New developments in QCD-based kinetic transport theory

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Initial Stages 2021

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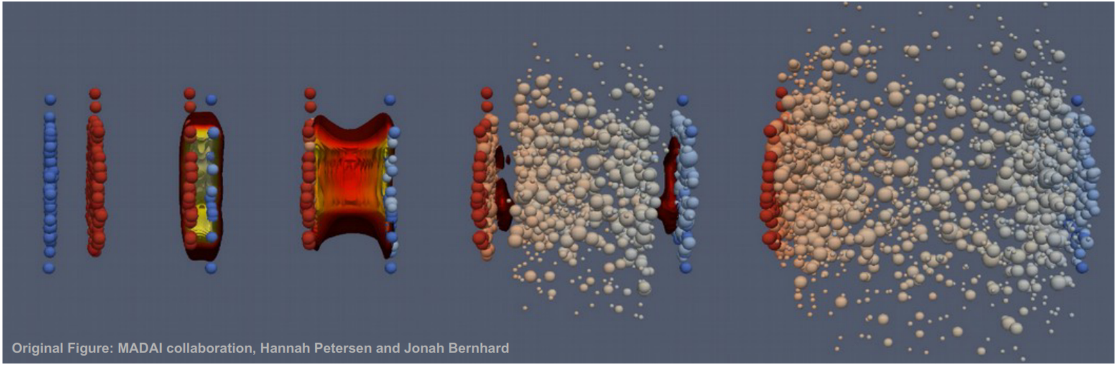
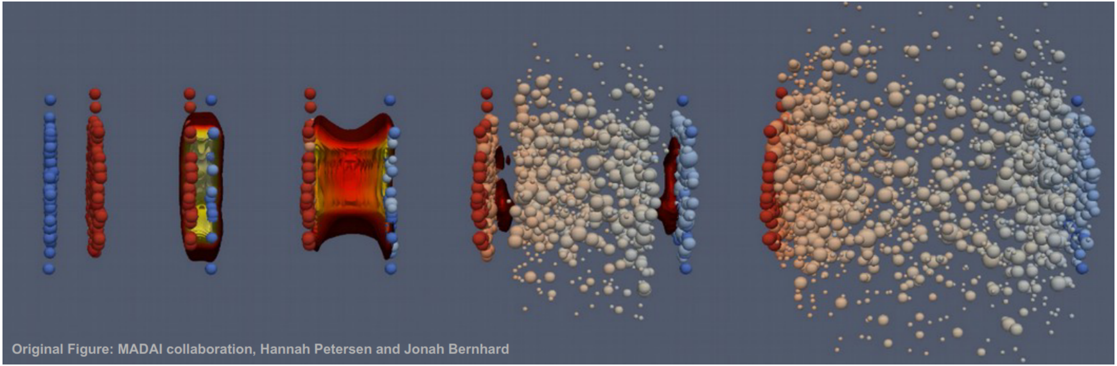
KENT STATE UNIVERSITY

Kinetic transport theory in ultra-relativistic heavy ion collisions



initial state preequilibrium Hydrodynamics Hadronization Hadronic cascade

Kinetic transport theory in ultra-relativistic heavy ion collisions



initial state preequilibrium Hydrodynamics Hadronization Hadronic cascade

QCD kinetic tranport theory

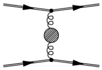
Formalism

QCD medium at high temperatures: Effective kinetic theory AMY JHEP0301 (2003) 030

$$P^\mu \partial_\mu f_{q,g}(\mathbf{p}) = -C[f(\mathbf{p})], \qquad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p} \qquad g, q(u, d, s, \bar{u}, \bar{d}, \bar{s}))$$

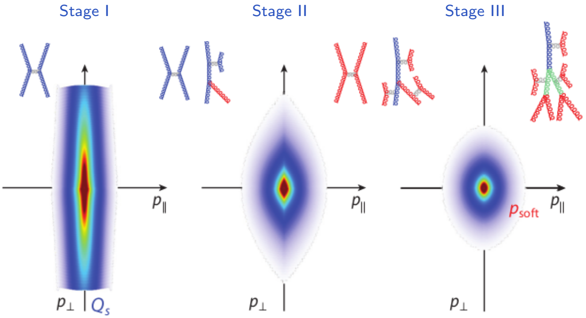
$$\frac{df_{q,g}(\mathbf{p})}{d\tau} - \frac{p_z}{\tau}\partial_{p_z} f_{q,g}(\mathbf{p}) = -\mathcal{C}_{\mathbf{2\leftrightarrow 2}}[f_{q,g}(\mathbf{p})] - \mathcal{C}_{\mathbf{1\leftrightarrow 2}}[f_{q,g}(\mathbf{p})] \qquad \underline{0 + 1\text{d Bjorken}}$$

