Simulating real-time dynamics of hard probes in nuclear matter on a quantum computer

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LBNL (Quantum Information)
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Experiments measure how cross-sections of hard probes are modified in heavy-ion collisions compared to proton-proton collisions.

\[ R_{AA} = \frac{1}{\left\langle N_{\text{coll}} \right\rangle} \frac{dN^{\text{PbPb}}}{dN^{\text{pp}}} \]

**Jets**

Jet yields are suppressed due to “energy loss” to the dense medium.

**Heavy quarks**

Heavy quark bound pairs (quarkonium) are “melted” by the hot medium.
Hard probes — theory

- In vacuum: calculate scattering of asymptotic states using perturbative QCD
  Note that there is no sense of “time evolution”

- In medium: must combine probe evolution with hydrodynamic evolution of the QGP
In heavy-ion collisions, the modifications of the probe due to its evolution through the QGP are typically put in “by hand”, rather than a true real-time evolution.

**Medium-modified parton shower**

- Majumder PRC 88 (2013)
- ...
Real-time dynamics of QCD

Typical methods in lattice QCD have a sign problem and use imaginary instead of real time

$$\int e^{iLt} \quad t \to it$$

Can instead use the Hamiltonian formulation of QCD

- Large Hilbert space can in principle be simulated by quantum computers
- Theoretical formulation ongoing: gauge choice, difficult color algebra, …

see e.g. Jordan, Lee, Preskill `11, Preskill `18, Klco, Savage et al. `18-`20, Cloet, Dietrich et al. `19

Quantum computing may allow for a solution of the real-time dynamics of QCD
Quantum computing

Superposition and entanglement

\[ |\psi\rangle = \sum_{i=1}^{2^N} a_i |\psi_i\rangle \]

For \( N \) qubits, there are \( 2^N \) amplitudes

e.g. \[ |\psi\rangle = a_1 |000\rangle + a_2 |001\rangle + a_3 |010\rangle + a_4 |011\rangle + a_5 |100\rangle + a_6 |101\rangle + a_7 |110\rangle + a_8 |111\rangle \]

If one can control this high-dimensional space, e.g. with appropriate interference of amplitudes, then one can potentially achieve exponential speedup of certain computations

- It is expected that quantum computers can solve some classically hard problems with exponential speedup
- These include a number of highly impactful problems such as quantum simulation
Superconducting circuits have become \( O \left( 100 \, \mu s \right) \), long enough to perform \( O \left( 10 \, – \, 100 \right) \) two-qubit operations. e.g. Kjaergaard et al. ’20

And a variety of others:

- Trapped ions
- Optical lattice
- Photonics
- Topological
- ...
Quantum devices

Superconducting circuits
- IBM Q
- Google
- Rigetti
- ...:

And a variety of others:
- Trapped ions
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- ...:

Superconducting circuit qubit coherence times have become $O(100 \, \mu s)$, long enough to perform $O(10 - 100)$ two-qubit operations (e.g. Kjaergaard et al. ’20).

The dream: universal, fault-tolerant digital quantum computer
- Shor’s and Grover’s algorithm
- Quantum error correction
- Shor, Preskill, Kitaev, Zoller ...

Noisy Intermediate Scale Quantum (NISQ) era
- Decoherence, limited number of qubits, imperfect gates
- Aim: achieve quantum advantage without full quantum error correction
- Martinis et al. (2019), Zhong et al. (2020)
A near-term approach: Open quantum systems

Study the real time dynamics of the quantum evolution of probes in the nuclear medium (LHC/RHIC/EIC)

**Subsystem** - Jet/heavy-flavor

**Environment** - Nuclear matter

\[ H(t) = H_S(t) + H_E(t) + H_I(t) \]
A near-term approach: Open quantum systems

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**Subsystem** - Jet/heavy-flavor

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\[ H(t) = H_S(t) + H_E(t) + H_I(t) \]

The time evolution is governed by the von Neumann equation:

\[ \frac{d}{dt} \rho^{(\text{int})}(t) = -i \left[ H^{(\text{int})}_I(t), \rho^{(\text{int})}(t) \right] \]

