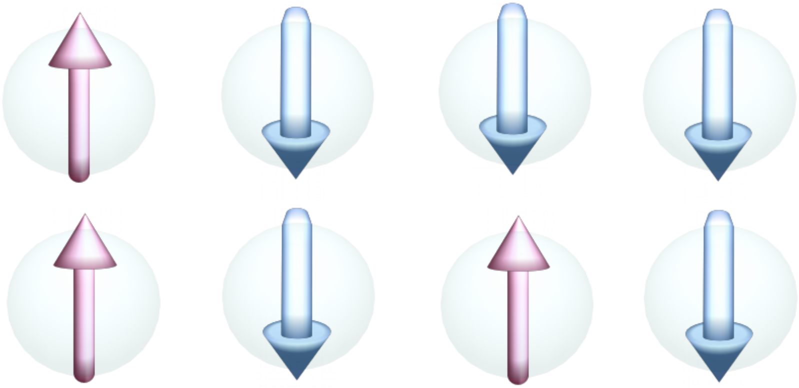


General problem:

How to map the Hilbert space $\mathcal{H}$ and $\hat{H}$ on to the qubits & gates?

Talk outline

- Hamiltonian SU(2) lattice gauge theory
- Plaquette ladder
- Mapping to qubits
- Hardware results from IBM Tokyo
- Summary & future directions

Hamiltonian lattice gauge theory

Canonical quantization, temporal gauge

$U_{n,i} \to \hat{U}_{n,i}$ link variables: **matrices** of **operators**

Gauge transformations: $\hat{U}_{n,i} \to \Omega_n \hat{U}_{n,i} \Omega^\dagger_{n+e_i}$

$$[E^a_{L/R}, E^b_{L/R}] = if^{abc} E^c_{L/R}$$
$$[E^a_R, U] = +UT^a$$
$$[E^a_L, U] = -T^a U$$

Left, right electric fields to generate left, right rotations.

$$\hat{H}_E = \frac{g^2}{2} \sum_{n,i} \hat{E}^a_{n,i} \hat{E}^a_{n,i} \qquad \hat{H}_B = -\sum_n \frac{1}{2g^2} \left(\hat{\Box}_n + \hat{\Box}^\dagger_n \right)$$

$$\hat{\Box}_n \equiv \mathrm{tr}(\hat{U}_{n,\Box})$$

gauge invariant Casimir

Hilbert space

More info: Zohar & Burrello, PRD 91, 054506 (2015)

coordinate-like basis

$$\hat{U}_{ij}\left|g\right\rangle = D_{ij}(g)\left|g\right\rangle \qquad g \in G$$

momentum-like basis

$$E^3_L\left|j,m,m'\right\rangle = -m\left|j,m,m'\right\rangle$$

$$E^3_R\left|j,m,m'\right\rangle = m'\left|j,m,m'\right\rangle$$

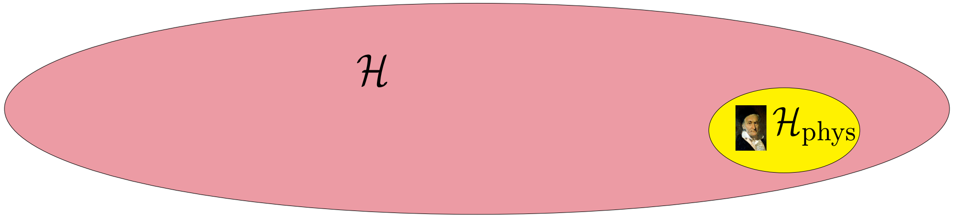
$$\begin{aligned} E^a_L E^a_L\left|j,m,m'\right\rangle &= E^a_R E^a_R\left|j,m,m'\right\rangle \\ &= j(j+1)\left|j,m,m'\right\rangle \end{aligned}$$

$$\langle g|j,m,m'\rangle = \sqrt{\frac{d_j}{|G|}}D^{(j)}_{m,m'}(g)$$

group element state

irrep state

Plus Gauss law constraints



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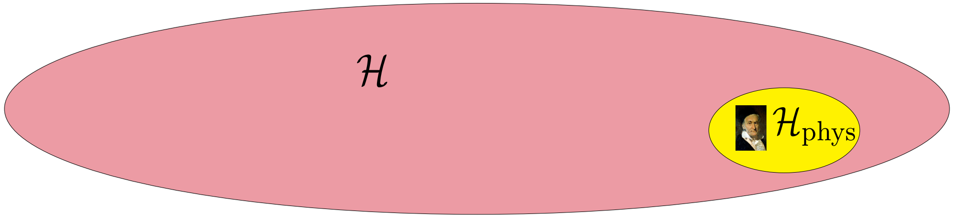
Plus Gauss law constraints

$$\hat{\mathcal{G}}^a = \nabla \cdot \mathbf{E}^a - \rho^a = 0$$

lattice discrete "divergence"



$|j\ m\ m'\rangle$ in practice

- Truncation not ideal for qubit registers

|j m m’> in practice

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$$|j, m, m'\rangle \,, \quad 0 \le j \le J \qquad\qquad d_j = (2j+1)^2$$

