New developments in QCD-based kinetic transport theory

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Initial Stages 2021
Jan 15, 2021
Kinetic transport theory in ultra-relativistic heavy ion collisions

- initial state
- preequilibrium
- Hydrodynamics
- Hadronization
- Hadronic cascade

Original Figure: MADAI collaboration, Hannah Petersen and Jonah Bernhard
Kinetic transport theory in ultra-relativistic heavy ion collisions

Original Figure: MADA1 collaboration, Hannah Petersen and Jonah Bernhard

Initial state  Preequilibrium  Hydrodynamics  Hadronization  Hadronic cascade

QCD kinetic transport theory
Formalism
QCD medium at high temperatures: Effective kinetic theory

\[ P^\mu \partial_\mu f_{q,g}(p) = -C[f(p)], \]

\[ f_{q,g} \propto \frac{dN_{g,q}}{d^3xd^3p}, \quad g, q(u, d, s, \bar{u}, \bar{d}, \bar{s}) \]

\[ \frac{df_{q,g}(p)}{d\tau} - \frac{p_z}{\tau} \partial_{p_z} f_{q,g}(p) = -C_2 \leftrightarrow 2[f_{q,g}(p)] - C_1 \leftrightarrow 2[f_{q,g}(p)] \]

At Leading order, transport at different momentum scales in C[f]

regulated by HTL

LPM included

"Bottom up thermalisation"

Relaxation time approximation: RTA


- Approach to equilibrium set by an equilibration rate.

\[-\frac{df_g(p)}{dT} + \frac{p_z}{\tau} \frac{\partial p_z}{\partial f_g(p)} = \frac{\mu u_{\mu}}{\tau_R} (f_g(p) - f_{eq}(p))\]

Popular approach ⇒ direct affect on transport coefficients calculations
(Gabriel Soares Rocha’ poster and flash talk)
Non-equilibrium dynamics
Non-equilibrium effects and Hydrodynamization

Uncertainty in our understanding of the regime applicability of fluid dynamics

- Collective behaviour at various system sizes and collision energies
- Emergence of hydrodynamics constitutive relations far from equilibrium in AdS/CFT holography
  

- Large order hydrodynamic gradient expansion can be divergent!
  
  Heller, Janik, Witaszczyk PRL 110, (2013)

- System lives most of its life-time in a state of out of equilibrium
  
  (Niemi, Denicol arxiv.1404.7327) (Noronha-Hostler, Noronha, Gyulassy PRC 93(2016))

  \[ Kn = \frac{\text{microscopic scale}}{\text{macroscopic scale}} > 1 \]

  especially true for small systems!

  see Christopher Plumperg’s poster for OO \( K_n \) plots!
The attractor: a better way to think of hydrodynamics

How universal are hydrodynamic attractors?

- Hydrodynamics as a universal attractor
  Heller and Spalinski. PRL.115 (2015)
- Memory loss of initial conditions
- Competition between expansion and interaction rate
- Onset of hydrodynamics is set by the decay of non-hydrodynamic modes
- Microscopic model dependent

\[
W = \frac{\tau}{\tau_R} = \frac{\tau_T}{4\pi \eta} = K_n^{-1}
\]
Attractors in different microscopic theories

Decay of non-hydro modes depends on the underlying microscopic theory

Kurkela, van der Schee, Wiedemann, Wu PRL. 124, (2020)

How can one access the underlying information?

see Jasmine Brewer’s talk

Strong coupling
Structure of non-hydrodynamics modes

Kurkela, Wiedemann, Wu, EPJ.C79,(2019)

Non-hydrodynamic excitations make large contribution to $v_2$ in small systems.

\[ G^{\mu\nu, \alpha\beta}_R (x; t) = \langle [T^{\mu\nu}(x, t), T^{\alpha\beta}(0, 0)] \rangle \]

- Small systems? see Aleksas Mazeliauskas's talk
- Collectivity arises in small system from microscopic interactions?
- Can we disentangle their effects from the hydrodynamical flow?
Universal scaling

Giuliano Giacalone, Aleksas Mazeliauskas, and Sören Schlichting

Phys. Rev. Lett. 123, 262301

Qualitatively similar description from different microscopic theories

\[ \tilde{\omega} = \frac{\tau}{\tau_R} = \frac{\tau T}{4\pi \eta} = K_n^{-1} \]
Non-equilibrium attractor beyond hydrodynamics?

