Initial Stages 2021

On the way to collectivity in rarely interacting systems

Nina Kersting

In collaboration with
Nicolas Borghini
Hendrik Roch

UNIVERSITÄT BIELEFELD

BGTS

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

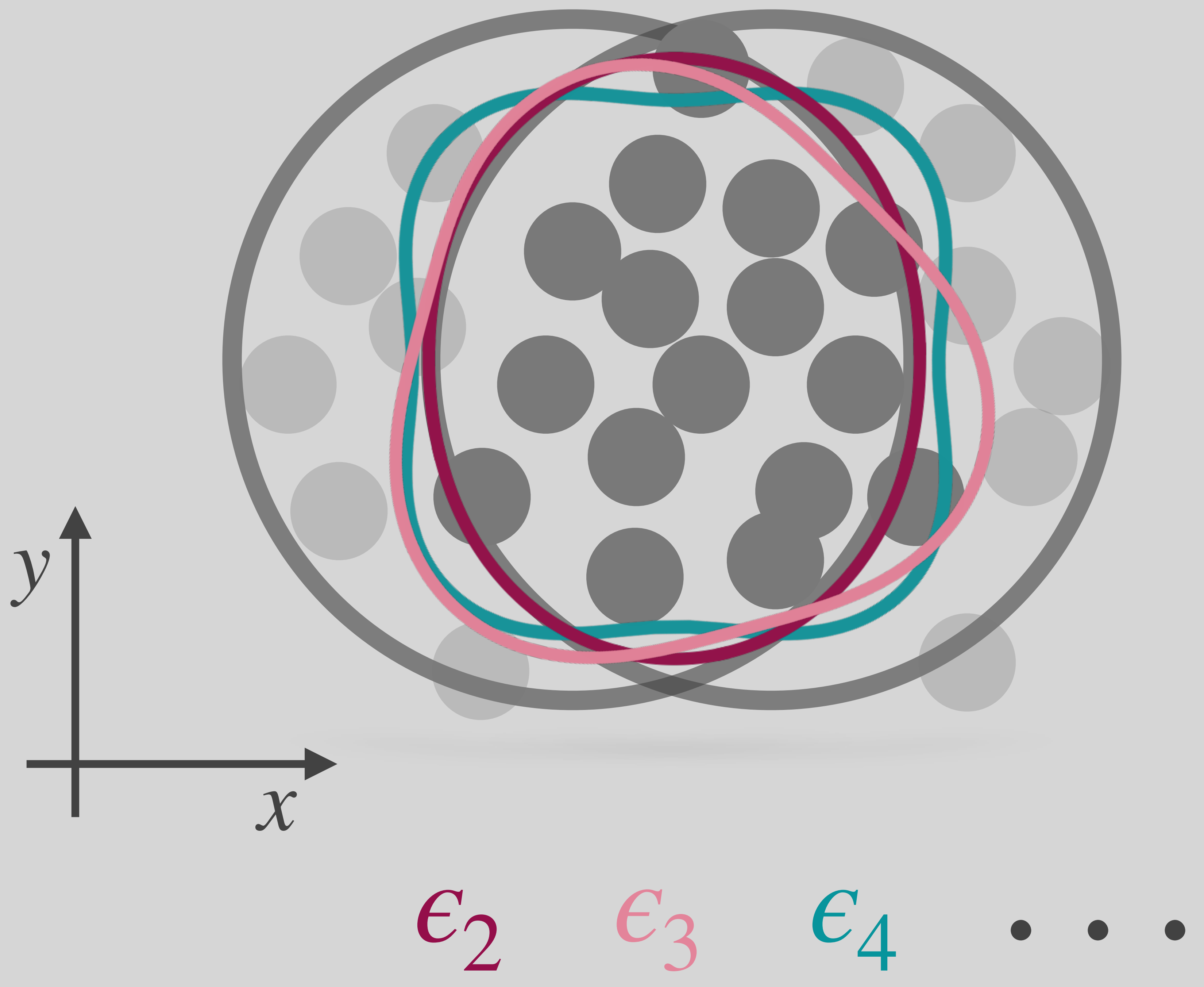
CRC-TR 211
Strong-interaction matter under extreme conditions