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$$|j, m, m'\rangle\,, \quad 0 \leq j \leq J \qquad\qquad d_j = (2j+1)^2$$

$$\Rightarrow \dim(\mathcal{H}) = (8/3)(J+1/2)(J+3/4)(J+1) \neq 2^n$$

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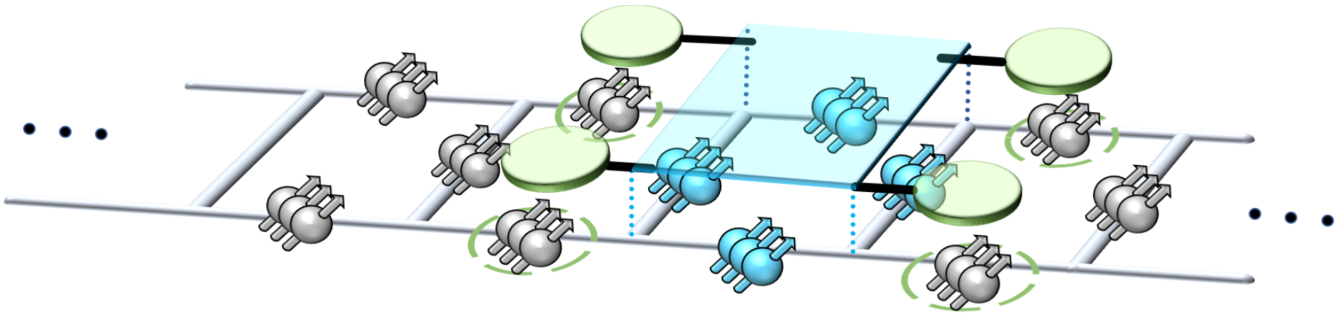
actual photograph of quantum programmer reacting to |jmm'>

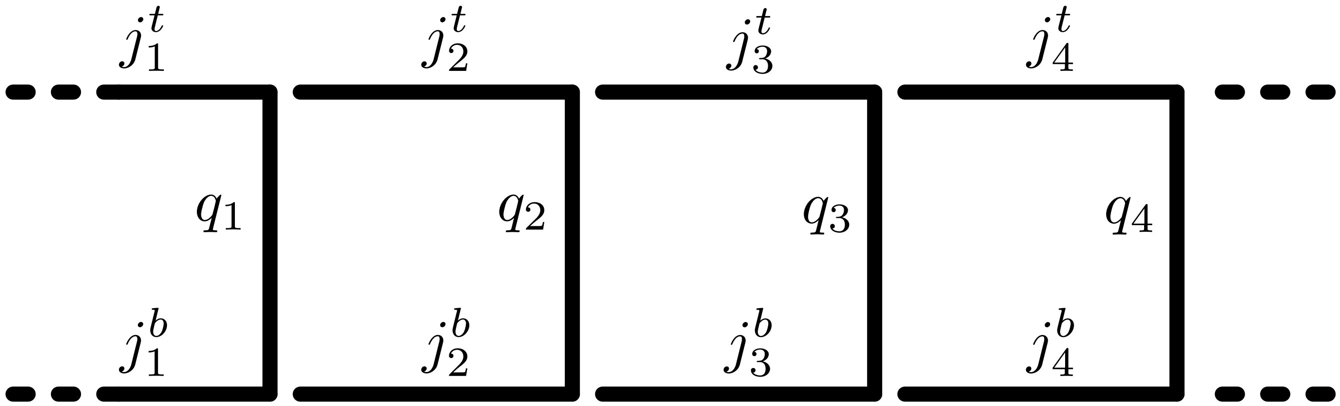
System choice



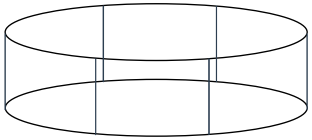
We consider a periodic **string of plaquettes** or “ladder”

- ✓ “1d,” but has H_B
- ✓ 3-pt vertices ↔ unique singlet at vertex for specified j’s
- ✓ Arbitrary length