In the Markovian limit, the subsystem is described by a **Lindblad equation**

\[ \frac{d}{dt} \rho_S = -i [H_S, \rho_S] + \sum_{j=1}^{m} \left( L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2} \rho_S L_j^\dagger L_j \right) \]

\[ \rho_S = \text{tr}_E[\rho] \]
Open quantum systems: Quarkonia

The evolution of quarkonia in the QGP can be described by the Lindblad equation

\[ \text{Akamatsu, Rothkopf et al. '12-'20, Brambilla et al. '17-'20} \]
\[ \text{Yao, Mueller, Mehen '18-'20, Sharma, Tiwari '20} \]

"Simple" system: reduces to quantum mechanics (NRQCD)

Currently various approximations are considered

- Markovian limit
  \[ \text{Blaizot, Escobedo '18, Yao, Mehen '18, '20} \]
- Small coupling of system and environment
- Semi-classical transport
Open quantum systems: Quarkonia

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NRQCD + semiclassical approach vs. full quantum evolution

Quantum treatment has important phenomenological consequences

Survival probability of the vacuum state

Bjorken expanding QGP \( T_0 = 475 \) MeV

Sharma, Tiwari '20

Akamatsu, Rothkopf et al. '12-'20, Brambilla et al. '17-'20
Yao, Mueller, Mehen '18-'20, Sharma, Tiwari '20
Open quantum systems: Jet broadening

First steps in the direction of jet physics

Markovian master equation describes evolution of jet density matrix:

\[ \partial_t P(Q,t) = -R(Q)P(Q,t) + \int dq K(Q,q)P(q,t) \]

where the probability to be in a given momentum state is:

\[ P(Q,t) = \langle Q|\rho_S(t)|Q \rangle \]
Quantum simulation

It is exponentially expensive to simulate an $N$-body quantum system on a classical computer: $2^N$ amplitudes!

But a quantum computer can naturally simulate a quantum system

**State preparation**  \[ |\psi_S\rangle \]

**Time evolution**  \[ e^{-iH_S \Delta t} \]

**Measurement**

Evolution in time steps $\Delta t = t/N_{\text{cycle}}$

**Time evolution of closed systems**

- Quantum simulation of the Schrödinger equation
- The evolution is unitary and time reversible
Non-unitary evolution

In open quantum systems, the subsystem evolution is non-unitary

\[
\frac{d}{dt} \rho_S = -i [H_S, \rho_S] + \sum_{j=1}^{m} \left( L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2} \rho_S L_j^\dagger L_j \right)
\]

The Stinespring dilation theorem

Any allowed quantum operation can be written as a unitary evolution acting on a larger space (after coupling to appropriate ancilla), and reducing back to the subsystem.
Quantum simulation of open quantum systems

Toy model setup

Two-level system in a thermal environment

\[ H_S = -\frac{\Delta E}{2} Z \]

\[ H_E = \int d^3x \left[ \frac{1}{2} \Pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 \right] \]

\[ H_I = gX \otimes \phi(x = 0) \]

\[ \rho_E = \frac{e^{-\beta H_E}}{\text{Tr}_E e^{-\beta H_E}}. \]

Pauli matrices \( X, Y, Z \), interaction strength \( g \)

Lindblad operators

\[ L_j \sim g(X \mp iY) \]

\[ j = 0, 1 \]

\[ J = \begin{pmatrix} 0 & L_0^\dagger & L_1^\dagger & 0 \\ L_0 & 0 & 0 & 0 \\ L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

Quantum circuit: Lindblad evolution
Quantum circuit synthesis

Approximate unitary operations with a compiled circuit of one- and two-qubit gates

Optimization problem w/unitary loss function

qsearch Siddiqi et al. `20

Single qubit

CNOT

IBM Q

10 CNOT gates/cycle

Error mitigation

Readout error
Constrained matrix inversion IBM Q qiskit-ignis

Unfolding
Nachman, Urbanek, de Jong, Bauer `19

Gate error
Zero-noise extrapolation of CNOT noise using Random Identity Insertions
He, Nachman, de Jong, Bauer `20
Quantum simulation of open quantum systems

Real-time evolution

$P_0(t)$ describes fraction that remains in “bound state”
Similar to $t$-dependent $R_{AA}$

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Similar to $t$-dependent $R_{AA}$

