CGC and Low X - the Overview

Alex Kovner

University of Connecticut

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GENYA LEVIN IS 80!

Genya and QCD

A very incomplete and somewhat biased sample:

Levin-Frankfurt Additive Quark Model (1965)
Gribov-Levin-Ryskin (GLR) (1983): Saturation $\equiv$ CGC
Levin Tuchin law (1999) - approach to saturation
Kharzeev-Levin-Nardi (KLN) (2004) - pocket formula for energy and centrality dependence of multiplicity in CGC

... ...

Soft physics at high energy: how to consistently restore Froissart bound
Particle correlations in CGC
From CGC to consistent Reggeon Field Theory

A sonnet for Genya's 80th

The S is very high, the x is low,
Some gluons - they are soft, and some are slow.
The ticks of lightcone time are way too fast,
The seas of QCD are rough and vast,

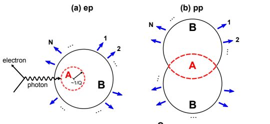
And danger lurks below the gluon clouds
So far outside the unitary bounds.
The sail will saturate with gluons wee,
There's condensate on glass! So hard to see!

And still your boat sails on through shockwaves dense,
Through GLRs and KLNs with grit and sense.
Adventure, purpose, passion chart your course,
We wish you Genya to remain that force!

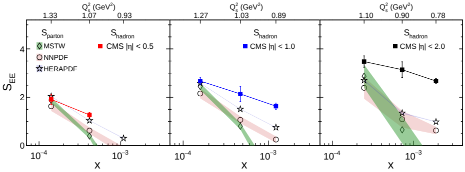
Like light-like Wilson line in lightcone gauge,
Forever straight and fast, defying age!

Entanglement and the parton model

D. Kharzeev and E. Levin, Phys.Rev.D 95 (2017) 11, 114008
E. Gotsman and E. Levin, Phys.Rev.D 102 (2020) 7, 074008
O. Baker and D. Kharzeev, Phys.Rev.D 98 (2018) 5, 054007
Z.Tu, D. Kharzeev and T. Ullrich, Phys.Rev.Lett. 124 (2020) 6, 062001
Why are partons in DIS incoherent? Entanglement with the rest of the proton?
If so, maybe Entanglement entropy = Entropy of partons = Entropy of hadrons?
Entanglement entropy in saturation $\mathbf{S_E = \ln(xG(x))}$, $\quad \mathbf{\frac{d}{dY} S_E = \Delta}$
Reduced density matrix $\hat{\rho}_A \propto 1$ on a subspace fo dimension $\sim \Delta Y$ - maximally incoherent.



$$S_A = -tr[\hat{\rho}_A \ln \hat{\rho}_A] = S_B = -tr[\hat{\rho}_B \ln \hat{\rho}_B] \stackrel{?}{=} S_{\text{hadron}} = -\sum \mathrm{P(N)} \ln \mathrm{P(N)}$$



This looks good.

Entanglement and the parton model

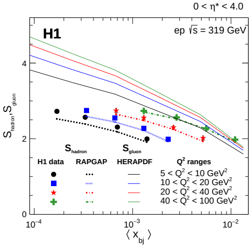
Theoretical studies:

J. Berges, S. Floerchinger and R. Venugopalan,JHEP 04 (2018) - entropy in particle production.

N. Armesto, F. Dominguez, A.K., M. Lublinsky and V. Skokov, JHEP 05 (2019) 025: (extended) JIMWLK: $\frac{d}{dY} S_E = 2\Delta$- density matrix $\hat{\rho}_A$ becomes "more diagonal" with Y.

H. Duan, K. Akkaya, A. K. and V. Skokov, Phys.Rev.D 101 (2020) 3, 036017: not entaglement entropy S_E, but the "entropy of ignorance" S_I (associated with incomplete set of measurements). In a calculable CGC model $S_I \neq S_E$, but not very different.

