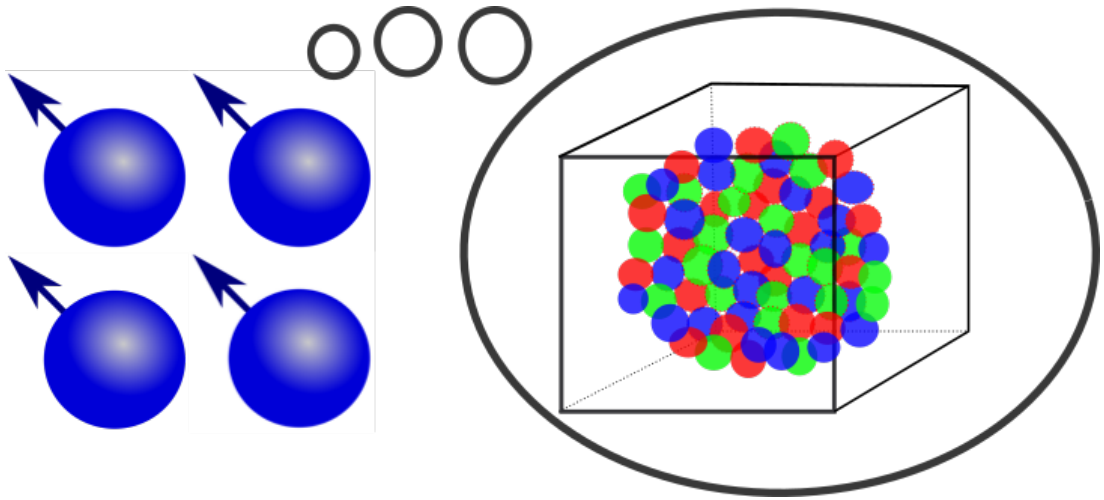
Quantum Simulation for Heavy-ion Physics



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in collaboration with Tom Cohen, Henry Lamm, and Scott Lawrence

based on papers in preparation (2101.xxxxx and 2102.xxxxx)

11 January 2021 at IS2021

UNIVERSITY OF MARYLAND

Nearly Perfect Liquid

In the middle of heavy-ion collision, the fireball is a **nearly perfect liquid**!

200 A GeV Au+Au collisions serve a nearly perfect quark-gluon liquid

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[2] *Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA*
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(Dated: October 29, 2018)

$$\frac{1}{4\pi} < \frac{\eta}{s} < \frac{2.5}{4\pi} \text{ when } \mathrm{T_c} < \mathrm{T} \lesssim 2\mathrm{T_c}$$

Can we confirm this result from **first principles, i.e. QCD**?

Viscosity from First Principles

In underlying microscopic theory, shear viscosity is defined with

correlators of the energy-momentum tensor T_{ij}

- Green-Kubo formula

$$\eta(T) = \lim_{\omega\to 0}\lim_{\vec{k}\to 0}\frac{\pi}{\omega}\int_0^\infty dt\int_V d\vec{x}e^{i(\omega t-\vec{k}\cdot\vec{x})}\langle\phi(T)|\left[T_{13}(t,\vec{x}),\,T_{13}(0,0)\right]|\phi(T)\rangle$$

- Alternatively

$$\int_V d\vec{x}e^{i\vec{k}\cdot\vec{x}}\langle\phi(T)|\left[T_{01}(t,\vec{x}),\,T_{01}(0,0)\right]|\phi(T)\rangle \sim e^{-\frac{\eta k^2}{\epsilon}t},\ (\epsilon:\text{energy density})$$

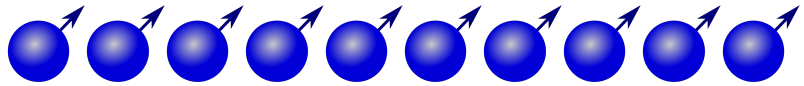
Near the phase transition, medium is **strongly-coupled**.
Non-perturbative calculation of Real-time correlators on a **lattice**?

- Lattice QCD in $3+1d$ on a **classical** computer?
  $\rightarrow$ Sign problem (could be removed[1])
- Lattice QCD in $3d$ on a **quantum** computer?
  $\rightarrow$ Quantum Computer is a quantum system evolved in real-time

Viscosity is *natural* in a quantum simulation.

