The memory of initial momentum eccentricity in far-from-equilibrium QGP

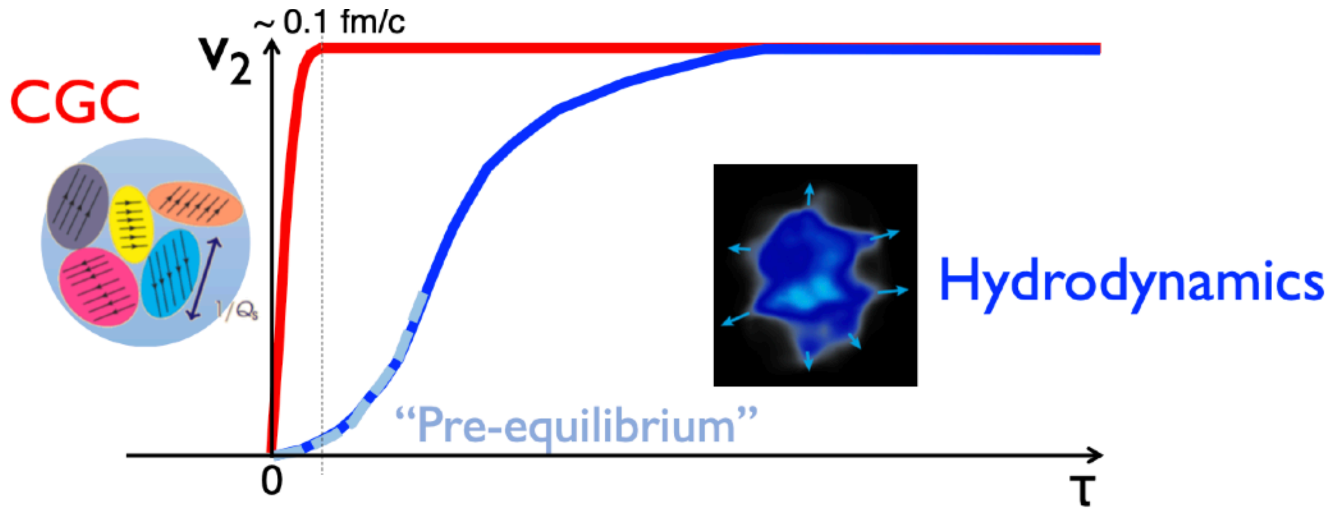


Fig. from Wei Li's slides

Two potential contribution to final stage momentum space eccentricity v_m :

Initial momentum space eccentricity generated by quantum fluctuations v_m^I

Initial position space eccentricity ϵ_n. (the conversion of ϵ_n into v_n due to hydrodynamic has been studied extensively.)

This work: the fate of v_m^I based on kinetic theory.

Yi Yin (IMP, CAS) on behalf of Jasmine Brewer (MIT->CERN), Li Yan (Fudan U.)Weiyao Ke (LBNL& UC Berkeley)

Initial stages, Jan.10-14, 2021

Set-up

Assuming QGP can be described by kinetic theory and consider the Fourier decomposition of particle transverse energy density distribution at mid-rapidity.

$$\frac{dE_\perp}{d\eta d\phi_p} = V_0(\tau) + 2\left[V_2(\tau)\cos(2\phi) + V_3(\tau)\cos(3\phi) + \ldots\right]$$

We use kinetic equation under isotropization time approximation (ITA) to describe the evolution of single particle distribution of gluons.

Kurkela-Wiedemann-Wu EPJC 19'

$$\partial_\tau F + \frac{1}{\tau}(4\cos^2\theta - \cos\theta\sin\theta\,\frac{\partial}{\partial\cos\theta})\,F = -\,\frac{u^\mu p_\mu}{p\tau_R}\left(F - F_{\text{eq}}\right),$$

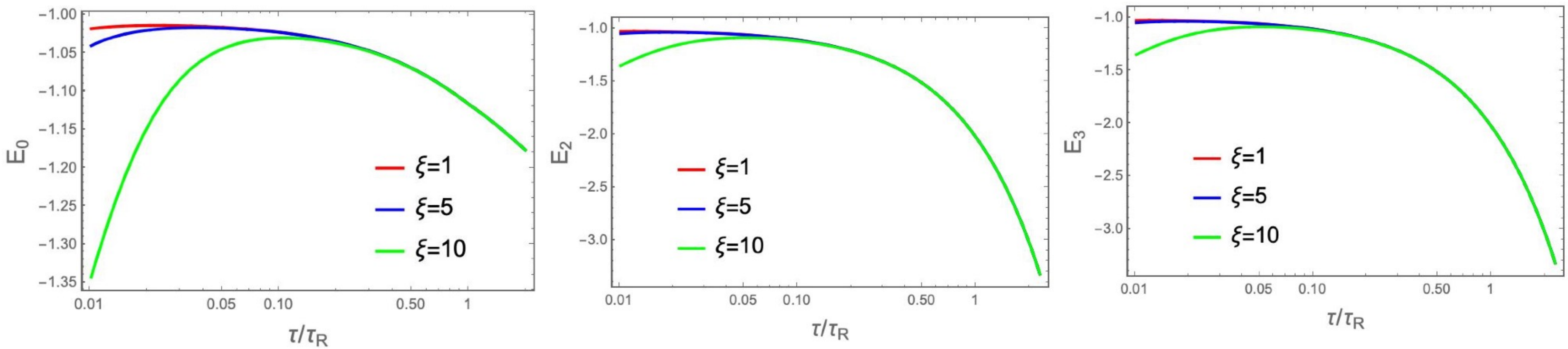
$$F(\phi,\cos\theta) \equiv \frac{1}{2\pi^2}\int dp\,p^3\,f(\phi,\cos\theta,p)$$

