Chromoelectric Distribution Function of Nuclear Matter Probed by Quarkonium

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arXiv: 2009.02408

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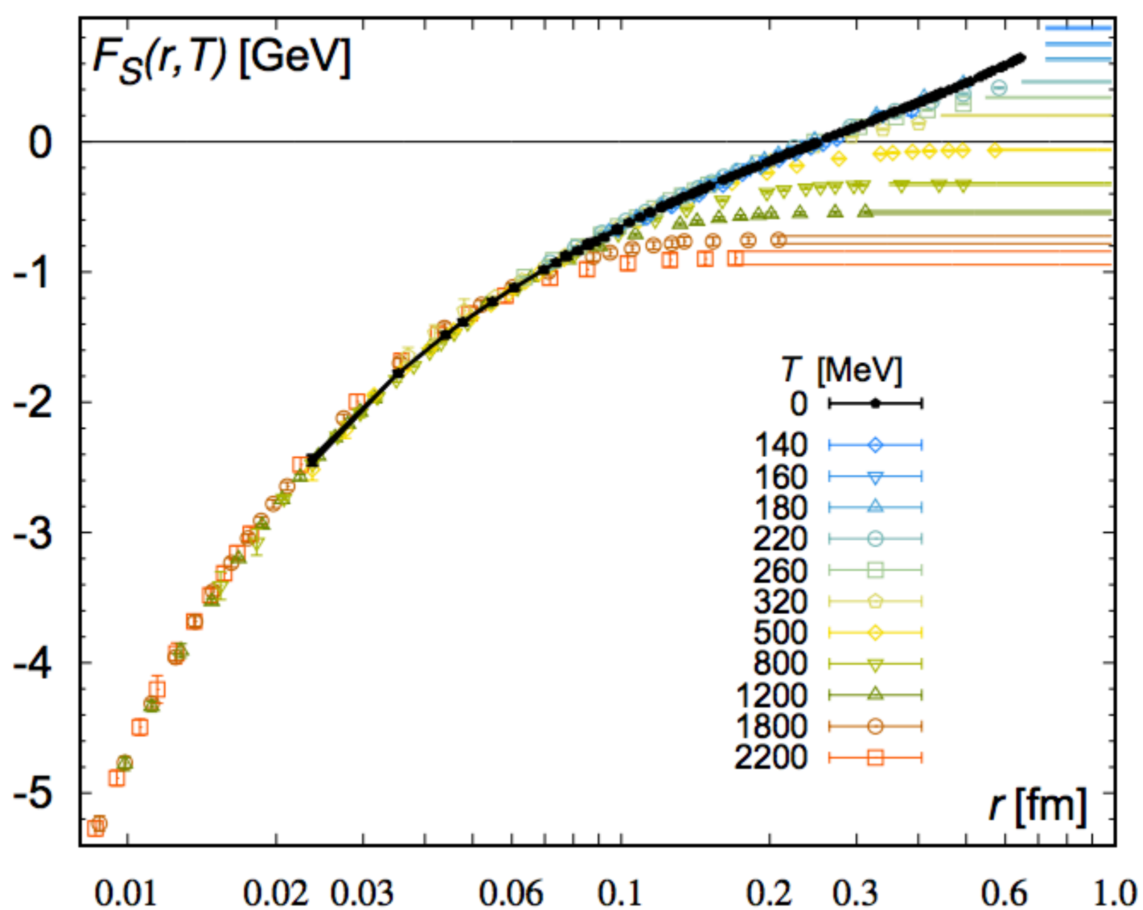
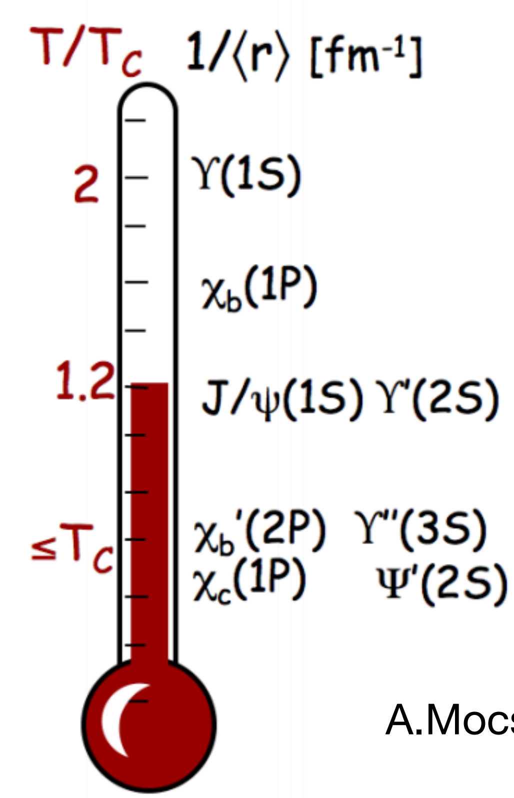
Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened



