Charge conservation in initial conditions and relativistic hydrodynamics

(relativistic fluid dynamics at large net-charge densities)

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What you will see in this talk

✓ Motivation

✓ Fluid-dynamical modeling of heavy-ion collisions

✓ Fluid dynamics at finite net-charge: some challenges

✓ Conclusions and perspectives
- QCD phase diagram (thermodynamic properties)
- Non equilibrium phenomena (transport properties)
QCD matter at large densities

- Heavy ion collisions:

- Neutron star merger:
Neutron star mergers

- High temperatures are also achieved in these natural events
  \[ T_{\text{max}} \sim 50 \text{ MeV} \]

- Simulations by Frankfurt group


Snapshot of temperature in equatorial plane at three different times
Heavy-Ion Collisions: *Produce and study QCD matter near (local) equilibrium*

**Assumption:** *fluid-dynamical expansion (valid for BES?)*

Measured: hadrons, leptons, and photons

Properties of matter must be reverse-engineered
Current theoretical description

1) Initial state and “pre-equilibrium” dynamics:
   • description of early-time dynamics and “thermalization”
   • initial condition for hydrodynamic evolution

   (approach) thermalization (?)

2) Fluid-dynamical expansion of QGP and Hadron Gas
   • Phase transition
   • Matter described by EoS and transport coefficients
     shear and bulk viscosity, charge diffusion, relaxation times ...

   fluid elements converted to particles

3) Transport description of Hadron Gas
   • Late stage description using the hadron resonance gas
     model – using cross sections and decay probabilities
During last 10 years, developed mostly at $n_B = 0$

- Focus has **not** been in extracting EoS
  - Extraction of transport properties (shear and bulk viscosities)
  - Understanding the initial state

- Fluid-dynamical models have evolved dramatically in the last 15 years:
  - inclusion of dissipation (2006),
  - event-by-event fluctuations (2010),
  - sub-nucleonic fluctuations (2012), ...
Relativistic fluid dynamics

Effective theory describing the dynamics of a system over long-times and long-distances

- Conservation laws
- Equation of state
- Simple constitutive relations
Basics of fluid dynamics (Landau frame)

Conservation laws

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Net charge conservation

$$\partial_\mu N^\mu_s = 0$$
$$\partial_\mu N^\mu_e = 0$$
$$\partial_\mu N^\mu_b = 0$$

strangeness
electric charge
Baryon number

Tensor decomposition

$$N^\mu_q = n_q u^\mu + n_q$$
$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (P_0 + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

net-charge diffusion
4-current

Bulk viscous pressure

Shear stress tensor

Projection operator: $$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$
Equation of state

Thermodynamic pressure: \( P_0 = P_0(T, \mu_b, \mu_e, \mu_s) \) not known!

Taylor expansion up to 4th order:

\[
\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi_{l,m,n}^{B,Q,S}}{l!m!n!} \left( \frac{\mu_B}{T} \right)^l \left( \frac{\mu_Q}{T} \right)^m \left( \frac{\mu_S}{T} \right)^n
\]

- matched to hadron resonance gas model at small \( T \)
- matched to Stefan-Boltzmann limit at large \( T \)

Prescription employed by:
Monnai, Schenke, Shen, PRC 100, 024907 (2019)
Noronha-Hostler, Parotto, Ratti, Stafford, PRC 100, 064910 (2019)
Equation of state

**Thermodynamic pressure:** \( P_0 = P_0(T, \mu_b, \mu_e, \mu_s) \)

**Taylor expansion up to 4th order:**
\[
\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{x^{B,Q,S}_{l,m,n}}{l!m!n!} \left( \frac{\mu_B}{T} \right)^l \left( \frac{\mu_Q}{T} \right)^m \left( \frac{\mu_S}{T} \right)^n
\]

- matched to *hadron resonance gas* model at small T
- matched to Stefan-Boltzmann limit at large T

**Added a critical point:**

See Travis Dore’s talk for the effects on hydro
Relativistic Navier-Stokes theory

Shear Viscosity
(Resistance to deformation)

\[ \pi^{\mu \nu} = 2\eta \nabla \langle \mu u^\nu \rangle \]

\( \eta (T, \mu_q) \)

Bulk Viscosity
(Resistance to expansion)

\[ \Pi = -\zeta \nabla_\mu u^\mu \]