[1] S. Lawrence and YY in preparation

A Quantum Computer - Qubits and Gates



Qubits are quantum spins

The Hilbert space: 2^N-dimensional for N qubits

$$|\psi\rangle = a|0000000\rangle + b|1000000\rangle + c|0100000\rangle + \cdots$$

Once you do measurement, ψ collapses to one of those basis state

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Gates apply to qubits and change the state on qubits

- 1 -qubit gates in matrix form...

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},\ T = \begin{pmatrix} e^{i\pi/8} & 0 \\ 0 & e^{-i\pi/8} \end{pmatrix}$$

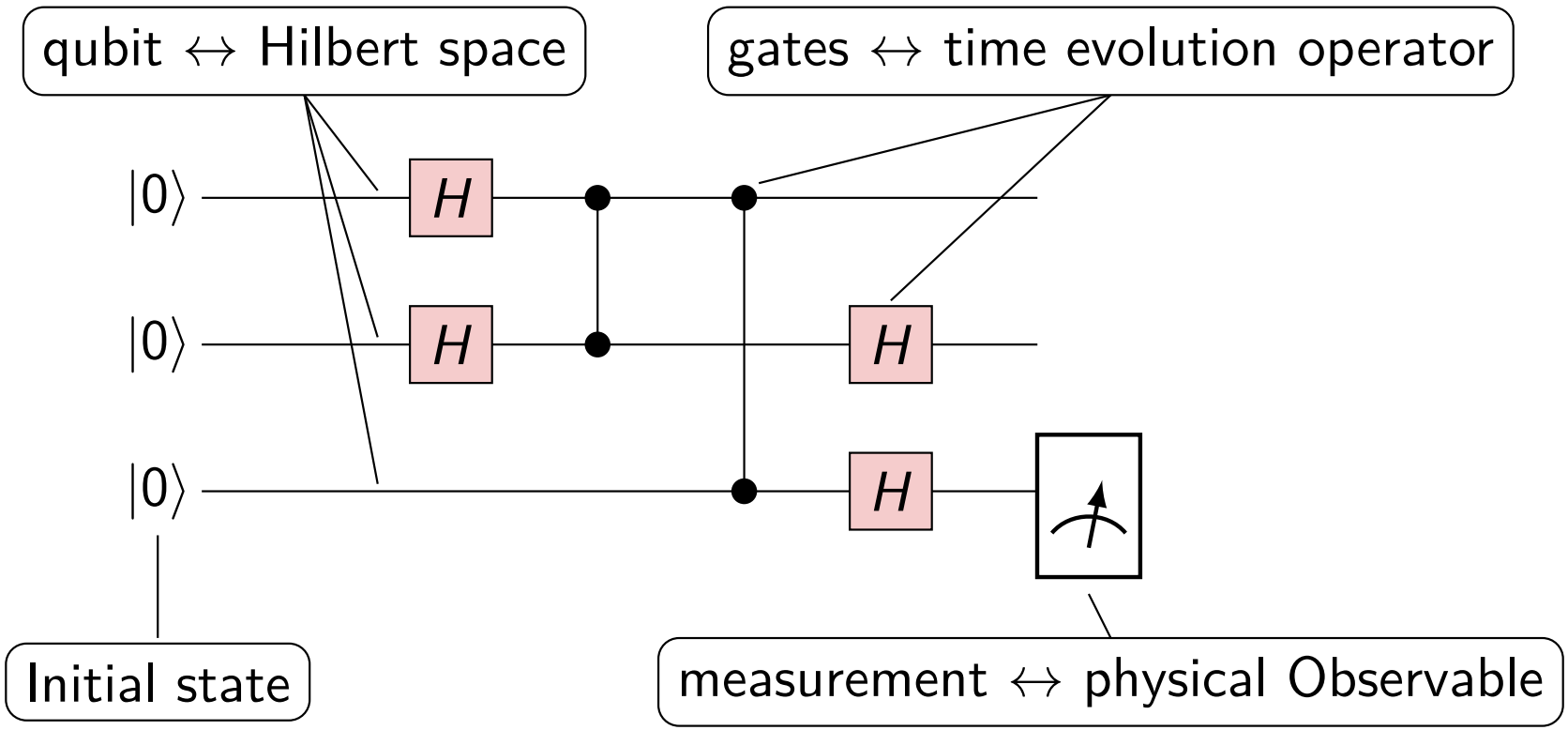
- 2 -qubit gates in matrix form ... example Controlled-not (CNOT)

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|00\rangle \mapsto |00\rangle\,,\quad |01\rangle \mapsto |01\rangle$$
$$|10\rangle \mapsto |11\rangle\,,\quad |11\rangle \mapsto |10\rangle$$