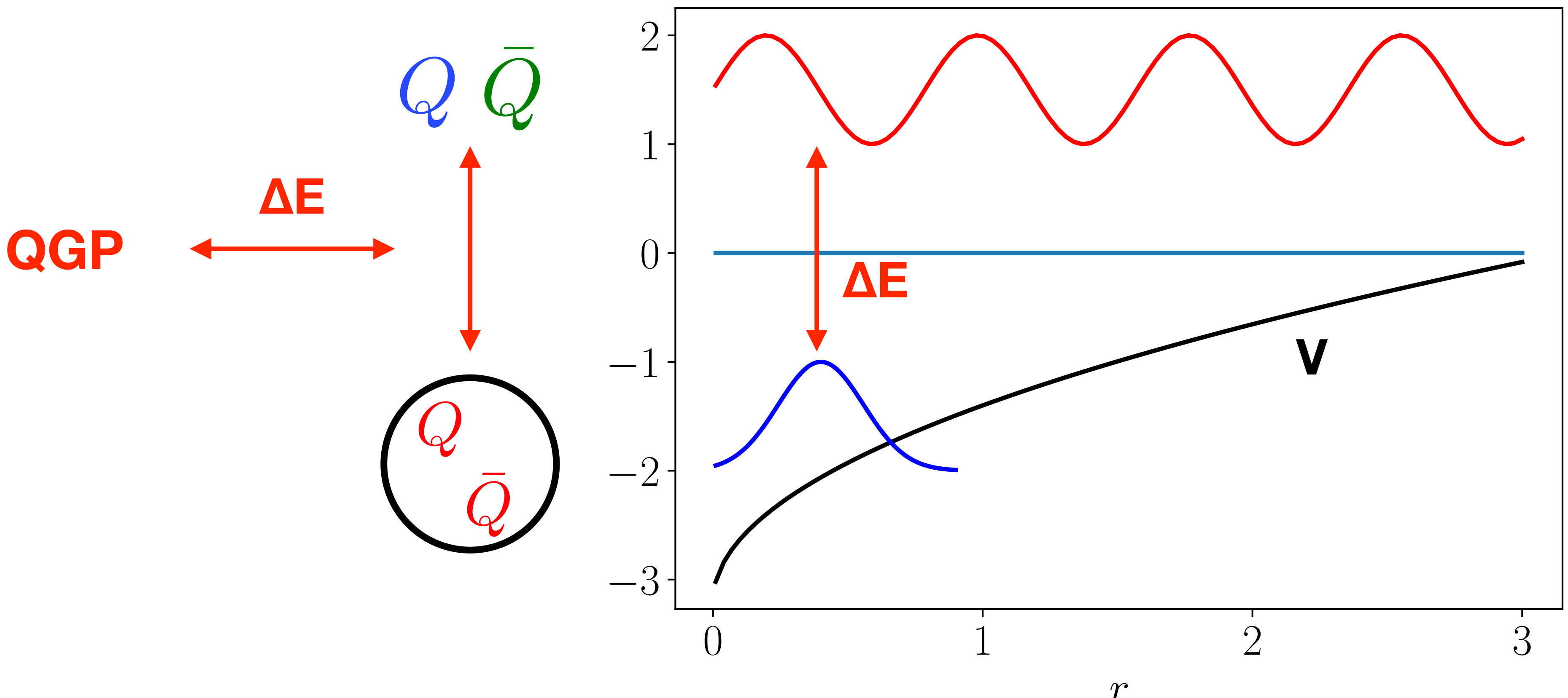
1804.10600, A.Bazavov, N.Brambilla
P.Petreczky, A.Vairo, J.H.Weber



A.Mocsy, 0811.0337

Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer
- **Dynamical screening**: related to imaginary potential, **dissociation** induced by dynamical process, lead to suppression even when T(QGP) < melting T
- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, crucial for phenomenology and theory consistency



Simple physics picture of thermometer does not work

What QGP properties are we probing by measuring quarkonium?

This talk:

In certain limit, we are probing chromoelectric distribution functions of QGP/nuclear medium

Leading-power, all-order construction, gauge invariant

Two tools: open quantum systems, effective field theory

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$$\rho(t=0) = \rho_S \otimes \rho_E$$

Subsystem & environment

(Heavy quark pairs & QGP)

