How to connect physics of spin with hydrodynamics

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The VIth International Conference on the Initial Stages of High-Energy Nuclear Collisions

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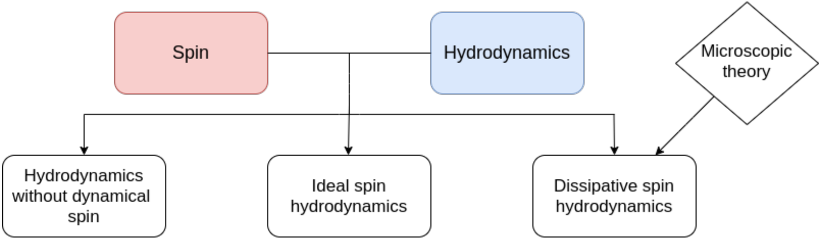
Spin	-	**Hydrodynamics**
Quantum		Classical

- Why do we want to connect?
- How can we connect?
- This talk: Construction of spin hydrodynamics



Focus on hydrodynamics for massive spin-1/2 particles from kinetic theory

NOT COVERED:

Massless case: Chiral kinetic theory

Son, Yamamoto, PRD87, 085016 (2013)
Chen, Son, Stephanov, Yee, Yin, PRL113, 182302 (2014)
Chen, Son, Stephanov, PRL115, 021601 (2015)
Hidaka, Pu, Yang, PRD95, 091901 (2017), PRD97, 016004 (2018)
Huang, Shi, Jiang, Liao, Zhuang, PRD98, 036010 (2018)
Gao, Liang, Wang, Wang, PRD98, 036019 (2018)
Yang, PRD98, 076019 (2018)
Gao, Pang, Wang, PRD100 (2019)

Chiral hydrodynamics

Buzzegoli, Becattini, JHEP 12 (2018) 002
Shi, Gale, Jeon, arXiv:2008.08618

Spin-1 particles

Chernodub, Cortijo, Landsteiner, PRD98, 065016 (2018)
Huang, Mitkin, Sadofyev, Speranza, JHEP 10 (2020) 117
Hattori, Hidaka, Yamamoto, Yang, arXiv:2010.13368

Entropy analysis

Hattori, Hongo, Huang, Matsuo, Taya, PLB795, 100 (2019)
Fukushima, Pu, arXiv:2010.01608
Li, Stephanov, Yee, arXiv:2011.12318

Lagrangian formulation

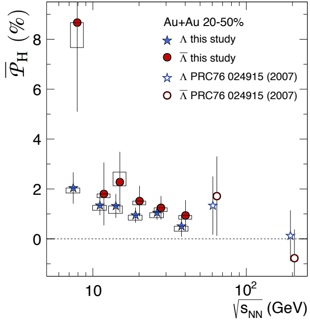
Montenegro, Tinti, Torrieri, PRD96, 076016 (2017)
Montenegro, Torrieri, PRD100 (2019) 5, 056011, PRD102 (2020) 3, 036007

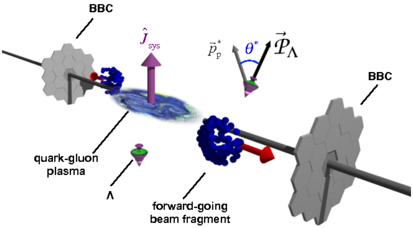
Zubarev approach

Becattini, Tinti, PRD84 (2011) 025013, PRD87 (2013) 2, 025029
Becattini, Florkowski, Speranza, PLB789, 419 (2019)
Becattini, arXiv:2003.01406

- Weak decay of Λ hyperons: daughter particles emitted preferably along polarization direction $\mathcal{P}$.
- Angular distribution of emitted momenta in hyperon rest frame
  B.I. Abelev, I. Selyuzhenkov. et al. (STAR), PRC76, 024915 (2007)

$$\frac{dN}{d\cos\theta_\star} = \frac{1}{2}\left(1 + \alpha|\mathcal{P}|\cos\theta_\star\right)$$





L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

$\Longrightarrow$ Polarization of Λ hyperons along global angular momentum!

Recent measurements at LHC energies:

see poster by Debojit Sarkar
S. Acharya et al. (ALICE), PRC 101, 044611 (2020)

How do particles get polarized?

