Matching Conditions Across Time Evolution Stages of Ultrarelativistic Heavy Ion Collisions

Duke UNIVERSITY

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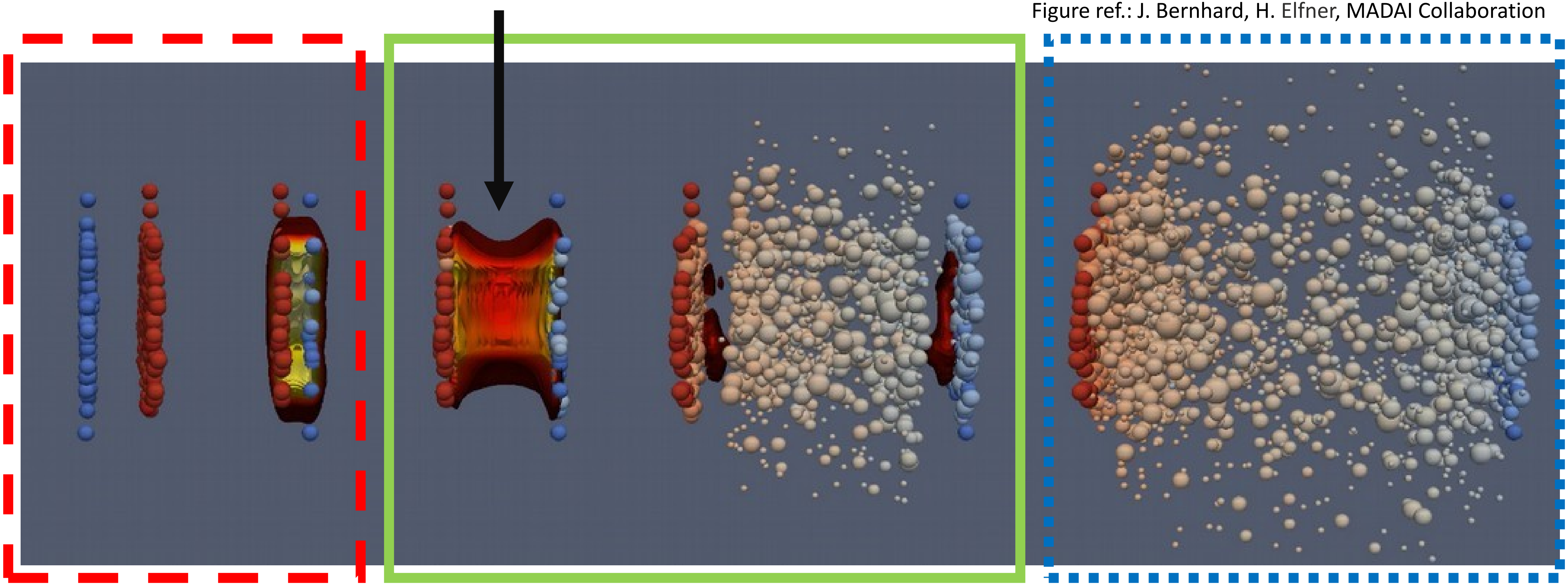
Live from Durham, North Carolina