Unitary evolution → **Time reversible**

$$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$$

Subsystem & environment

Trace out (integrate out) environment

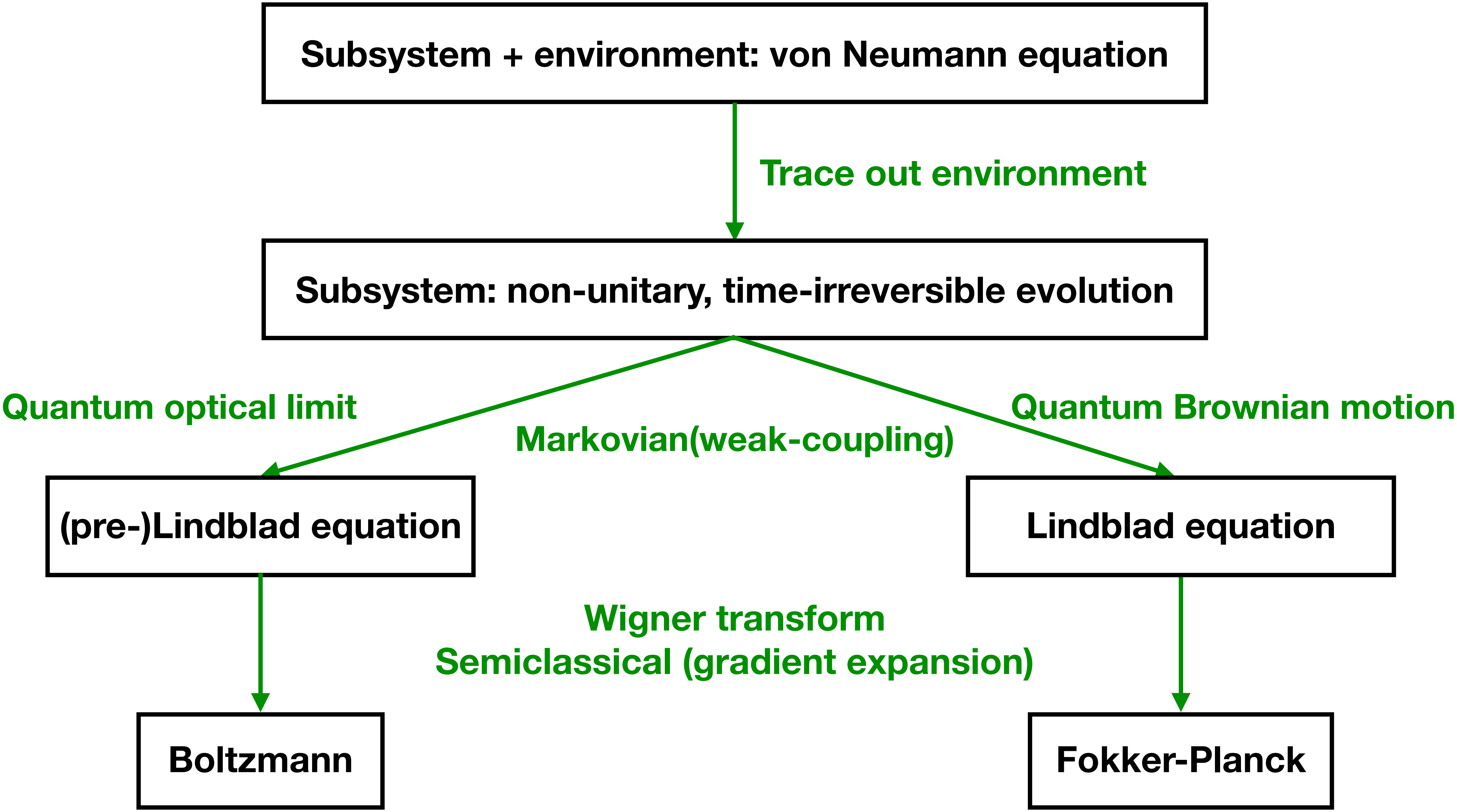
$$\rho_S(t=0)$$

Subsystem

Non-unitary → **Time irreversible**

$$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$$

From Open Quantum System to Semiclassical Transport



Wigner transform $$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1 \middle| \rho_S(t) \middle| \boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1 \right\rangle$$

Separation of Scales and pNRQCD

Separation of scales

$$M \gg Mv \gg Mv^2,\ T,\ \Lambda_{QCD}$$

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium

QCD

Hard M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Soft Mv — perturbative / non-perturbative matching

potential NRQCD

hep-ph/9907240, hep-ph/0410047,
N.Brambilla A.Pineda, J.Soto, A.Vairo

Ultrasoft

$Mv^2 \sim 500\ \mathrm{MeV}$

T

Hard $p_h \sim M(1,1,1,1)$

Soft $p_s \sim Mv(1,1,1,1)$

Ultrasoft $p_{us} \sim Mv^2(1,1,1,1)$

Leading Power

- **Nonrelativistic & multipole expansions: v & r**

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + \boxed{V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.})} + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$

- **Leading power in v :**

  No center-of-mass kinetic energy

  Potential simple, no hyperfine splitting

$$H_{s,o} = \frac{(i\boldsymbol{\nabla}_{\mathrm{rel}})^2}{M} + V^{(0)}_{s,o}$$

  Quarkonium wavefunction simple $|H\rangle = |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + \cdots = |Q\bar{Q}\rangle$