At Leading order, transport at different momentum scales in C[f]



regulated by HTL

LPM included



"Bottom up thermalisation"

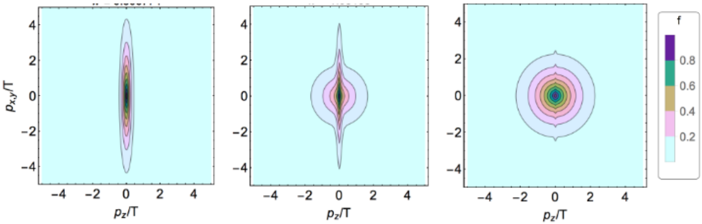
Baier, Mueller, Schiff, and Son (2001); J.Berges, M.Heller, A.Mazeliauskas and R.Venugopalan arxiv.2005.12299 (2020); Schlichting, Teaney, Ann. Rev. of Nuc Part. Sci.(2019); Arnold, P. Gorda, T. Iqbal, S. JHEP. 2020, 53

Relaxation time approximation: RTA

J.L. Anderson, H.R. Witting Physica, 74(1974)

- Approach to equilibrium set by an equilibration rate.

$$-\frac{df_g(\mathbf{p})}{d\tau} + \frac{p_z}{\tau}\partial_{p_z} f_g(\mathbf{p}) = \frac{p^\mu u_\mu}{\tau_R}(f_g(\mathbf{p}) - f_g^{eq}(\mathbf{p}))$$



M. Strickland. JHEP.2018,128 (2018)

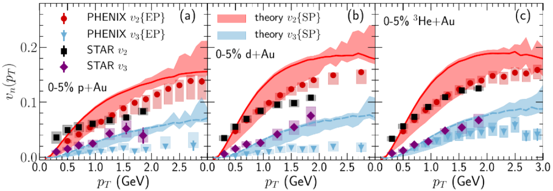
Popular approach $\Rightarrow$ direct affect on transport coefficients calculations
(Gabriel Soares Rocha' poster and flash talk)

Non-equilibrium dynamics

Non-equilibrium effects and Hydrodynamization

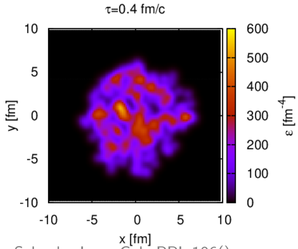
Uncertainty in our understanding of the regime applicability of fluid dynamics

Schenke.Chun.Prithwish PRB 803 (2020)



- Collective behaviour at various system sizes and collision energies
- Emergence of hydrodynamics constitutive relations far from equilibrium in AdS/CFT holography

Chesler,Yaffe PRD 82,(2010) Heller,Janik,Witaszczyk PRL.108,(2012)



Schenke,Jeon, Gale PRL.106()

- Large order hydrodynamic gradient expansion can be divergent!

Heller, Janik, Witaszczyk PRL. 110,(2013)

- System lives most of its life-time in a state of out of equilibrium

(Niemi,Denicol arxiv.1404.7327) (Noronha-Hostler,Noronha,Gyulassy PRC 93(2016))

$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}} > 1$$

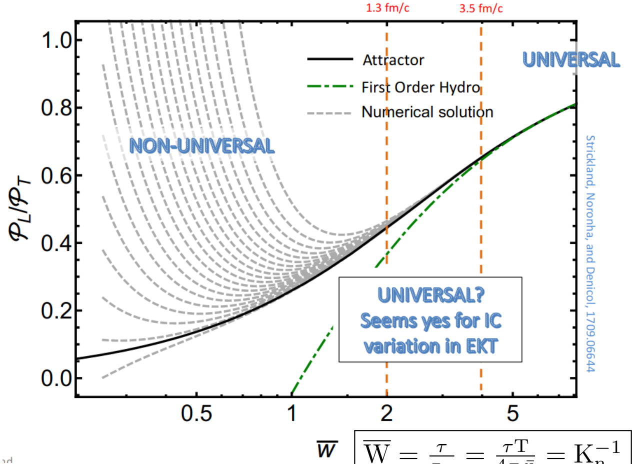
especially true for small systems!.

see Christopher Plumperg's poster for OO $\mathrm{K_n}$ plots!

The attractor: a better way to think of hydrodynamics

How universal are hydrodynamic attractors?