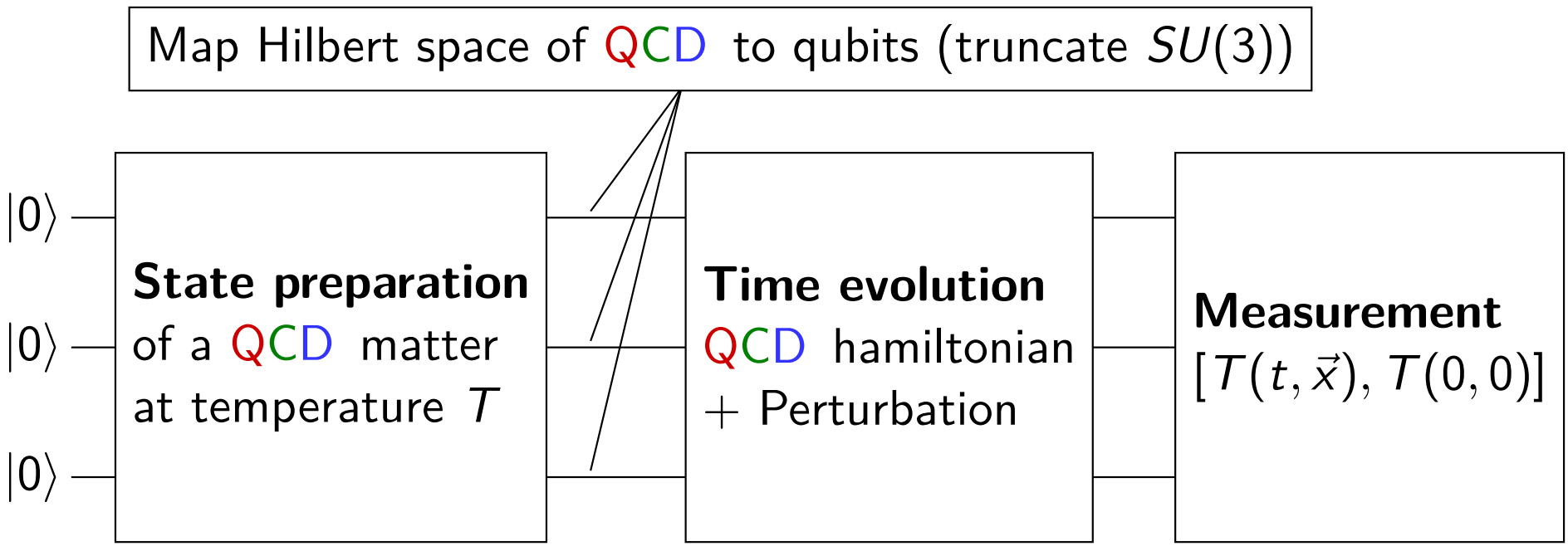
Building Blocks of Quantum Simulation of QFT on a Lattice

Quantum Computer is a quantum system evolved in real-time



Ingredients for Viscosity of QCD Matter[2]