Almaalol, Kurkela, Strickland PRL. 125, (2020)
General moments of the distribution function

M. Strickland, JHEP2018, 128; 1809.01200.

Solving higher moments of the Boltzmann equation

\( N_C = 3 \) in 0 + 1d Bjorken flow, 2D grid \( \{x_i, p_j\} \) with 250 \( \times \) 2000 grid points

(Kurkela and Zhu PRL 115, 182301 (2015))

A general moment of the distribution function is defined by

\[
\mathcal{M}^{nm}[f] \equiv \int dP (p.u)^n (p.z)^{2m} f(x,p)
\]

The corresponding equilibrium values using a Bose distribution,

\[
\mathcal{M}^{eq}_{nm} = \frac{T^{n+2m+2} \Gamma(n + 2m + 2) \zeta(n + 2m + 2)}{2\pi^2 (2m + 1)}
\]

the hydrodynamics degrees of freedom are

\( \mathcal{M}^{10} = \) number density

\( \mathcal{M}^{20} = \) energy density

\( \mathcal{M}^{01} = \) longitudinal pressure

Study the deviations from equilibrium

\[
\overline{\mathcal{M}}^{nm}(\tau) \equiv \frac{\mathcal{M}^{nm}(\tau)}{\mathcal{M}^{eq}_{nm}(\tau)}
\]

. 
Initial distribution \[-\frac{df_p}{dT} = C_{1\leftrightarrow 2}[f_p] + C_{2\leftrightarrow 2}[f_p] + C_{\exp}[f_p].\]

**Thermal Romatschke-Strickland**

\[f_{0,RS}(p) = f_{Bose}\left(\sqrt{p^2 + \xi_0 p_z^2}/\Lambda_0\right)\]

Anisotropy parameter \((-1 < \xi_0 < \infty)\)

\(\Lambda_0\) is set by Landau matching


**Non-thermal CGC**

\[f_{0,CGC}(p) = \frac{2A}{\lambda} \frac{\tilde{\Lambda}_0}{\sqrt{p^2 + \xi_0 p_z^2}} \exp -\frac{2}{3}(p^2 + \xi_0 \tilde{p}_z^2)/\tilde{\Lambda}_0^2\]

The initial scale \(\tilde{\Lambda}_0\) is related to the saturation scale \(\tilde{\Lambda}_0 = \langle p_T \rangle_0 \approx 1.8 Q_s\)

\(A\) is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation


Non-equilibrium QCD attractor at high temperature

DA, Kurkela, Strickland PRL. 125, (2020)

Non-equilibrium evolution becomes insensitive to initial conditions at very early times

Forward attractor

Pressure anisotropy

\( \mathcal{M}^{01}[f] = \langle p^{-1} p_z^2 \rangle \)

\( \mathcal{M}^{21}[f] = \langle p^1 p_z^2 \rangle \)

\( \mathcal{M}^{33}[f] = \langle p^2 p_z^6 \rangle \)

\[ \tau_R(\tau) = 4\pi \bar{\eta}/T(\tau) \]

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<th>( t/t_R )</th>
<th>( \tau )</th>
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</table>

EKT evolution (RS)

EKT evolution (CGC)

1st-order gradient exp

D. Almaalol - Kent State University - 16
Non-equilibrium QCD attractor at high temperature

DA, Kurkela, Strickland PRL. 125, (2020)

An attractor for the momentum phase space distribution function

- Pullback attractor
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

Pressure anisotropy

\[ \mathcal{M}^{01}[f] = \langle p^{-1}p_z^2 \rangle \]

\[ \mathcal{M}^{21}[f] = \langle p^1p_z^2 \rangle \]

\[ \mathcal{M}^{33}[f] = \langle p^2p_z^6 \rangle \]
Beyond Bjorken flow?
Beyond $0 + 1d$: transverse dynamics

Kurkela, van der Schee, Wiedemann Wu, PRL.124(2020)

What remains universal is the early-time dynamics and not what follows from hydrodynamization