- Non-central heavy-ion collisions: large orbital angular momentum
- Conversion of orbital angular momentum into spin

  $\Longrightarrow$ Global rotation leads to polarization

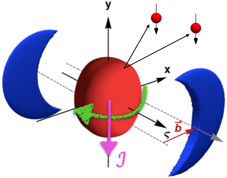


Figure from W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

- Estimate vorticity from thermal approach in global equilibrium

  F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, PRC 95 (2017) 5, 054902

$$\omega \approx T(\mathcal{P}_{\Lambda} + \mathcal{P}_{\bar{\Lambda}})$$

Quark-gluon plasma is the "most vortical fluid ever observed"

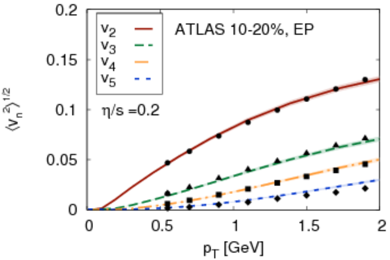
L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

$$\omega \approx (9 \pm 1) \times 10^{21} \mathrm{s}^{-1}$$

Great Red Spot of Jupiter $10^{-4}\,\mathrm{s}^{-1}$

- How to describe spin dynamics in heavy-ion collisions?
- "Standard model" for dynamical evolution of quark gluon plasma:
  Hydrodynamics
  See e.g.
  U. Heinz, R. Snellings, Ann. Rev. Nucl. Part. Sci. 63, 123 (2013)
  C. Gale, S. Jeon, B. Schenke, International Journal of Modern Physics A, 28.11 (2013) 1340011
  ...

  $\Longrightarrow$ very successful at reproducing experimental data, e.g. flow harmonics



C. Gale, S. Jeon, B. Schenke, P. Tribedy, R. Venugopalan, PRL110 (2013) 1, 012302

More about hydrodynamics in many talks at this conference

- Can we use hydrodynamics also for description of polarization effects?

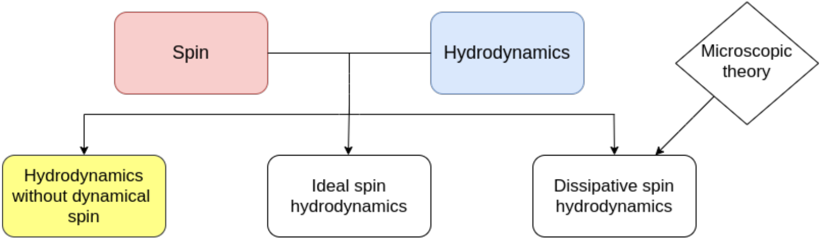
- Spin equilibration time slow enough on time scales of heavy-ion collisions for spin dynamics to be important
  J.I. Kapusta, E. Rrapaj, S. Rudaz, PRC 101, 024907, 2004.14807, PRC 101, 031901 (2020)
  A. Ayala, D. De La Cruz, S. Hernández-Ortíz, L. Hernández, J. Salinas, PLB801, 135169 (2020)
  A. Ayala, D. de la Cruz, L. Hernández, J. Salinas, arXiv:2003.06545

Goal

Incorporate spin into hydrodynamics

$\Longrightarrow$ Need dynamical variable which describes spin degrees of freedom

- **Conserved quantities:**
  charge current
  $$\partial_\mu N^\mu = 0$$
  energy-momentum tensor
  $$\partial_\mu T^{\mu\nu} = 0$$
  + **total** angular-momentum tensor
  $$J^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + \hbar S^{\lambda,\mu\nu}$$
  orbital part   spin tensor
  $$\partial_\lambda J^{\lambda,\mu\nu} = 0 \quad \Longrightarrow \quad \hbar \partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$
  $$a^{[\mu} b^{\nu]} \equiv a^\mu b^\nu - a^\nu b^\mu$$
- Polarization described by Pauli-Lubanski vector for particles with momentum p
  F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, AP338, 32 (2013)
  F. Becattini, arXiv:2004.04050
  E. Speranza, NW, arXiv:2007.00138
  L. Tinti, W. Florkowski, arXiv:2007.04029
  $$\Pi_\mu = -\frac{1}{2m} \epsilon_{\mu\nu\alpha\beta} p^\nu \int_\Sigma d\Sigma_\lambda J^{\lambda,\alpha\beta}$$


Spin
Hydrodynamics
Microscopic theory
Hydrodynamics without dynamical spin
Ideal spin hydrodynamics
Dissipative spin hydrodynamics

- **Assumption:** local thermodynamic equilibrium, polarization at freeze-out determined by thermal vorticity

$$\varpi_{\mu\nu} = -\frac{1}{2}\partial_{[\mu}\beta_{\nu]}$$

with $\beta^\mu \equiv (1/T)u^\mu$, T temperature, u^μ fluid velocity

- Expectation value of Pauli-Lubanski vector on freeze-out hypersurface
F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, AP338, 32 (2013)

$$\langle \Pi^\mu(p) \rangle = -\frac{\hbar^2}{8m}\epsilon^{\mu\nu\alpha\beta} p_\nu \frac{\int_{\Sigma_{FO}} d\Sigma_\lambda \, p^\lambda f_F (1 - f_F) \varpi_{\alpha\beta}(x)}{\int_{\Sigma_{FO}} d\Sigma_\lambda \, p^\lambda f_F}$$