- Hydrodynamics as a universal attractor
  Heller and Spalinski. PRL.115 (2015)
- Memory loss of initial conditions
- Competition between expansion and interaction rate
- Onset of hydrodynamics is set by the decay of non-hydrodynamic modes
- Microscopic model dependent

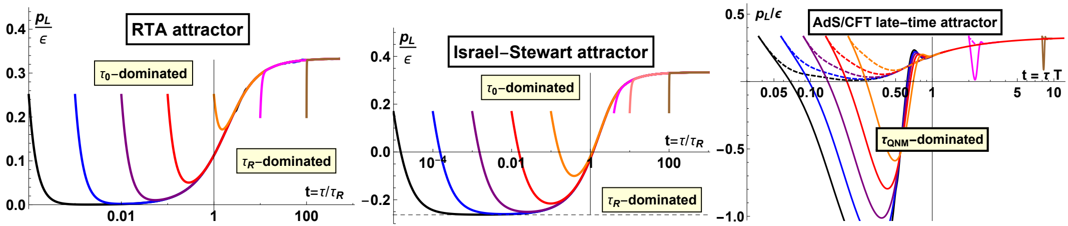


$$\boxed{\overline{\mathrm{W}} = \frac{\tau}{\tau_R} = \frac{\tau \mathrm{T}}{4\pi\,\bar{\eta}} = \mathrm{K}_\mathrm{n}^{-1}}$$

Attractors in different microscopic theories

Kurkela,van der Schee,Wiedemann,Wu PRL.124,(2020)

Decay of non-hydro modes depends on the underlying microscopic theory



Strong coupling

How can one access the underlying information?

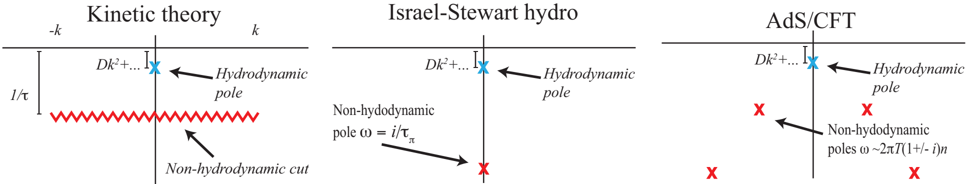
see Jasmine Brewer's talk

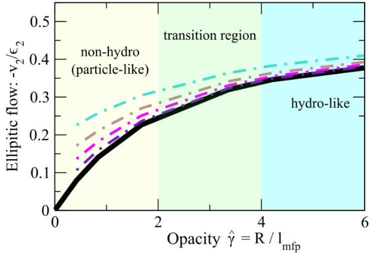
Structure of non-hydrodynamics modes

Kurkela,Wiedemann,Wu, EPJ.C79,(2019)

Non-hydrodynamic excitations make large contribution to v_2 in small systems.

$$G_R^{\mu\nu,\alpha\beta}(x;t) = \langle [T^{\mu\nu}(x,t),\, T^{\alpha\beta}(0,0)] \rangle$$



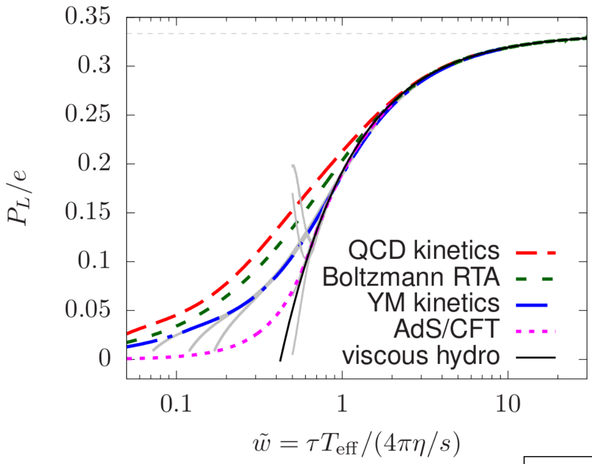


- Small systems? see Aleksas Mazeliauskas's talk
- Collectivety arises in small system from microscopic interactions?.
- Can we disentangle their effects from the hydrodynamical flow?

Universal scaling

Giuliano Giacalone, Aleksas Mazeliauskas, and Sören Schlichting Phys. Rev. Lett. 123, 262301

Qualitatively similar description from different microscopic theories



$$\boxed{\tilde{\omega} = \frac{\tau}{\tau_R} = \frac{\tau T}{4\pi\,\bar{\eta}} = \mathrm{K_n^{-1}}}$$

Non-equilibrium attractor beyond hydrodynamics?