Initial condition: ξ parametrizes the initial an-isotropy.

$$f_I(p,\theta,\phi) = C_0\,\frac{Q_s}{p}\,\frac{e^{-\frac{2}{3}\frac{p^2}{Q_0^2}\left[1+(\xi^2-1)\cos^2\theta\right]}}{\sqrt{1+(\xi^2-1)\cos^2\theta}}\left[1 + 2\sum_m v_m^I\cos(m\phi)\right]$$

By solving ITA kinetic equation numerically, we can watch the evolution of V_m .

Attractor behavior for the evolution of V_m



Consider the change rate of (unnormalized) V_m:

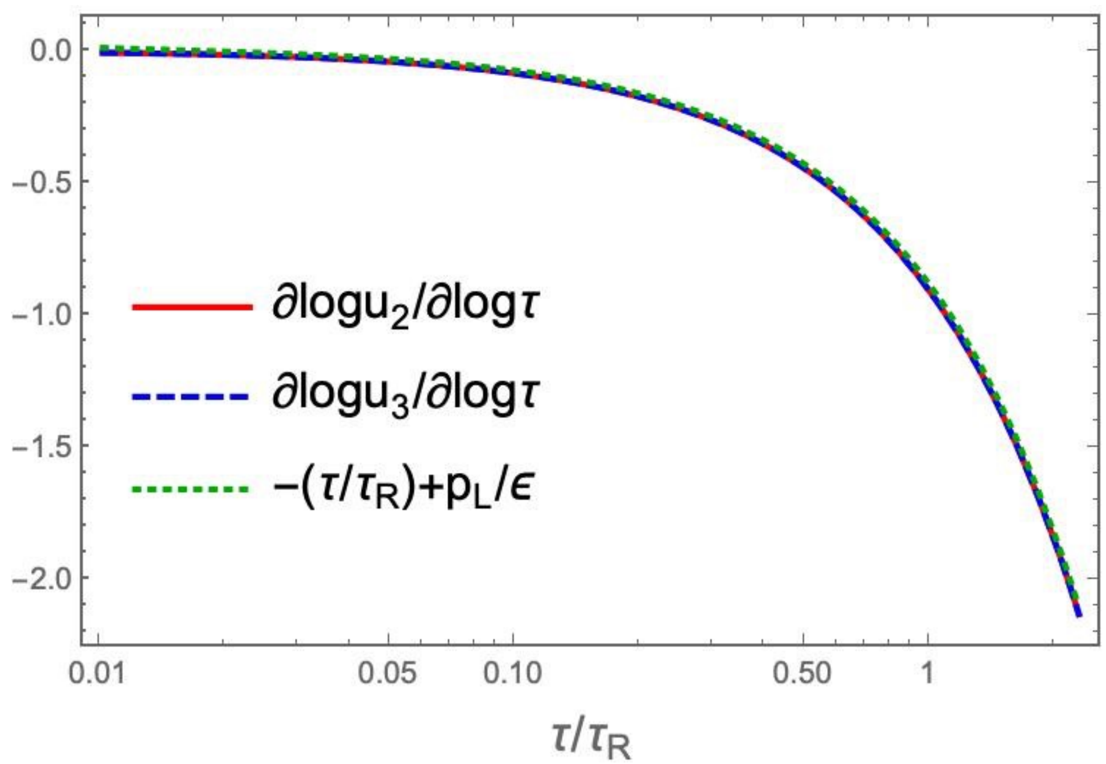
$$E_m(\tau) \equiv -\frac{\tau\partial_\tau V_m}{V_m}$$