- **Leading (nontrivial) power in r :**

  Weak coupling between quarkonium and QGP: quarkonium small in size $r \sim \frac{1}{Mv}$

  Dipole interaction $O^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}S \sim rT \sim \frac{T}{Mv}$

- **Boltzmann equation at leading-power, leading-order in g** XY, T.Mehen 1811.07027

  Dissociation and recombination rates depend on QGP via

$$\mathrm{Tr}_E\big(\rho_E E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\big) = \langle E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\rangle_T$$ **Not gauge invariant !**

All-Order Construction: Resum A0

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\Big({\rm S}^\dagger(i\partial_0 - H_s){\rm S} + \boxed{{\rm O}^\dagger(iD_0 - H_o){\rm O}} + V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.}) + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots\Big)$$

Octet—A0 interaction not suppressed by v or r

Need resum A0 to all orders at leading power

Field redefinition:

$$\begin{aligned} {\rm O}(\boldsymbol{R},\boldsymbol{r},t) &= \mathcal{W}_{[(\boldsymbol{R},t),(\boldsymbol{R},t_0)]}\widetilde{\rm O}(\boldsymbol{R},\boldsymbol{r},t) \\ \widetilde{E}_i(\boldsymbol{R},t) &= \mathcal{W}_{[(\boldsymbol{R},t_0),(\boldsymbol{R},t)]}E_i(\boldsymbol{R},t) \\ \mathcal{W}_{[(\boldsymbol{R},t_f),(\boldsymbol{R},t_i)]} &= \mathcal{P}\exp\left(ig\int_{t_i}^{t_f} ds\,\mathcal{A}_0(\boldsymbol{R},s)\right) \end{aligned}$$

New form of dipole interaction:

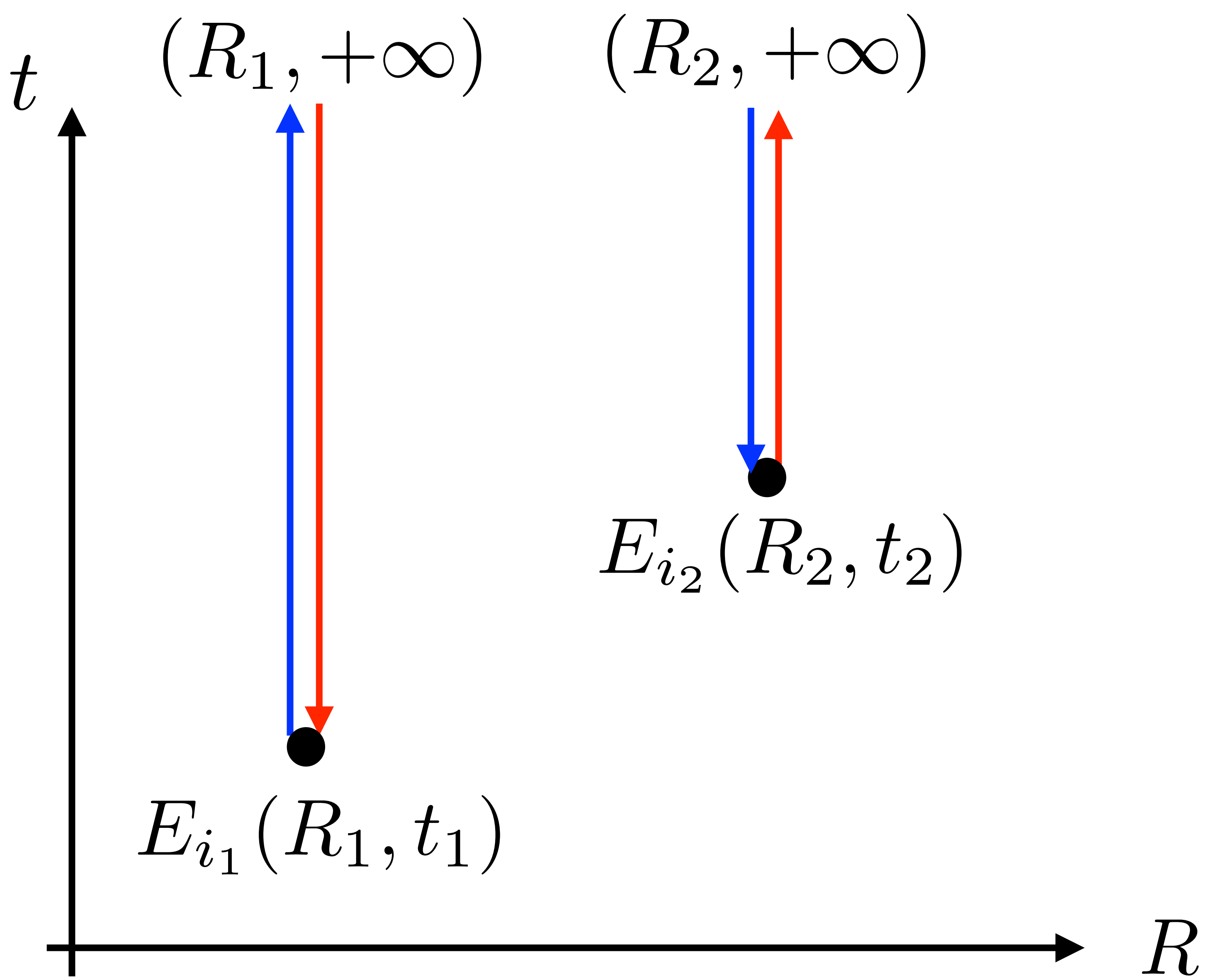
$$g\int d^3r\,{\rm Tr}\Big(\widetilde{\rm O}^\dagger r_i \widetilde{E}_i {\rm S} + {\rm S}^\dagger r_i \widetilde{E}_i^\dagger \widetilde{\rm O}\Big)$$