H1 Collaboration(V. Andreev et al.) 2011.01812 [hep-ex]:



Does not looks so good: $S_{hadrons} < S_{Gluons}$. But for gluons this is S_I, not S_E, and rigorously $S_E \leq S_I$.

Right or wrong, but interesting!

WHAT IS CGC?

The idea: Gribov, Levin, Ryskin 1983.

Gluon density grows with $1/x$, or energy. In a dense hadron the average density of partons (or color fields) defines a natural transverse scale, saturation momentum $Q_S^2 \propto \rho$. It is the "typical momentum" of gluons in the WF, and determines the bulk of physical properties of the hadron (on scales smaller than the hadron size).

Q_S dominates inclusive observables.

"Modern" framework: a gluon probe scatters on such a hadron eikonally with a "pure phase" scattering matrix $S(x) = \mathcal{P} e^{ig\int dx^- T^a A^{+a}(x)}$.

With energy the density and Q_s grow, and the S-matrix evolves.

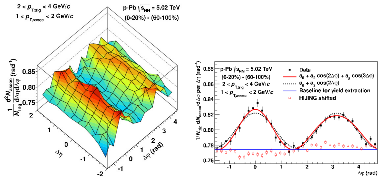
For a dipole probe $D(\vec{x},\vec{y}) = Tr[S^\dagger(\vec{x})S(\vec{y})]$ the energy evolution at LO - the BK equation

$$\frac{d}{dY}D(x,y) = -\frac{\bar{\alpha}_s}{\pi}\int d^2z \frac{(x-y)^2}{(x-z)^2(y-z)^2}\left[D(x,y) - D(x,z)D(y,z)\right]$$

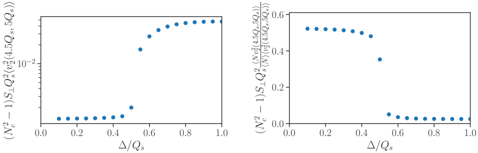
Other observables evolve as well - functional JIMWLK equation for all.

Initial state effects in particle correlations.

Famous ridge correlations aka elliptic flow in p-p and p-A.

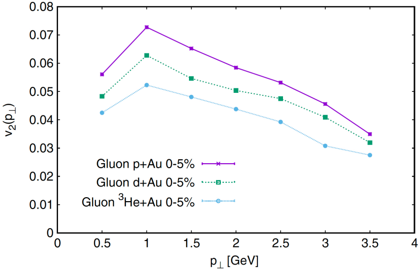


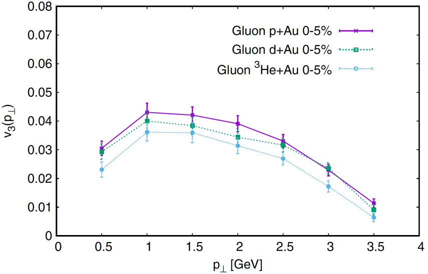
Is it a final state effect (hydro, kinetic theory) or initial state effect (CGC)?
CGC contains several effects that lead to angular correlations: Classical ("color domains", "local density gradients") and Quantum ("Bose enhancement", gluon "HBT"), but could be erased in final state.
Some new studies, e.g.: interesting characteristic stricture of v_2 and v_2-multiplicity correlations as a function of bin width: T. Altinoluk, N. Armesto, A.K., M. Lublinsky and V. Skokov, e-Print: 2012.01810 [hep-ph] (Tolga Altinoluk's talk)



Initial state effects in particle correlations.

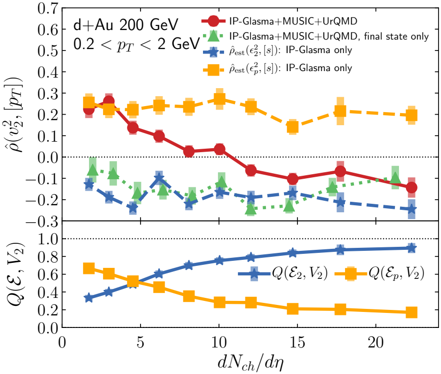
Some features are qualitatively correct, but the systematics of the correlations does not fit. The most comprehensive study in the dense-dilute framework: M. Mace, V. Skokov, P. Tribedy and R. Venugopalan, Phys.Rev.Lett. 121 (2018) 5, 052301, Phys.Rev.Lett. 123 (2019) 3, 039901 (erratum); Phys.Lett.B 788 (2019) 161-165, Phys.Lett.B 799 (2019) 135006 (erratum). The hierarchy of v_2 and v_3 for p-A, d-A and He^3-A is reversed, v_4 is too large.