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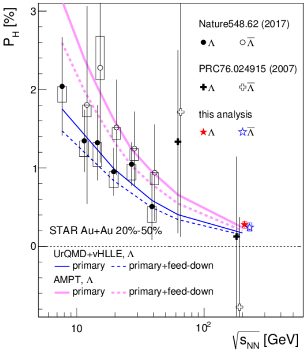
Λ polarization along global angular momentum:
Theoretical prediction agrees with experiment

Becattini, Piccinini, Rizzo, PRC77, 024906 (2008);
Becattini, Csernai, Wang, PRC88, 034905 (2013);
Becattini, Inghirami, Rolando, Beraudo, Del Zanna, De Pace, Nardi, Pagliara, Chandra, Eur. Phys. J. C75, 406 (2015);
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC95, 054902 (2017);
Karpenko, Becattini, Eur. Phys. J. C77, 213 (2017);
Pang, Petersen, Wang, Wang, PRL117, 192301 (2016);
Xie, Wang, Csernai, PRC95, 031901 (2017)



Adam et al. (STAR), PRC98, 014910 (2018)

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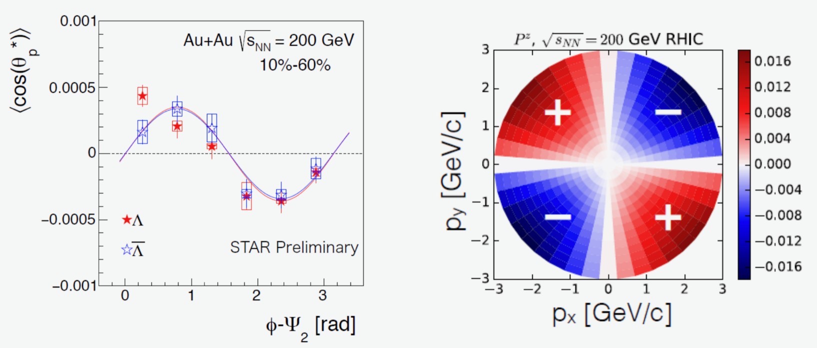
However:

Polarization along beam direction has opposite sign



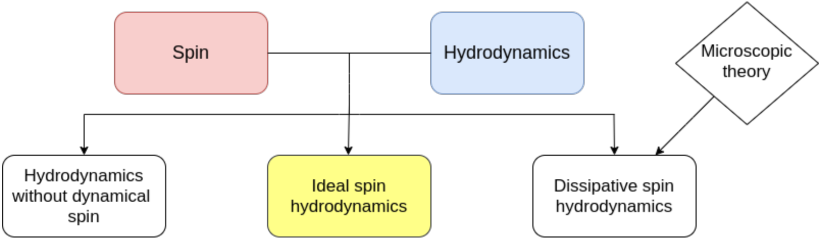
Adam et al. (STAR), PRL123, 132301 (2019)

Becattini, Karpenko, PRL120, 012302 (2018)

✗

- How can disagreement between experiment and theory be explained?
- Replace thermal vorticity by something else (projected thermal vorticity, T-vorticity, ...)?
  W. Florkowski, A. Kumar, R. Ryblewski, A. Mazeliauskas, PRC100, 054907 (2019)
  H.Z. Wu, L.G. Pang, X.G. Huang, Q. Wang, Phys. Rev. Research. 1, 033058 (2019)
- Include Λ hyperons from secondary decays?
  F. Becattini, G. Cao, E. Speranza, Eur. Phys. J. C79, 741 (2019)
  X.L. Xia, H. Li, X.G. Huang, H.Z. Huang, PRC100, 014913 (2019)
- Use chiral kinetic theory to describe polarization?
  Y. Sun, C.M. Ko, PRC99, 011903 (2019)
  S.Y.F. Liu, Y. Sun, C.M. Ko PRL125 (2020)
- No agreement in literature reached up to now
- Dynamical description of spin degrees of freedom?