The Time Evolution Stages of a Heavy Ion Collision

The quark-gluon plasma proper

Figure ref.: J. Bernhard, H. Elfner, MADAI Collaboration

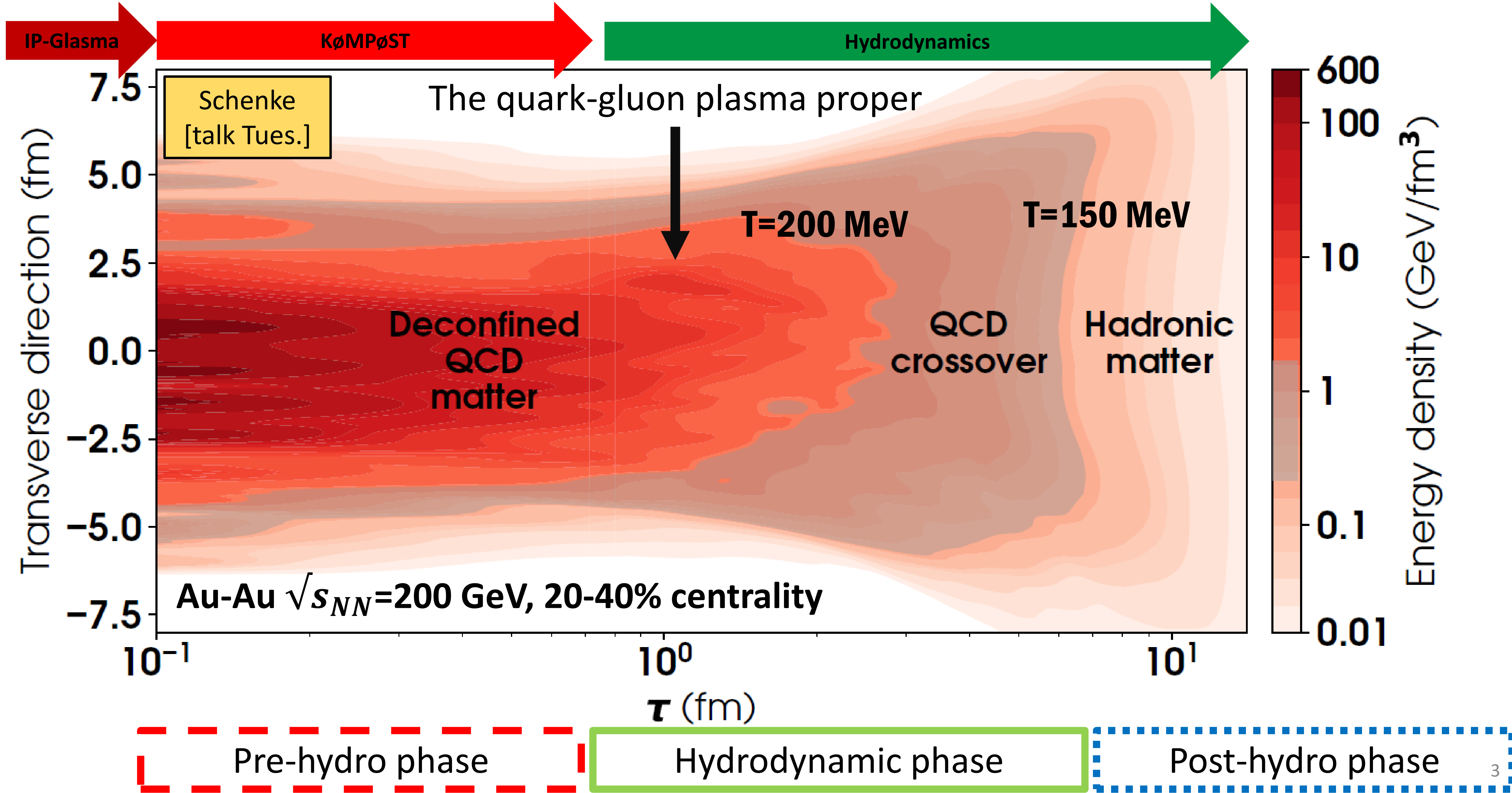


Pre-hydro phase

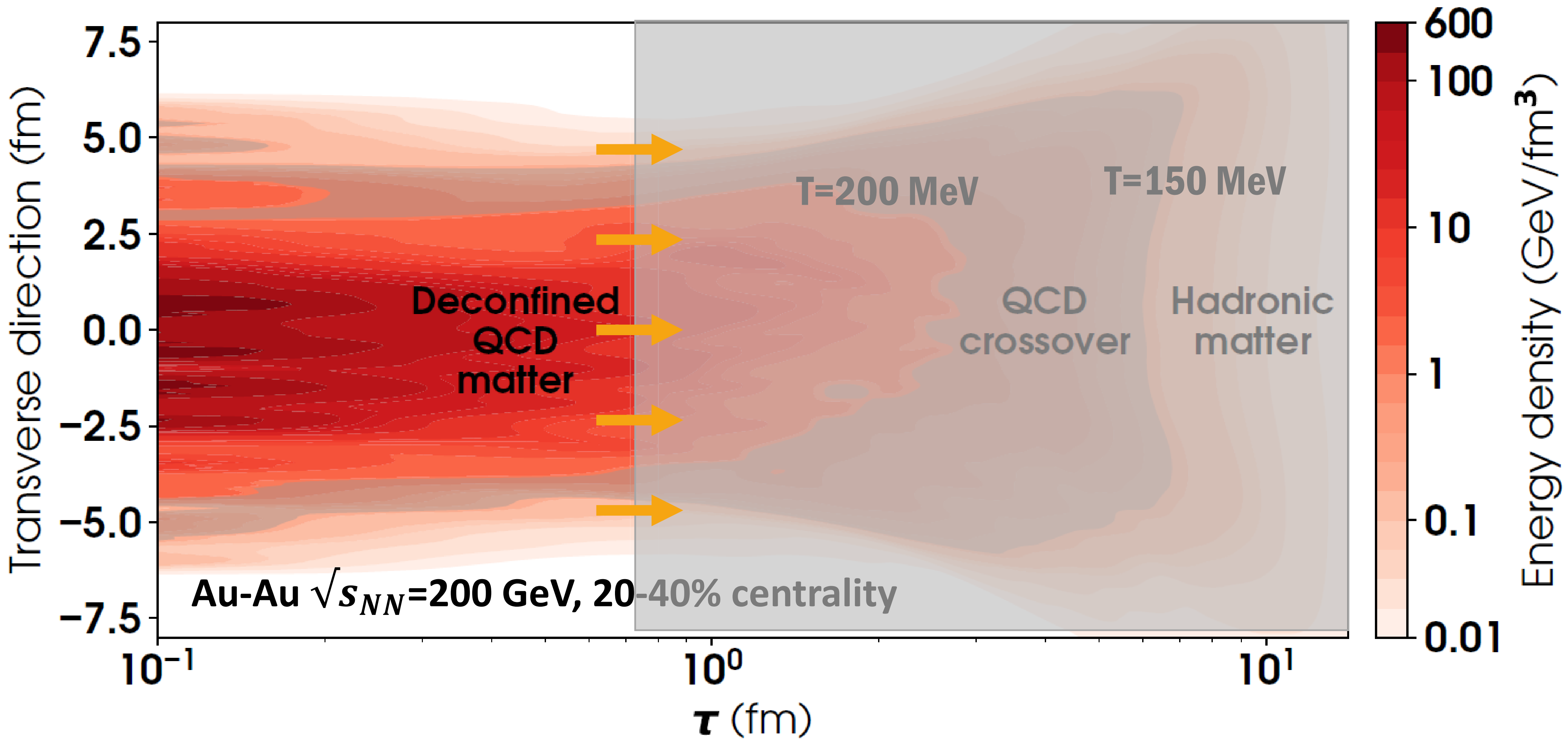
Hydrodynamic phase

Post-hydro phase

More Realistic Time Evolution Stages of a Heavy Ion Collision

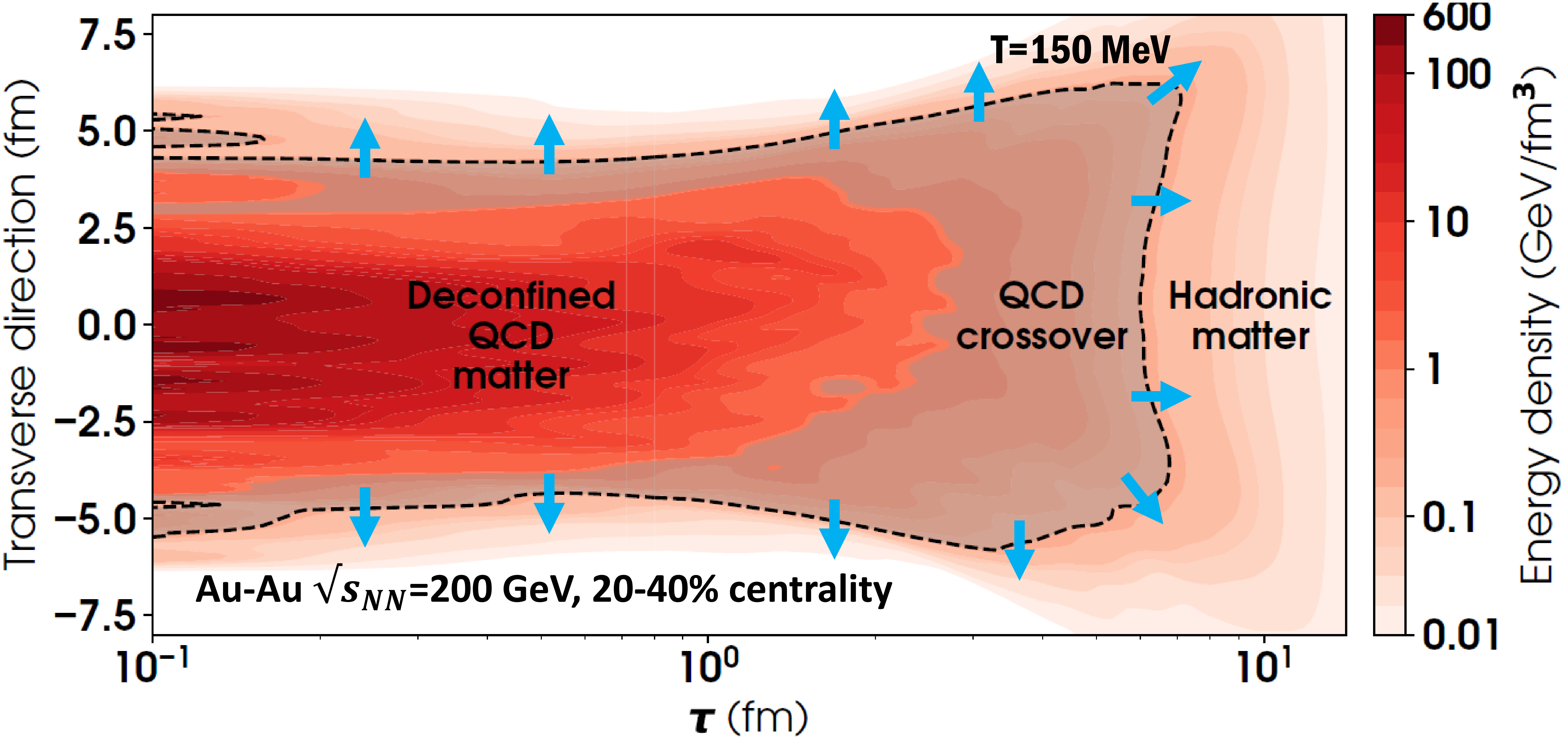


More Realistic Time Evolution Stages of a Heavy Ion Collision



Pre-hydro phase | Hydrodynamic phase | Post-hydro phase

More Realistic Time Evolution Stages of a Heavy Ion Collision



Pre-hydro phase | Hydrodynamic phase | Post-hydro phase

Hydrodynamics as the anchor ⚓

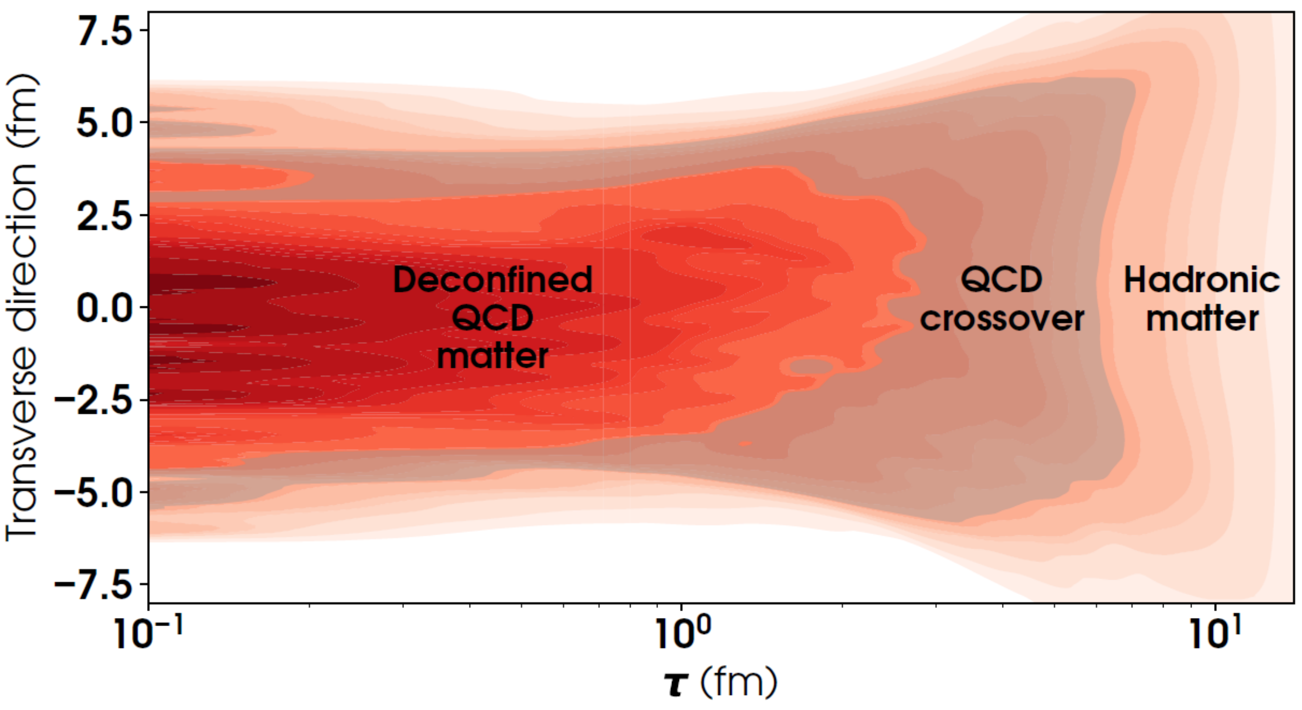
Initial stage → Pre-hydro

Hydrodynamics

"Afterburner" → Post-hydro

Important:
our understanding of hydrodynamics
is also still evolving