Initial state effects in particle correlations

Still initial state is important. Recent study: disentangling effects of initial momentum anisotropy (a.k.a. "CGC initial state") and geometry (a.k.a "hydro") G. Giacalone, B. Schenke, C. Shen, Phys.Rev.Lett. 125 (2020) 19, 192301



CGC in EIC

DIS is a clean environment. Observable signals at EIC? Semi inclusive measurements.

A lot of activity on TMD's in the CGC framework: translation between the TMD and CGC (Wilson line) language. (Peter Taels' plenary)

In CGC two "variants" of TMD, or rather "Wigner function"

Dipole: $D(x,y) = \langle Tr[S(x)S^\dagger(y)]\rangle$

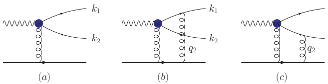
Weizsacker-Williams: $G^{WW}_{ij}(x,y) = \langle S^\dagger(x)\partial_i S(x) S^\dagger(y)\partial_j S(y)\rangle$

Both can be probed in DIS. But inclusive DIS probes only $\int_{x+y} D(x,y) \approx \int_{x+y} G^{WW}_{ii}(x,y)$.

But dependence on impact parameter, orientation and polarization is interesting. Probes the transverse structure of the target.

These can be probed in semi inclusive DIS.

Exploring Linearly Polarized gluons - dijet production.



$G_{ij}^{WW} = \frac{1}{2}\delta_{ij}G - \frac{1}{2}[\delta_{ij} - \frac{k_i k_j}{k^2}]h_\perp$

$h_\perp$ -linearly polarized TMD.

At leading order (large k) $G = h_\perp$, in saturation $G \neq h_\perp$, $\quad h_\perp \to 0$

Study inclusive dijet elliptic anizotropy v_2:

$\vec{q}_\perp = \vec{k}_1 + \vec{k}_2$ -momentum imbalance

$\vec{P}_\perp \approx \vec{k}_1 - \vec{k}_2$ - total dijet momentum

$\frac{d\sigma}{dP_\perp dq_\perp} \propto G(q) + \cos(2\phi_{Pq})h_\perp(q)$ - in "correlation limit"

A. Dumitru, T. Lappi and V. Skokov, Phys.Rev.Lett. 115 (2015) 25, 252301

A. Dumitru, V. Skokov and T. Ullrich, Phys.Rev.C 99 (2019) 1, 015204

Further studies away from "correlation limit" plus diffractive dijets:

F. Salazar and B. Schenke, Phys.Rev.D 100 (2019) 3, 034007

H. Mantysaari, N. Mueller, F. Salazar and B. Schenke, Phys.Rev.Lett. 124 (2020) 11, 112301

At leading order anisotropy of order of several %, decreasing from p to A and decreasing with energy.

At high $P_\perp$ the picture is unfortunately scrambled by Sudakov emissions, which overwhelm the intrinsic correlations:

Y. Hatta, N. Mueller, T. Ueda and F. Yuan, Phys. Lett. B 802, 135211 (2020)

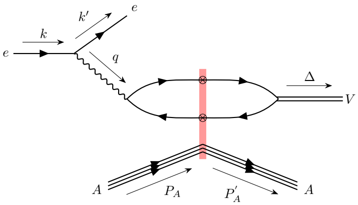
Y. Hatta, Bo-Wen Xiao, F. Yuan(LBNL, NSD) and J. Zhou, e-Print: 2010.10774 [hep-ph]