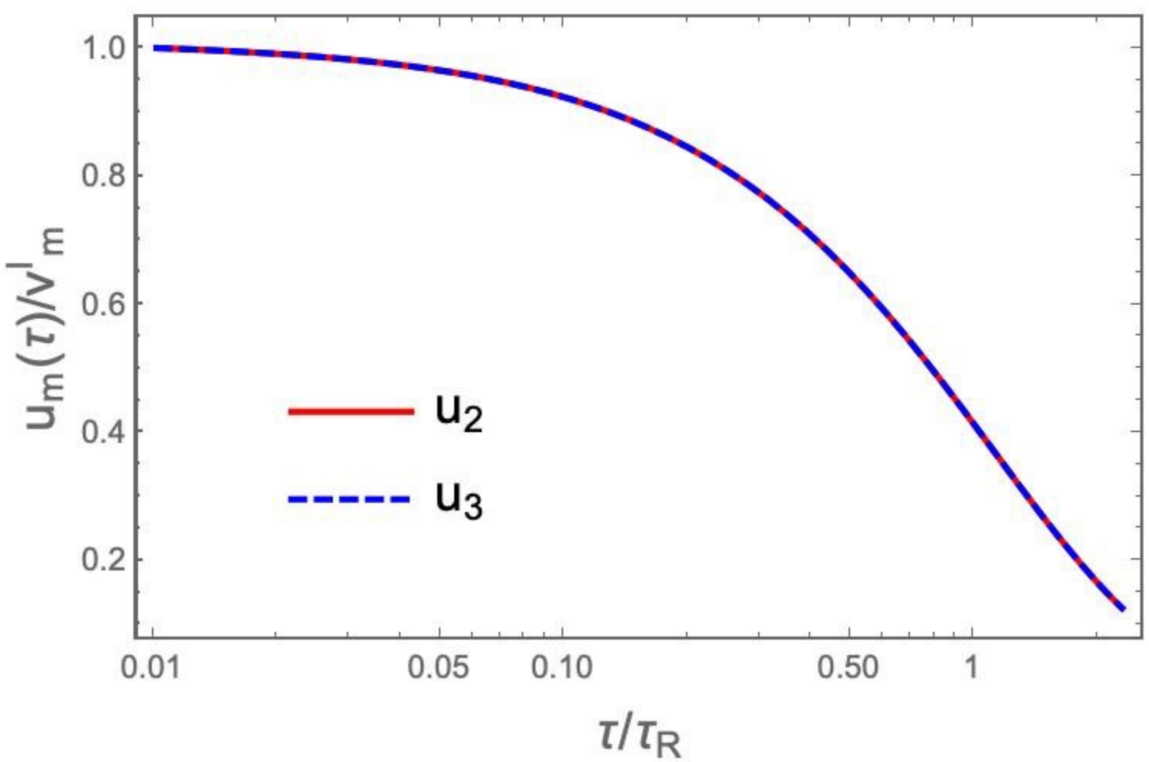
The change rate E_m is insensitive to initial an-isotropy ξ, generalizing the conventional notion of attractor behavior.

E_0 is related to the change rate of energy density, i.e. $E_0 = -(1 + p_L/\epsilon)$. The insensitivity of p_L/ϵ and hence that of E_0 to ξ is known as the "attractor behavior".

When $\tau \leq \tau_R$, $E_2 \approx E_3 \sim E_0$, meaning V_2, V_3 evolves as slow as energy density.

The evolution of the normalized harmonics





Consider ratio $u_m = V_m/V_0$, which is a proxy to momentum space eccentricity v_m.

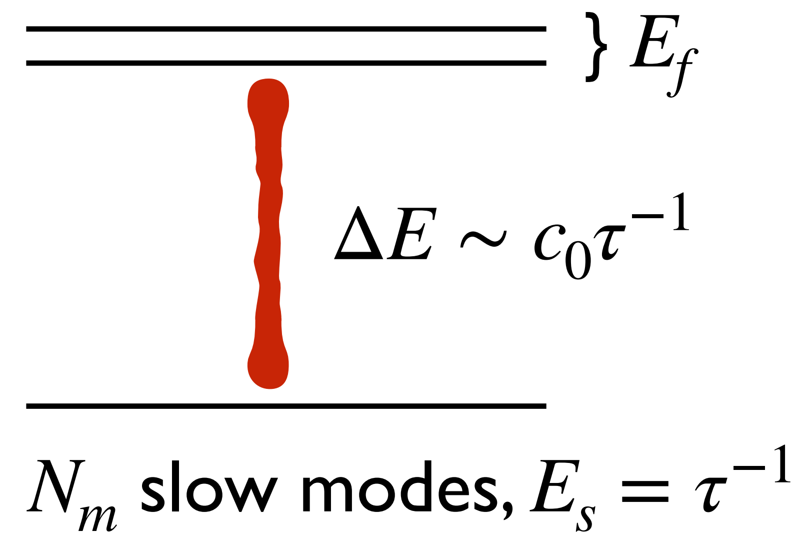
We find u_m changes slowly up to $\tau = \tau_R$

$$\frac{\partial \log u_{m\geq 2}}{\partial \log \tau} = (E_m - E_0) \approx -\frac{\tau}{\tau_R} + \frac{p_L}{\epsilon}$$

Usual 2nd order hydrodynamic eqns do not describe the evolution of u_m properly.

Primordial slow modes and the evolution of V_m

Based on "adiabatic analysis" (see Brewer's plenary talk this Friday for more details), we find for each V_m , there is an associated slow mode at early times.



Those "primordial slow modes" represent specific shapes in phase space.

Those "primordial slow modes" are important at early stages as far as the typical gradient is smaller than the gap, i.e. ,

$$k/\Delta E < 1$$

The modes associated with $m \geq 2$ are not slow modes in hydrodynamics. The descriptions of $m \geq 2$ primordial slow modes might be important for small systems.

Summary

We analyzed the evolution of momentum space eccentricity V_m in far-from-equilibrium QGP based on kinetic theory.