Almaalol,Kurkela, Strickland PRL. 125,(2020)

General moments of the distribution function

M. Strickland, JHEP2018, 128; 1809.01200.

Solving higher moments of the Boltzmann equation

$\mathrm{N_C} = 3$ in $0+1\mathrm{d}$ Bjorken flow, 2D grid $\{x_i, p_j\}$ with 250×2000 grid points

(Kurkela and Zhu PRL 115, 182301 (2015))

- A general moment of the distribution function is defined by

$$\boxed{\mathcal{M}^{nm}[f] \equiv \int dP\,(p.u)^n\,(p.z)^{2m}\,f(x,p)}$$

- The corresponding equilibrium values using a Bose distribution,

$$\mathcal{M}^{nm}_{\mathrm{eq}} = \frac{T^{n+2m+2}\Gamma(n+2m+2)\zeta(n+2m+2)}{2\pi^2(2m+1)}$$

- the hydrodynamics degrees of freedom are

$$\begin{aligned} \mathcal{M}^{10} &= \quad \text{number density} \\ \mathcal{M}^{20} &= \quad \text{energy density} \\ \mathcal{M}^{01} &= \quad \text{longitudinal pressure} \end{aligned}$$

- Study the deviations from equilibrium

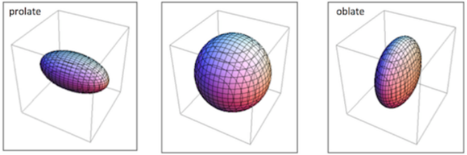
.

$$\boxed{\overline{\mathcal{M}}^{nm}(\tau) \equiv \frac{\mathcal{M}^{nm}(\tau)}{\mathcal{M}^{nm}_{\mathrm{eq}}(\tau)}}$$

Initial distribution $-\frac{d\mathbf{f_p}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{2\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{\exp}[\mathbf{f_p}].$

▶ Thermal Romatschke-Strickland

$$\boxed{\mathrm{f}_{0,\mathrm{RS}}(\mathbf{p}) = f_{\mathrm{Bose}}\left(\sqrt{\mathbf{p}^2 + \xi_0 p_z^2}/\Lambda_0\right)}$$



$-1 < \xi < 0$ $\quad$ $\xi = 0$ $\quad$ $\xi > 0$

anisotropy parameter $(-1 < \xi_0 < \infty)$

Λ_0 is set by Landau matching

Romatschke,Strickland,PRD68, (2003)

▶ Non-thermal CGC

$$\boxed{\mathrm{f}_{0,\mathrm{CGC}}(\mathbf{p}) = \frac{2A}{\lambda}\frac{\tilde{\Lambda}_0}{\sqrt{\mathbf{p^2} + \xi_0 p_z^2}} \exp^{-\frac{2}{3}(\mathbf{p}^2 + \xi_0 \hat{p}_z^2)/\tilde{\Lambda}_0^2}}$$

The initial scale $\tilde{\Lambda}_0$ is related to the saturation scale $\tilde{\Lambda}_0 = \langle \mathrm{p_T} \rangle_0 \approx 1.8\,\mathrm{Q_s}$

A is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation

A. Kurkela and Y. Zhu, Phys. Rev. Lett.115, 182301(2015)

T. Lappi, Phys. Lett.B703, 325-330 (2011)

Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

Non-equilibrium evolution becomes insensitive to initial conditions at very early times

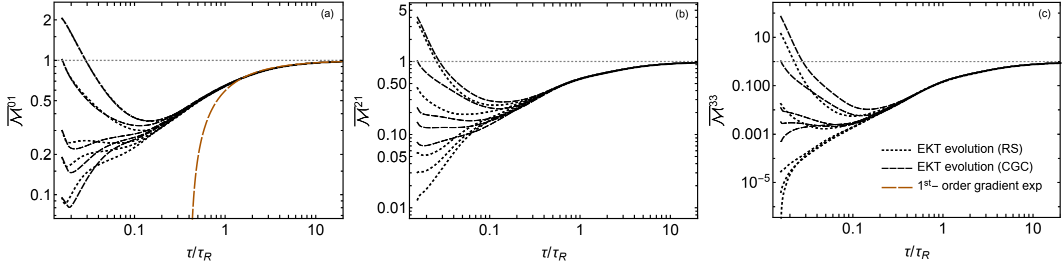
$\tau_{\mathrm{R}}(\tau) = 4\pi\bar{\eta}/\mathrm{T}(\tau)$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Forward attractor

Pressure anisotropy

$\mathcal{M}^{01}[f] =< p^{-1}p_z^2 >$ $\quad$ $\mathcal{M}^{21}[f] =< p^{1}p_z^2 >$ $\quad$ $\mathcal{M}^{33}[f] =< p^{2}p_z^6 >$



Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

An attractor for the momentum phase space distribution function

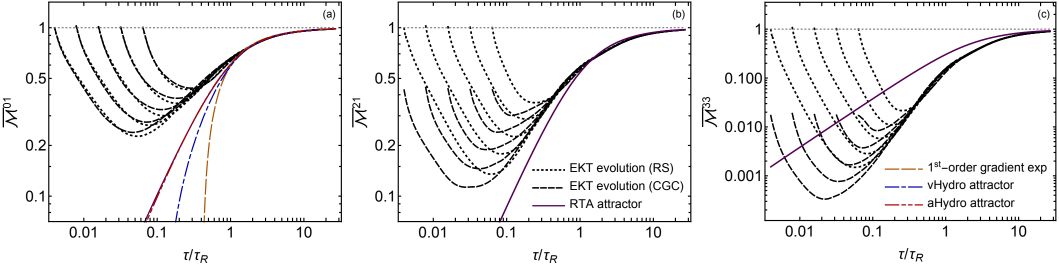
$$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Pullback attractor
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

Pressure anisotropy

$\mathcal{M}^{01}[f] =< p^{-1}p_z^2 >$ $\mathcal{M}^{21}[f] =< p^{1}p_z^2 >$ $\mathcal{M}^{33}[f] =< p^{2}p_z^6 >$



Beyond Bjorken flow?

Beyond $0+1\mathrm{d}$: transverse dynamics

Kurkela, van der Schee, Wiedemann Wu, PRL.124(2020)

What remains universal is the early-time dynamics and not what follows from hydrodynamization

- Which aspects of the attractor behavior are accessible in collisions with a finite transverse extent and realistic transverse gradients

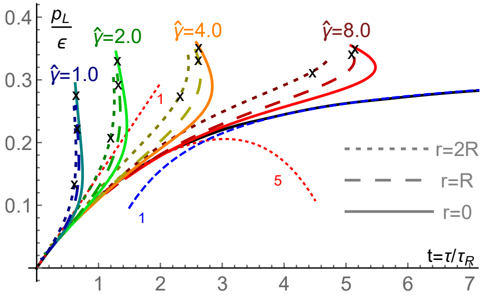
Single model parameter

$$-C[F] = -\frac{(-v_\mu u^\mu)}{\tau_{iso}}(F - F_{iso})\,,$$

$$\tau_{iso} = \frac{1}{\gamma \varepsilon^{1/4}}$$

$$\hat{\gamma} = R^{3/4}\gamma(\,\varepsilon_0\tau_0)^{1/4}$$

see Wiedemann's talk



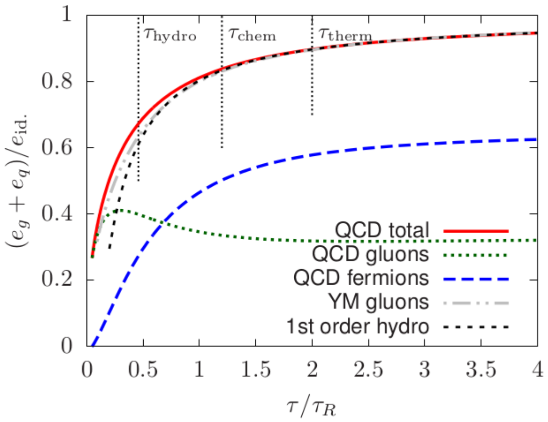
Chemical equilibration?

QGP chemical equilibration

Kurkela.Mazeliauskas.PRL, 122,(2019)

$$\boxed{\tau_{\text{hydro}} < \tau_{\text{chem}} < \tau_{\text{therm}}}$$

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced through fusion $\mathrm{gg} \to \mathrm{q\bar{q}}$ and splitting $\mathrm{g} \to \mathrm{q\bar{q}}$
- energy transfer from glounic to quark sectors



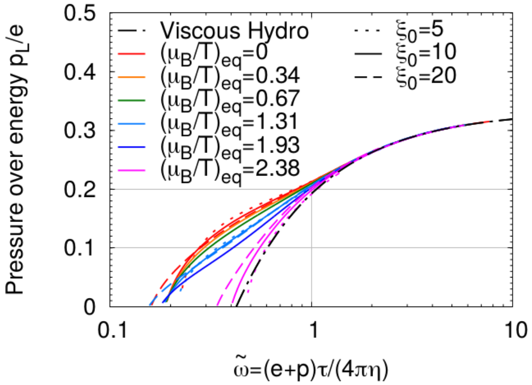
QGP chemical equilibration

Giacalone,Mazeliauskas,Schlichting PRL.123(202)

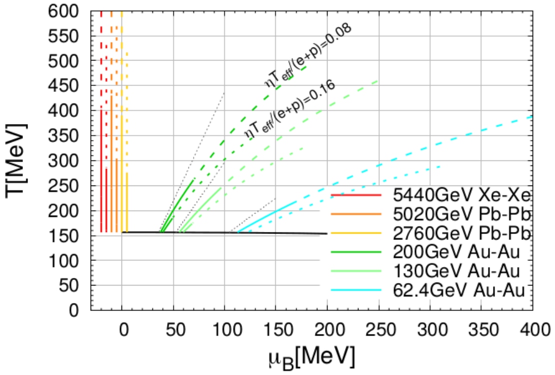
Du, Schlichting (2012.09068), (2012.09079)