DFG

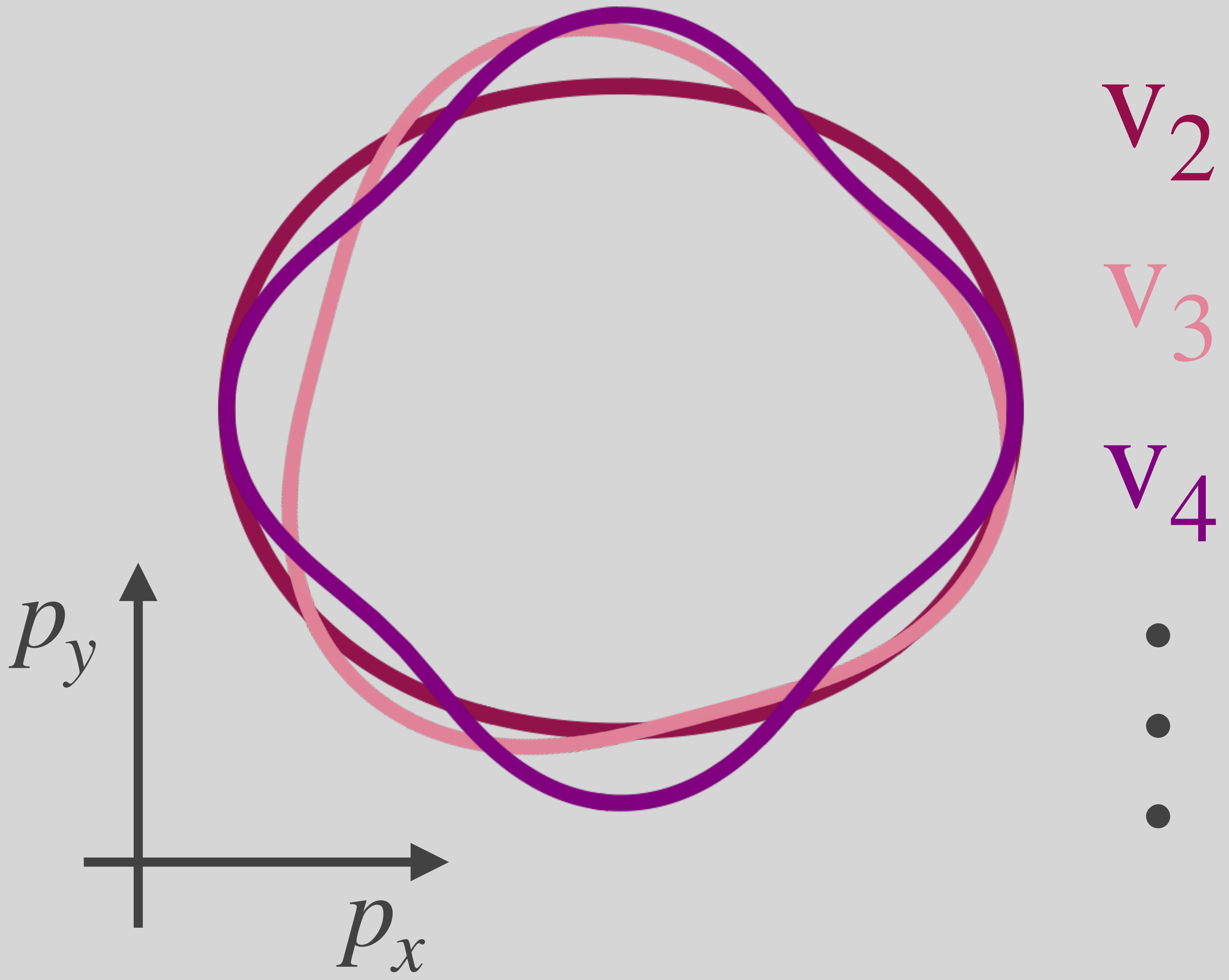
Motivation

Link between initial eccentricities and flow harmonics

