Monte Carlo for initial energy density with correlated fluctuations

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Introduction

One of the most important aspects of a heavy-ion collision is the system's behavior at early stages. The Color-Glass Condensate is our most successful effective theory at describing the system's state after the collision.

The initial energy density is fundamental to the evolution of the QGP, and its mean value has been determined using the CGC framework[1]. Not only that, but also its two-point function, which states the system's covariance.

Goal

We set ourselves to generate events respecting said one and two-point functions proposed by the CGC framework.
Differently from other event generators [2][3], our implementation is such that, if the state-of-the-art theory changes tomorrow, the program is ready to accommodate it.
We can parameterize different unknown components and test how their values change the generated events.

The 1-point function

As proposed[1], below we explicitly write the expected value for the mean of the initial energy density following a heavy-ion collision:

$$\langle \epsilon_0 (\vec{x}_\perp) \rangle = \frac{C_F}{g^2} \bar{Q}_{s1}^2 (\vec{x}_\perp) \, \bar{Q}_{s2}^2 (\vec{x}_\perp) \left(4\pi \partial^2 L (0_\perp)\right)^2 \quad (1)$$

Defining:

$$\bar{Q}_s^2 (\vec{x}_\perp) \left(-4\pi \partial^2 L (0_\perp)\right) = Q_{s0}^2 \, T_s(\vec{x}_\perp)$$

$$T_s(x, y) = \int \frac{\rho_0}{1 + exp \frac{\sqrt{x^2+y^2+z^2}-R}{\chi}} dz \; ; \; \rho_0^{-1} = \int \frac{1}{1 + exp \frac{\sqrt{z^2}-R}{\chi}} dz$$

$R = 6.62 \; fm$ and $\chi = 0.546 \; fm$, which are the radius and skin depth parameters used in the Fermi parametrization of the ^{208}Pb.
$Q_{s0} = 1.24 \; GeV$ is the saturation scale at the center of the nucleus

The 2-point function

$$
\begin{aligned}
&\left\langle \epsilon_0\left(\vec{x}_\perp\right)\epsilon_0\left(\vec{y}_\perp\right)\right\rangle - \left\langle \epsilon_0\left(\vec{x}_\perp\right)\right\rangle\left\langle \epsilon_0\left(\vec{y}_\perp\right)\right\rangle = \\
&\frac{1}{9g^4r^8}\Bigg[-2\left(4-\frac{B^2}{A^2}\right)\left(p_1q_2+p_2q_1\right)+4\left(16+\frac{B^4}{A^4}\right)p_1p_2+2q_1q_2 \\
&+4\left(4+\frac{B^2}{A^2}\right)\left(4\pi\partial^2 L\left(0_\perp\right)\right)^2\left(\left[\bar{Q}_{s1}^4\left(Q_{s2}^2r^2-4+4e^{-\frac{Q_{s2}^2r^2}{4}}\right)\right]+\left[1\leftrightarrow 2\right]\right) \\
&+\left(16+8\frac{B^2}{A^2}+\frac{B^4}{A^4}\right)\Bigg(\Bigg[\frac{91}{8}-\frac{134}{5}e^{-\frac{Q_{s1}^2r^2}{4}}+\frac{81}{100}e^{-\frac{2Q_{s1}^2r^2}{3}}\left(\frac{3}{2}e^{-\frac{2Q_{s2}^2r^2}{3}}+5-8e^{-\frac{Q_{s2}^2r^2}{4}}\right) \\
&+\frac{r^4}{2}Q_{s1}^2Q_{s2}^2-4r^2Q_{s1}^2\left(1-e^{-\frac{Q_{s2}^2r^2}{4}}\right)+\frac{832}{50}e^{-\frac{\left(Q_{s1}^2+Q_{s2}^2\right)r^2}{4}}\Bigg]+\left[1\leftrightarrow 2\right]\Bigg)\Bigg]
\end{aligned}
\tag{2}
$$

$$
\left\langle \epsilon_0\left(\vec{x}_\perp\right)^2\right\rangle - \left\langle \epsilon_0\left(\vec{x}_\perp\right)\right\rangle^2 = \frac{3C_F}{2N_cg^4}\left(4\pi\partial^2 L\left(0_\perp\right)\right)^4\bar{Q}_{s1}^4\bar{Q}_{s2}^4
$$

$$
\begin{aligned}
p_{1,2} &\equiv e^{-\frac{Q_{s1,2}^2r^2}{4}}\left(Q_{s1,2}^2r^2+4\right)-4 \\
q_{1,2} &\equiv e^{-\frac{Q_{s1,2}^2r^2}{4}}\left(Q_{s1,2}^4r^4+8Q_{s1,2}^2r^2+32\right)-32
\end{aligned}
$$