Chromoelectric Distribution Function of QGP

$$g_{i_1 i_2}^{E++}(t_1, t_2, \boldsymbol{R}_1, \boldsymbol{R}_2) = \left\langle E_{i_1}(\boldsymbol{R}_1, t_1) \mathcal{W}_{[(\boldsymbol{R}_1, t_1),(\boldsymbol{R}_1, +\infty)]} \mathcal{W}_{[(\boldsymbol{R}_2, +\infty),(\boldsymbol{R}_2, t_2)]} E_{i_2}(\boldsymbol{R}_2, t_2) \right\rangle_T$$

Wilsons not connected at infinite time!

For gauge invariance, need spatial gauge link



Wilson Lines at Infinite Time

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r\,\text{Tr}\Big(\text{S}^\dagger(i\partial_0 - H_s)\text{S} + \text{O}^\dagger(iD_0 - H_o)\text{O} + V_A(\text{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\text{S} + \text{h.c.}) + \frac{V_B}{2}\text{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \text{O}\} + \cdots\Big)$$

Coulomb interaction between octet heavy quark pair included in potential

But Coulomb between octet center-of-mass motion and medium not considered

For Coulomb modes $p_c^\mu \sim A_c^\mu \sim M(v^2, v, v, v)$

$$\int \mathrm{d}^3r\,\text{Tr}\Big(\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(iD_0 + \boxed{\frac{\boldsymbol{D}_{\boldsymbol{R}}^2}{4M}} + \frac{\nabla_{\boldsymbol{r}}^2}{M} - V_o(\boldsymbol{r}) + \cdots\Big)\text{O}(\boldsymbol{R},\boldsymbol{r},t)\Big)$$

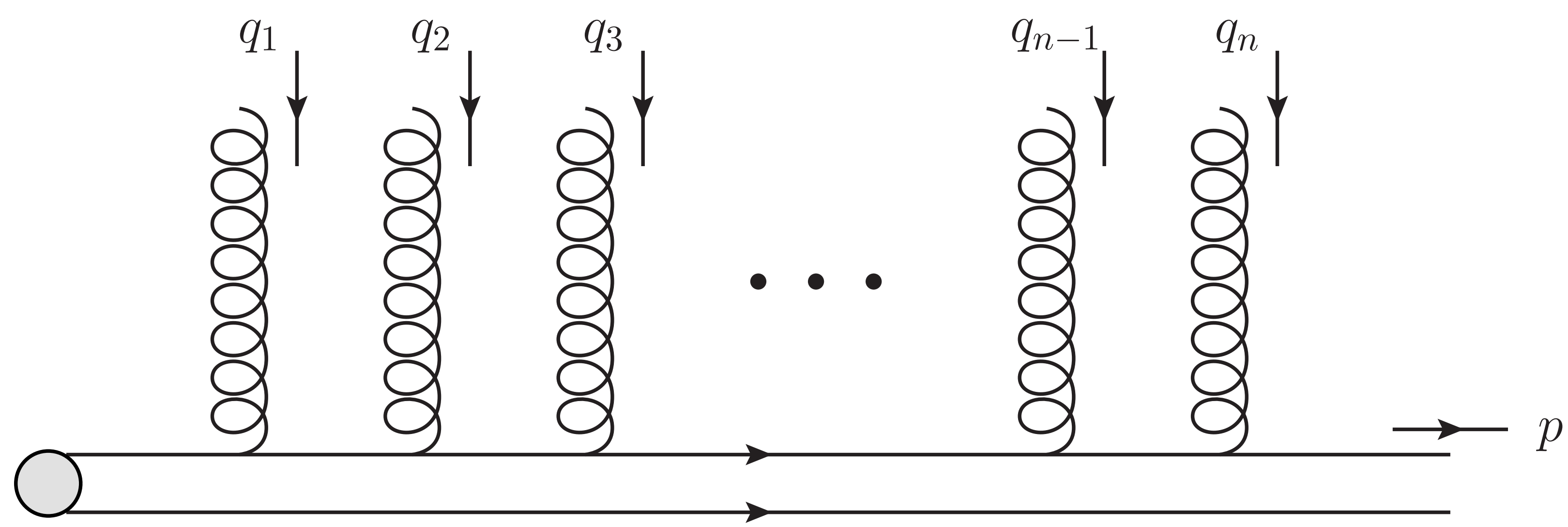
C.m. kinetic term same order as D0, so leading power in v