Initial state
Eccentricities
Position space



Final state
Anisotropies
Momentum space

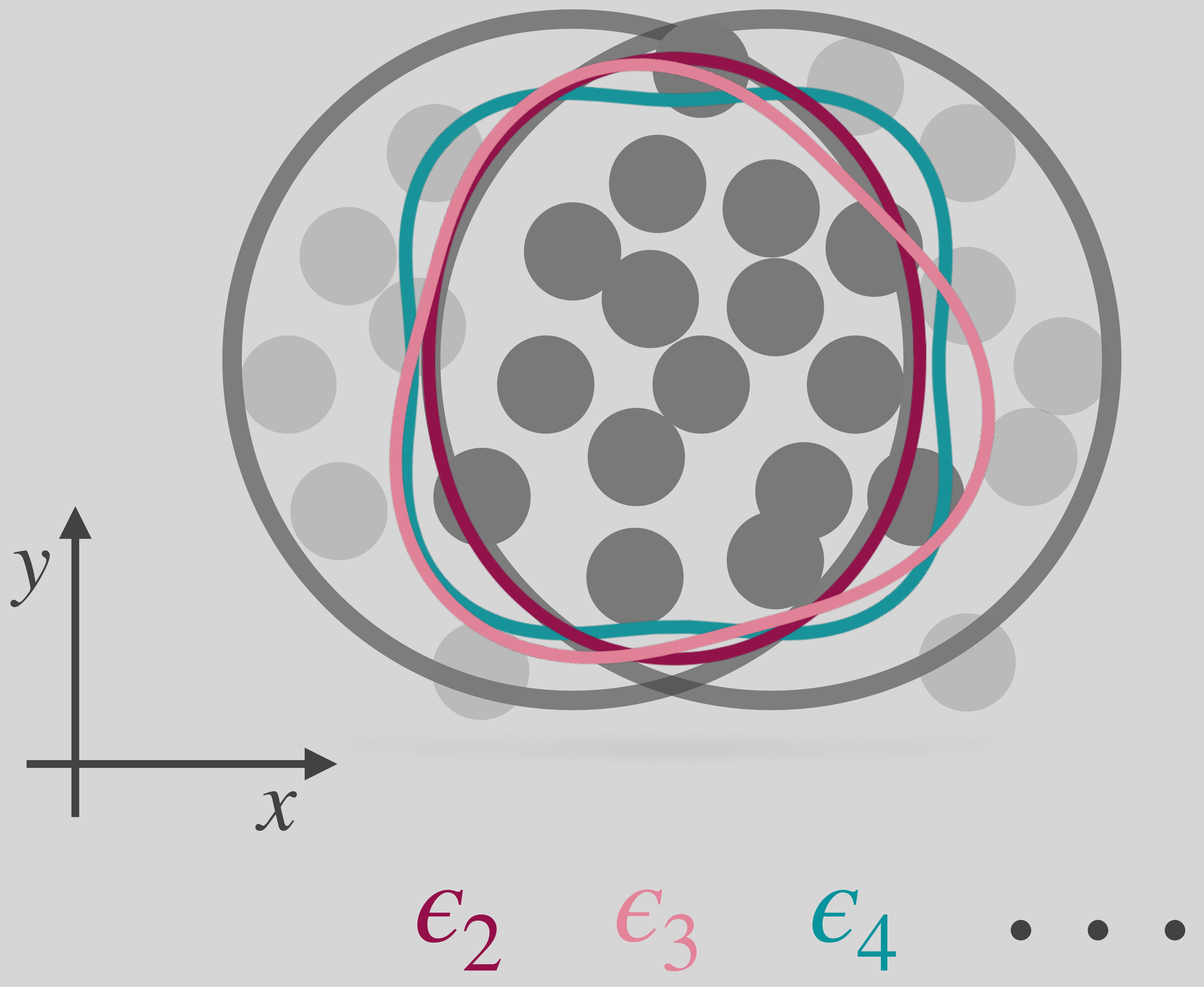