Gauge singlet basis



periodically wrapped



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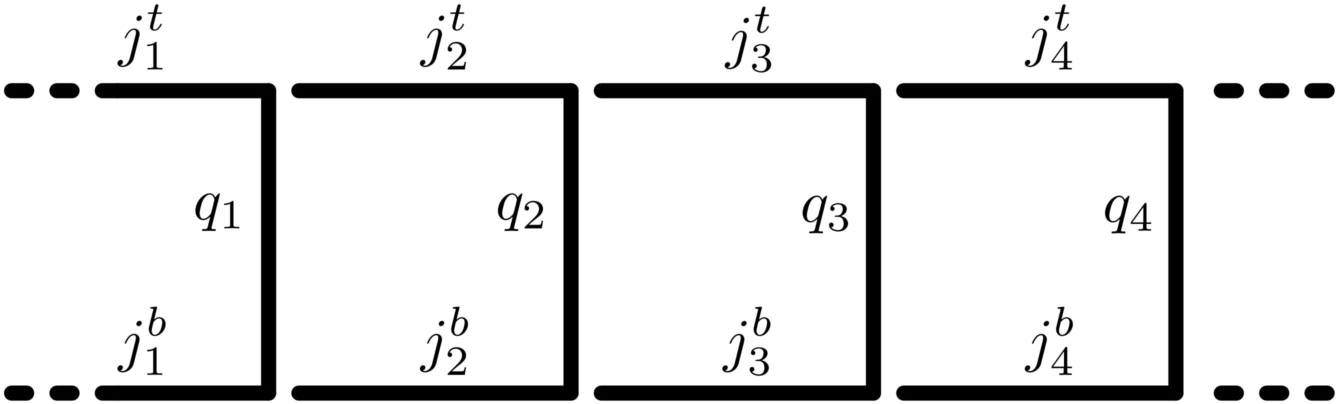
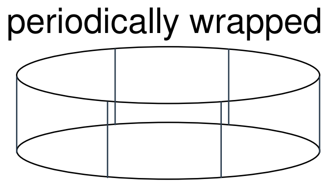
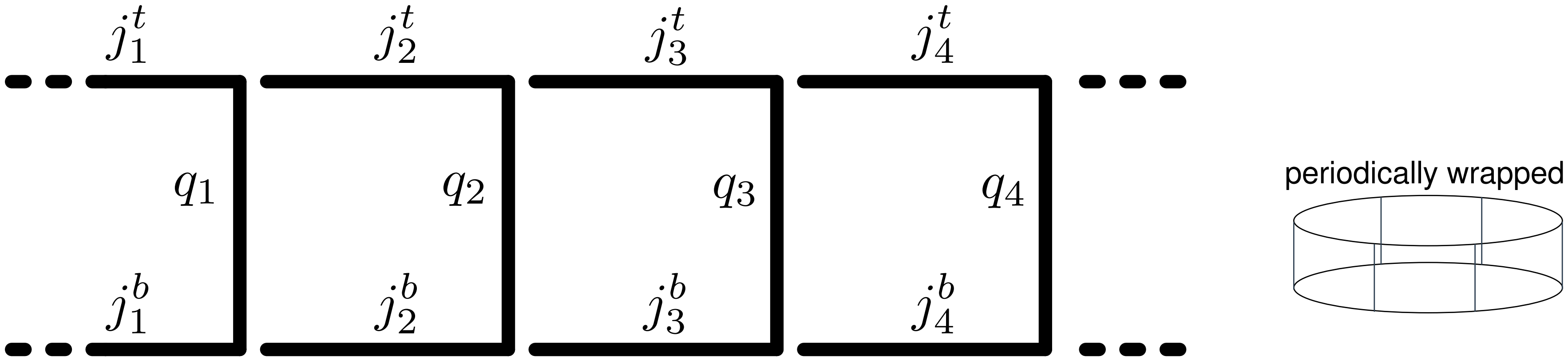




Fully gauge invariant state of lattice with definite link angular momenta:

Gauge singlet basis



Fully gauge invariant state of lattice with definite link angular momenta:

$$|\chi\rangle = \mathcal{N} \sum_{\{m\}} \prod_{i=1}^{L} \langle j_i^t, m_{i,R}^t, j_{i+1}^t, m_{i+1,L}^t | q_i, m_{q_i}^t \rangle$$

} CG's to form singlets at "top" vertices

$$\langle j_i^b, m_{i,R}^b, j_{i+1}^b, m_{i+1,L}^b | q_i, m_{q_i}^b \rangle$$

} CG's to form singlets at "bottom" vertices

$$|j_i^t, m_{i,L}^t, m_{i,R}^t\rangle \otimes |j_i^b, m_{i,L}^b, m_{i,R}^b\rangle \otimes |q_i, m_{q_i}^t, m_{q_i}^b\rangle$$

} Kets going around each "staple"

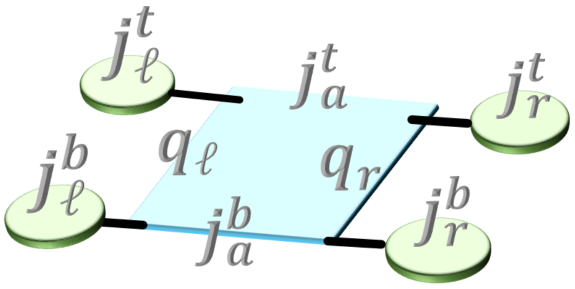
- Just using angular momentum addition (Clebsch-Gordan coefficients) to form singlets

Matrix elements of *H*

- **Non-diagonal elements derive from link operators in H_B : ***

$$\hat{U}_{\alpha\beta}|j,a,b\rangle = \sum_{\oplus J} \sqrt{\frac{\dim(j)}{\dim(J)}}\ |J, a+\alpha, b+\beta\rangle$$