Noronha [talk Mon.]



Why hydrodynamics?

- Quark-gluon plasma is strongly-coupled
- Clear collective effects in heavy ion collisions
- Phenomenological success at describing measurements

Transitioning from the initial stage to hydrodynamics

Initializing hydrodynamics

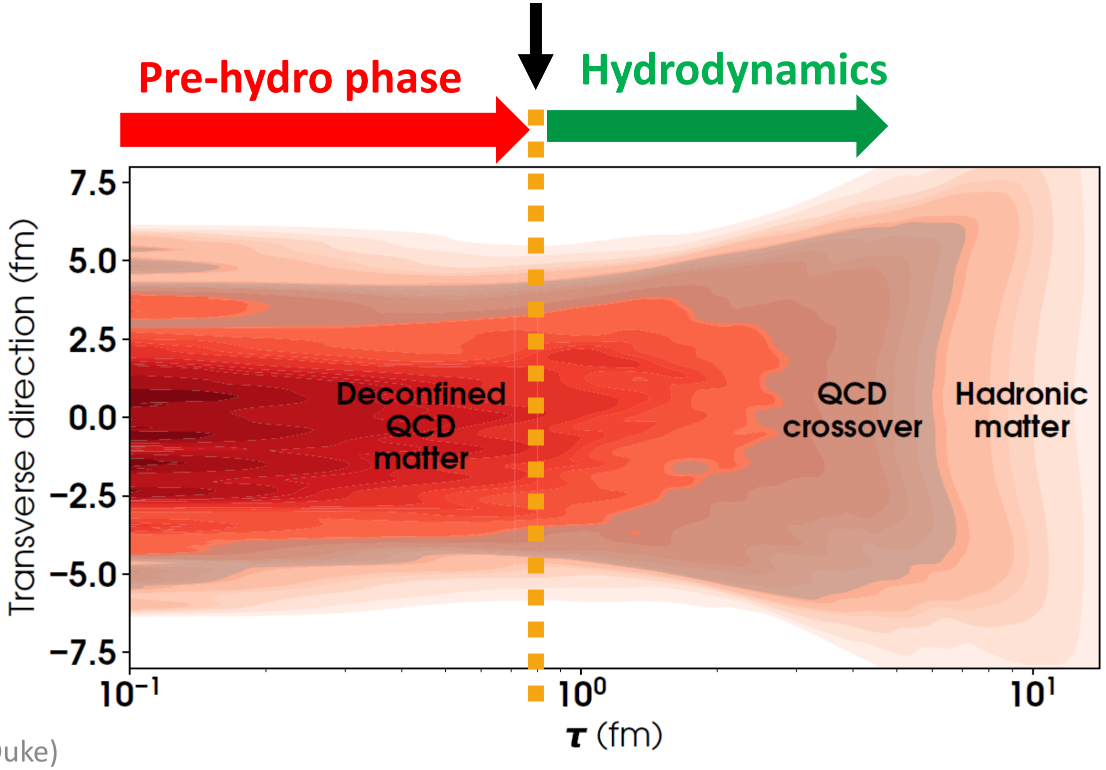
- Hydrodynamics describes the evolution of the energy-momentum tensor $T^{\mu\nu}$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

  with
  - ϵ the energy density
  - u^μ the flow velocity
  - $\pi^{\mu\nu}$ the shear tensor
  - Π the bulk pressure

- 2nd order hydrodynamics requires the initialization of all four fields independently: $\epsilon, u^\mu, \pi^{\mu\nu}, \Pi$ on an « initial condition hypersurface »
- $T^{\mu\nu}(\tau = \tau_0, \vec{x})$ can in principle be parametrized, with caveats