$$\langle \phi(T)| \left[T(t,\vec{x}), T(0,0)\right] |\phi(T)\rangle$$



[2] H. Lamm, S. Lawrence and YY in preparation

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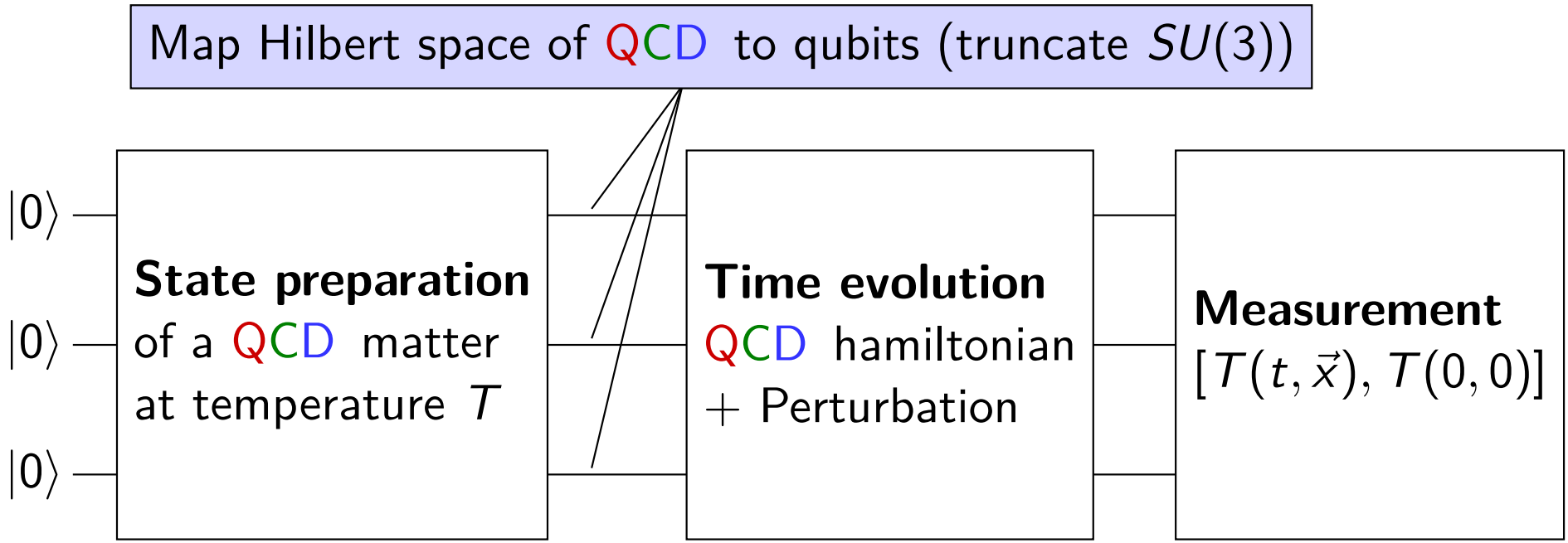

$SU(3)$: **Infinite** dimensional Hilbert space. $\rightarrow$ Need to truncate!
A proposal: Use the largest crystal-like subgroup of $SU(3)$, $S(1080)$ $\rightarrow$ **11 qubits/link**.[3]

[3] A. Alexandru et al., Phys.Rev.D 100(2019)11, 114501

Ingredients for Viscosity of QCD Matter

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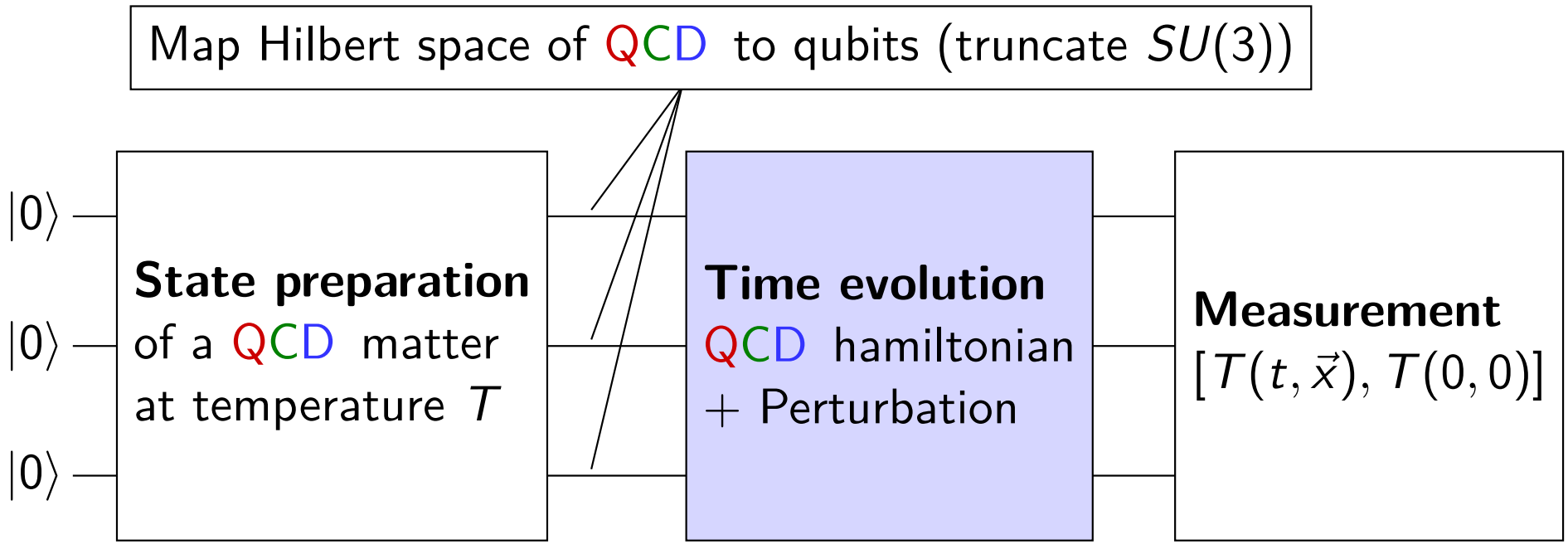