Non-equilibrium corrections dominate the system's lifetime at lower beam energies

see Xiaojian Du's poster and Also (Travis Dore's talk)



Slower isotropization with quarks



moderate dependence on μ_B/T

Non-equilibrium effects at Freeze out

Almaalol,Kurkela, Strickland PRL. 125,(2020)

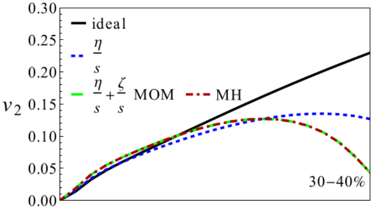
Non-equilibrium effects at freezeout

(Almaalol,Kurkela, Strickland PRL 125,(2020))

- δf can be computed for a particular form of $C[f]$

  (Dusling,Moore,Teaney PRC 81, (2008))

- Large δf corrections at freezeout directly affect the anisotropic flow $v_2(p_T)$ (Noronha-Hostler, Noronha,Grassi PRC 90 (2014))



The *quadratic ansatz* ($\alpha = 0$)

$$\frac{\delta f_{(i)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{3\overline{\Pi}}{16T^2}(p^2 - 3p_z^2)$$

$$\overline{\Pi} = \Pi/\epsilon = 1/3 - T^{zz}/\epsilon$$

The *LPM ansatz* ($\alpha = 0.5$)

$$\frac{\delta f_{(ii)}}{f_{\rm eq}(1+f_{\rm eq})} = \frac{16\overline{\Pi}}{21\sqrt{\pi}\,T^{3/2}}\left(p^{3/2} - \frac{3p_z^2}{\sqrt{p}}\right)$$

$$\overline{\Pi} = \Pi/\epsilon = 1/3 - T^{zz}/\epsilon$$

The *aHydro freeze-out ansatz*

$$f(p) = f_{\rm Bose}\left(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda\right)$$

$$\overline{\mathcal{M}}^{nm}_{\rm aHydro}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{\mathcal{H}^{nm}(\alpha)}{[\mathcal{H}^{20}(\alpha)]^{(n+2m+2)/4}}$$

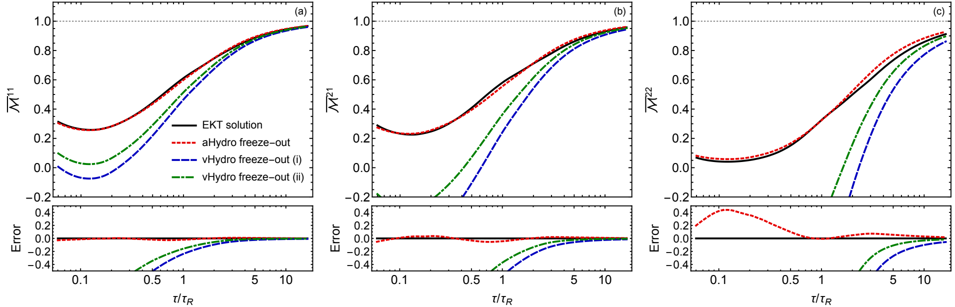
Insights into the freezeout perscription

(Almaalol,Kurkela,Strickland PRL 125, (2020))

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Disagreement increases for higher moments and for earlier times.
- Good agreement between aHydro ansatz and EKT at all times

$\mathcal{M}^{11}[f] = < p_z^2 >$ $\qquad$ $\mathcal{M}^{21}[f] = < p^1 p_z^2 >$ $\qquad$ $\mathcal{M}^{22}[f] = < p^1 p_z^4 >$



For earlier implementation: (Pratt,Torrieri PRC 82(2010)) (Weller, Romatchke PLB 774 (2017))

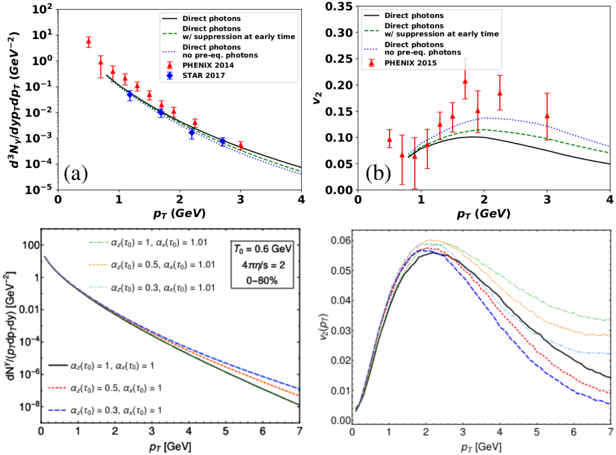
Electromagnetic probes

Electromagnetic probes: Photon and dilepton production

Paquet,Shen, Schenke, Gale. arxiv.2002.05191(2020)