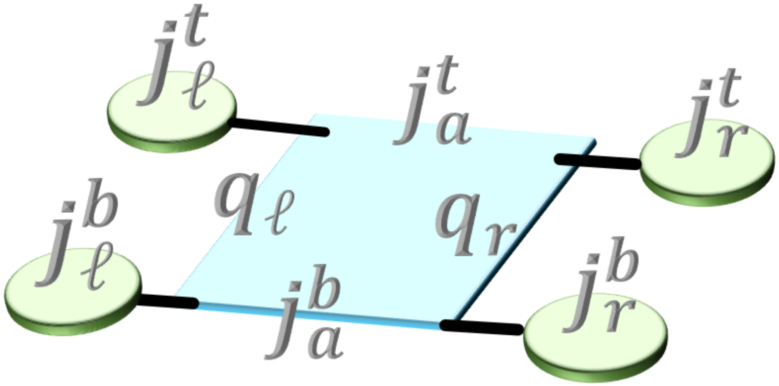
$$\times\ \langle j, a, \frac{1}{2}, \alpha | J, a+\alpha\rangle\langle j, b, \frac{1}{2}, \beta | J, b+\beta\rangle$$



each link op going round a plaquette "adds" ½-unit of angular momentum: $J = j \pm 1/2$

- **This is all the info needed to compute matrix ||*H* || w.r.t. singlet states**

Plaquette operator & gauge-variant completion



- With reduced/singlet basis, matrix elements of $\hat{\square}$ depend on plaquette's j's, as well as adjacent j's
- Still have disallowed states
  - Action of plaquette op on disallowed space is arbitrary

    → "Gauge-variant completion" (GVC): Only bother reproducing correct matrix elements between allowed states

Specific truncation, size

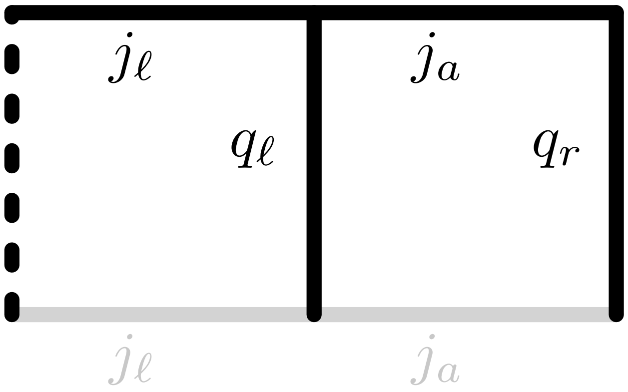
For our simulation:

- Cutoff Λ_j = ½
- Length L=2

\+ Simplifications
→ Four 'active' links

Four qubits represent state

$$|j_l\ q_l\ j_a\ q_r\rangle$$



$$|j=0\rangle \rightarrow |0\rangle\,, \quad |j=1/2\rangle \rightarrow |1\rangle$$

Specific truncation, size

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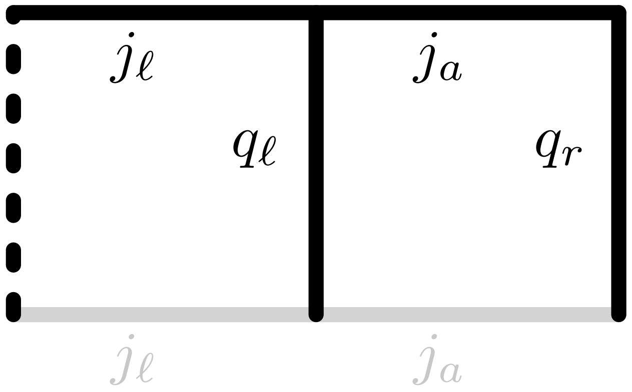
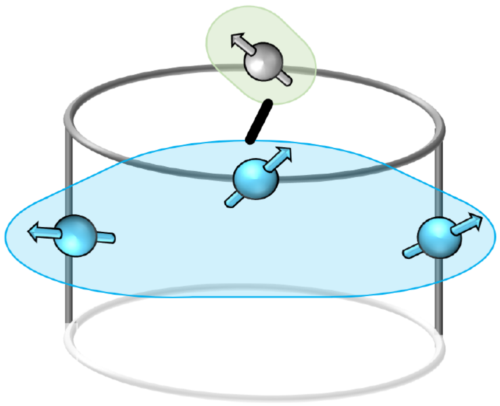


$$|j=0\rangle \to |0\rangle\,, \quad |j=1/2\rangle \to |1\rangle$$



GVC of plaquette operator

$$\hat{\Box}^{(1/2)} = \Pi_0 XXX + \frac{1}{4}\Pi_1 XXX$$

Trotter-Suzuki time evolution

Trotter-Suzuki time evolution

Time evolution operator replaced by Trotter-Suzuki approximation

$$e^{-i\ \Delta t(H_E+H_B)} \simeq e^{-i\ \Delta t H_E} e^{-i\ \Delta t \Box_1/(2g^2)} e^{-i\ \Delta t \Box_2/(2g^2)}$$

Try: *t* spread over one Trotter step, two Trotter steps, … starting from strong-coupling vacuum (all j=0)