Dynamical spin in ideal case



W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, PRC 97, no. 4, 041901 (2018)
W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, PRD 97, no. 11, 116017 (2018)
W. Florkowski, F. Becattini, and E. Speranza, APB 49, 1409 (2018)

- Idea: promote spin tensor $S^{\lambda,\mu\nu}$ to additional dynamical variable
- 10 equations of motion: 4 usual hydro + 6 due to total angular momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \qquad \hbar\,\partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$

- Ideal case: 10 unknowns: 4 + 6 additional independent fields (spin potential)

$$\beta^\mu \qquad \Omega^{\mu\nu}$$

- β^μ and $\Omega^{\mu\nu}$ evolve separately
- $\Omega^{\mu\nu}$ determines polarization, in general independent of thermal vorticity

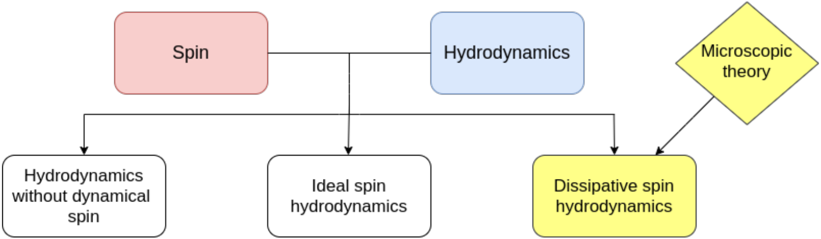
$$\Omega^{\mu\nu} \neq \varpi^{\mu\nu}$$

- Numerical calculations, imposing Bjorken symmetry

Pauli-Lubanski vector at freeze-out in boost-invariant background

see poster by Rajeev Singh
W. Florkowski, A. Kumar, R. Ryblewski, R. Singh, PRC99 (2019)
R. Singh arXiv:2001.05592, 2009.07067

- Studies of spin evolution based on conformal symmetry
R. Singh, G. Sophys, R. Ryblewski, arXiv:2011.14907


Spin
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- Dissipation
  - $\Longrightarrow$ More degrees of freedom given by dissipative currents
  - $\Longrightarrow$ Additional equations of motion needed
  - $\Longrightarrow$ Need microscopic information
- Starting point for deriving kinetic theory: quantum field theory
- In the following: spin-1/2 particles, Dirac theory

$$\mathcal{L}_D(x) = \frac{i\hbar}{2}\,\bar{\psi}(x)\gamma\cdot\overleftrightarrow{\partial}\,\psi(x) - m\,\bar{\psi}(x)\psi(x)$$

- Noether's theorem $\Longrightarrow$ canonical energy-momentum tensor

$$T_C^{\mu\nu} = \frac{i\hbar}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi$$

and canonical spin tensor

$$S_C^{\lambda,\mu\nu} = -\frac{1}{2}\epsilon^{\lambda\mu\nu\alpha}\bar{\psi}\gamma_\alpha\gamma_5\psi$$

- $T_C^{\mu\nu}$ not symmetric for free fields $\Longrightarrow$ canonical spin tensor not conserved

✗ Physical picture: spin density changed only by interactions

✗ Canonical global spin

$$S_C^{\mu\nu} \equiv \int_\Sigma d\Sigma_\lambda S_C^{\lambda,\mu\nu}$$

no Lorentz tensor

- **In general:** Hypersurface-integrated quantities transform as tensors under Lorentz transformations only if integrand conserved

For proof see e.g.
E. Speranza, NW, arXiv:2007.00138

- Definition of $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ depends on choice of pseudo-gauge.
  F. W. Hehl, Rept. Math. Phys. 9, 55 (1976)
  F. Becattini, W. Florkowski, and E. Speranza, PLB789, 419 (2019)
  E. Speranza, NW, arXiv:2007.00138
  L. Tinti, W. Florkowski, arXiv:2007.04029
- Pseudo-gauge transformation:

$$T'^{\mu\nu} = T_C^{\mu\nu} + \frac{\hbar}{2}\partial_\lambda(\Phi^{\lambda,\mu\nu} + \Phi^{\nu,\mu\lambda} + \Phi^{\mu,\nu\lambda}),$$
$$S'^{\lambda,\mu\nu} = S_C^{\lambda,\mu\nu} - \Phi^{\lambda,\mu\nu} + \hbar\,\partial_\rho Z^{\mu\nu,\lambda\rho}$$

$\Longrightarrow$ equations of motion invariant

$$\partial_\mu T'^{\mu\nu} = 0 \qquad \partial_\lambda J'^{\lambda,\mu\nu} = 0$$

and total charges invariant

$$P^\nu \equiv \int_\Sigma d\Sigma_\mu T^{\mu\nu} = \int_\Sigma d\Sigma_\mu T'^{\mu\nu}$$
$$J^{\mu\nu} \equiv \int_\Sigma d\Sigma_\lambda J^{\lambda,\mu\nu} = \int_\Sigma d\Sigma_\lambda J'^{\lambda,\mu\nu}$$