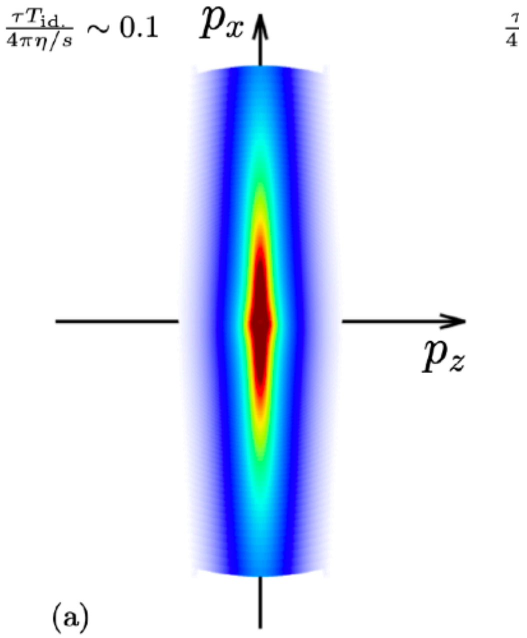
At early stages $\tau \leq \tau_R$, V_m changes approximately at the same rate as that of energy density.

The memory of initial momentum space eccentricity would last much longer than hydrodynamics analysis.

A dynamical model which describes the evolution of momentum space eccentricity might be important for the study of small colliding systems.

Back-up

Shapes of "equal-probability surface" as collective modes



(Figs from Kurkela et al, PRC 19')

Define "equal-probability surface": $p^n f(\overrightarrow{p}) = \text{const.}$

The primordial slow mode in m=0 sector can be visualized as to a highly anisotropic "equal-probability surface"

Primordial slow modes: some specific shapes of "equal-probability surface" which evolve relative slowly in fast longitudinal expansion environment.

C.f. "Chiral Metric Hydrodynamics" where the shapes of Fermi surface are treated as slow collective modes

D. Son, 19

Moments

characterizing phase space distribution

$$L_{n,\pm m,l}\left(\tau, \overrightarrow{x}\right) \equiv \int_{\overrightarrow{p}} p^{n-3}\ \left(Y_{l,m}(\theta,\phi) \pm Y_{l,-m}(\theta,\phi)\right) f(\overrightarrow{p};\tau,\overrightarrow{x}),$$

can be related to $T^{\mu\nu}$ (mass dimension $n = 4$),

$m = 0$: energy density T^{00} $(l = 0)$ and longitudinal momentum density T^{z0} $(l = 1)$

$m = \pm 1$: transverse momentum density T^{x0}, T^{y0} $(l = 1)$

m=2 : eccentricity of the distribution.

Define a larger vector $\psi = \left(L_{n,\pm m,l}, \ldots,\right)$ $\Longleftrightarrow$ $f(\overrightarrow{x},\tau)$

Studying the evolution of ψ is equivalent to studying the evolution of distribution function.

The evolution equation for ψ

$$\partial_\tau f(\vec{p};\tau,\vec{x}) = -\left[\hat{p}\cdot\frac{\partial}{\partial\vec{x}} + \frac{p_z}{\tau}\partial_{p_z}\right] f(\vec{p};\tau,\vec{x}) - \hat{C}[f]$$

decomposition

$$\partial_\tau \psi = -\left(H_D + \frac{1}{\tau}H_F\right)\psi - \text{ collisions}\,.$$

$$H_F = \begin{pmatrix} 4/3 & 2/3 & \ldots\ldots \\ 8/15 & 38/21 & \ldots\ldots \\ \ldots & \ldots & \ldots\ldots \end{pmatrix}$$

Considering the class of collision integrals that equation for ψ can be recast into the form:

$$\partial_\tau \psi = -\left(H_D + \frac{1}{\tau}H_F + H_C\right)\psi = -H(\tau)\,\psi\,.$$

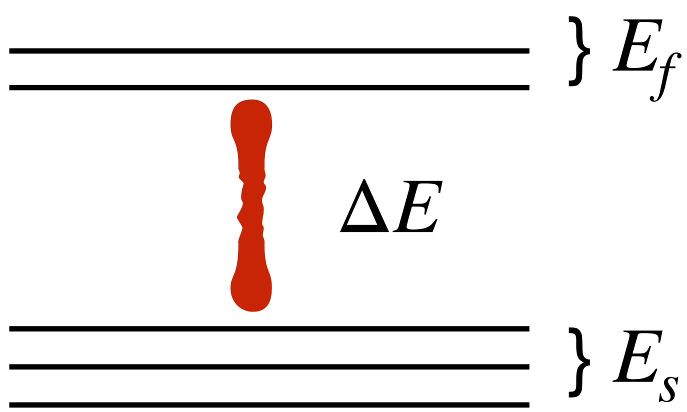
Slow modes

The instantaneous eigenvalue of non-Hermitian matrix $H(\tau)$ can be complex. ($ReE \geq 0$ because of expansion and collision),

$$H(\tau)\phi_n(\tau) = E_n(y)\phi_n(\tau)$$

Slow modes: assuming H has a collection of low-lying instantaneous eigenmodes which are gapped from other modes.