- Which aspects of the attractor behavior are accessible in collisions with a finite transverse extent and realistic transverse gradients

Single model parameter

$$-C[F] = -\left(\frac{-v_{\mu}u^{\mu}}{\tau_{iso}}\right)(F - F_{iso}),$$

$$\tau_{iso} = \frac{1}{\gamma \varepsilon^{1/4}}$$

$$\hat{\gamma} = R^{3/4} \gamma (\varepsilon_0 \tau_0)^{1/4}$$

see Wiedemann’s talk
Chemical equilibration?
QGP chemical equilibration

Kurkela.Mazeliauskas.PRL, 122,(2019)

$\tau_{\text{hydro}} < \tau_{\text{chem}} < \tau_{\text{therm}}$

- QCD transport of $N_f = 3$ massless fermions.
- Quarks are dynamically produced through fusion $gg \rightarrow q\bar{q}$ and splitting $g \rightarrow q\bar{q}$
- energy transfer from gluonic to quark sectors
QGP chemical equilibration

Giacalone, Mazeliauskas, Schlichting PRL 123(202)

Du, Schlichting (2012.09068), (2012.09079)

Non-equilibrium corrections dominate the system’s lifetime at lower beam energies

see Xiaojian Du’s poster and Also (Travis Dore’s talk)

Slower isotropization with quarks

moderate dependence on $\mu_B/T$
Non-equilibrium effects at Freeze out

Almaalol, Kurkela, Strickland PRL. 125, (2020)
Non-equilibrium effects at freezeout

(Almaalol, Kurkela, Strickland PRL 125, (2020))

- $\delta f$ can be computed for a particular form of $C[f]$
  (Dusling, Moore, Teaney PRC 81, (2008))
- Large $\delta f$ corrections at freezeout directly affect the anisotropic flow $v_2(p_T)$ (Noronha-Hostler, Noronha, Grassi PRC 90 (2014))

The quadratic ansatz ($\alpha = 0$)

$$
\frac{\delta f^{(i)}}{f_{eq}(1 + f_{eq})} = \frac{3\Pi}{16T^2}(p^2 - 3p_z^2)
\Pi = \Pi / \epsilon = 1/3 - T^{zz} / \epsilon
$$

The LPM ansatz ($\alpha = 0.5$)

$$
\frac{\delta f^{(ii)}}{f_{eq}(1 + f_{eq})} = \frac{16\Pi}{21\sqrt{\pi}T^{3/2}}\left(\frac{p^{3/2} - 3p_z^2}{\sqrt{p}}\right)
\Pi = \Pi / \epsilon = 1/3 - T^{zz} / \epsilon
$$

The aHydro freeze-out ansatz

$$
f(p) = f_{\text{Bose}}\left(\sqrt{p^2 + \xi p_z^2} / \Lambda\right)
\mathcal{M}^{nm}_{\text{aHydro}}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{H^{nm}(\alpha)}{[H^{20}(\alpha)]^{(n+2m+2)/4}}
$$
Insights into the freezeout prescription
(Almaalol, Kurkela, Strickland PRL 125, (2020))

- Disagreement increases for higher moments and for earlier times.
- Good agreement between $a_{Hydro}$ ansatz and EKT at all times

For earlier implementation:
(Pratt, Torrieri PRC 82(2010) (Weller, Romatchke PLB 774 (2017))
Electromagnetic probes
Electromagnetic probes: Photon and dilepton production


Kasmaei, Strickland PRD 102(2019)

Sensitivity of electromagnetic probes to initial state see B. Schenke’s talk and Also (J. Paquet’s talk)
Conclusions

▶ QCD kinetic theory played a significant role in understanding the equilibration of the QCD medium at high temperature
▶ EKT provided insights into different stages of the ultra-relativistic heavy ion collisions evolution
▶ Potential insight into understanding emergence of macroscopic quantities from the underlying microscopics picture
▶ Possible future improvement in KoMPoST?
▶ Transport coefficients?
Thank you for your attention!
hydrodynamic model results are dependent on initialization time, and different hydrodynamic codes regulate these extreme initial conditions in different ad hoc ways

\[ T_{\mu\nu}(\tau_{\text{hydro}}, x) = T_{\mu\nu}(\tau_{\text{EKT}}, x) + \delta T_{\mu\nu}(\tau_{\text{EKT}}, x'). \]

the subsequent hydrodynamic evolution becomes independent of the hydrodynamic initialization time!!
Scaling and Entropy production