Pseudo-gauge transformations change global spin

Different splitting into spin and orbital angular momentum

Various choices of pseudo-gauge

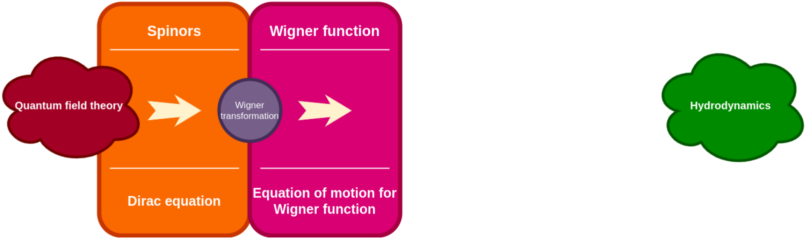
E. Speranza, NW, arXiv:2007.00138 (Review)

- Belinfante choice:
  Spin tensor vanishes, energy-momentum tensor symmetric
  $\Longrightarrow$ Spin degrees of freedom hidden in orbital angular momentum
  $\Longrightarrow$ No dynamical description of spin with independent spin potential

- Hilgevoord-Wouthuysen (HW) choice:
  J. Hilgevoord and S. Wouthuysen, Nuclear Physics 40, 1 (1963)
  $\Longrightarrow$ Spin tensor conserved for free fields
  $\Longrightarrow$ Global spin is Lorentz tensor,
  reduces to canonical global spin in particle rest frame
  $\Longrightarrow$ Covariant generalization of canonical spin
  $\Longrightarrow$ Consistent with Frenkel theory, $P_\mu S^{\mu\nu}_{HW} = 0$

- De-Groot-Van-Leeuwen-Van-Weert (GLW) choice:
  same global spin as HW, but different spin tensor
  S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

Pseudo-gauge transformations also important for spin physics, see e.g.
E. Leader and C. Lorce, Phys. Rept. 541, 163 (2014)



- Wigner function: Quantum analogue of classical distribution function
- Wigner transformation of two-point function:
  H.-Th. Elze, M. Gyulassy, and D. Vasak, Ann. Phys. 173 (1987) 462

$$W_{\alpha\beta}(x,p) = \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p\cdot y} \left\langle : \bar{\Psi}_\beta \left(x + \frac{y}{2}\right) \Psi_\alpha \left(x - \frac{y}{2}\right) : \right\rangle$$

- Dirac equation implies equation of motion for Wigner function
  S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)
  D. Vasak, M. Gyulassy, and H. T. Elze, AP 173, 462 (1987)

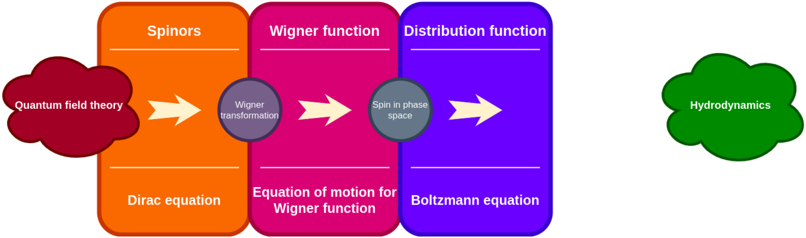
- Expand Wigner function up to first order in gradients (equivalent to $\hbar$ expansion).

- Solve equation of motion for Wigner function order by order.

$\Longrightarrow$ Kinetic theory with spin

- Free-streaming case with electromagnetic fields studied in

NW, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, PRD100, 056018 (2019)
J.-H. Gao and Z.-T. Liang, PRD100, 056021(2019)
K. Hattori, Y. Hidaka, and D.-L. Yang, PRD100, 096011 (2019)
Y.-C. Liu, K. Mameda, and X.-G. Huang, Chin.Phys.C 44 (2020) 9, 094101



- In order to account for spin dynamics enlarge phase space
  J. Zamanian, M. Marklund, and G. Brodin, NJP 12, 043019 (2010)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$f(x,p) \to f(x,p,\mathfrak{s})$$

- Exact relation between $f(x,p,\mathfrak{s})$ and Wigner function
  NW, E. Speranza, X.l. Sheng, Q. Wang, D.H. Rischke, arXiv:2005.01506

Boltzmann equation

$$p \cdot \partial f(x,p,\mathfrak{s}) = \mathfrak{C}[f]$$

- Need expression for collision term to solve Boltzmann equation

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, R. Ryblewski, arXiv:2002.03937, 2008.10976

- Relaxation-time approximation:

$$p \cdot \partial f(x, p, \mathfrak{s}) = p \cdot u \, \frac{f_{eq}(x, p, \mathfrak{s}) - f(x, p, \mathfrak{s})}{\tau_{eq}}$$