The 2-point function

Defining, in the MV Model[4]:

$$r = |\vec{x}_\perp - \vec{y}_\perp|$$

$$-4\pi\partial_\perp^2 L(0_\perp)_{\mathrm{MV}} = \lim_{r\to 0}\left[\ln\left(\frac{4}{m^2 r^2}\right)\right]$$

$$Q_s^2(r, \vec{x}_\perp)_{\mathrm{MV}} \approx \bar{Q}_s^2(\vec{x}_\perp)\ln\left(\frac{4}{m^2 r^2}\right)$$

$$Q_s^2(r_\perp, \vec{x}_\perp) \overset{r\to 0}{=} \bar{Q}_s^2(\vec{x}_\perp)\left(-4\pi\partial^2 L(0_\perp)\right)$$

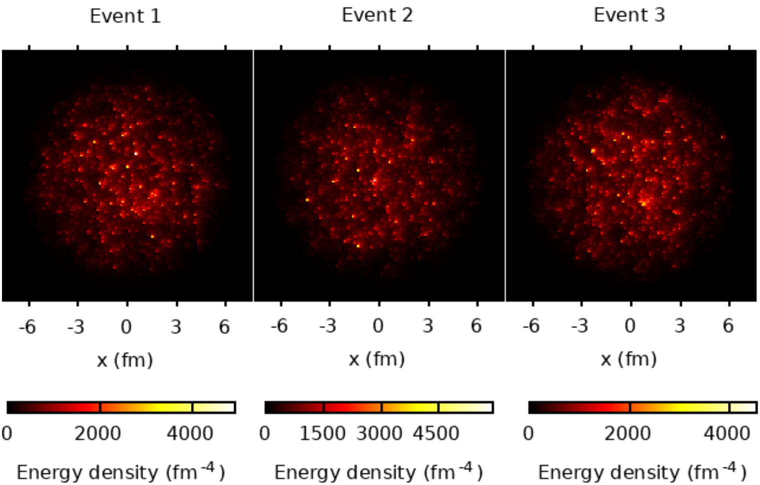
$$\frac{B^2}{A^2} = \frac{4}{\ln\left(\frac{4}{m^2 r^2}\right)^2}$$

Infrared Cutoff (m)

$$\frac{1}{Q_{s0}} \ll \frac{1}{m} \ll R\ ;\ r < \frac{1}{m}$$

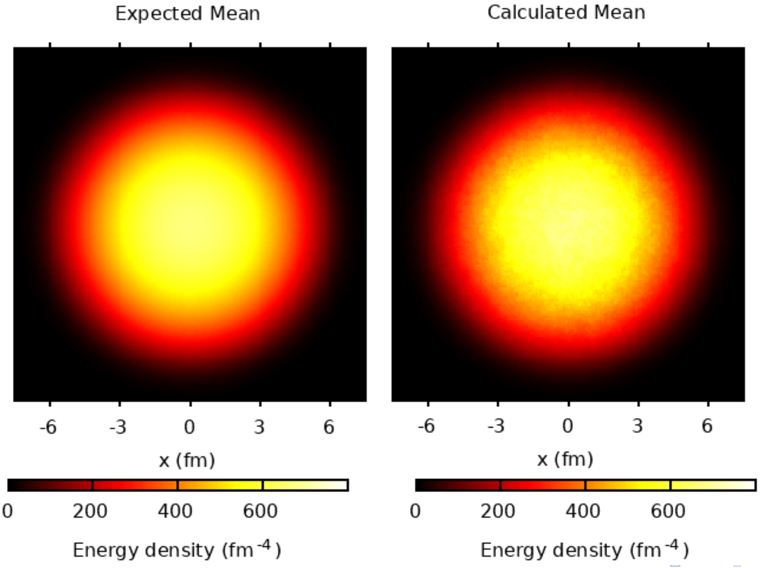
Generated events ($m = 0.14 \ GeV$)