Trotter-Suzuki time evolution

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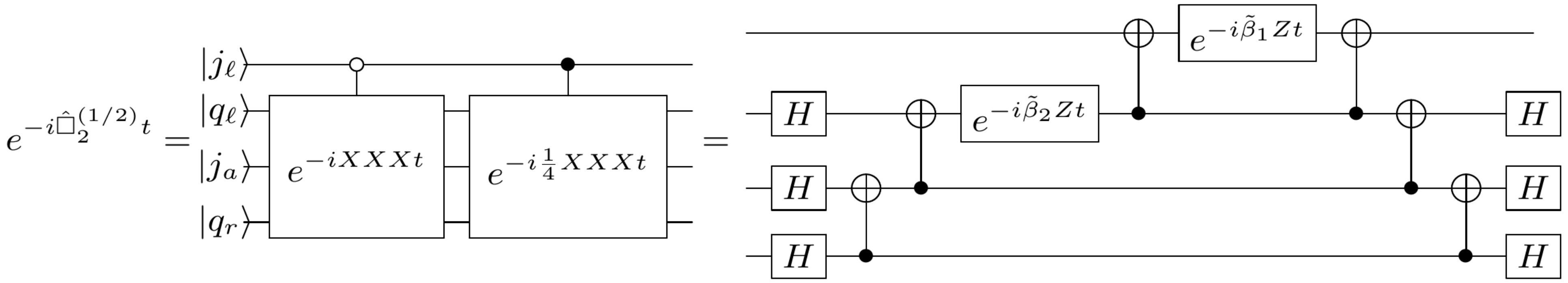
Try: *t* spread over one Trotter step, two Trotter steps, …
starting from strong-coupling vacuum (all j=0)

$$\tilde{\beta}_2 = (5/8)t, \quad \tilde{\beta}_1 = (3/8)t,$$



- Circuit doesn't introduce further systematics
- Trotterization respects gauge constraints

Data processing

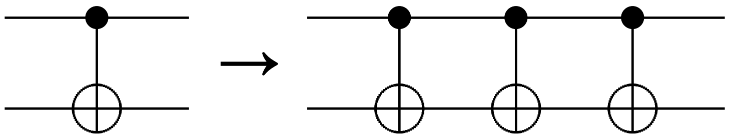
From IBM: Probabilities measured in computational basis

1) Constrained inversion → pre-measurement probabilities

- Needed because of measurement errors

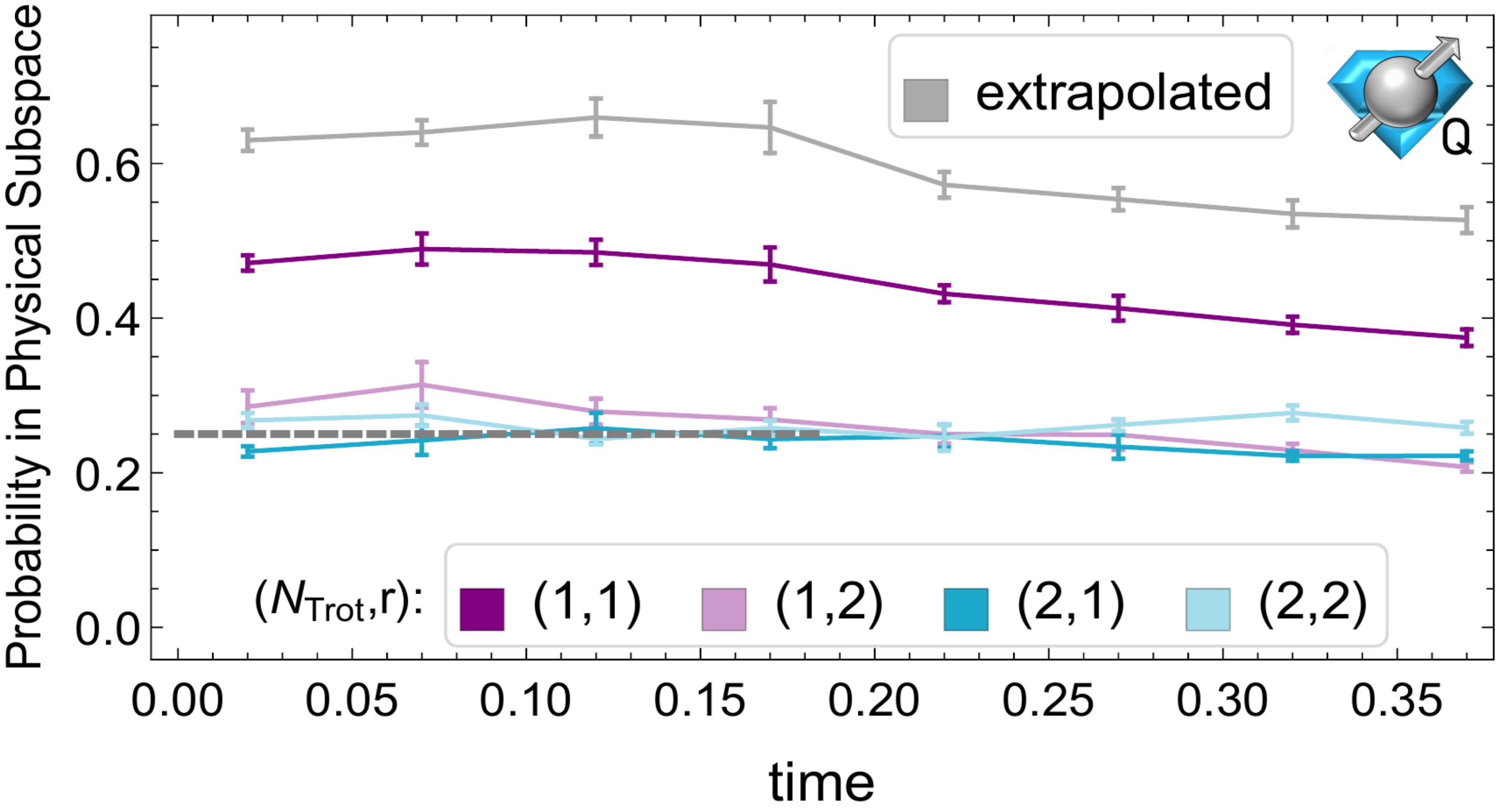
2) Run simulation with superfluous CNOT pairs inserted