- Obtain equation of motion for spin potential $\Omega^{\mu\nu}$ from

$$\partial_\lambda S^{\lambda,\mu\nu}_{GLW} = 0$$

- Spin potential $\Omega^{\mu\nu}$ independent of thermal vorticity

- **Physical situation:** **local** interaction:
  all distribution functions evaluated at same space-time point
  $\Longrightarrow$ suitable to study spin-diffusion effects

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NW, E. Speranza, X.l. Sheng, Q. Wang, D.H. Rischke, arXiv:2005.01506

- For conversion of vorticity into polarization:
  need to consider orbital angular momentum in microscopic collisions

- Sum of spin and orbital angular momentum conserved

- Difference between ingoing and outgoing orbital angular momentum in collision
  $\Longrightarrow$ Net spin changes during collision

- Expand collision term up to first order in gradients

$$\mathfrak{C}[f] = \mathfrak{C}_l[f] + \hbar\, \mathfrak{C}_{nl}[f]$$

Local contribution + **Nonlocal** contribution

NW, E. Speranza, X.l. Sheng, Q. Wang, D.H. Rischke, arXiv:2005.01506

- Intuitive result of quantum-field-theory derivation:

Nonlocal collision term

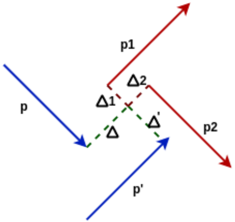
$$\mathfrak{C}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' \, \mathcal{W} \times [f(x+\Delta_1, p_1, \mathfrak{s}_1) f(x+\Delta_2, p_2, \mathfrak{s}_2) - f(x+\Delta, p, \mathfrak{s}) f(x+\Delta', p', \mathfrak{s}')]$$

$$d\Gamma \equiv d^4p\, \delta(p^2 - m^2) \frac{\sqrt{p^2}}{\sqrt{3}\pi} d^4\mathfrak{s}\, \delta(\mathfrak{s}^2 + 3) \delta(p \cdot \mathfrak{s})$$

Collision nonlocal, particle positions displaced by

$$\Delta^\mu \equiv -\frac{\hbar}{2m(p \cdot \hat{t} + m)} \epsilon^{\mu\nu\alpha\beta} p_\nu \hat{t}_\alpha \mathfrak{s}_\beta$$

with $\hat{t}^\mu \equiv (1, 0, 0, 0)$



$p + p' \to p_1 + p_2$

- Collision term can be alternatively calculated from perturbative QCD
  S. Li, H.U. Yee, PRD 100, 056022 (2019)

  or from Kadanoff-Baym equation
  D.L. Yang, K. Hattori, Y. Hidaka, JHEP 07 (2020) 070
  Z. Wang, X. Guo, P. Zhuang, arXiv:2009.10930

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- Equilibrium condition in kinetic theory: Collision term has to vanish
- Ansatz for distribution function
  F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, AP. 338, 32 (2013)
  W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, PRC 97, no. 4, 041901 (2018)
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$f_{eq}(x,p,\mathfrak{s}) = \frac{1}{(2\pi\hbar)^3}\exp\left[\alpha(x) - \beta(x)\cdot p + \frac{\hbar}{4}\Omega_{\mu\nu}(x)\Sigma_{\mathfrak{s}}^{\mu\nu}\right]$$

- Dipole-moment tensor $\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha\mathfrak{s}_\beta$
- **Local** collision term vanishes for any α, β^μ, $\Omega^{\mu\nu}$.
  $\Longrightarrow$ spin polarization not determined by vorticity
  W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)
- Conditions for vanishing of **nonlocal** collision term: global equilibrium
  NW, E. Speranza, X.l. Sheng, Q. Wang, D.H. Rischke, arXiv:2005.01506

$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0, \qquad \partial_\mu\alpha = 0,$$
$$\Omega_{\mu\nu} = \varpi_{\mu\nu} \equiv -\frac{1}{2}\partial_{[\mu}\beta_{\nu]} = \text{const.}$$

NW, E. Speranza, X.l. Sheng, Q. Wang, D.H. Rischke, arXiv:2005.01506

- $\Longrightarrow$ Confirm known result from statistical quantum field theory:
  In equilibrium spin potential equal to thermal vorticity.
  F. Becattini, PRL 108, 244502 (2012)

- Interpretation: Non-vanishing vorticity converts orbital angular momentum into spin through nonlocal collisions