Kasmaei, Strickland PRD 102(2019)

Sensitivity of electromagnetic propes to initial state see B. Schenke's talk and Also (J.Paquet's talk)



Conclusions

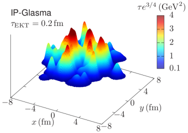
- QCD kinetic theory played a significant role in understanding the equilibration of the QCD medium at high temperature
- EKT provided insights into different stages of the ultra-relativistic heavy ion collisions evolution
- Potential insight into understanding emergence of macroscopic quantities from the underlying microscopics picture
- Possible future improvement in KoMPoST?
- Transport coefficients?

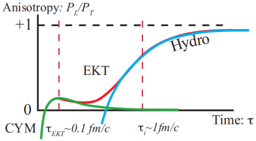
Thank you for your attention!

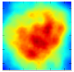
KoMPoST

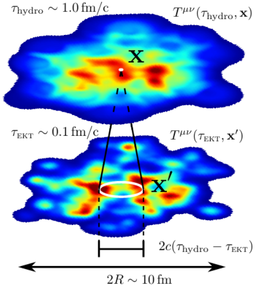
A. Kurkela, A. Mazeliauskas, J.F. Paquet, S. Schlichting, D. Teaney, Phys.Rev.Lett. 122 (2019) 12, 122302

- hydrodynamic model results are dependent on initialization time, and different hydrodynamic codes regulate these extreme initial conditions in different ad hoc ways









$$T^{\mu\nu}(\tau_{\text{EKT}}, \mathbf{x}') = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\text{EKT}}) + \delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\text{EKT}}, \mathbf{x}').$$

$$f_{\mathbf{x},\mathbf{p}} = \bar{f}_{\mathbf{p}} + \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, \delta f_{\mathbf{k},\mathbf{p}}\, e^{i\mathbf{k}\cdot\mathbf{x}}.$$

$$T^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x}) = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\text{hydro}}) + \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\text{hydro}})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\text{EKT}})} \times$$
$$\times \int d^2\mathbf{x}'\, G^{\mu\nu}_{\alpha\beta}\left(\mathbf{x}, \mathbf{x}', \tau_{\text{hydro}}, \tau_{\text{EKT}}\right) \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\text{EKT}}, \mathbf{x}').$$

the subsequent hydrodynamic evolution becomes independent of the hydrodynamic initialization time !!

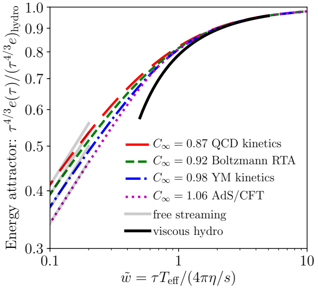
Scaling and Entropy production

Giuliano Giacalone, Aleksas Mazeliauskas, and Sören Schlichting Phys. Rev. Lett. 123, 262301;P. Hanus, A. Mazeliauskas, and K. Reygers, Phys. Rev.C100, 064903 (2019)

- Macroscopic description from hydrodynamics attractors. Integrate equations of motion to find energy attractor

$$\partial_\tau e = -\frac{e}{\tau} + \frac{P_L}{e}; \qquad \frac{P_L}{e} = f\left[\tilde{w} = \frac{\tau T_{\text{eff}}}{4\pi\eta/s}\right] \Rightarrow \frac{e(\tau)\tau^{4/3}}{e_{\text{hydro}}\tau_{\text{hydro}}^{4/3}} = \mathcal{E}\left(\tilde{w} = \frac{T_{\text{eff}}(\tau)\tau}{4\pi\eta/s}\right)$$

$$\boxed{(s\tau)_{\text{hydro}} = \frac{4}{3} C_\infty^{3/4} \left(4\pi \frac{\eta}{s}\right)^{1/3} \left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/3} (e\tau)_0^{2/3},}$$



$$\frac{dN_{\text{ch}}}{d\eta} \approx \frac{1}{J} A_\perp \, (s\tau)_{\text{hydro}} \frac{N_{\text{ch}}}{S} \, .$$

where $A_\perp \approx \pi R^2 \; S/N_{\text{ch}} \approx 7$ is a constant of hadron gas

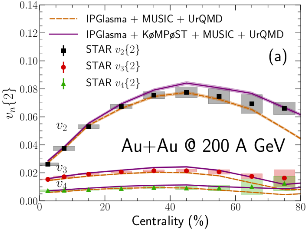
$$\frac{dE_\perp}{d\eta} \approx A_\perp \, (e\tau)_0 \, .$$

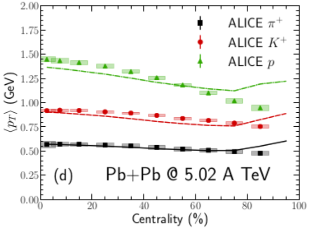
Macroscopic description from hydrodynamics attractors!