Example of
« initial condition hypersurface » :
$\tau = \tau_0 = 0.8$ fm

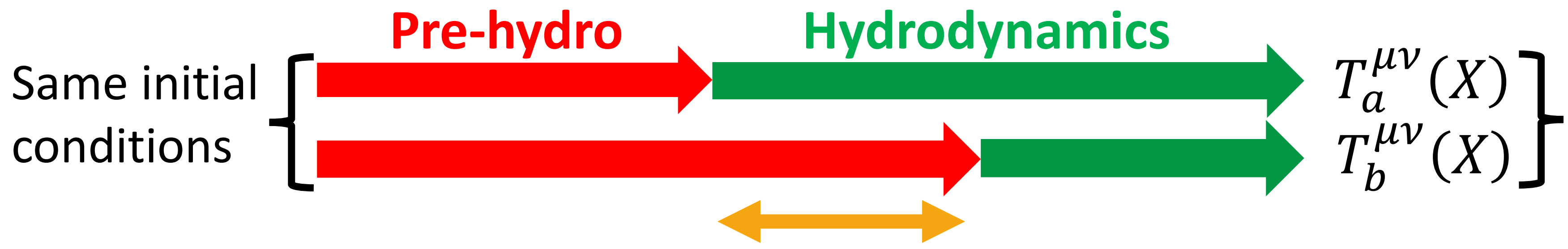


Initializing hydrodynamics vs transitioning to hydrodynamics

- Models of the pre-hydro do not generally approach hydrodynamics:
  - IP-Glasma, EKRT, PYTHIA , AMPT, URQMD/SMASH, Glauber/Trento+free-streaming, …

Lober [poster session] | L. Du [talk Wed.] | Ryu [poster session]

- Practical test of smooth transition to hydrodynamics:



Smooth if

$$T_a^{\mu\nu}(X) \approx T_b^{\mu\nu}(X)$$

(or observables of interests do not change)

Smooth transition to hydro. rare in phenomenological studies, but would be beneficial:

- Sign of **theoretical consistency** of model
- Reduce **theoretical uncertainty** of model

Matching to 0+1D (boost-invariant) hydrodynamics

Schlichting
[Poster session]

- In 0+1D hydro, we can characterize $T^{\mu\nu}$ with single component: energy density
- 0+1D dynamical models with smooth transition to hydrodynamics:
  - Kinetic theory (gluons, QCD, RTA) or AdS/CFT

Conclusion:

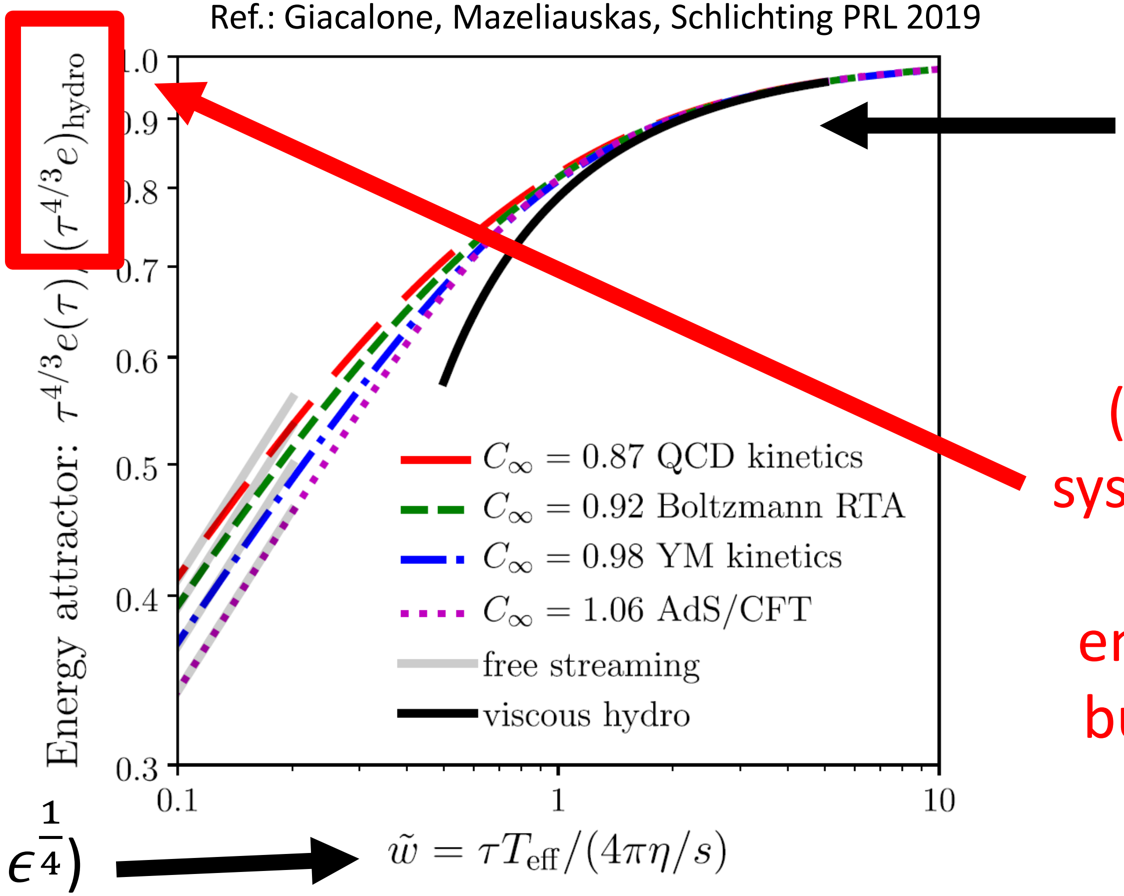
Properly scaled 0+1D systems approach hydro similarly

Timescale necessary to converge to hydro depends:

- Strength of interaction $\left(\frac{\eta}{s} \sim \frac{1}{\alpha_s^2}\right)$
- Energy density of the system

(or "effective temperature" $T_{eff} \propto \epsilon^{\frac{1}{4}}$)

Ref.: Giacalone, Mazeliauskas, Schlichting PRL 2019



All systems converge to 0+1D viscous hydro

(note: not all systems have the same final energy density, but this can be rescaled)

Matching to 2+1D hydrodynamics: the “KøMPøST” approach

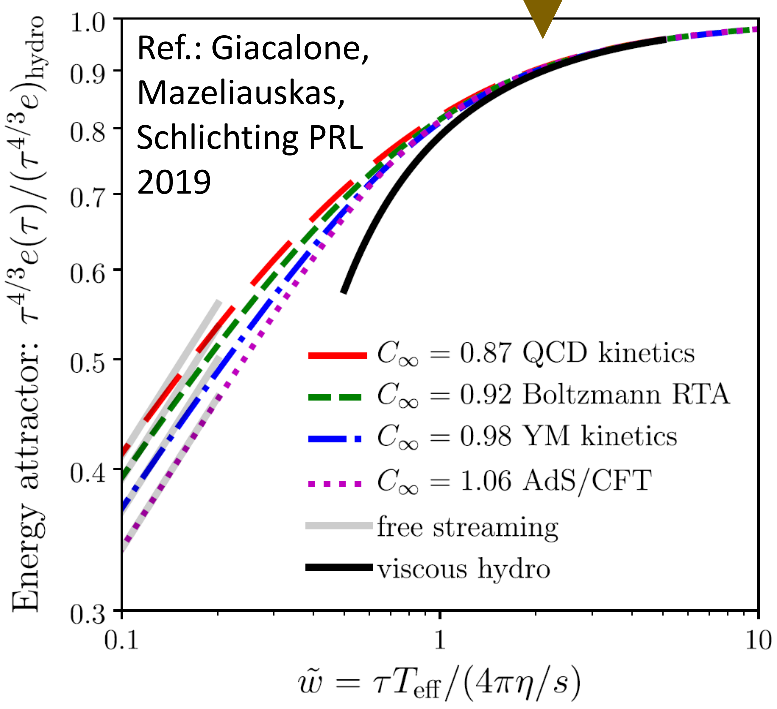
- Take a 2+1D pre-hydro system: how does it approach hydrodynamics?