DVCS and Diffractive Vector meson production

DVCS and Diffractive vector meson production:

Y. Hatta, Bo-Wen Xiao, F. Yuan, Phys. Rev. D 95, 114026 (2017)

H. Mantysaari, K. Roy, F. Salazar, B. Schenke, e-Print: 2011.02464 [hep-ph]



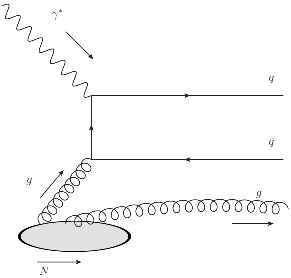
Correlations between the electron plane and the momentum of the meson.

$$D_Y(r_\perp; b_\perp) = D_{Y;0}(r_\perp; b_\perp) + 2D_{Y;2}(r_\perp; b_\perp)cos(2\phi_{rb}) + ...$$

The effect comes from the "skin" region of the target. In MV model the effect is $v_2 \sim 2 - 10\%$, and much smaller for Au target. Also decreases with energy.

Triple jet correlations: Accessing Bose enhancement?

Here is another idea (H.Duan, A.K., Ming Li and V. Skokov, in progress)



A dijet and a third jet in the direction of the proton (nucleus). The cross section is sensitive to double gluon TMD - so directly sensitive to gluon Bose enhancement that should lead to v_2 between the dijet and the third jet.

NLO CGC - Impact factors

NLO precision is important to definitively confirm or rule out saturation.

At NLO one has to calculate the impact factor and the evolution equation.

Both are complicated calculations.

Dipole impact factor (for DIS)

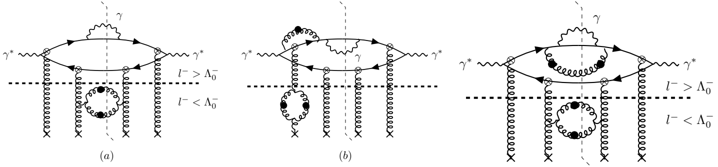
I. Balitsky and G. A. Chirilli, Phys. Rev. D83 (2011) 031502, Phys. Rev. D 87 (2013) no. 1 014013;

G. Beuf, Phys. Rev. D 85 (2012) 034039, Phys. Rev. D94 (2016) no. 5 054016, Phys. Rev. D96 (2017) no. 7 074033;

H. Hanninen, T. Lappi and R. Paatelainen, Annals Phys. 393 (2018) 358;

Last year - a tour de force NLO impact factor for dijet+photon production:

K. Roy and R. Venugopalan, Phys.Rev.D 101 (2020) 3, 034028;



NLO CGC - Evolution

BFKL at NLO is unstable. The same is true for BK or JIMWLK - large transverse logarithms.

NLO BK and JIMWLK derived in

Ian Balitsky and G. Chirilli, Phys.Rev.D 77 (2008) 014019; Phys.Rev.D 88 (2013) 111501; A.K., M. Lublinsky and Y. Mulian JHEP 08 (2014) 114; M. Lublinsky and Y. Mulian, JHEP 05 (2017) 097

Numerical demonstration of instability:

T. Lappi and H. Mantysaari, Phys.Rev.D 91 (2015) 7, 074016

Proposals for resumation of large logs:

G. Beuf, Phys.Rev.D 89 (2014) 7, 074039;

E. Iancu, J.D. Madrigal, A.H. Mueller, G. Soyez and D.N. Triantafyllopoulos, Phys.Lett.B 744 (2015) 293-302

The main idea: impose simultaneous ordering in k^+ and k^- on emissions in the wave function.

NLO CGC - Evolution

The new interesting developement

B. Ducloué, E. Iancu, A.H. Mueller, G. Soyez and D.N. Triantafyllopoulos, JHEP 04 (2019) 081

Upshot - the realization that the energy evolution is the evolution in k^- (the light cone frequency - or the longitudinal momentum of the target) and not k^+ (the longitudinal momentum of the projectile). k^- is physically the right parameter plus the evolution is more stable.