Implement e^{-iHt} with local gates[4], where $H_{S(1080)} = \sum_L K_L + \sum_P V_P$

Trotterization: $e^{-iHt} = \left[e^{-iH\epsilon}\right]^{t/\epsilon} \approx \left[e^{-i\epsilon K_1} e^{-i\epsilon K_2} \cdots e^{-i\epsilon V_1} e^{-i\epsilon V_2} \cdots\right]^{t/\epsilon}$

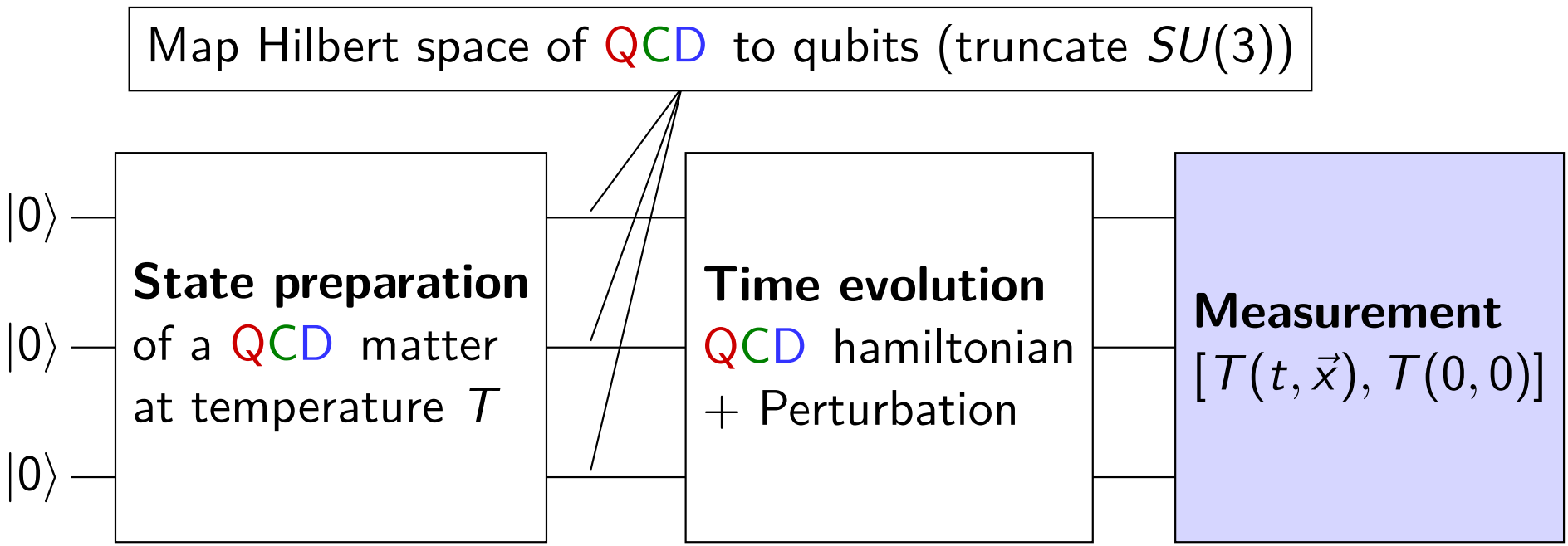
Circuits for $e^{-i\epsilon K_L}, e^{-i\epsilon V_P}$ via primitive gates.[5]

[4] H. Lamm, S. Lawrence and YY, Phys.Rev.D 100(2019)3, 034518

[5] H. Kumar, S. Lawrence in preparation

Ingredients for viscosity of QCD matter

$$\langle \phi(T)| \left[T(t,\vec{x}), T(0,0) \right] |\phi(T)\rangle$$



Measurement of Correlators (Linear Response)

Measure $\langle [T(t,x), T(0,0)] \rangle = \langle \psi | \left[e^{iHt} T_{01}(x) e^{-iHt}, T_{01}(0) \right] | \psi \rangle$