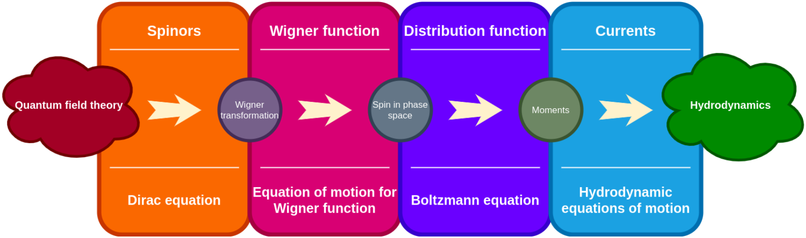
Initial state

Fluid unpolarized

$\Downarrow$ **Equilibration through nonlocal collisions**

Final state

Fluid polarized along vorticity



- Currents can be expressed through distribution function, e.g. HW tensors
  NW, E. Speranza, X.l. Sheng, Q. Wang, D.H. Rischke, arXiv:2005.01506

$$T^{\mu\nu}_{HW} = \int d\Gamma p^\mu p^\nu f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)\ ,$$

$$S^{\lambda,\mu\nu}_{HW} = \int d\Gamma p^\lambda \left(\frac{1}{2}\Sigma^{\mu\nu}_{\mathfrak{s}} - \frac{\hbar}{4m^2} p^{[\mu}\partial^{\nu]}\right) f(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)$$

NW, E. Speranza, X.l. Sheng, Q. Wang, D.H. Rischke, arXiv:2005.01506

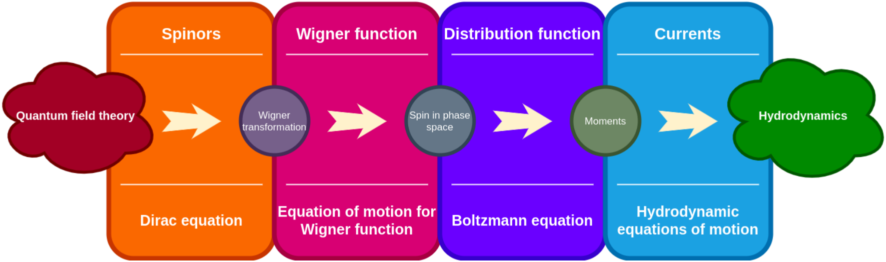
- Using Boltzmann equation

$$\partial_\mu T^{\mu\nu}_{HW} = \int d\Gamma \, p^\nu \, \mathfrak{C}[\mathfrak{f}] = 0 \, ,$$

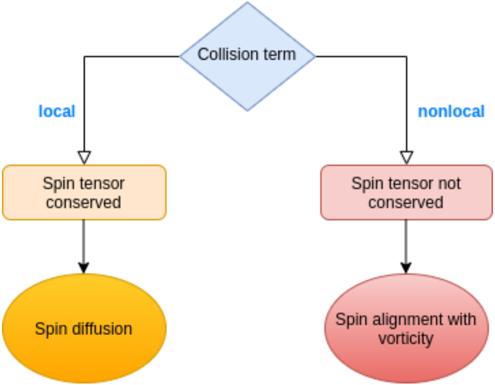
$$\hbar \, \partial_\lambda S^{\lambda,\mu\nu}_{HW} = \int d\Gamma \, \frac{\hbar}{2} \Sigma^{\mu\nu}_{\mathfrak{s}} \, \mathfrak{C}[\mathfrak{f}] = T^{[\nu\mu]}_{HW} = \mathcal{O}(\hbar^2)$$

- Energy-momentum conserved in a collision
- Spin not conserved in nonlocal collisions $\Leftrightarrow$ $T^{[\nu\mu]}_{HW} \neq 0$
  $\Rightarrow$ Conversion between spin and orbital angular momentum
- $T^{[\nu\mu]}_{HW} = 0$
  (i) for local collisions, as spin is collisional invariant
  (ii) in global equilibrium, as collision term vanishes
- Nonlocal collisions out of global equilibrium $\Longrightarrow$ dynamics dissipative
- Similar result obtained from Lagrangian formulation of spin hydrodynamics: Dissipation necessary for causality
  D. Montenegro, G. Torrieri, PRD102 (2020) 3, 036007

- Incorporated spin into hydrodynamics: from microscopic to macroscopic



- For conversion of vorticity into spin: Need nonlocal collision term



- ✓ Assumption of equilibrated spin:
  Prediction of polarization along global angular momentum confirmed by experiment

- ✓ First numerical calculations with dynamical spin in ideal case

- ✓ First steps towards dissipative spin hydrodynamics:
  Derivation of kinetic theory from quantum field theory, including nonlocal collision term
  Progress in deriving dissipative corrections to spin tensor

✗ Disagreement between theory and experiment for polarization along beam direction

? Hydrodynamics from kinetic theory with nonlocal collisions still in progress

? Different pseudo-gauges used in literature
$\rightarrow$ How do they affect physical quantities?

? Importance of dissipation for spin dynamics needs to be better understood

? More realistic numerical calculations with dynamical spin need to be done

? How do initial conditions affect results?