- $(\mathrm{CNOT})^2 = 1$, but introduces extra noise



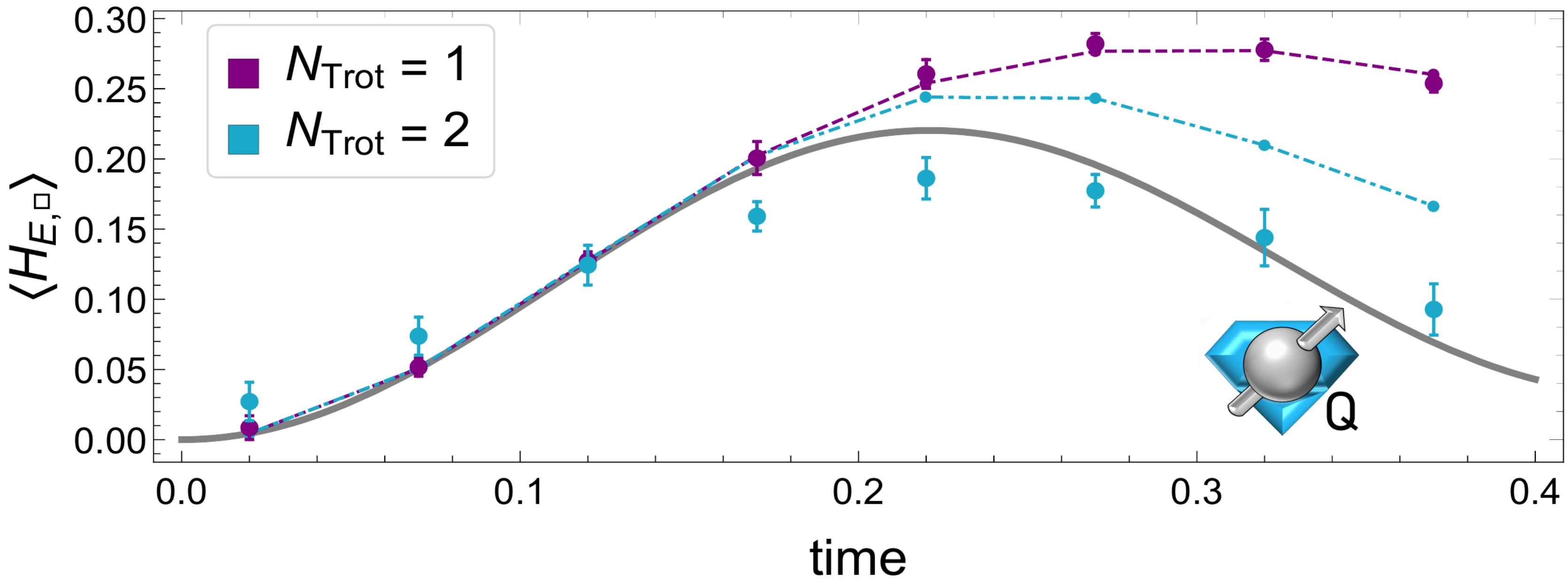
3) Extrapolate pre-measurement probabilities to zero CNOT noise

Probability extrapolation



- Errors into disallowed space mitigated for $N_{\text{Trot}} = 1$
- Coherence is lost for $N_{\text{Trot}} > 1$

Test observable: Plaquette electric energy



- Measure: Electric energy encircling one plaquette
  - Using extrapolated probability densities
- Compare to ideal Trotterized simulation outcome
- $N_{\text{Trot}} = 1$ gets it right within uncertainties!

Summary of hardware results

- First simulation of a truncated SU(2) system done on existing IBM hardware
- Used gauge theory constraints + NISQ-era tricks to mitigate subset of errors
- Low enough circuit depth → Can extract an observable

Generalizations, future directions

- Higher cutoff
  - All links now active
  - More interesting GVC
- Higher dimensions
  - 3-point vertices important
  - More qubits, more gates, more noise

    → Maybe not today, but soon?
- SU(3)
  - Schwinger bosons (I. Raychowdhury et al.) may be helpful for computing reduced matrix elements

FIN

Thank you for your attention!