1. **Add small perturbation** $T_{01}(0)\delta(t)$ to the QCD Hamiltonian

$$H' = H_{QCD} + \epsilon T_{01}(0)\delta(t)$$

2. **Time-evolve** the initial state $\psi(0) = \psi$ with H' till time t

$$|\psi(t)\rangle = e^{-iHt} e^{-i\epsilon T_{01}(0)} |\psi\rangle$$

3. **Measure** $T_{01}(x)$ on the state $|\psi(t)\rangle$

$$\langle T_{01}(x) \rangle = \langle \psi(0) | e^{i\epsilon T_{01}(0)} e^{iHt} T_{01}(x) e^{-iHt} e^{-i\epsilon T_{01}(0)} | \psi(0) \rangle$$

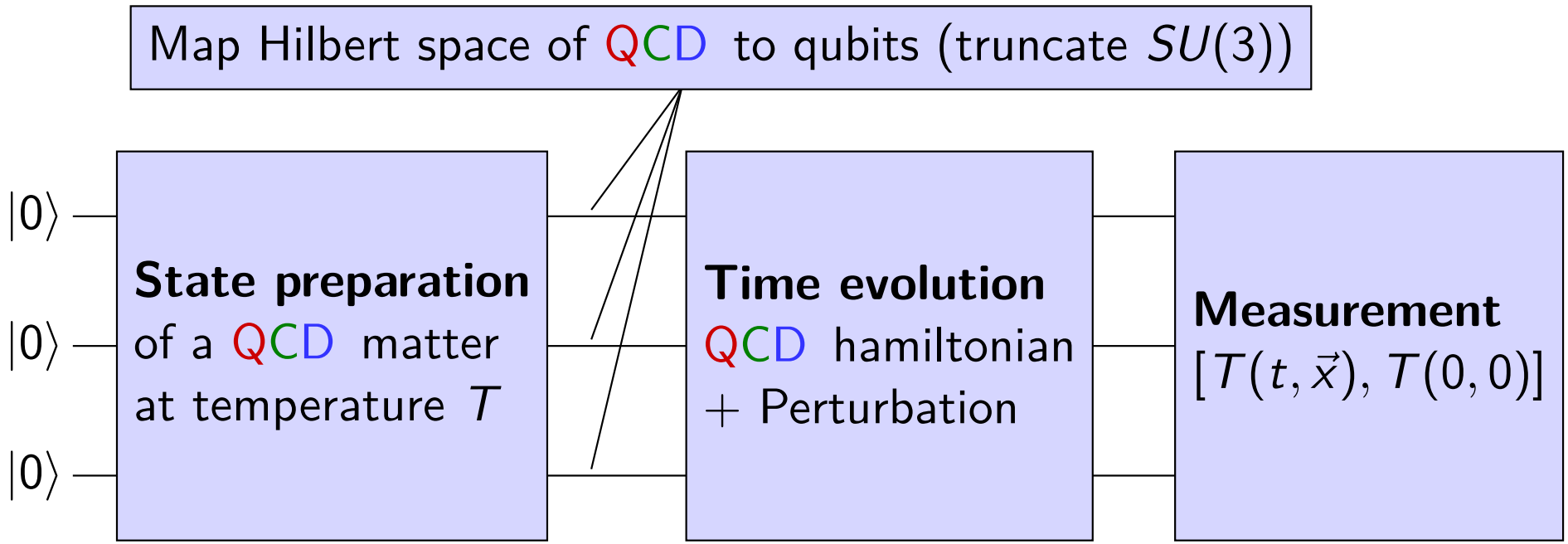
4. **Take derivative** of $\langle T_{01}(x) \rangle$ with respect to ϵ and then $\epsilon \to 0$

$$\lim_{\epsilon \to 0} \frac{\partial}{\partial \epsilon} \langle T_{01}(x) \rangle = -i \langle \psi | \left[T_{01}(t,x), T_{01}(0,0) \right] | \psi \rangle$$

Just as in experiment!