We generate 1000 events at cellsize $= 0.1 \ fm$



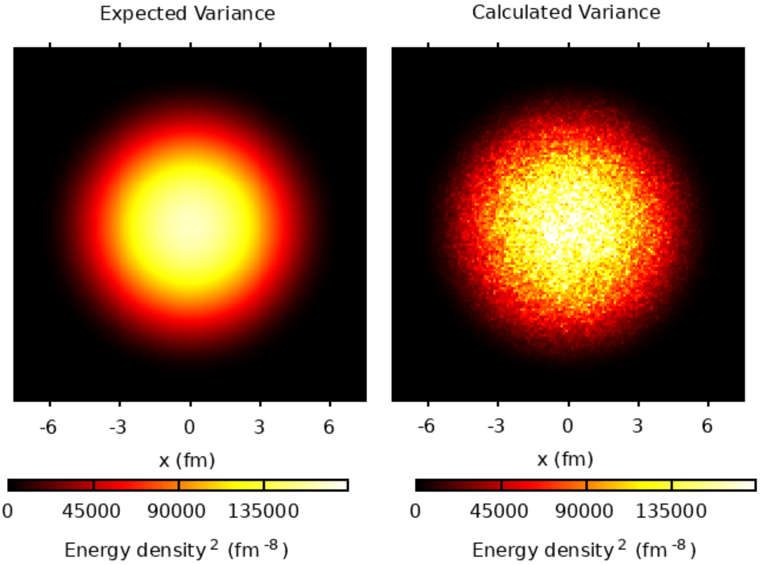
1-point function ($m = 0.14\ GeV$)

We compare the expected and calculated mean from the events



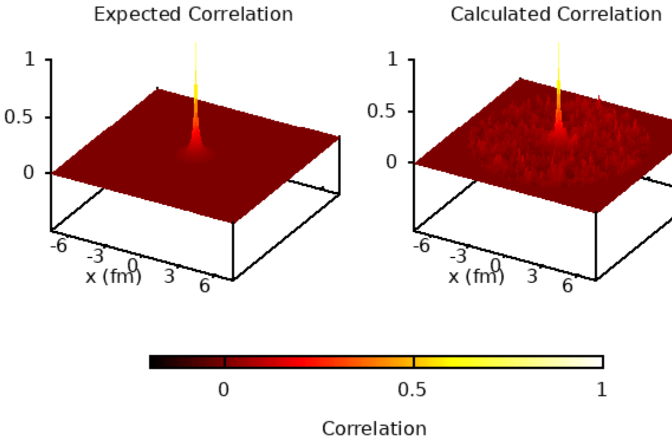
2-point function ($m = 0.14\ GeV$)

We compare the expected and calculated variance from the events



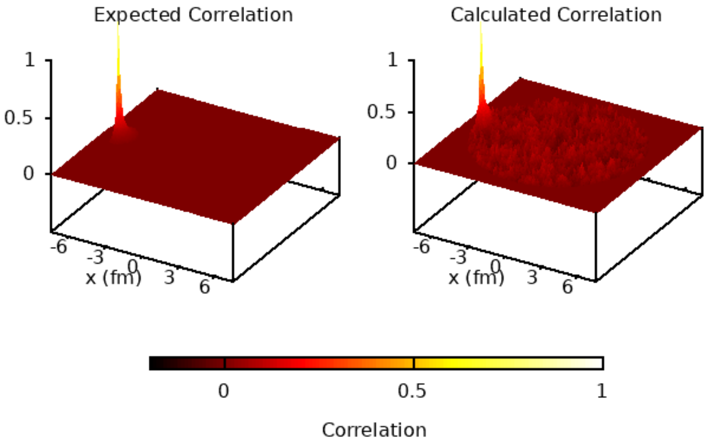
2-point function ($m = 0.14\ GeV$)

... and provide a visual comparison of the correlations between the center of coordinates and the rest of the surface...