Questions?

Helpful conversations with: D. Kaplan, I. Raychowdhury, E. Zohar

JRS was supported by DOE Grant No. DE-FG02-00ER41132, and by the National Science Foundation Graduate Research Fellowship under Grant No.1256082.

Plaquette operator matrix elements

$$
\begin{aligned}
&\langle \chi_{\ldots,j_\ell^{t,b},q_{\ell f},j_{af}^{t,b},q_{rf},j_r^{t,b},\ldots} | \hat{\square} | \chi_{\ldots,j_\ell^{t,b},q_{\ell i},j_{ai}^{t,b},q_{ri},j_r^{t,b},\ldots} \rangle = \\
&\qquad \sqrt{\dim(j_{ai}^t)\dim(j_{af}^t)\dim(j_{ai}^b)\dim(j_{af}^b)} \\
&\qquad \times \sqrt{\dim(q_{\ell i})\dim(q_{\ell f})\dim(q_{ri})\dim(q_{rf})} \\
&\qquad \times (-1)^{j_\ell^t+j_\ell^b+j_r^t+j_r^b+2(j_{af}^t+j_{af}^b-q_{\ell i}-q_{ri})} \\
&\times 0.9 \begin{Bmatrix} j_\ell^t & j_{ai}^t & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_\ell^b & j_{ai}^b & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^b \end{Bmatrix} \begin{Bmatrix} j_r^t & j_{ai}^t & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_r^b & j_{ai}^b & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^b \end{Bmatrix}
\end{aligned}
$$

Plaquette operator matrix elements

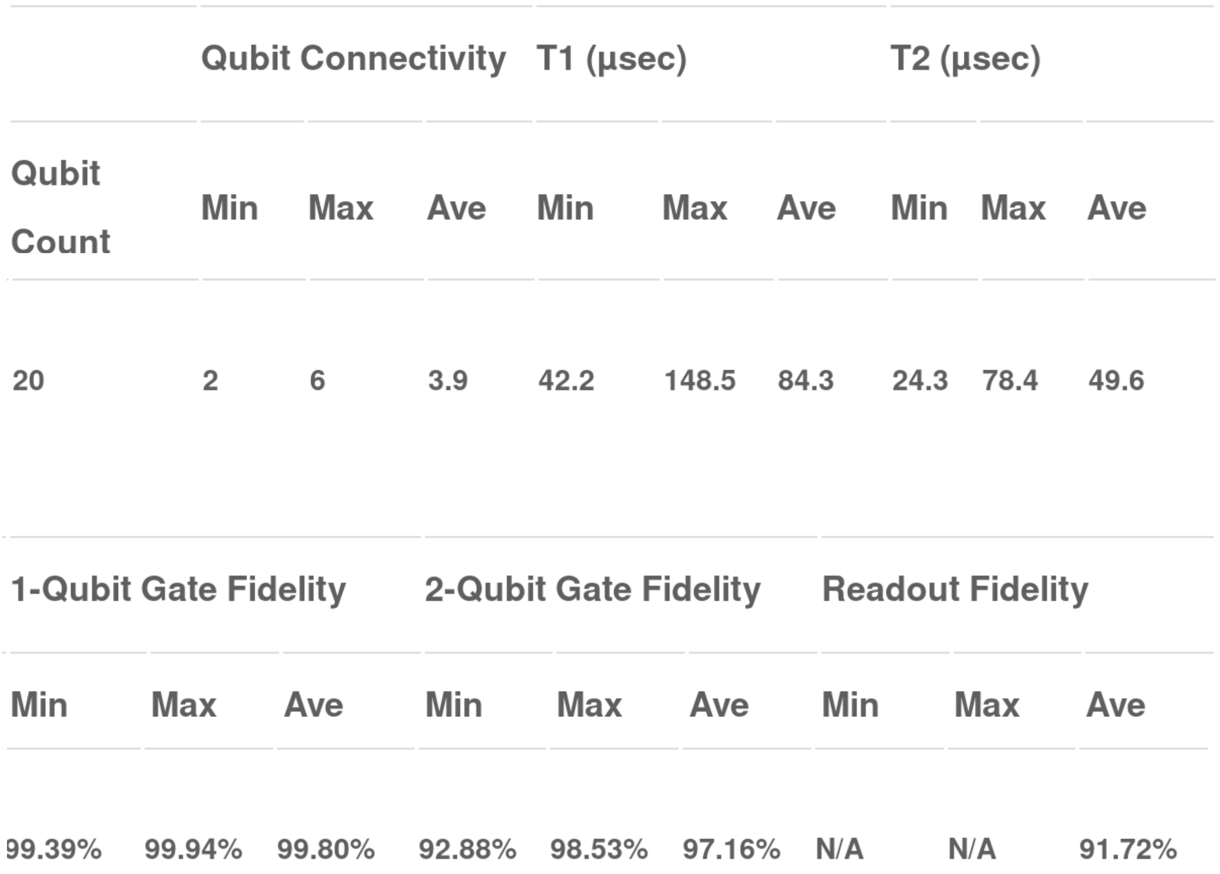
$\langle j_{\ell f}\ q_{\ell f}\ j_{af}\ q_{rf}\ j_{rf} \vert \hat{\square}^{(1/2)} \vert j_{\ell i}\ q_{\ell i}\ j_{ai}\ q_{ri}\ j_{ri} \rangle$	
$\langle 00000 \vert \hat{\square}^{(1/2)} \vert 0\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}0 \rangle, \langle 0\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}0 \vert \hat{\square}^{(1/2)} \vert 00000 \rangle$	1
$\langle 000\tfrac{1}{2}\tfrac{1}{2} \vert \hat{\square}^{(1/2)} \vert 0\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2} \rangle, \langle 0\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2} \vert \hat{\square}^{(1/2)} \vert 000\tfrac{1}{2}\tfrac{1}{2} \rangle$	$\frac{1}{2}$
$\langle \tfrac{1}{2}\tfrac{1}{2}000 \vert \hat{\square}^{(1/2)} \vert \tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}0 \rangle, \langle \tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}0 \vert \hat{\square}^{(1/2)} \vert \tfrac{1}{2}\tfrac{1}{2}000 \rangle$	$\frac{1}{2}$
$\langle \tfrac{1}{2}0\tfrac{1}{2}0\tfrac{1}{2} \vert \hat{\square}^{(1/2)} \vert \tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2} \rangle, \langle \tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2} \vert \hat{\square}^{(1/2)} \vert \tfrac{1}{2}0\tfrac{1}{2}0\tfrac{1}{2} \rangle$	$\frac{1}{4}$