$P_0(t) [\text{fm/c}] (T = 300 \text{ MeV})$

- Runge–Kutta
- Thermal equilibrium

$E_{\Delta E}$

$S$

Similar to $t$-dependent $R_{AA}$ describes fraction that remains in “bound state”

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Quantum simulation of open quantum systems

Real-time evolution

$P_0(t)$ describes fraction that remains in “bound state”
Similar to $t$-dependent $R_{AA}$

The algorithm converges to Lindblad evolution with a small number of cycles

$E$  $\Delta E$

Simulator, $N_{cycle} = 1$
Simulator, $N_{cycle} = 3$
Runge–Kutta
Thermal equilibrium

$P_0(t)$

$t$ [fm/c] ($T = 300$ MeV)

$t$ [1/$T$]
Quantum simulation of open quantum systems

**Real-time evolution**

$P_0(t)$ describes fraction that remains in “bound state”

Similar to $t$-dependent $R_{AA}$

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**Graphical Representation**

- **IBM Q Vigo device**
  - **Simulation**
    - Uncorrected
    - Simulator, $N_{\text{cycle}} = 1$
    - Runge–Kutta
  - **Thermal equilibrium**

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*Source: arXiv:2010.03571*
Quantum simulation of open quantum systems

Real-time evolution

\( P_0(t) \) describes fraction that remains in “bound state”

Similar to \( t \)-dependent \( R_{AA} \)

\[
\begin{align*}
\text{IBM Q Vigo, } N_{\text{cycle}} = 1, g = 0.3 \\
\text{Uncorrected} & : \quad \text{Simulator, } N_{\text{cycle}} = 1 \\
\text{Readout corrected} & : \quad \text{Runge – Kutta} \\
\text{Readout + RIIM corrected} & : \quad \text{Thermal equilibrium}
\end{align*}
\]

IBM Q Vigo device

Including CNOT gate error correction gives good agreement

Random Identity Insertion Method (RIIM) 
Bauer, He, de Jong, Nachman `20

Proof of concept
Summary

Real-time evolution of hard probes in heavy-ion collisions can be formulated as an open quantum system, and encoded in a quantum algorithm. This allows to go beyond semiclassical approximations in current models.

Proof of concept that these systems can be simulated on current and near-term quantum computers, specifically using NISQ era digital quantum computing.

Future steps:
- Extension toward QCD
- Explore different digital/analog devices
- More efficient quantum algorithms and error mitigation
Quantum advantage

**Last year**

**Article**

Quantum supremacy using a programmable superconducting processor

Martinis et al. (2019)

53-qubit superconducting circuit device

Algorithm: sampling of random circuits

$O(10^3)$ times faster than best classical supercomputers

**Last month**

Quantum computational advantage using photons

Han-Sen Zhong$^{1,2}$, Hui Wang$^{1,2}$, Yu-Hao Deng$^{1,2}$, Ming-Cheng Chen$^{1,2}$, Li-Chao Peng$^{1,3}$, Yi-Han Luo$^{1,2}$, Jian Qin$^{1,2}$, Dian Wu$^{1,2}$, Xing Ding$^{1,2}$, Yi Hu$^{1,2}$, Peng Hu$^{1,2}$, Xiao-Yan Yang$^{1,2}$, Wei-Jun Zhang$^{1,2}$, Hao Li$^{1,2}$, Yuxuan Li$^{1,2}$, Xiao Jiang$^{1,2}$, Lin Gan$^{1,2}$, Guangwen Yang$^{1,2}$, Lixing You$^{1,2}$, Zhen Wang$^{1,2}$, Li Li$^{1,2}$, Nai-Le Liu$^{1,2}$, Chao-Yang Lu$^{1,2}$, Jian-Wei Pan$^{1,2}$

Science (2020)

Photonic device — special-purpose

Algorithm: boson sampling

Claim: $O(10^{14})$ times faster than best classical supercomputers
Constrained matrix inversion

IBM Q qiskit-ignis

Prepare states by applying bit-flip X gates and read out

Unfolding

Nachman, Urbanek, de Jong, Bauer `19

ibmq_vigo device
Error mitigation

**Readout error**

Constrained matrix inversion

IBM Q qiskit-ignis

**Gate error**

Zero-noise extrapolation of CNOT noise using Random Identity Insertions

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Circuit 1

Circuit 2

Circuit 3

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