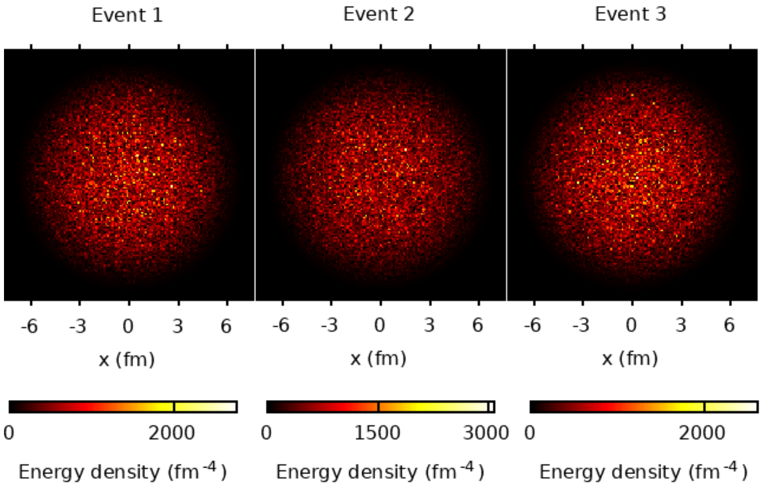
2-point function ($m = 0.14\ GeV$)

... along with a visual comparison of the correlations between a peripheral position and the surface.



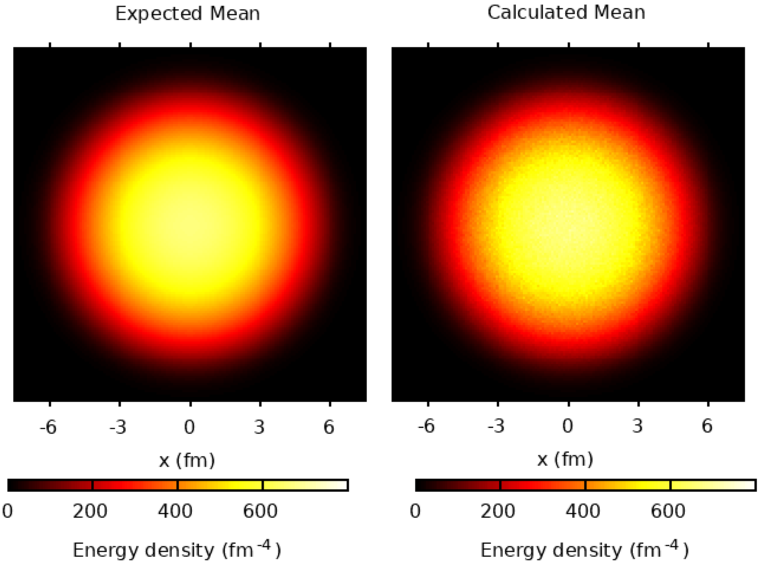
Generated Events (uncorrelated ($m \to \infty$))

1000 uncorrelated events



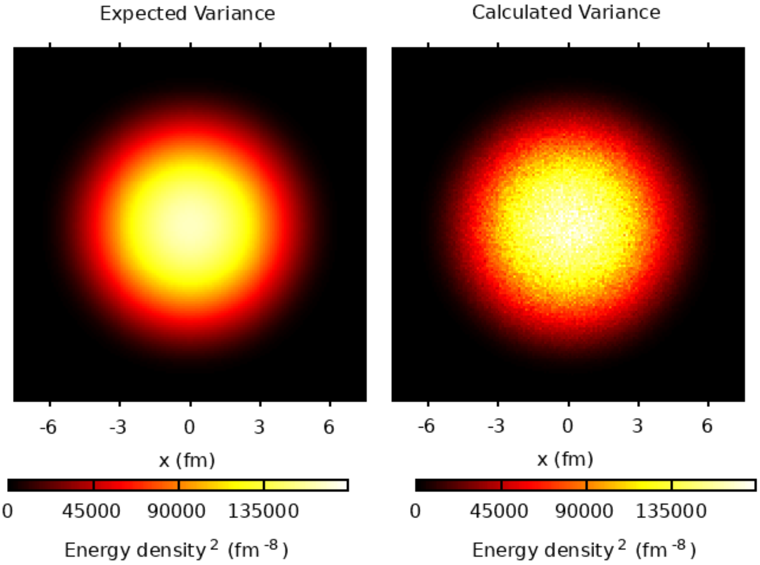
1-point function (uncorrelated ($m \to \infty$))