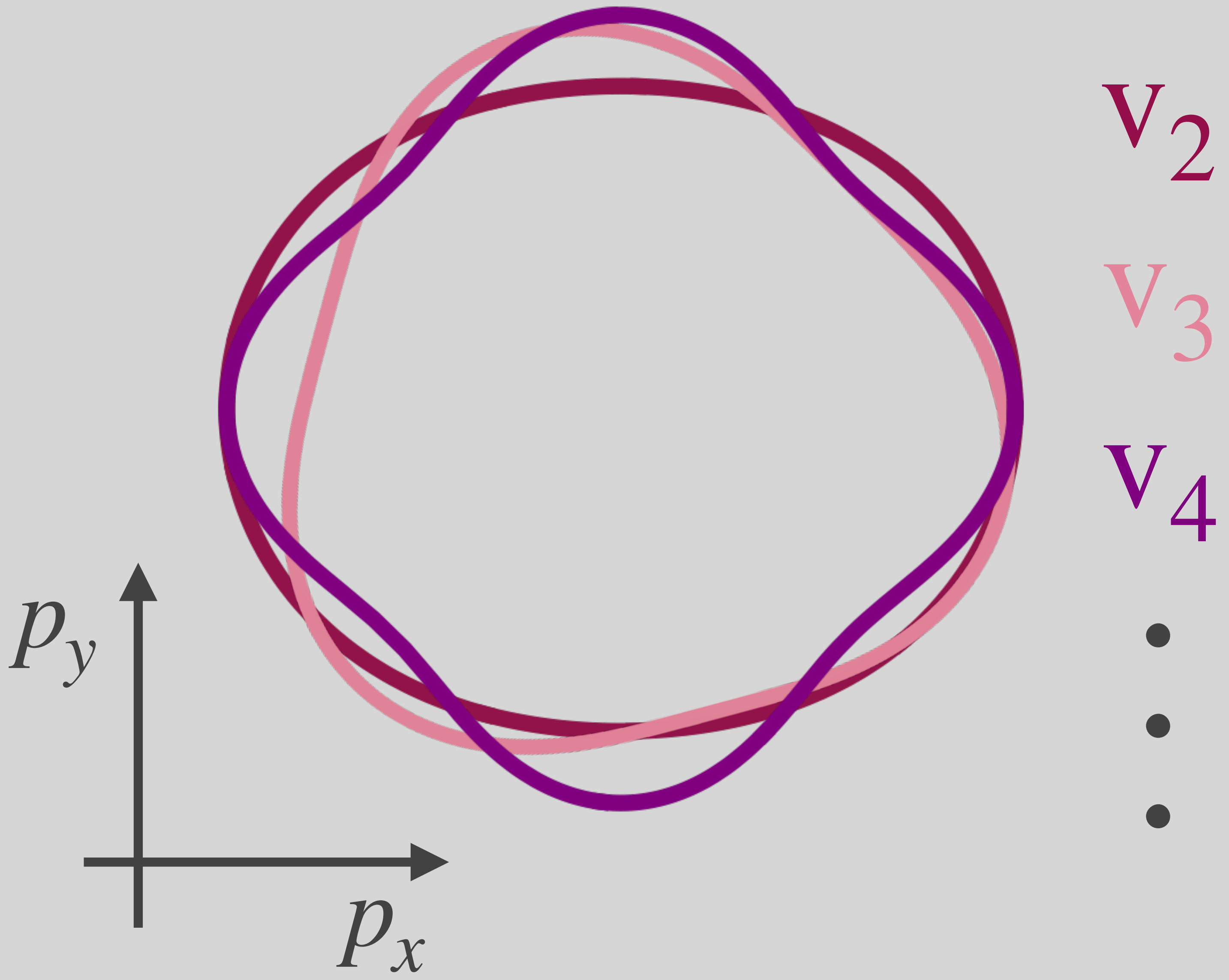
Link between initial eccentricities and flow harmonics