? **Can dissipative corrections to Pauli-Lubanski vector solve sign puzzle?**

Backup

- Idea for free fields:
  Apply Noether's theorem to Klein-Gordon Lagrangian for spinors
  J. Hilgevoord and S. Wouthuysen, Nuclear Physics 40, 1 (1963)

$$\mathcal{L}_{KG} = \frac{1}{2m}(\hbar^2 \partial_\mu \bar{\psi} \partial^\mu \psi - m^2 \bar{\psi}\psi)$$

- Result:

$$T^{\mu\nu}_{HW} = \frac{1}{m}\int d^4p \left[p^\mu p^\nu + \frac{\hbar^2}{4}\partial^\mu\partial^\nu - \frac{\hbar^2}{4} g^{\mu\nu}\partial^2\right] \mathrm{Tr}(W)$$

$$S^{\lambda,\mu\nu}_{HW} = \frac{1}{2m}\int d^4p\, p^\lambda \mathrm{Tr}(\sigma^{\mu\nu} W)$$

$\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

- Energy-momentum tensor symmetric for free fields.
- Conserved (nonzero) spin tensor.

- Nonrelativistic spin operator given by Pauli matrices: $\frac{1}{2}\boldsymbol{\sigma}$
- How to generalize to relativistic theory?
- Spin vector $\boldsymbol{S}$ connected to global spin by

$$S^{ij} = \epsilon^{ijk} S^k.$$

  Obviously no Lorentz tensor.
- Make this covariant:

$$S_n^{\mu\nu} = -\epsilon^{\mu\nu\alpha\beta} n_\alpha S_\beta$$

  Spin defined in the frame moving with four-velocity $n^\mu \Longleftrightarrow n_\mu S_n^{\mu\nu} = 0$.
- Different choices of pseudo-gauge: different choices of frame vector.
  M. H. L. Pryce, Proc. Roy. Soc. Lond., A195:62–81, 1948
  C. Lorcé, Eur. Phys. J. C (2018) 78:785
  E. Speranza, NW, arXiv:2007.00138 (2020)
- One preferred reference frame for massive particles: rest frame
  $\Longrightarrow$ Frenkel theory, $p_\mu S^{\mu\nu} = 0$

- Dirac equation with general interaction:

$$(i\hbar\gamma \cdot \partial - m)\,\psi = \hbar\rho$$

- Equation of motion for Wigner function
  S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)
  D. Vasak, M. Gyulassy, and H. T. Elze, AP 173, 462 (1987)

$$\left[\gamma \cdot \left(p + i\,\frac{\hbar}{2}\partial\right) - m\right] W = \hbar\,\mathcal{C}$$

collision term

$$\mathcal{C}_{\alpha\beta} \equiv \int \frac{d^4 y}{(2\pi\hbar)^4}\, e^{-\frac{i}{\hbar}p\cdot y} \left\langle : \bar{\psi}_\beta(x_1)\rho_\alpha(x_2) : \right\rangle \;.$$

$x_{1,2} \equiv x \pm y/2$

- Idea: Expand Wigner function and collision term up to first order in gradients (equivalent to $\hbar$ expansion).

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- Starting point:
  S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Weert, Relativistic Kinetic Theory. Principles and Applications (North-Holland, 1980)

$$p \cdot \partial W = C$$

with

$$C_{\alpha\beta} = \frac{i}{2}\int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \left\langle [\bar{\rho}(x_1)(-i\hbar\gamma\cdot\overleftarrow{\partial} + m)]_\beta \psi_\alpha(x_2) - \bar{\psi}_\beta(x_1)[(i\hbar\gamma\cdot\partial + m)\rho(x_2)]_\alpha \right\rangle$$

- Expand ensemble average in initial n-particle scattering states

$$C_{\alpha\beta} = \frac{(2\pi\hbar)^6}{2(4m^4)} \sum_{r_1,r_2,s_1,s_2} \int d^4p_1 d^4p_2 d^4u_1 d^4u_2$$

$$\times\, {}_{\text{in}}\langle p_1 - \frac{1}{2}u_1, p_2 - \frac{1}{2}u_2; r_1, r_2|\Phi_{\alpha\beta}(p)|p_1 + \frac{1}{2}u_2, p_2 + \frac{1}{2}u_2; s_1, s_2\rangle_{\text{in}}$$

$$\times \prod_{j=1}^{2} \bar{u}_{s_j}(p_j + \frac{1}{2}u_j)\left[W(x,p_j)\delta^{(4)}(u_j) - i\hbar(\partial^\mu_{u_j}\delta^{(4)}(u_j))\partial_{x\mu}W(x,p_j)\right] u_{r_j}(p_j - \frac{1}{2}u_j)$$

- Consider contribution from zeroth and first order in gradients.