We compare the expected and calculated mean



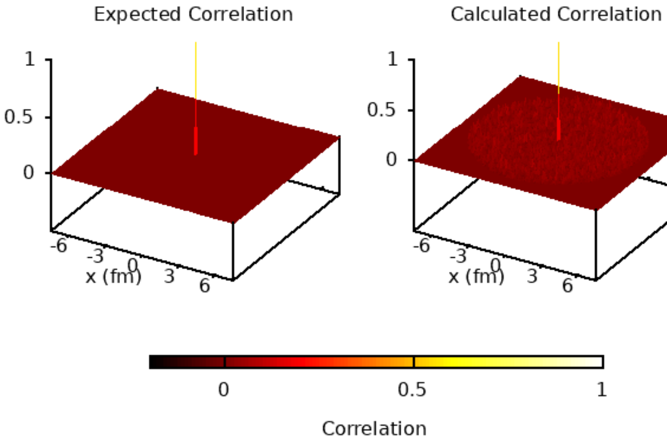
2-point function (uncorrelated ($m \to \infty$))

We compare the expected and calculated variance



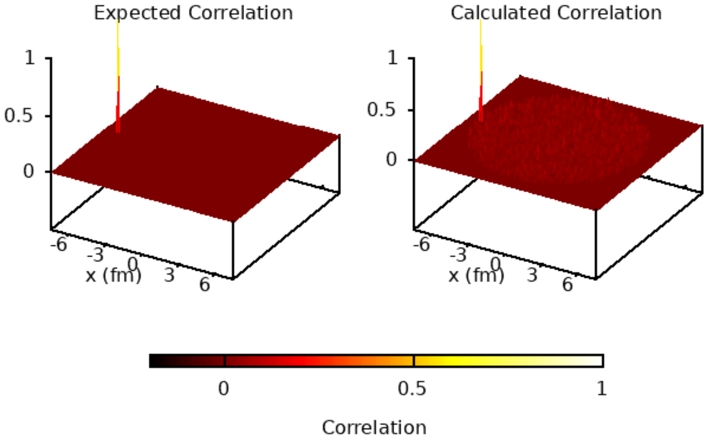
2-point function (uncorrelated ($m \to \infty$))

... and provide a visual comparison of the correlations between the center of coordinates and the rest of the surface...



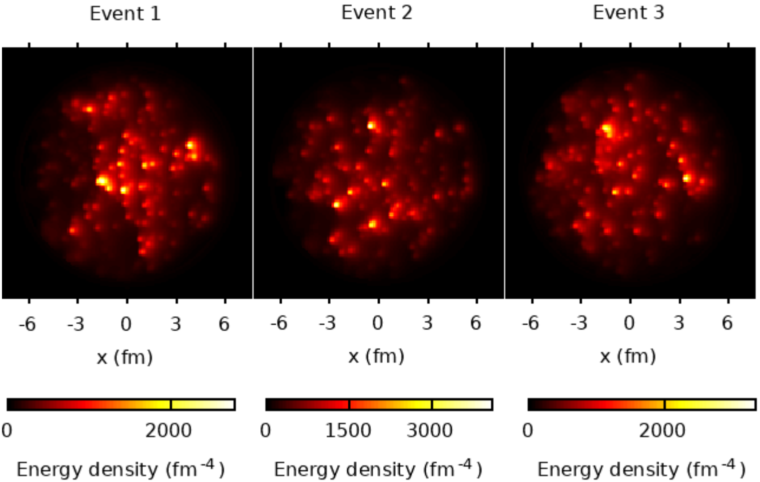
2-point function (uncorrelated ($m \to \infty$))

... along with a visual comparison of the correlations between a peripheral position and the surface.



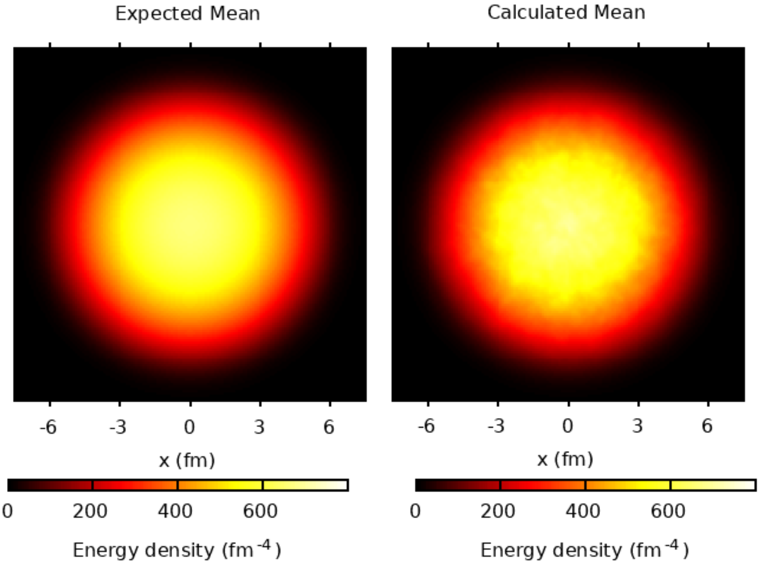
Generated Events (GBW Limit ($m \to 0$))