Initial state
Eccentricities
Position space

Theoretical
description

Final state
Anisotropies
Momentum space



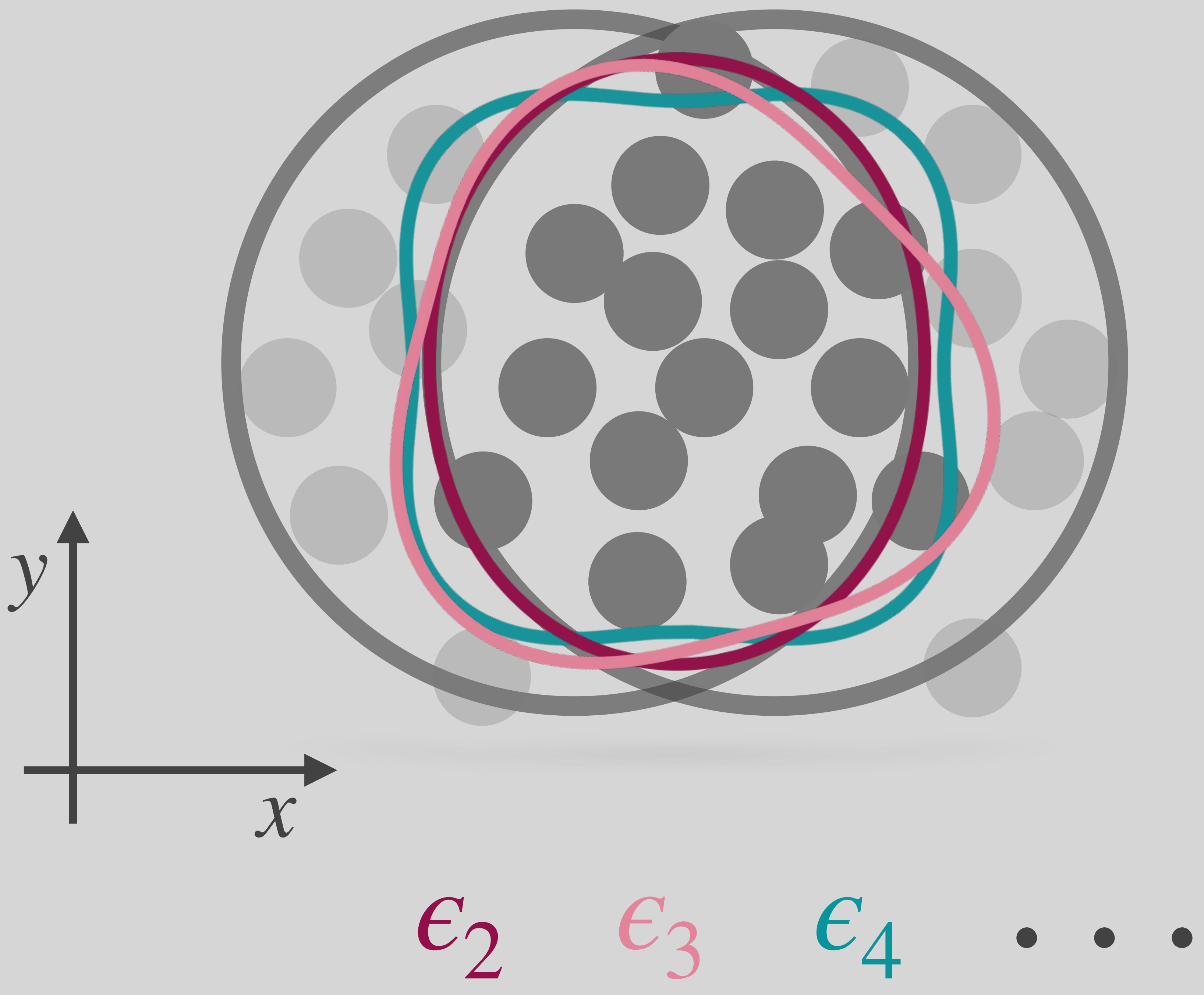


Link between initial eccentricities and flow harmonics