How Much Does the Simulation Cost?

$$\langle \phi(T) | \, [T(t,\vec{x}), T(0,0)] \, | \phi(T) \rangle$$

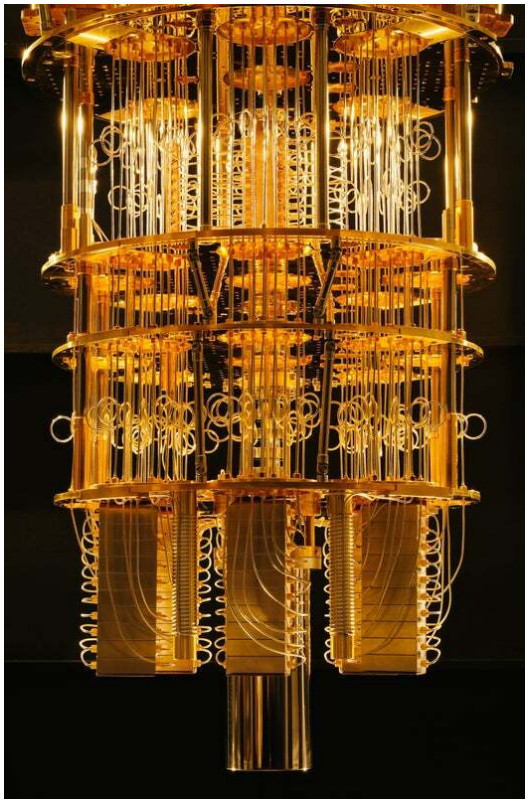


- Number of qubits $\sim N_{link} \times 11$
- Number of gates $\propto N_{link} \times$ number of time steps / correlator

So, suppose you want 20^3 lattice with lattice spacing $a = 0.1$fm

You need $\sim 3 \times 10^5$ qubits

Reality



Quantum computer available now is:

- 50 qubits
- circuit depth 20
- noisy..

We'll have to wait for **XX years** to calculate viscosity on a large lattice.

Well, do we need a large lattice?

What if we measure viscosity on a **small lattce**?
Let's study **finite-volume effects** in

1. $\mathcal{N} = 4$ Super Yang-Mills via AdS/CFT correspondence
2. Molecular dynamics simulation

Viscosity of $\mathcal{N} = 4$ Super Yang-Mills

Strong coupling and large N limit of $\mathcal{N} = 4$ **SYM** on the boundary

$\leftrightarrow$ **classical gravity** in AdS spacetime in the bulk

$G^{12,12}$ on the boundary $\leftrightarrow$ solution to **Einstein's Eq** in the bulk

Kubo relation gives the viscosity for $\mathcal{N} = 4$ SYM with:

$$-i\eta = \lim_{\omega \to 0} \frac{\partial G^{12,12}(\omega, k = 0)}{\partial \omega}$$

Leading order in ω, k: $G^{12,12}(\omega, k) = -\frac{i\omega}{4\pi} s \rightarrow \frac{\eta}{s} = \frac{1}{4\pi}$ [6]
Higher order in ω, k may give finite volume effect as k is discretized?

$$G^{12,12} = s\left(-\frac{i}{4\pi}\omega + \frac{1 - \ln(2)}{8\pi^2 T}\omega^2 - \frac{1}{8\pi^2 T}k^2 + \frac{i\ln(2)}{8\pi^3 T^2}\omega k^2 - \frac{C}{8\pi^3 T^2}\omega^3\right)$$

($C \sim 0.9$, numerically checked)
Finite volume effect: k can be taken to zero anyway, so **No correction**
This is a special feature of CFT.

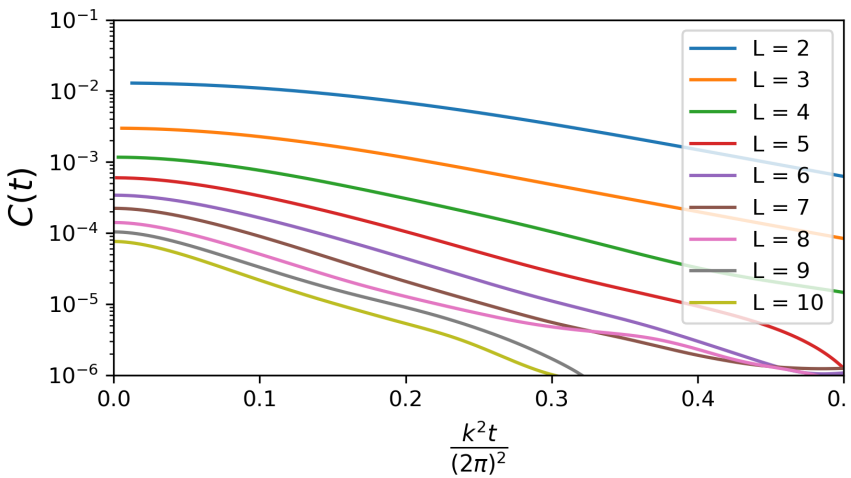
[6] G. Policastro et al., Phys.Rev.Lett 87(2001)081601