Schenke [talk Tues.]

- Better approximation [KøMPøST]: decompose $T^{\mu\nu}$ in 0+1D background + linear perturbation

$$T^{\mu\nu}(\tau_{\mathrm{hydro}}, \mathbf{x}) = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\mathrm{hydro}}) + \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\mathrm{hydro}})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\mathrm{EKT}})} \int d^2\mathbf{x}'\, G^{\mu\nu}_{\alpha\beta}(\mathbf{x}, \mathbf{x}', \tau_{\mathrm{hydro}}, \tau_{\mathrm{EKT}})\, \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\mathrm{EKT}}, \mathbf{x}')$$

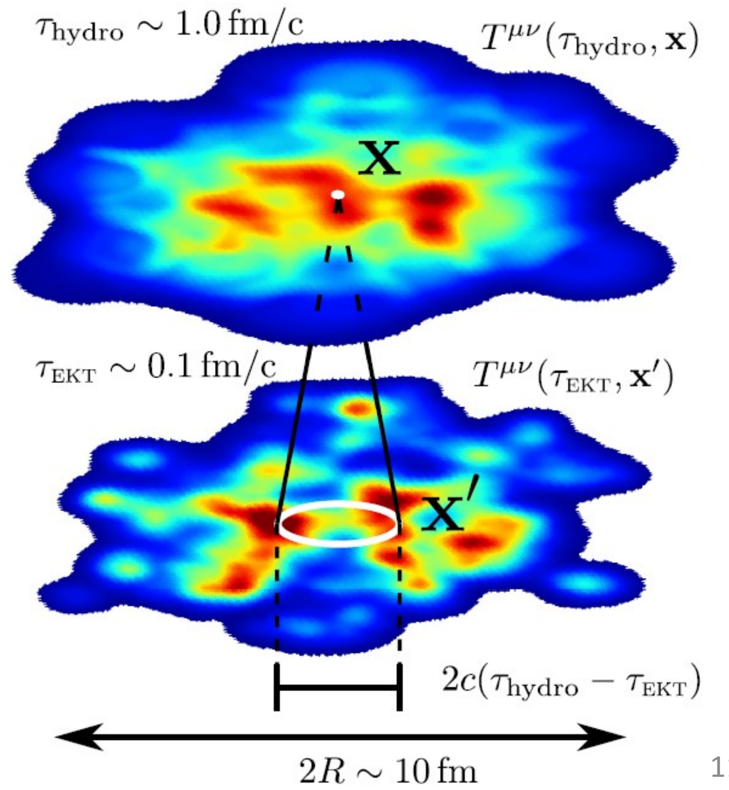
Ref.: Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney PRL2019, PRC2019



Response functions describing evolution of perturbations

- Can be evaluated with QCD kinetic theory
- Also exhibits scaling with interaction strength and energy density of system

[See also Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting, PRD 2020]



Matching to hydrodynamics: challenges

- Approach to hydrodynamics in higher dimensions still under investigation

  See e.g. Kurkela, van der Schee, Wiedemann, Wu, PRL 2020 (Kinetic theory, cylindrical symmetry);
  Behtash, Cruz-Camacho, Martinez PRD 2018 (Kinetic theory, Gubser symmetry); Denicol, Noronha PRD 2019 (Hydro, Gubser symmetry);
  Romatschke JHEP 2017 (Hydro, Semi-peripheral optical Glauber)
  See also van der Schee, Paul Romatschke & Pratt, PRL 2013;

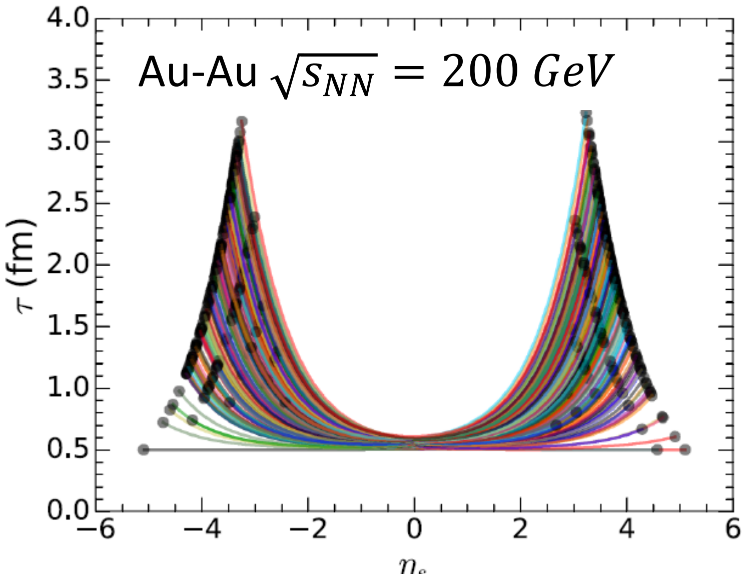
- Additional challenges of small systems, high rapidities and lower $\sqrt{s_{NN}}$ [e.g. beam energy scan]
  - Hydrodynamic initialization hypersurface becomes more complex
  - Additional conserved quantities to initialize, e.g. baryon density

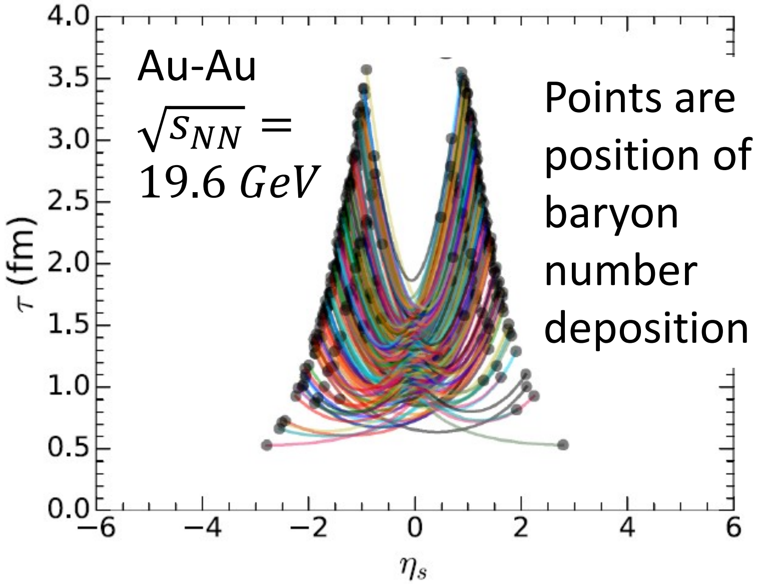
Dore [talk Thurs.]

Denicol [talk Mon.]