Write out c.m. kinetic term

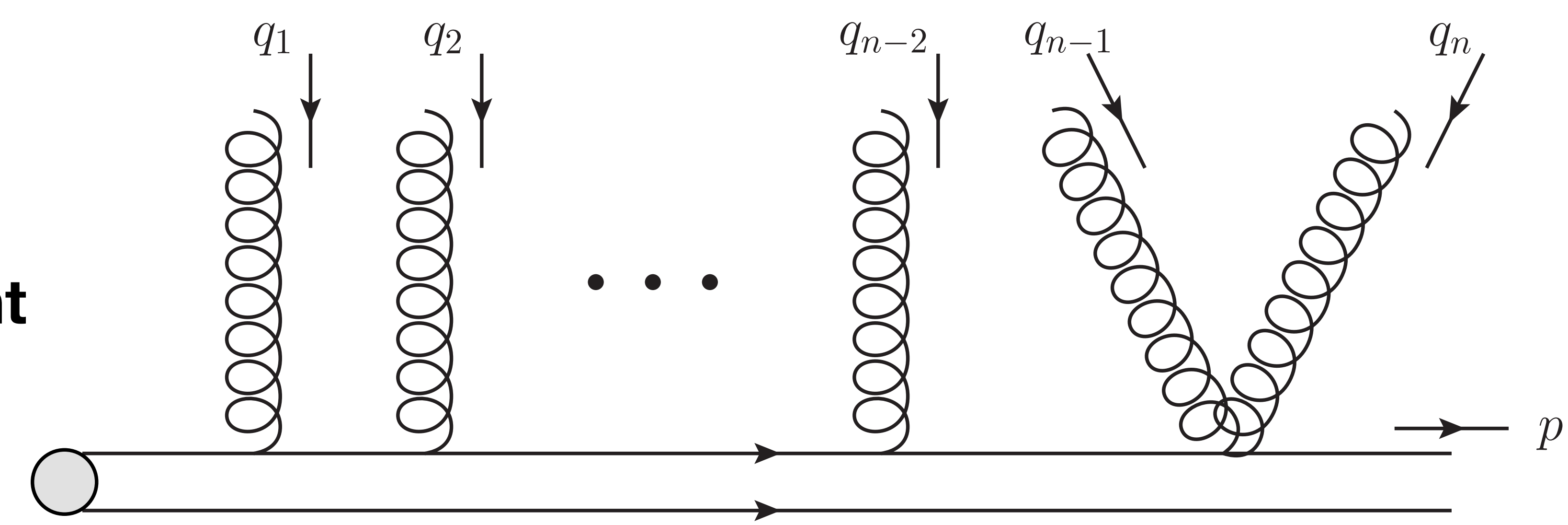
$$\int \mathrm{d}^3r\,\text{Tr}\Big(\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\frac{\nabla_{\boldsymbol{R}}^2}{4M}\text{O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{ig}{4M}\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(\boldsymbol{A}(\boldsymbol{R},t)\cdot\nabla_{\boldsymbol{R}} + \nabla_{\boldsymbol{R}}\cdot\boldsymbol{A}(\boldsymbol{R},t)\Big)\text{O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{g^2}{4M}\text{O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{A}^2(\boldsymbol{R},t)\text{O}(\boldsymbol{R},\boldsymbol{r},t)\Big).$$