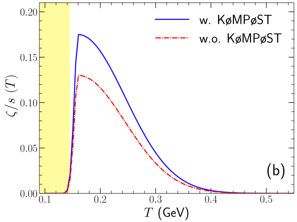
KoMPoST : future improvement

- Inclusion of kompost influence the extraction of transport coefficients see (B. Schenke's talk)

  Paquet,Shen, Schenke, Gale. arxiv.2002.05191(2020)

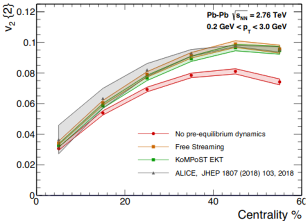






- Insensitivity of PCA to the pre-equilibrium stage see (T.Nunes' talk)

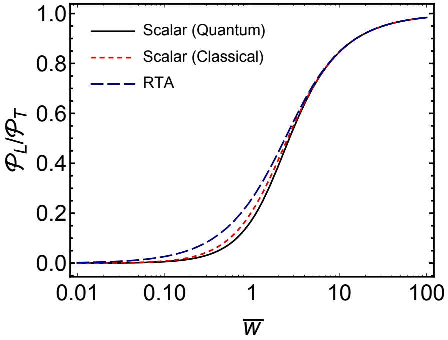
  da Silva. Chinellato, Hippert , Serenone,Takahashi,Denicol,Luzum,Noronha arxiv.2006.02324 (2020)



$\Rightarrow$ Call for non-conformal treatment

Kinetic based hydrodynamics equations

Moment method



Almaalol, Alqahtani, Strickland PRC 99, (2019)

- Popular approach in phenomenolgy
- Direct impact on transpot coefficients! (Gabriel's poster and flash talk)

Landau Matching

$$\partial_\mu n^\mu = \mathcal{C}$$
$$\partial_\mu T^{\mu\nu} = \mathcal{C}^\nu$$

$$\mathcal{C}_{RTA} = \frac{u^\mu . p^\mu}{\tau_\mathrm{R}} \left[f_\mathrm{eq}(p/T) - f_p\right]$$

$$\partial_\mu n^\mu = \frac{1}{\tau_\mathrm{R}} \left[n_\mathrm{eq} - n\right]$$
$$\partial_\mu T^{\mu\nu} = \frac{1}{\tau_\mathrm{R}} \left[\epsilon_\mathrm{eq} - \epsilon\right]$$

Kinetic based hydrodynamics equations

Moment integral operator

$$\hat{\mathcal{O}}_n \, g = \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n}[g] \equiv \int dP \, p^{\mu_1} p^{\mu_2} \cdots p^{\mu_n} \, g(p) n^{th}$$

⇓

$$p^\mu \partial_\mu f_p = C[f_p]$$

⇓

$$\partial_\mu I^{\mu\nu_1\nu_2\cdots\nu_n} = \mathcal{C}^{\nu_1\nu_2\cdots\nu_n}$$

$$I^{\mu\nu_1\nu_2\cdots\nu_n} \equiv \int dP \, p^\mu p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n} \, f$$

$$\mathcal{C}^{\nu_1\nu_2\cdots\nu_n} \equiv \int dP \, p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n} \, C[f]$$

Landau Matching

$$\partial_\mu n^\mu = \mathcal{C}$$

$$\partial_\mu T^{\mu\nu} = \mathcal{C}^\nu$$

⇓

$$\mathcal{C}_{RTA} = \frac{u^\mu . p^\mu}{\tau_{\mathrm{R}}} \left[f_{\mathrm{eq}}(p/T) - f_p \right]$$

⇓

$$\partial_\mu n^\mu = \frac{1}{\tau_{\mathrm{R}}} \left[n_{\mathrm{eq}} - n \right]$$

$$\partial_\mu T^{\mu\nu} = \frac{1}{\tau_{\mathrm{R}}} \left[\epsilon_{\mathrm{eq}} - \epsilon \right]$$

Insights into far from equilibrium transport coefficients

Kamata, Martinez, Plaschke, Ochsenfeld, Schlicting PRD 102 (2020)

Both RTA and QCD capture the macroscopic response of the system similarly!

- How microscopic details affect macroscopic quantities far from equilibrium?
- "renormalized transport coefficients"?