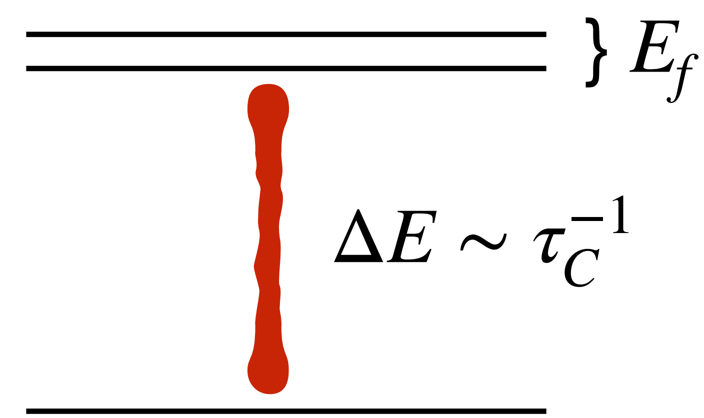
The dynamics should be dominated by those slow modes under certain conditions.



Late time limit and hydro. modes

H has four independent slow modes with $E_s = 0$ In long time and small gradient limit.

$$\left(H_F + \frac{1}{\tau} H_D + H_C \right) \to H_C$$



4 slow modes, $E_s = 0$

Each slow mode contains only one non-zero components associated with conserved densities. They are hydrodynamics modes.

$$\left(0, \ldots, T^{00}, 0, \ldots\right) \qquad \left(0, \ldots, T^{0i}, 0, \ldots\right)$$

Non-hydro. modes are gapped from hydro. modes by $1/\tau_C$.

The corrections due to finite expansion rate and gradient: expressible as an expansion in $k/\Delta E, \omega/\Delta E$ (match to the gradient expansion).

Primordial slow modes

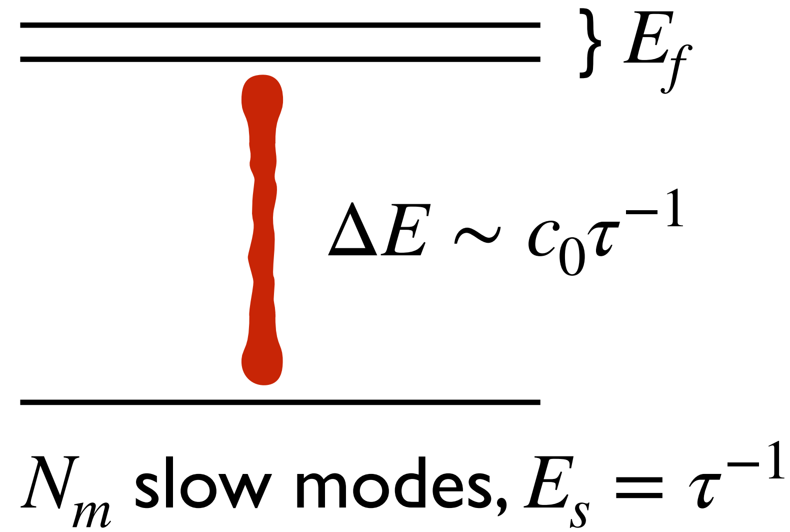
In early time limit $\tau \ll \tau_C$, free-streaming Hamiltonian dominates

$$\left(\frac{1}{\tau}H_F + H_D + H_C\right) \to \frac{1}{\tau}H_F$$

H_F: one slow mode per spin (m)

Even under longitudinal parity,

Gapped from other modes by $1/\tau$.

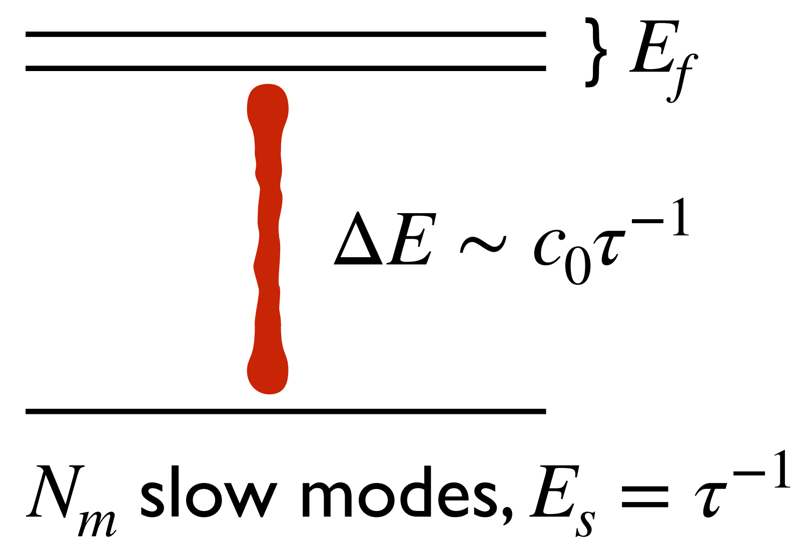


NB: H_F is even under longitudinal parity and commutes with ∂_ϕ

The dominance of primordial slow modes

The evolution will be dominated by the primordial slow modes ϕ_s^P when

$$\tau_I \ll \tau \ll \tau_C$$



The contribution from "faster mode" will decay as power-law in τ:

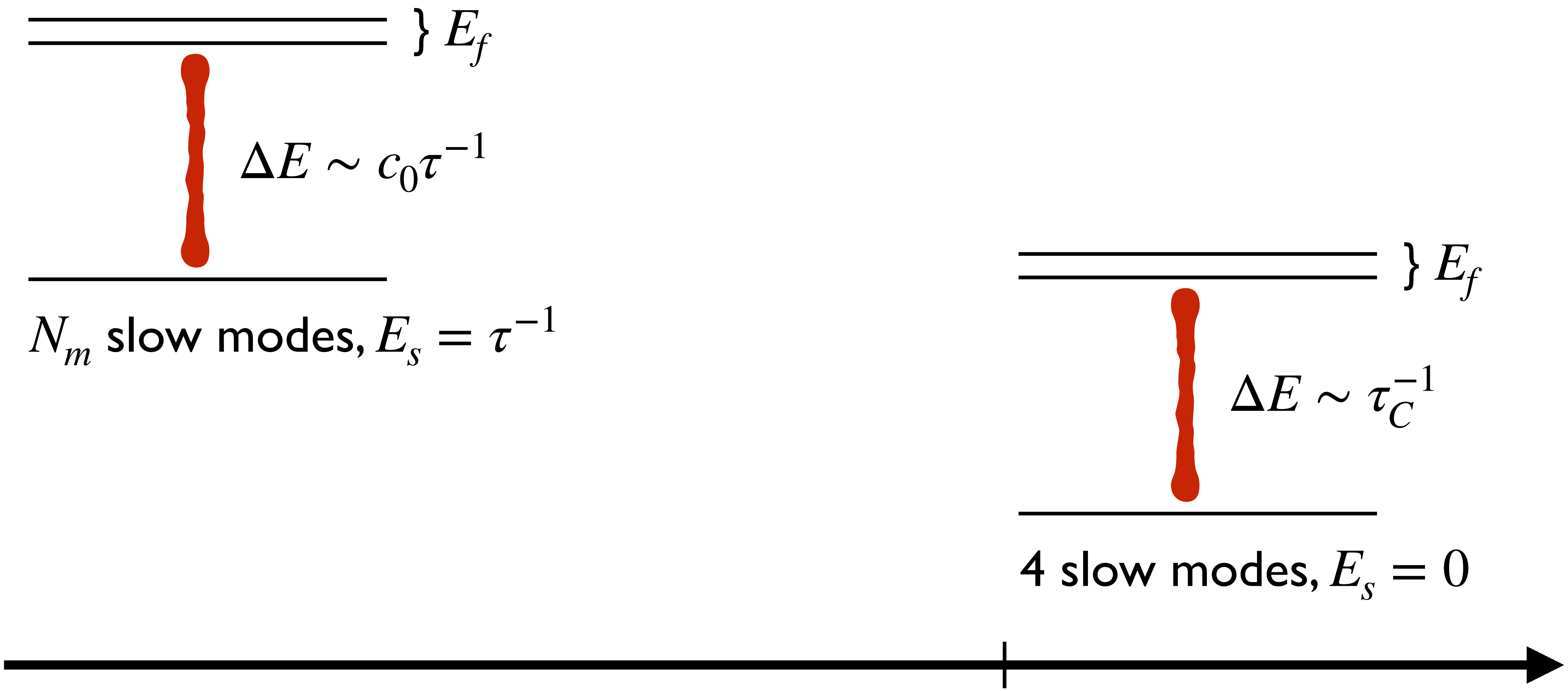
$$\psi(\tau) = \sum_s b_s(\tau)\phi_s + \sum_f b_f(\tau)\phi_j \qquad \frac{b_f(\tau)}{b_s(\tau)} \sim e^{-\int d\tau' \Delta(\tau')} = \left(\frac{\tau}{\tau_I}\right)^{-c_0},$$

power-law decay of fast modes ⇒ insensitivity to initial condition (attractor behavior).
see also Bin-Kurkela-Wiedermann-Wilke, PRL 19

For weakly coupled QGP in high energy density limit, τ_I and τ_C is separated parametrically