Wilson Lines at Infinite Time: Resum Coulomb

Single Coulomb attachment



Double Coulomb attachment

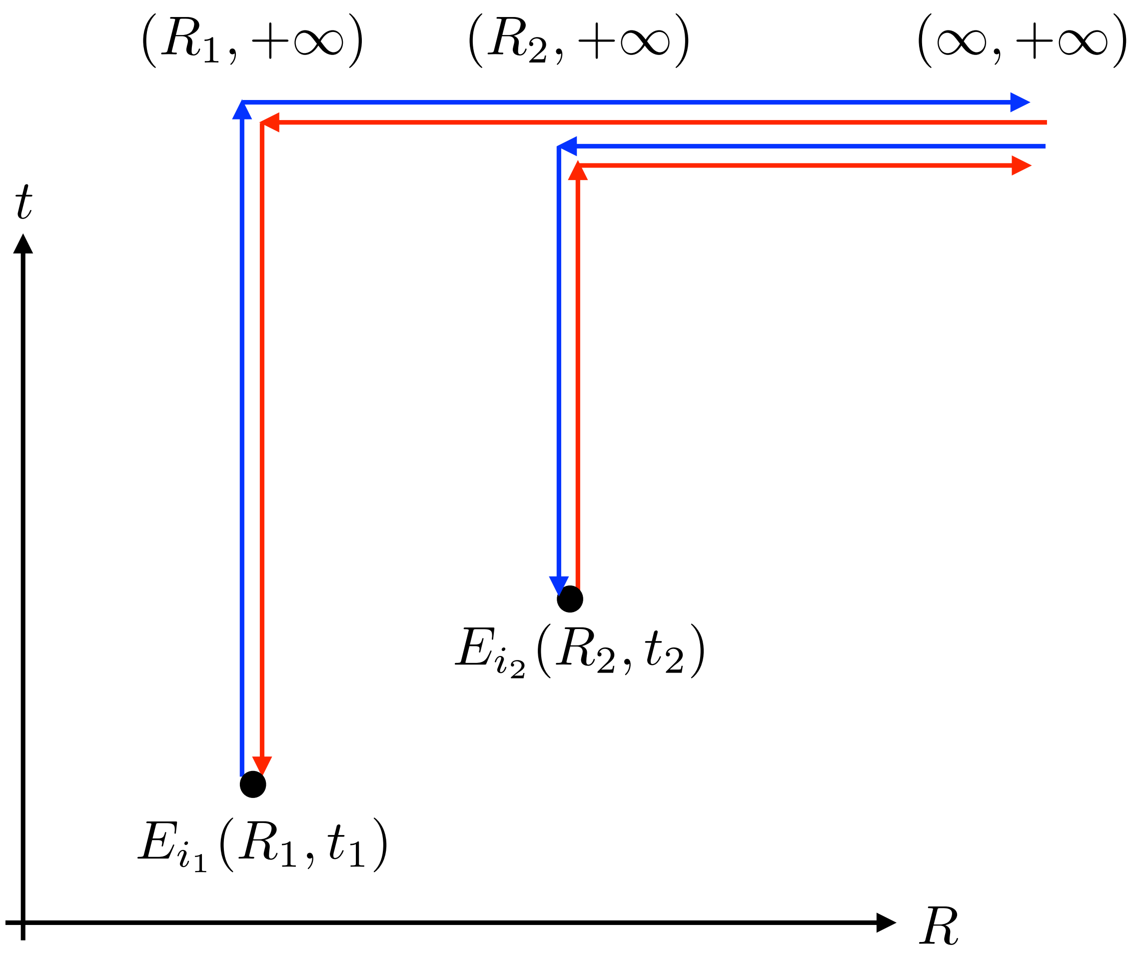


Calculations have some similarity with A.V. Belitsky, X. Ji, F. Yuan hep-ph/0208038