\( \zeta (T, \mu_q) \)

Net-Charge Diffusion

\[ \eta^\mu_q = \kappa_q \nabla^\mu \frac{\mu_q}{T} \]

\( \kappa_q (T, \mu_q) \)
Relativistic Navier-Stokes theory

**Shear Viscosity**
(Resistance to deformation)

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**Bulk Viscosity**
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\Pi = -\zeta \nabla_\mu u^\mu
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**Net-Charge Diffusion**

\[
\eta_q^\mu = \kappa_q \nabla^\mu \frac{\mu_q}{T}
\]

How does the critical point affect these coefficients?
First calculations with viscosity (2015)

Finite baryon number, isospin, and electric charge must be included

Effective EoS is employed

Constant $\eta/s$ is extracted separately for each collision energy
• proxy for temperature and chemical potential dependencies

$\eta/s$ estimated


$v_2$ as a function of energy described

$\eta/s$ vs collision energy

Karpenko et al, PRC91 (2015) 6, 064901

UrQMD + Hydro + UrQMD
Recent calculation with $\eta/s(T,\mu_B)$:
C. Shen and S. Alzhrani, PRC 102 (2020) 1, 014909

$\frac{\eta}{s}(T, \mu_B) = \left( \frac{\eta}{s} \right)_0 f_T(T) \left( \frac{e + P}{T s} \right) f_{\mu_B}(\mu_B)$

See poster by Sahr Alzhrani

Our systems have at least 3 conserved charges: baryon number, strangness, electric charge

\[ j^\mu_q = \kappa_q \nabla^\mu \alpha_q \]

\[ \alpha = \mu / T \]

The diffusion currents are coupled!

\[
\begin{pmatrix}
    j^\mu_B \\
    j^\mu_Q \\
    j^\mu_S \\
\end{pmatrix} =
\begin{pmatrix}
    \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\
    \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\
    \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \\
\end{pmatrix}
\cdot
\begin{pmatrix}
    \nabla^\mu \alpha_B \\
    \nabla^\mu \alpha_Q \\
    \nabla^\mu \alpha_S \\
\end{pmatrix}
\]

M. Greif et al, PRL 120 (2018) no.24, 242301
Our systems have at least 3 conserved charges: baryon number, strangeness, electric charge

- **first estimates from kinetic theory**
- provide information on effective degrees of freedom of QCD
- off-diagonal terms are related to conductivities

M. Greif et al, PRL 120 (2018) no.24, 242301

Our systems have at least 3 **conserved charges**: baryon number, strangeness, electric charge

**Effects explored in 1+1D simulations**

J. Fotakis *et al*, PRD 101 (2020) 7, 076007

![Net baryon number](image1)

![Net strangeness](image2)
Initial Condition - sources

- Interpenetration time of the two nuclei is not negligible
- medium created: *freshly produced unthermalized particles* + *dissipative fluid*

Conservation laws are supplemented by source terms

\[ \partial_\mu T^{\mu\nu} = J_\text{source}^\nu \]
\[ \partial_\mu J^\mu = \rho_\text{source} \]

M. Okai, K. Kawaguchi, Y. Tachibana, and T. Hirano, PRC95, 054914 (2017)
C. Shen and B. Schenke, PRC97 (2018) 024907

See Chun Shen’s talk for more
See Lipei Du’s talk for more
Initial Condition - sources

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Challenges in the description of pre-equilibrium dynamics ...

See Sangwook Ryu’s poster for a study with SMASH
See Xiaojian Du’s poster for a study of attractors at finite $\mu$
net-charge fluctuations in *high energy* collisions

Global net-charge may be small, but may still fluctuate

Sea quarks distribution due to gluon splitting $g \rightarrow q\bar{q}$

Iccing  See Patrick Carzon’s poster for more

• May also expect some effects at large rapidity
Conclusions and outlook

Fluid-dynamical models that describe low energy heavy ion collisions are under construction – but appear to be able to fit the data

- Equation of state is not understood. What are its effects on the data? Can it really be extracted?

- Many transport coefficients appear at finite $\mu_B$. Very difficult to include and extract them...

  *but they may be crucial in identifying a phase transition*

**Hydrodynamics near critical point?** New theories may be required ... inclusion of fluctuations.