$$\tau_I \sim Q_s^{-1} \ll \tau_C \sim \alpha_s^{-x} Q_s^{-1}, \qquad x > 0$$

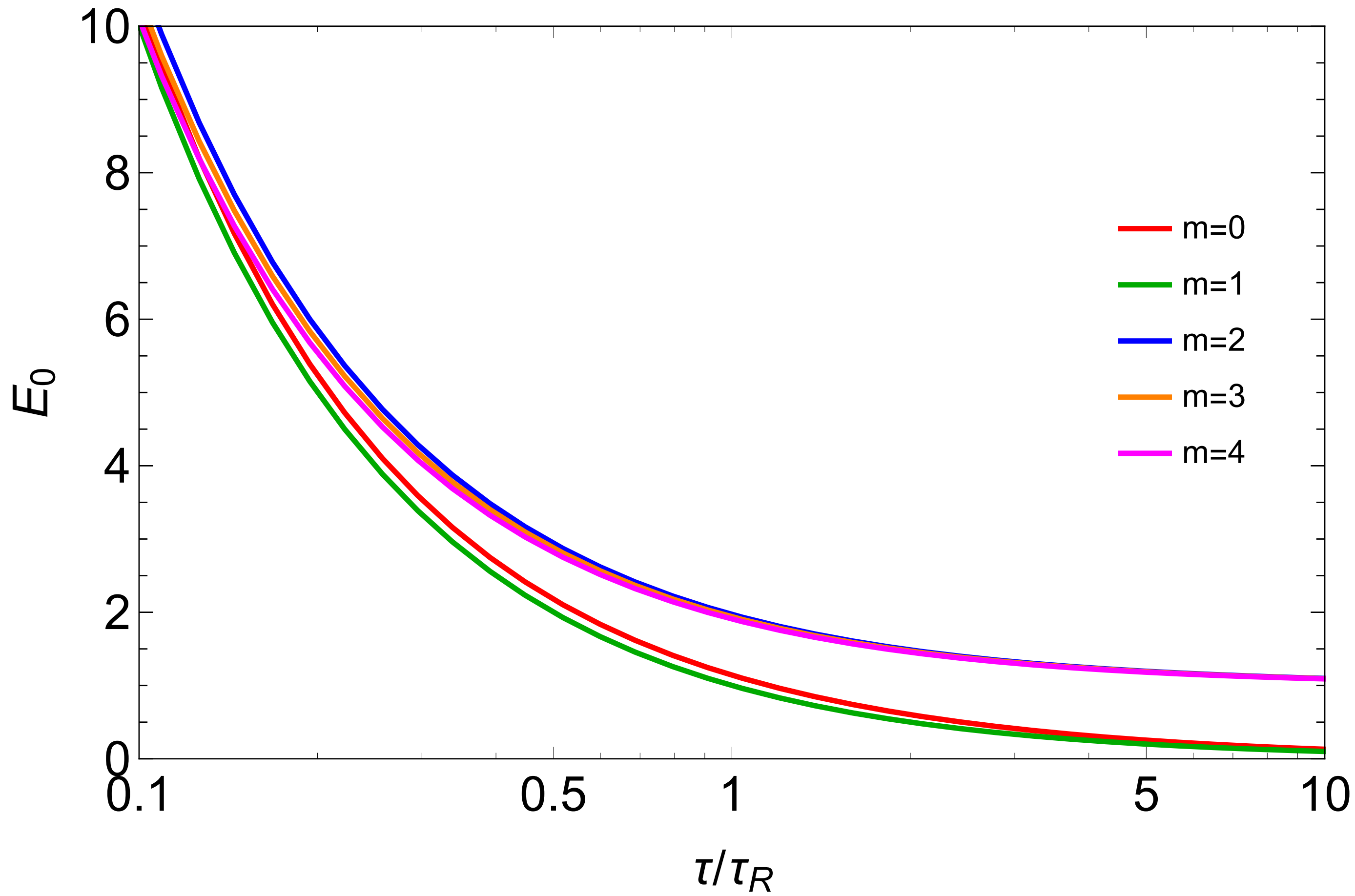
Question



Are $m = 0, \pm 1$ primordial slow modes related to hydro. modes?

The fate of higher spin $(m \geq 2)$ primordial modes ?