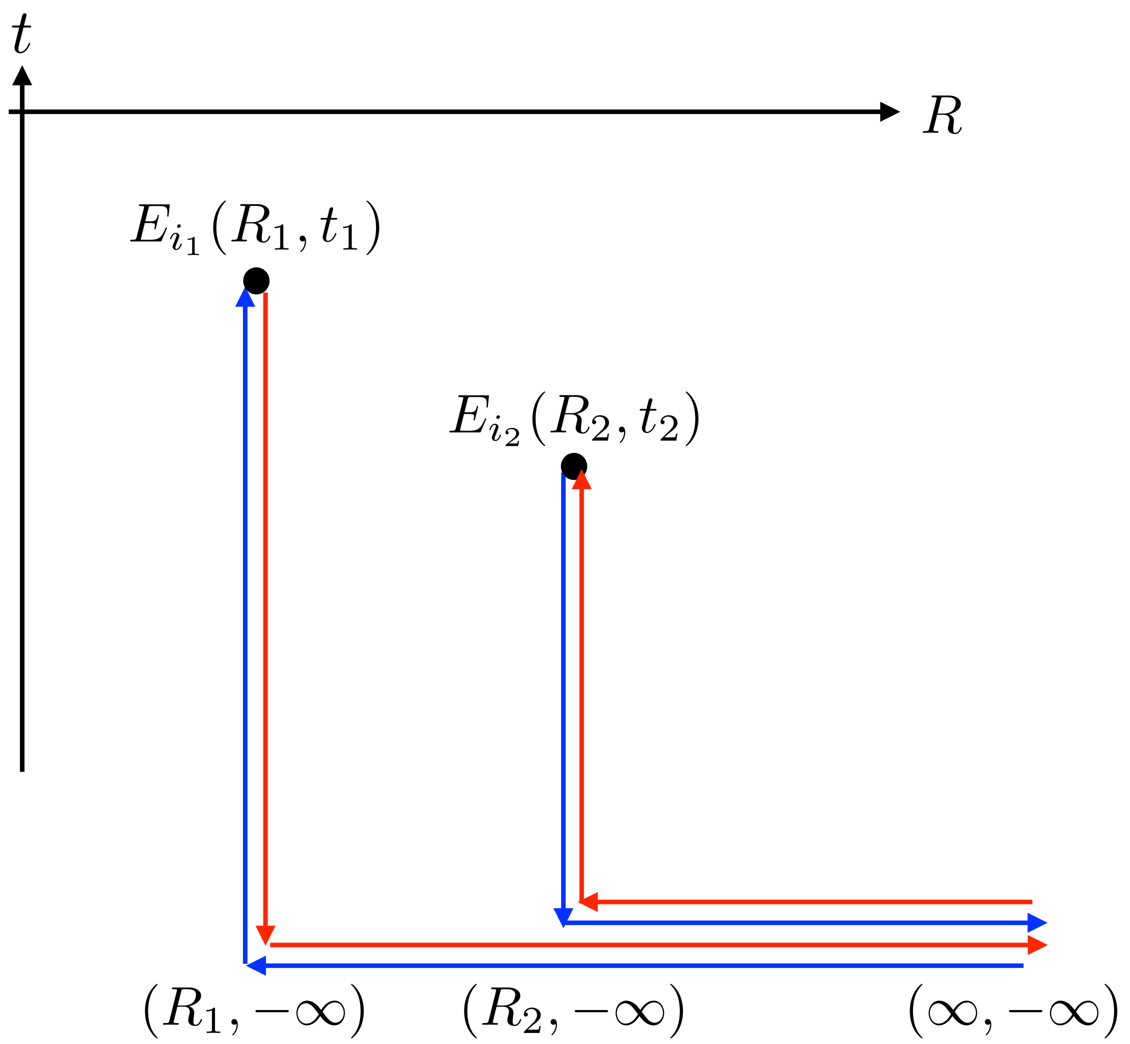
Chromoelectric Distribution Function of QGP

Staple shaped Wilson lines

For dissociation: final-state interaction



For recombination: initial-state interaction



Inclusive v.s. Differential Reaction Rates

- **Take dissociation rate as example**

$$R^{-}_{nl}(\boldsymbol{x},\boldsymbol{k},t) = \sum_{i_1,i_2}\int \frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}+\boldsymbol{q})\delta(E_{nl}-E_p+q^0)\, d^{nl}_{i_1,i_2}(\boldsymbol{p}_{\rm rel})g^{E++}_{i_1i_2}(q^0,\boldsymbol{q})$$

$$d^{nl}_{i_1i_2}(\boldsymbol{p}_{\rm rel}) \equiv g^2\frac{1}{N_c}\langle\psi_{nl}|r_{i_1}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r_{i_2}|\psi_{nl}\rangle$$

- **Inclusive rate**

$$R^{-}_{nl} = \int\frac{d^3p_{\rm rel}}{(2\pi)^3}\bar{d}_{nl}(\boldsymbol{p}_{\rm rel})G^{E++}\left(\frac{(\boldsymbol{p}_{\rm rel})^2}{M}-E_{nl}\right)$$

$$G^{E++}(q_0) = \int dt\, e^{-iq_0t}\,\langle E_i(t)\mathcal{W}_{[t,0]}E_i(0)\rangle$$

Momentum independent distribution has been constructed in N.Brambilla, M.A.Escobedo J.Soto, A.Vairo 1711.04515

Zero frequency limit = heavy quark diffusion coefficient

- **Differential rate**

$$(2\pi)^3\frac{dR^{-}_{nl}}{d^3p_{\rm cm}} = \int\frac{d^3p_{\rm rel}}{(2\pi)^3}\bar{d}_{nl}(\boldsymbol{p}_{\rm rel})g^{E++}\left(\frac{(\boldsymbol{p}_{\rm rel})^2}{M}-E_{nl},\boldsymbol{p}_{\rm cm}-\boldsymbol{k}\right)$$

Momentum dependent distribution

- **Similar to PDF v.s. TMDPDF, though different in time axis**

Summary

- What are we probing by measuring quarkonium?
- Open quantum + EFT: leading-power, all-order construction
- Reaction rates depend on chromoelectric distribution function
  - Inclusive rates depend on momentum independent distribution, straight-line Wilson line structure
  - Differential rates depend on momentum dependent distribution, staple-shape Wilson line structure
- Easily generalized to cold nuclear matter by replacing environment density matrix
- Calculate nonperturbatively and use as inputs for quantum simulation

see James Mulligan's talk