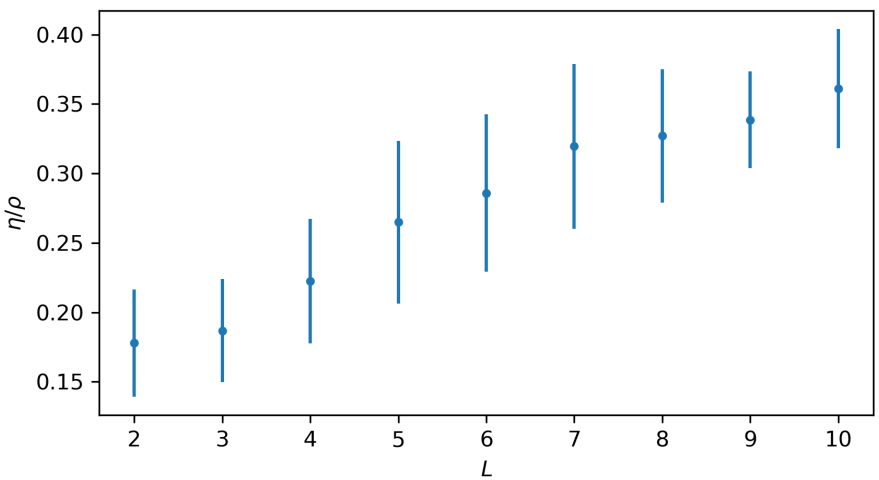
Viscosity from Molecular Dynamics Simulation

Transverse-current autocorrelation method[7]

1. Prepare a box (volume L^3) of particles in thermal equilibrium $|F(r)| = e^{-2r}$, repulsive force
2. Measure transverse momentum field $\vec{u}(\vec{k}, 0)$
3. Time-evolve the system via molecular dynamics simulation
4. Measure $\vec{u}(\vec{k}, t)$ and compute $\vec{u}(\vec{k}, t)\vec{u}(\vec{k}, 0)$

$$C(k,t) = \langle \vec{u}(\vec{k}, t)\vec{u}(\vec{k}, 0) \rangle \sim e^{-\frac{\eta k^2}{\rho} t} \quad (\rho : \text{mass density})$$





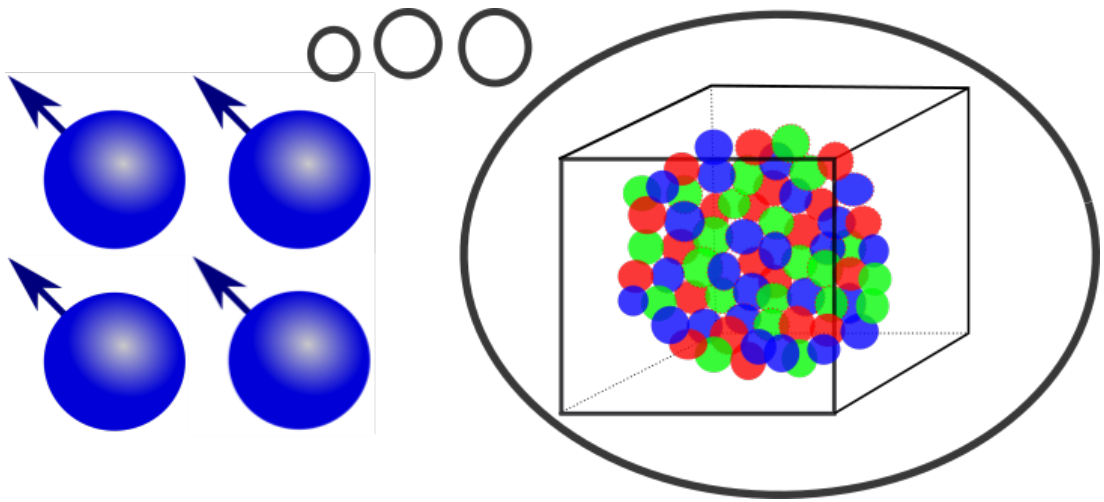
[7] B.Palmer, Phys.Rev.E 49(1994)359

Future Work?

While we wait for a "large enough" quantum computer to be build...

- State preparation methods' details
- Estimate of finite volume effects for more "QCD-like" system

Let's think what else we can do on a quantum computer!



Thank you!