Alzhrani [poster session]

Ref.: Shen and Schenke, PRC 2018

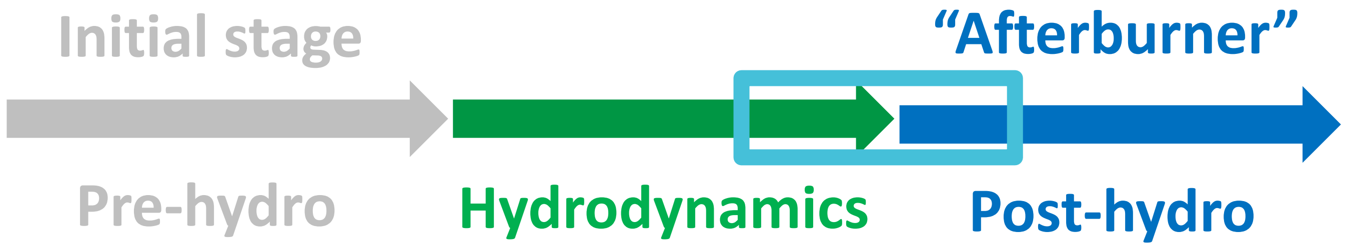




Non-trivial rapidity distribution of energy and baryon number

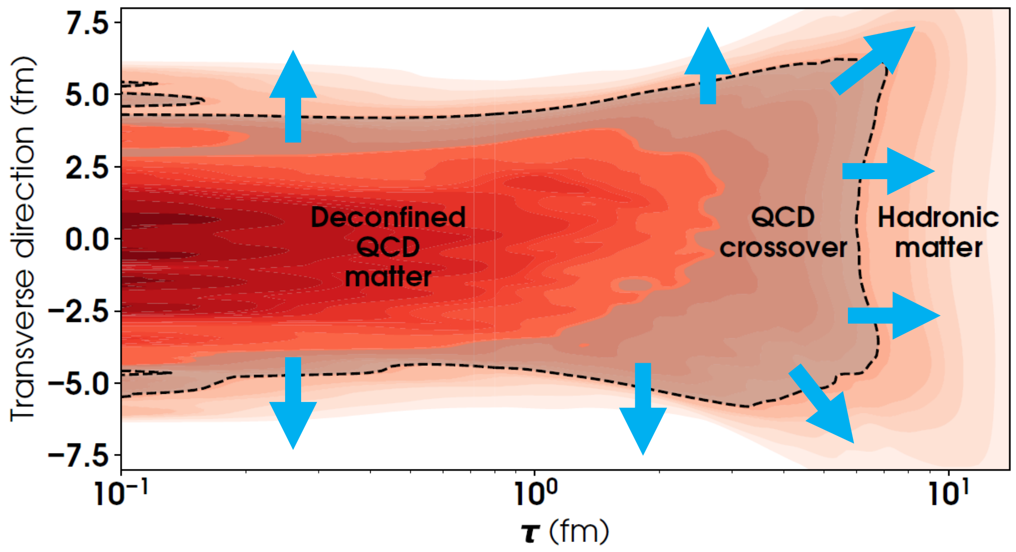
Shen [talk Wed.]

Transitioning from hydrodynamics to particles: particlization



When and how to transition from hydrodynamics

- When to transition from hydrodynamics?
  - Spacetime hypersurface?
  - Spacetime (hyper)volume?
  - Criteria?
    Temperature? Energy density?
    Knudsen number?



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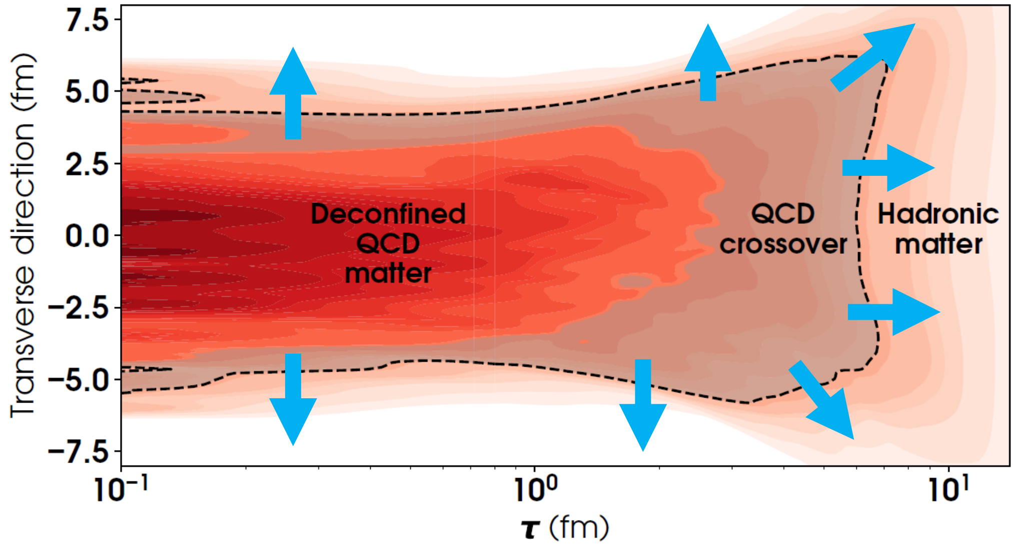

See e.g. Ahmad, Holopainen, Huovinen, PRC 2017; Oliinychenko, Huovinen, Petersen, PRC 2015 ; Eskola, Niemi, Ruuskanen PRC 2008; and references therein

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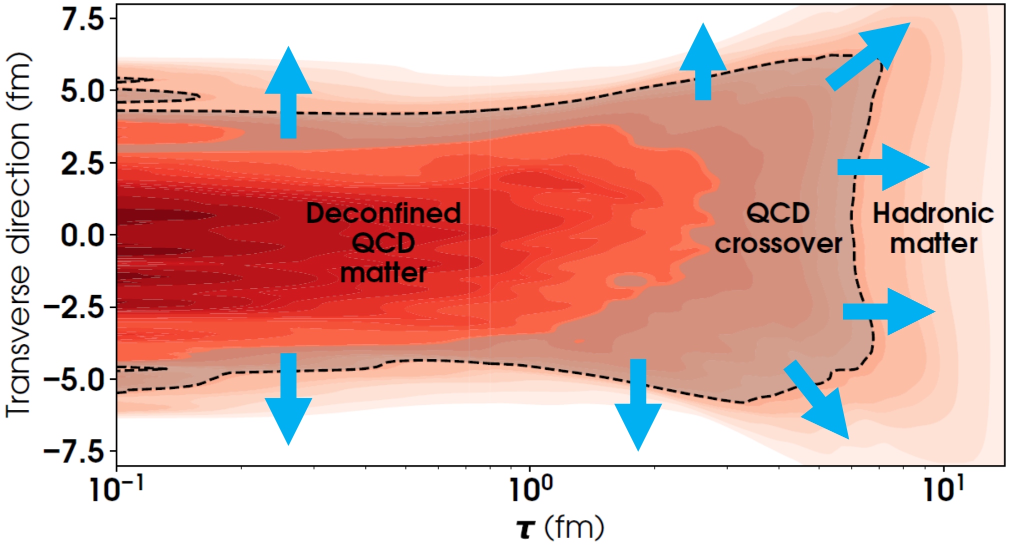