1000 events using the GBW Limit



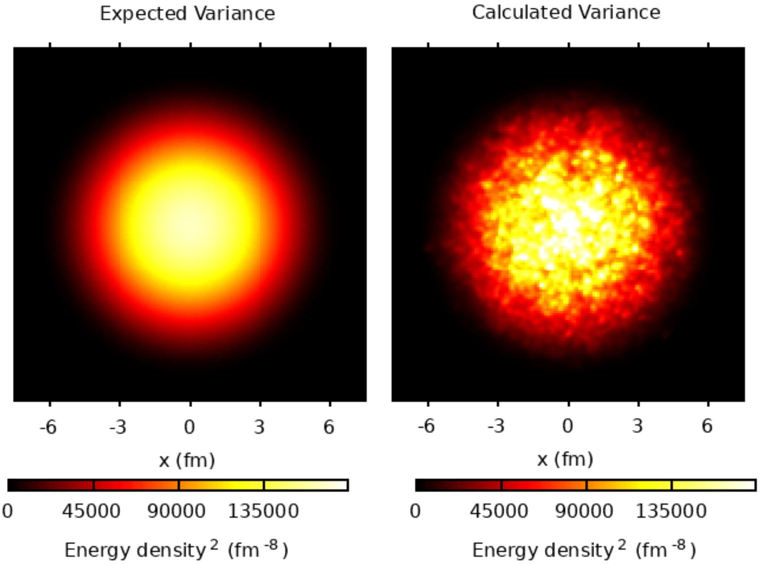
1-point function (GBW Limit ($m \to 0$))

We compare the expected and calculated mean



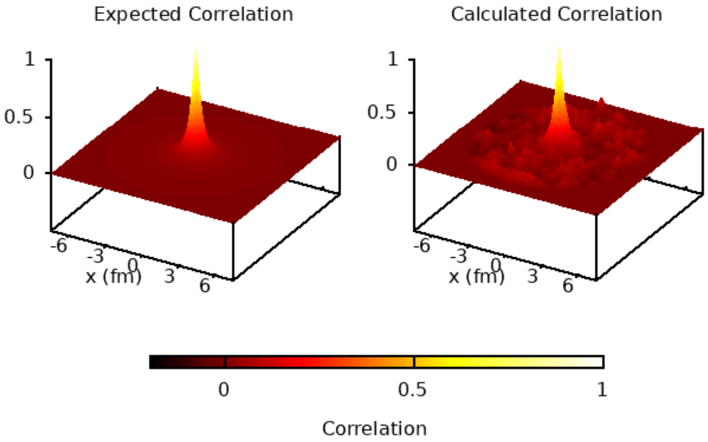
2-point function (GBW Limit ($m \to 0$))

We compare the expected and calculated variance



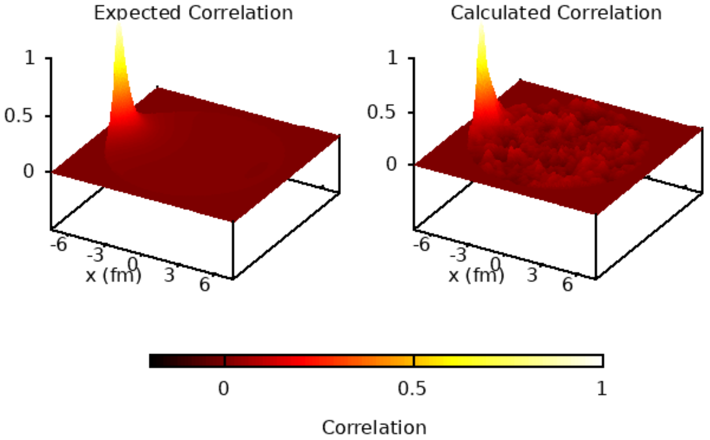
2-point function (GBW Limit ($m \to 0$))

... and provide a visual comparison of the correlations between the center of coordinates and the rest of the surface...



2-point function (GBW Limit ($m \to 0$))

... along with a visual comparison of the correlations between a peripheral position and the surface.