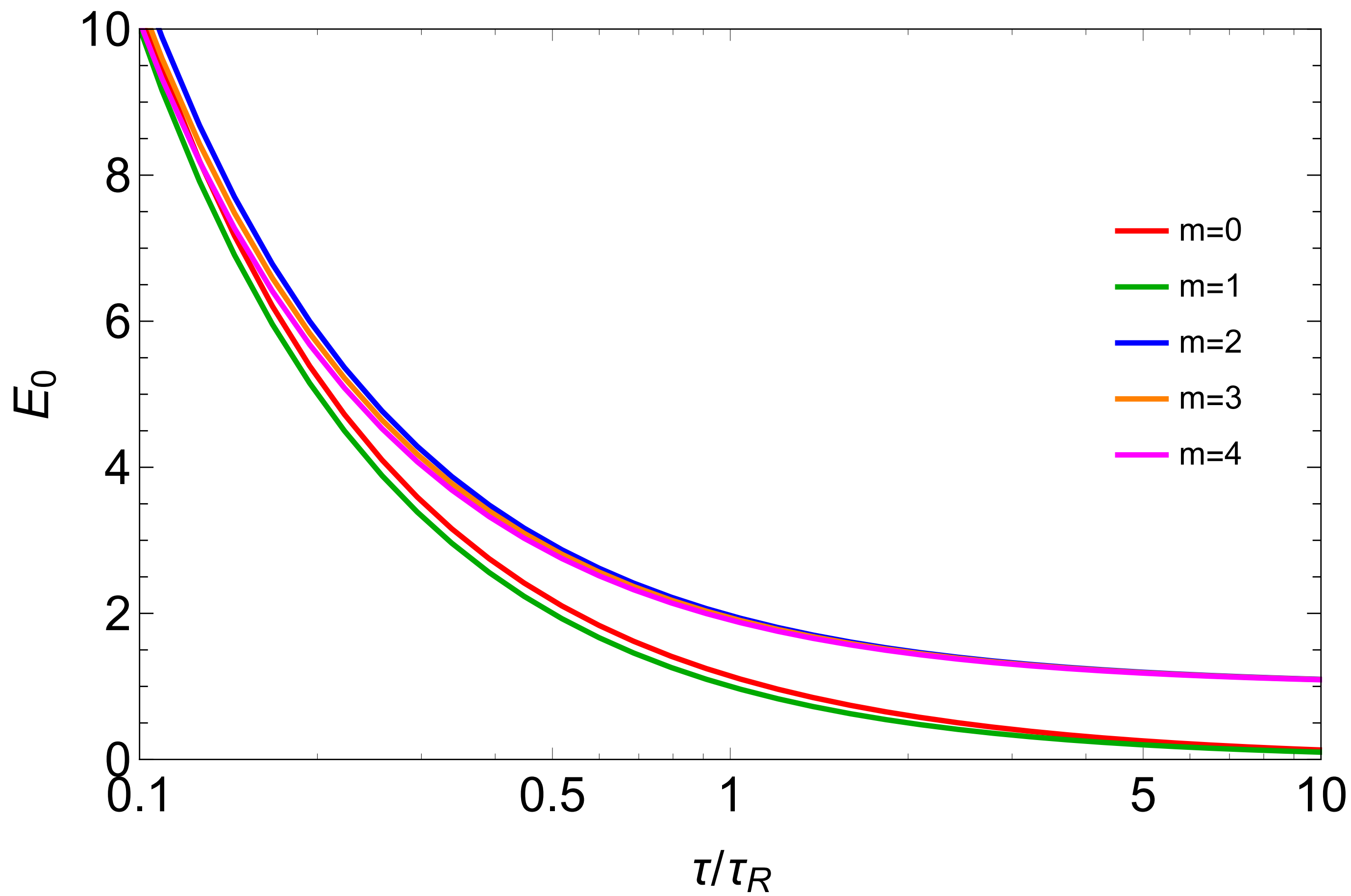
Fate of primordial slow modes



$m = 0, \pm 1$ primordial slow modes evolves to the corresponding hydro. modes

Higher spin $(m \geq 2)$ primordial modes undergo "mass-distinction" at some intermediate stage.

Finite gradient and $m \neq 0$



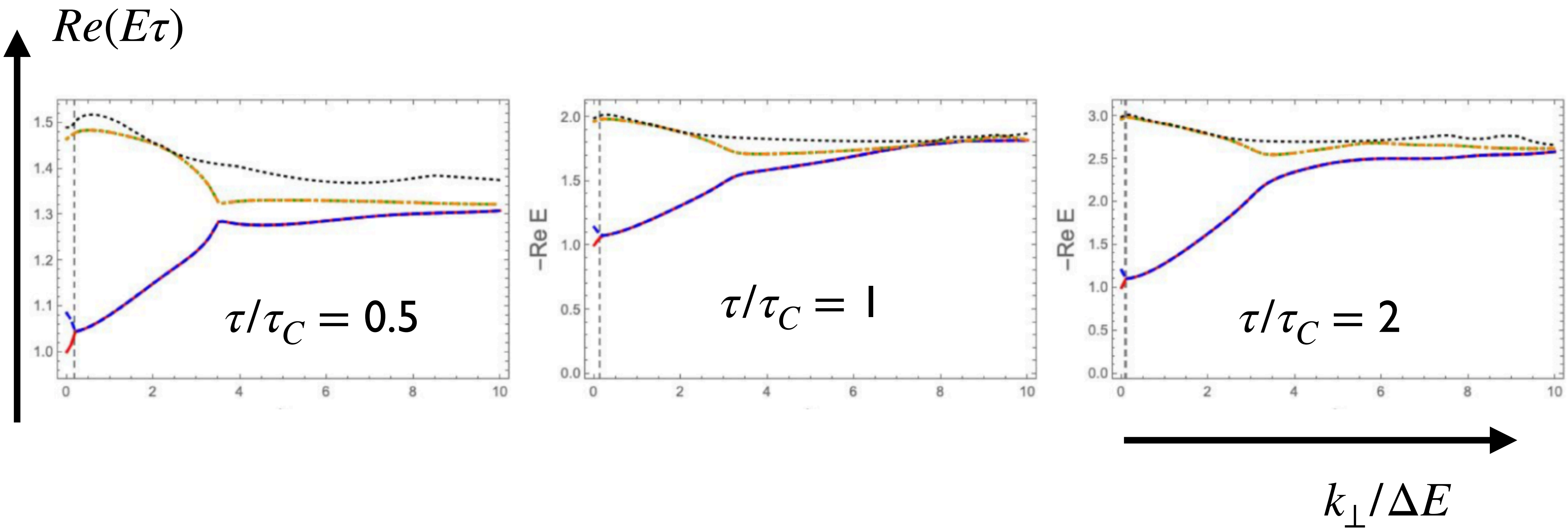
The discussion on adiabaticity is expected to be general.

To this point, one might expect that hydro. can be readily generalize to describe the evolution since far-from-equilibrium stages.

However, physics becomes much richer in the presence of gradient and $m > 0$ modes

Finite gradient ($k_\perp \neq 0$)

Brewer, Weiyao Ke, Li Yan and YY, in progress



Strong mixture among slow modes.

Slow modes dominate when

$$\frac{k}{\Delta E} < 1$$