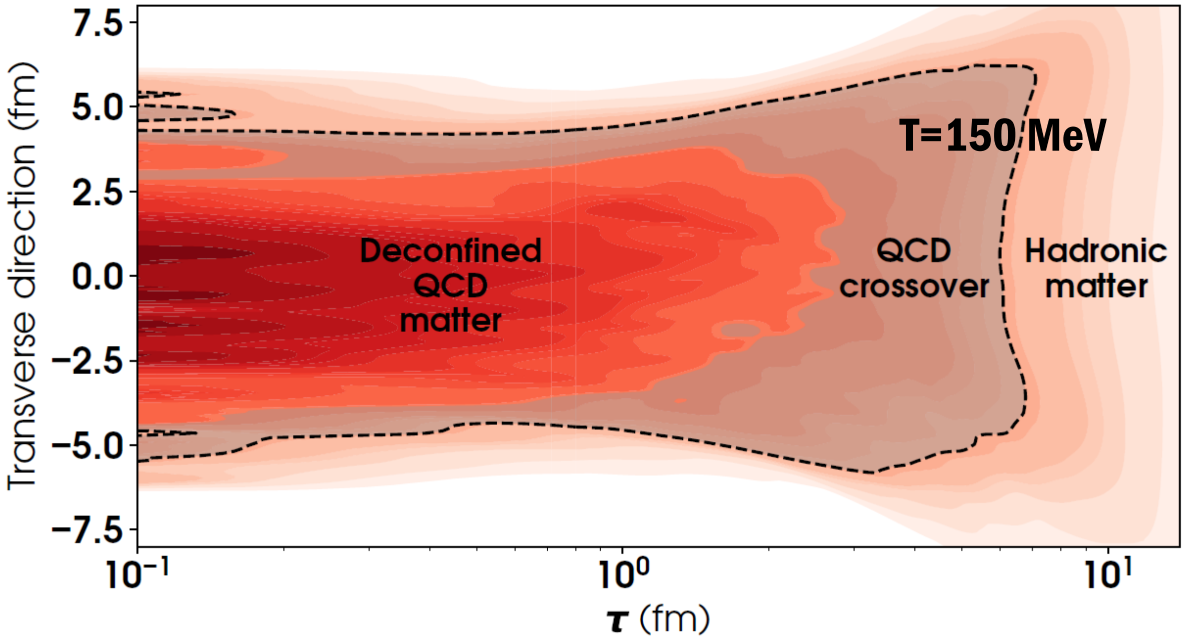
- See also:
  - Effect of matching the lattice equation of state to a hadron resonance gas — Auvinen [poster session]
  - Challenges with sampling the hadronic moment distribution
    e.g. Oliinychenko, Koch, PRL 2019; Oliinychenko, Shi, Koch, PRC 2020

- **What is the hadronic momentum distribution consistent with hydrodynamics?**

Transitioning from hydrodynamics: particlization

$$T^{\mu\nu} = \sum_n g_n \int \frac{d^3k}{(2\pi)^3 K^0} K^\mu K^\nu f_n(K)$$
$$= \epsilon u^\mu u^\nu - (P + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

$\Rightarrow f_n(K) = function(\epsilon, u^\mu, \Pi, \pi^{\mu\nu})$;
"n" is the hadron species



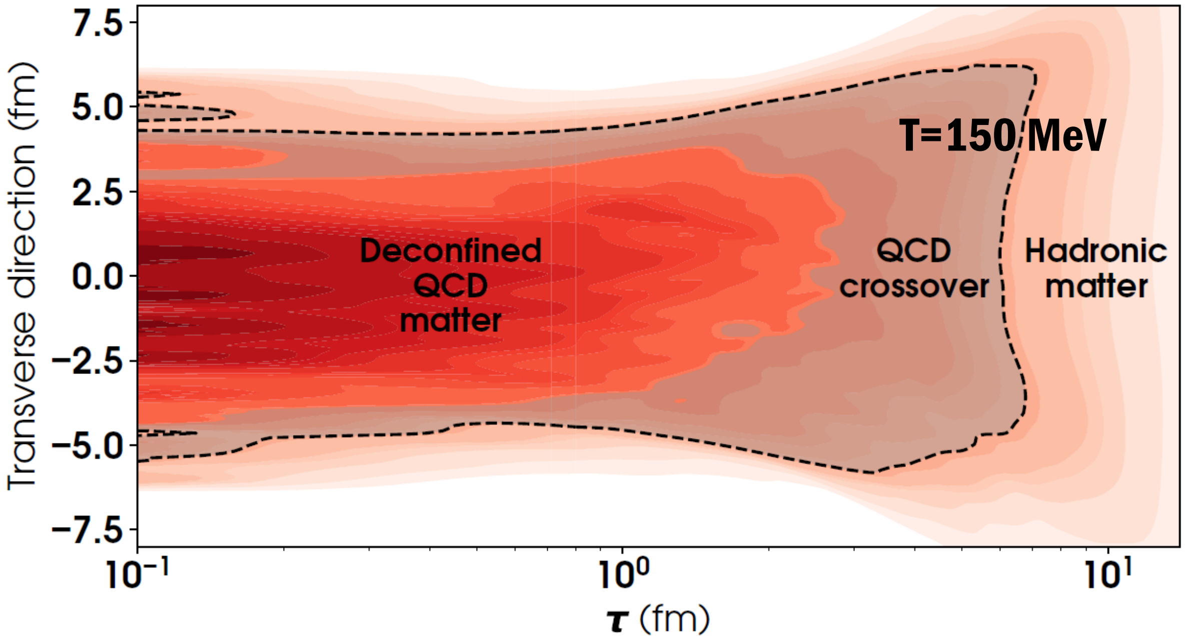
Fluid description	Hadronic momentum distribution
Ideal hydrodynamics & local thermal equilibrium	Equilibrium: Fermi-Dirac (baryons), Bose-Einstein (mesons)
Viscous hydrodynamics & deviation from equilibrium	?

Generic question for all quasi-particle descriptions

$$T^{\mu\nu} = \sum_n g_n \int \frac{d^3p}{(2\pi)^3 p^0} P^\mu P^\nu f_n(P)$$
$$= \epsilon u^\mu u^\nu - (P + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

$\Rightarrow f_n(K) = function(\epsilon, u^\mu, \Pi, \pi^{\mu\nu})$;
"n" is the hadron species

Photon and dilepton emission rates,
parton energy loss, etc,
often depend on $f(K)$:
the so-called "viscous corrections"



Almaalol [talk Fri.]

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Models of mapping viscous hydro to momentum distributions

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- "14 Moments (Grad)" model and related [quadratic in P for shear viscosity]:

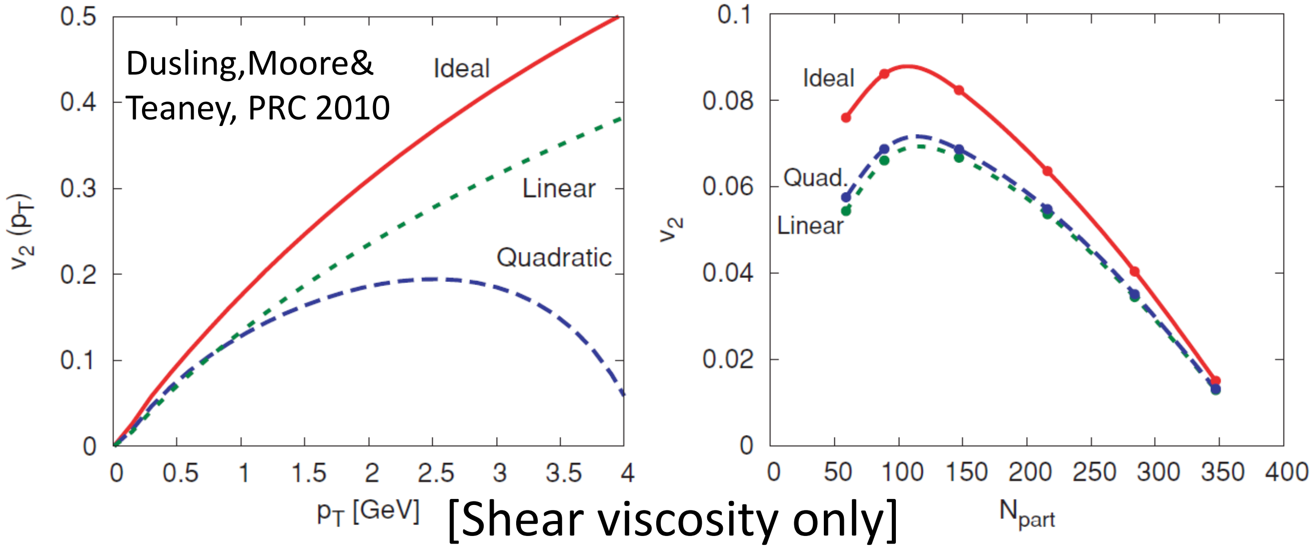
$$f(P) = f_{eq}(P) + f_{eq}(1 \pm f_{eq})(B_\Pi(P)\,\Pi + B_\pi\, P^\mu P^\nu \pi_{\mu\nu})$$

- "Chapman-Enskog – const Relaxation Time" [linear in P for shear viscosity]:

$$f(P) = f_{eq}(P) + f_{eq}(1 \pm f_{eq})\left(B'_\Pi(P)\Pi + B'_\pi \frac{P^\mu P^\nu \pi_{\mu\nu}}{P\cdot u}\right)$$

See e.g. McNelis, Everett & Heinz CPC 2021; Dusling, Moore & Teaney, PRC 2010 for discussion and primary references

Rocha [poster session]



[Shear viscosity only]

Models of mapping viscous hydro to momentum distributions

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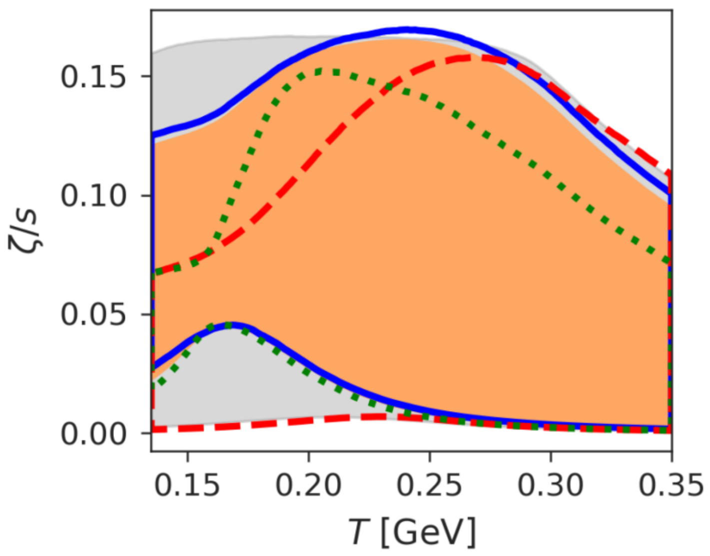
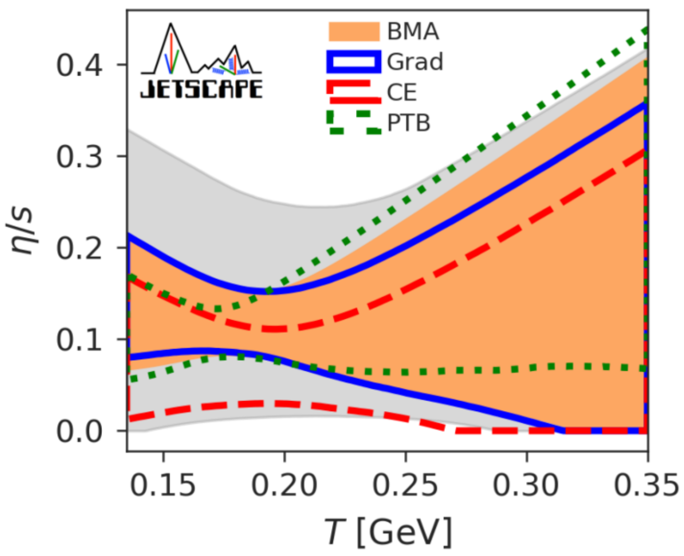
Recent developments:

- Quantifying effect of uncertainty on phenomeno. constraints on shear and bulk viscosity





Major theoretical uncertainties in phenomenological studies of heavy ion collisions

Everett [talk Wed.]

JETSCAPE Collaboration, arXiv:2010.03928

Models of mapping viscous hydro to momentum distributions

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Viscous hydrodynamics & deviation from equilibrium	?

<u>Recent developments:</u>

- Quantifying effect of uncertainty on phenomeno. constraints on shear and bulk viscosity
- New approach: Everett, Chattopadhyay & Heinz, arXiv:2101.01130
  Momentum distribution that maximizes the entropy given conditions on $T^{\mu\nu}$

$$T^{\mu\nu} = \sum_n g_n \int \frac{d^3P}{(2\pi)^3 P^0} P^\mu P^\nu f_n(P)$$

Entropy current: $s^\mu = -\sum_n g_n \int \frac{d^3P}{(2\pi)^3 P^0} P^\mu [f_n \ln f_n \mp (1 \pm f_n) \ln(1 \pm f_n)]$

What is $f_h(P)$ that maximizes the entropy? $\delta(s_\mu u^\mu)/\delta f_h = 0$

Models of mapping viscous hydro to momentum distributions

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Viscous hydrodynamics & deviation from equilibrium	?

Recent developments:

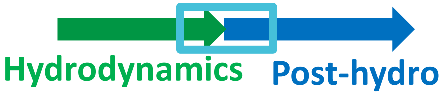
- Quantifying effect of uncertainty on phenomeno. constraints on shear and bulk viscosity
- New approach: $f_h(P)$ that maximizes the entropy
- Studying consistency of $f_h(P)$ ansatz with simplified transport theories
  - Molnar, arXiv:2012.1557:
    effect of bulk viscosity in 1-component gas with isotropic $2 \to 2$ interactions
  - Damodaran, Molnar, Barnaföldi, Berényi, and Nagy-Egri, PRC 2020:
    effect of shear viscosity in 0+1D parton cascade
  - See also
    Molnar & Wolff, PRC 2017
    Chakraborty & Kapusta, PRC 2017

Summary

Summary & outlook

- Considerable progress in understanding of how systems approach hydrodynamics
  - Understanding of hydrodynamics changing as well
  - Many results from simpler systems (e.g. conformal 0+1D)
  - For phenomenology, needs to better understand:
    - Higher dimension systems: 2+1D, non-boost-invariant 3+1D
    - Non-conformal systems
    - Approach to hydrodynamics in small and low $\sqrt{s_{NN}}$ systems

- Transition to hadron gas after hydrodynamics:
  - Mapping energy-momentum tensor to hadron momentum distribution still an outstanding problem



Questions?