Correlation Length

The code reproduces the different correlation length increasing as the infrared cutoff m decreases

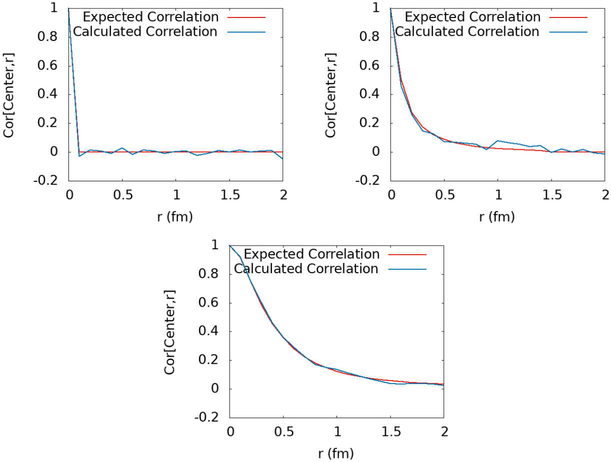


Figure: $m \to \infty$, $m = 0.14\ GeV$, $m \to 0$, respectively

Eccentricities

And the different values of m generate events with higher anisotropies.

m (GeV)	$\sqrt{\langle\epsilon_2^2\rangle}$	$\sqrt{\langle\epsilon_3^2\rangle}$	$\sqrt{\langle\epsilon_4^2\rangle}$	$\sqrt{\langle\epsilon_5^2\rangle}$
0	0.0688	0.0858	0.0947	0.1258
0.14	0.0290	0.0354	0.0448	0.0619
∞	0.0051	0.0065	0.0082	0.0112

Table: the eccentricities for the different infrared cutoff values

Conclusion

We have a fully functioning code that generates events obeying any 1 and 2-point functions.

The code will be publicly available later this year.

J. L. Albacete, P. Guerrero-Rodríguez and C. Marquet, JHEP **01**, 073 (2019) doi:10.1007/JHEP01(2019)073 [arXiv:1808.00795 [hep-ph]].

F. Gelis, G. Giacalone, P. Guerrero-Rodríguez, C. Marquet and J. Y. Ollitrault, [arXiv:1907.10948 [nucl-th]].

G. Giacalone, P. Guerrero-Rodríguez, M. Luzum, C. Marquet and J. Y. Ollitrault, Phys. Rev. C **100**, no.2, 024905 (2019) doi:10.1103/PhysRevC.100.024905 [arXiv:1902.07168 [nucl-th]].

L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233-2241 (1994) doi:10.1103/PhysRevD.49.2233 [arXiv:hep-ph/9309289 [hep-ph]].

Backup Slides with additional information

Error

For 5000 events at cellsize 0.25 *fm*, we calculate the error from our obtained mean, variance and correlation values.
For the mean, we have an average ratio error of 0.4(5)%
As for the variance, the average error is 5(4)%
When comparing the correlation surfaces, we have a 1.1(8)% fm^2 difference in average

Ultraviolet Cutoff

$$Q_s^2\left(r,\vec{x}_\perp\right)_{\mathrm{MV}} \approx \bar{Q}_s^2\left(\vec{x}_\perp\right)\ln\left(\frac{4}{m^2 r^2}\right)$$

$$Q_s^2\left(r,\vec{x}_\perp\right)_{\mathrm{MV}} \approx \bar{Q}_s^2\left(\vec{x}_\perp\right)\left(-4\pi\partial^2 L\left(0_\perp\right)\right)\frac{\ln\left(\frac{4}{m^2 r^2}\right)}{\left(-4\pi\partial^2 L\left(0_\perp\right)\right)} = Q_{s0}^2\ T_s(\vec{x}_\perp)\mathcal{L}_r$$

$$\mathcal{L}_r = \frac{\ln\left(\frac{4}{m^2 r^2}\right)}{\lim_{r'\to 0}\left[\ln\left(\frac{4}{m^2 r'^2}\right)\right]} \to \frac{\iint_{r_i-\frac{UV}{2}}^{r_i+\frac{UV}{2}} \ln\left(\frac{4}{m^2 r'^2}\right) dx' dy'}{\iint_{-\frac{UV}{2}}^{\frac{UV}{2}} \ln\left(\frac{4}{m^2 r''^2}\right) dx'' dy''}$$

For our events, we used $UV = 0.05\ fm$