TABLE I. Non-zero matrix elements of the $\Lambda_j = 1/2$ plaquette operator $\hat{\square}^{(1/2)}$ as calculated in Eq. (6) with $j^t_{\ell,a,r} = j^b_{\ell,a,r}$. All other matrix elements between physical states are zero.

For length $L > 2$:

$$\hat{\square}^{(1/2)} = \Pi_0 XXX \Pi_0 + \frac{1}{2}\Pi_0 XXX \Pi_1 + \frac{1}{2}\Pi_1 XXX \Pi_0 + \frac{1}{4}\Pi_1 XXX \Pi_1$$

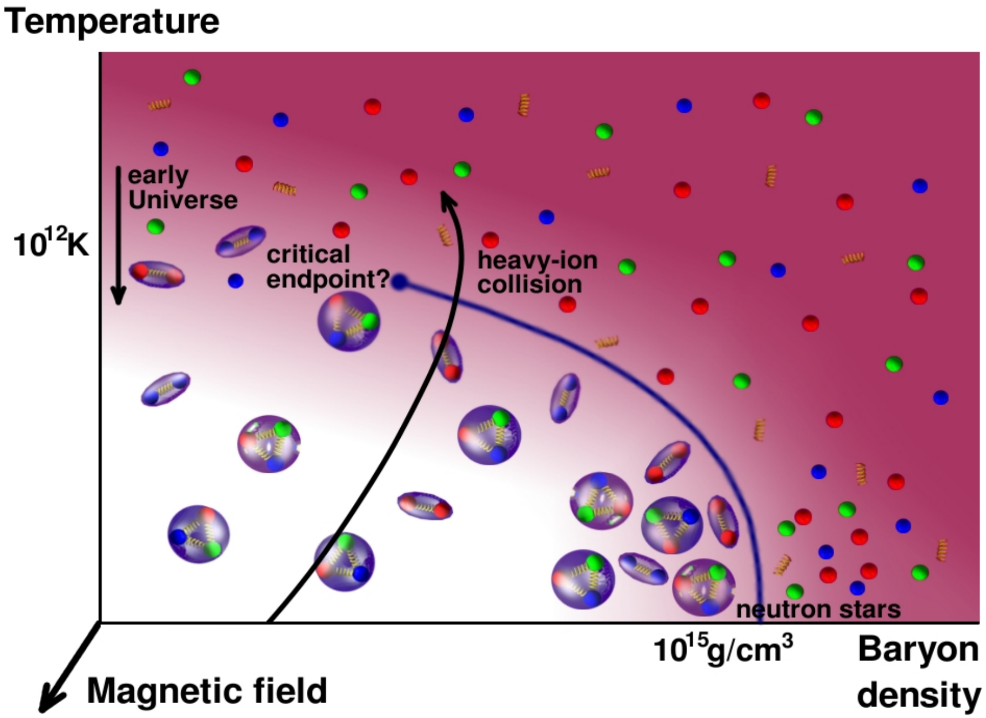
IBM Tokyo Q20 specs



	Qubit Connectivity			T1 (µsec)			T2 (µsec)		
Qubit Count	Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
20	2	6	3.9	42.2	148.5	84.3	24.3	78.4	49.6

1-Qubit Gate Fidelity			2-Qubit Gate Fidelity			Readout Fidelity		
Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
99.39%	99.94%	99.80%	92.88%	98.53%	97.16%	N/A	N/A	91.72%

Graphics credits



QCD phase diagram: Overview of recent lattice results - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Conjectured-QCD-phase-diagram_fig1_261701898 [accessed 23 Jan, 2019]

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