Macroscopic description from hydrodynamics attractors. Integrate equations of motion to find energy attractor

\[ \partial_\tau e = -\frac{e}{\tau} + \frac{P_L}{e} ; \quad \frac{P_L}{e} = f \left[ \bar{w} = \frac{\tau T_{\text{eff}}}{4\pi\eta/s} \right] \]

\[ (s\tau)_{\text{hydro}} = \frac{4}{3} C_\infty^{3/4} \left( \frac{4\pi\eta}{s} \right)^{1/3} \left( \frac{\pi^2}{30} \nu_{\text{eff}} \right)^{1/3} (e\tau)_0^{2/3} , \]

\[ \frac{dN_{\text{ch}}}{d\eta} \approx \frac{1}{J} A_\perp (s\tau)_{\text{hydro}} \frac{N_{\text{ch}}}{S} . \]

where \( A_\perp \approx \pi R^2 S / N_{\text{ch}} \approx 7 \) is a constant of hadron gas

\[ \frac{dE_\perp}{d\eta} \approx A_\perp (e\tau)_0 . \]

Macroscopic description from hydrodynamics attractors!
KoMPoST : future improvement

- Inclusion of kompost influence the extraction of transport coefficients see (B. Schenke’s talk)


- Insensitivity of PCA to the pre-equilibrium stage see (T. Nunes’ talk)


⇒ Call for non-conformal treatment
Kinetic based hydrodynamics equations

**Moment method**

- **Scalar (Quantum)**
- **Scalar (Classical)**
- **RTA**

![Graph](image)

- Popular approach in phenomenology
- Direct impact on transport coefficients! (Gabriel’s poster and flash talk)

**Landau Matching**

\[
\begin{align*}
\partial_\mu n^\mu &= C \\
\partial_\mu T^{\mu\nu} &= C^\nu \\
C_{RTA} &= \frac{u^\mu p^\mu}{\tau_R} \left[ f_{eq}(p/T) - f_p \right]
\end{align*}
\]

\[
\begin{align*}
\partial_\mu n^\mu &= \frac{1}{\tau_R} \left[ n_{eq} - n \right] \\
\partial_\mu T^{\mu\nu} &= \frac{1}{\tau_R} \left[ \epsilon_{eq} - \epsilon \right]
\end{align*}
\]

Kinetic based hydrodynamics equations

Moment integral operator

\[ \hat{O}_n g = \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n} [g] \equiv \int dP \, p^{\mu_1} p^{\mu_2} \cdots p^{\mu_n} \, g(p) n^{th} \]

\[ p^\mu \partial_\mu f_p = C[f_p] \]

\[ \partial_\mu I^{\mu_1 \nu_2 \cdots \nu_n} = C^{\nu_1 \nu_2 \cdots \nu_n} \]

\[ I^{\mu_1 \nu_2 \cdots \nu_n} \equiv \int dP \, p^{\mu_1} p^{\nu_2} \cdots p^{\nu_n} f \]

\[ C^{\nu_1 \nu_2 \cdots \nu_n} \equiv \int dP \, p^{\nu_1} p^{\nu_2} \cdots p^{\nu_n} \, C[f] \]

Landau Matching

\[ \partial_\mu n^\mu = C \]

\[ \partial_\mu T^{\mu \nu} = C^\nu \]

\[ C_{RTA} = \frac{u^\mu . p^\mu}{\tau_R} \left[ f_{eq}(p/T) - f_p \right] \]

\[ \partial_\mu n^\mu = \frac{1}{\tau_R} [n_{eq} - n] \]

\[ \partial_\mu T^{\mu \nu} = \frac{1}{\tau_R} [\epsilon_{eq} - \epsilon] \]
Insights into far from equilibrium transport coefficients

Kamata, Martinez, Plaschke, Ochsenfeld, Schlicting PRD 102 (2020)
Both RTA and QCD capture the macroscopic response of the system similarly!

▶ How microscopic details affect macroscopic quantities far from equilibrium?
▶ "renormalized transport coefficients"?