A milder instability still exists - due to emission of **small** dipoles - and it is taken care of by the ordering in k^+ as well as k^-.

$$\frac{\partial D_{xy}(\eta)}{\partial \eta} = \frac{\bar{\alpha}}{2\pi} \int_z \frac{(x-y)^2}{(x-z)^2(z-y)^2} \Theta(\eta - \delta_{xyz}) \left[D_{xz}(\eta - \delta_{xz;r}) D_{zy}(\eta - \delta_{zy;r}) - D_{xy}(\eta) \right].$$

$$\delta_{xyz} = \max\{\delta_{xz;r}, \delta_{zy;r}\}, \;\; \delta_{xz;r} \equiv \ln \frac{\max\{(x-z)^2, r^2\}}{(x-z)^2}, r = |x-y|$$

Evolution in $\eta = \log k^-$ is certainly the way to go. But I think this is not the end of the story - some important physics is still unaccounted for.

Numerically the new and older suggestions were used to fit the DIS HERA data G. Beuf, H. Hanninen, T. Lappi and H. Mantysaari, Phys.Rev.D 102 (2020) 074028 - all resumed equations seem to perform comparably. (Hanninen's talk)

Helicity and Orbital Angular Momentum at low x

Helicity evolution at low x within a CGC-like framework. One sub eikonal vertex in the scattering introduces dependence on the helicity of the target.
Y. Kovchegov, D. Pitonyak and M. D. Sievert, Phys. Rev. D95 (2017) 014033, Phys. Rev. Lett. 118 (2017) 052001, Phys. Lett. B772 (2017) 136–140, JHEP 10 (2017) 198; Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D99 (2019) 054032.
This is a doubly logarithmic evolution, so the helicity grows fast. In the linear regime in the large N_C limit.

$$\Delta q^S(Q^2, x) \propto \left(\frac{1}{x}\right)^{\frac{4}{\sqrt{3}}\sqrt{\frac{\bar{\alpha}_s}{2\pi}}}$$

and similar for ΔG.
Also OAM has been calculated in Y. V. Kovchegov, JHEP 1903, 174 (2019)
Does not agree with the solution of "IREE"
J. Bartels, B. I. Ermolaev and M. G. Ryskin, Z. Phys. C 70, 273 (1996), Z. Phys. C 72, 627 (1996)
Also does not lead to cancelation between helicity and OAM contributions to total spin
R. Boussarie, Y. Hatta and F. Yuan, Phys.Lett.B 797 (2019) 134817
So some unfinished business here...
Recently this evolution has been formulated in a form analogous to JIMWLK equation:
F. Cougoulic and Yu. Kovchegov, Phys.Rev.D 100 (2019) 11, 114020, Nucl.Phys.A 1004 (2020) 122051
This provides tools for studying the saturation corrections in the context of helicity TMD's

Is JIMWLK unitary?

JIMWLK is evolution for scattering amplitude S.
But the evolution originates from the evolution of the QCD Wave Function of the projectile and/or target. If we "unscramble" JIMWLK evolution for S and find evolution of Ψ, we should find at any rapidity **a wave function**, i.e. all partial probabilities must be positive $0 \leq P_i = \psi_i^* \psi_i \leq 1$ (s-channel unitarity).

Does it happen?

In a toy model in one transverse dimension

A. K., E. Levin and M. Lublinsky, JHEP 08 (2016) 031: toy JIMWLK generates negative probabilities in the "target wave function"! It can be amended to be unitary at any density (and reduce to JIMWLK where appropriate).

What about the real JIMWLK?

A.K., E. Levin, Ming Li, M. Lublinsky JHEP 09 (2020) 199: the target evolution also generates negative probabilities.

So JIMWLK is not *s*-channel unitary! Can we fix it? We know how to restore t-channel unitarity (A.K., E. Levin, Ming Li, M. Lublinsky, JHEP 10 (2020) 185,) but do not know yet whether it is enough for the s-channel unitarity.