Initial state
Eccentricities
Position space

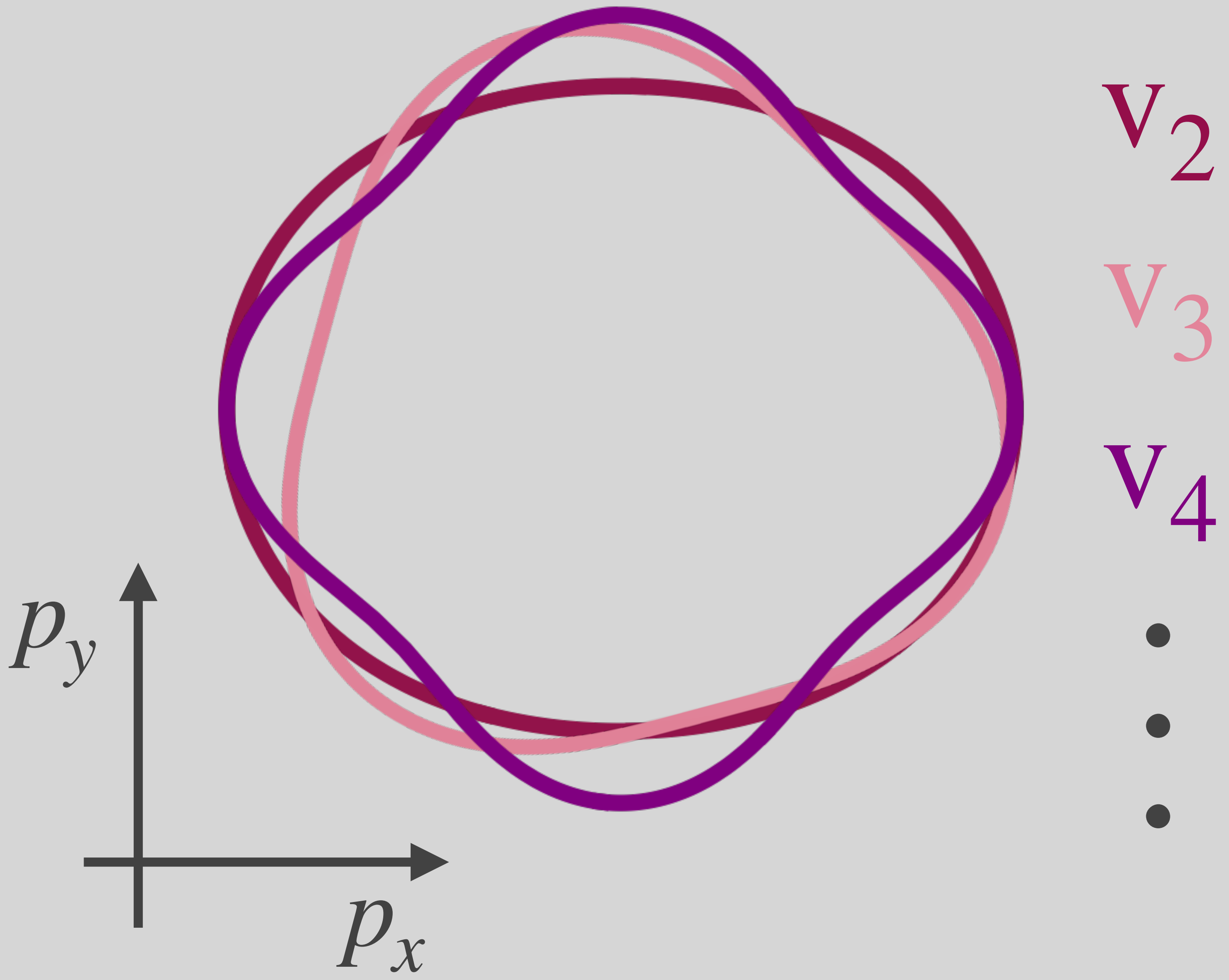
Theoretical
description

Final state
Anisotropies
Momentum space



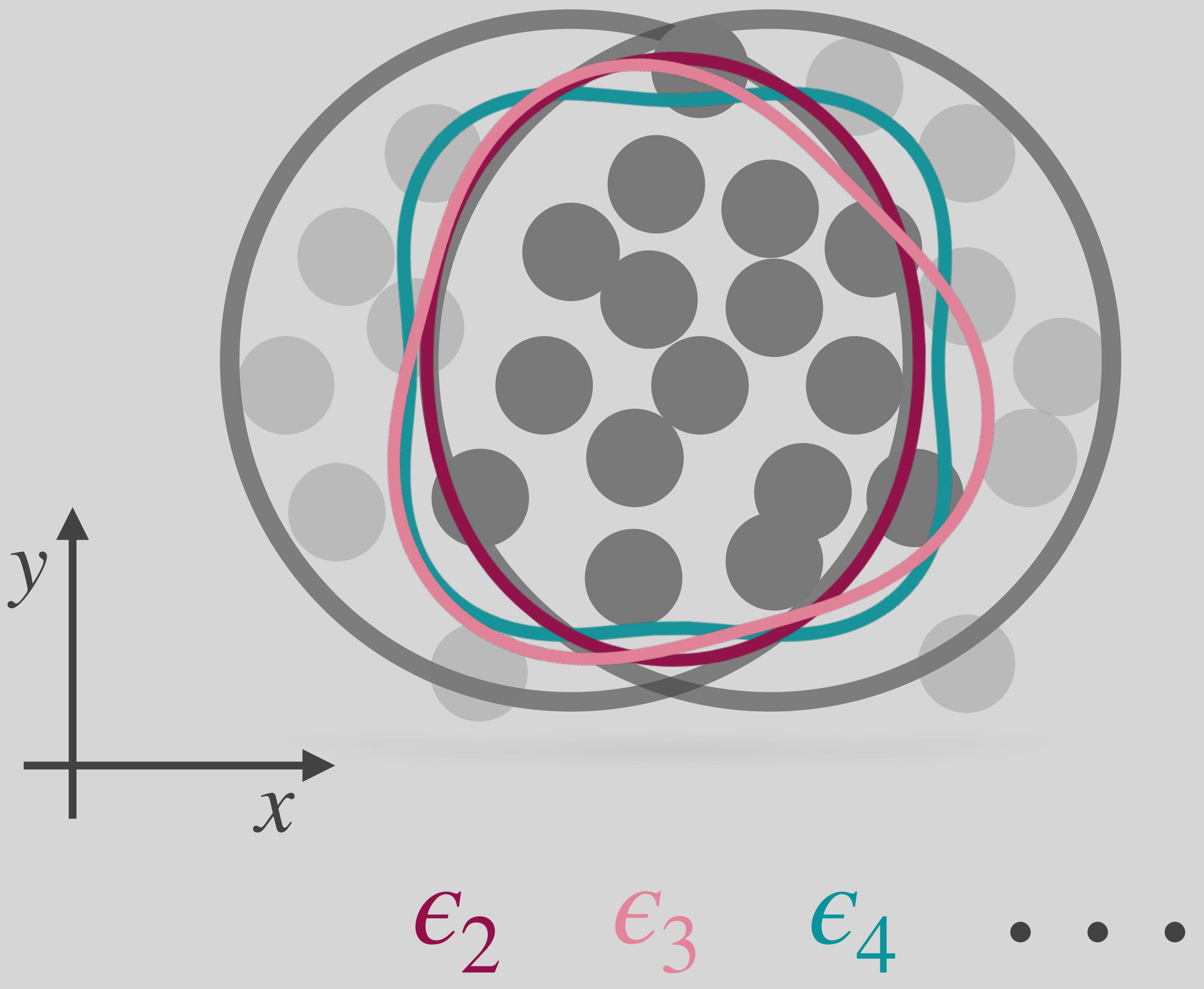
Hydrodynamics
Collective behaviour
$N_{\text{resc}} \gg 1$

Kinetic theory
Few collision regime
$N_{\text{resc}} < 1$



Link between initial eccentricities and flow harmonics

Initial state
Eccentricities
Position space

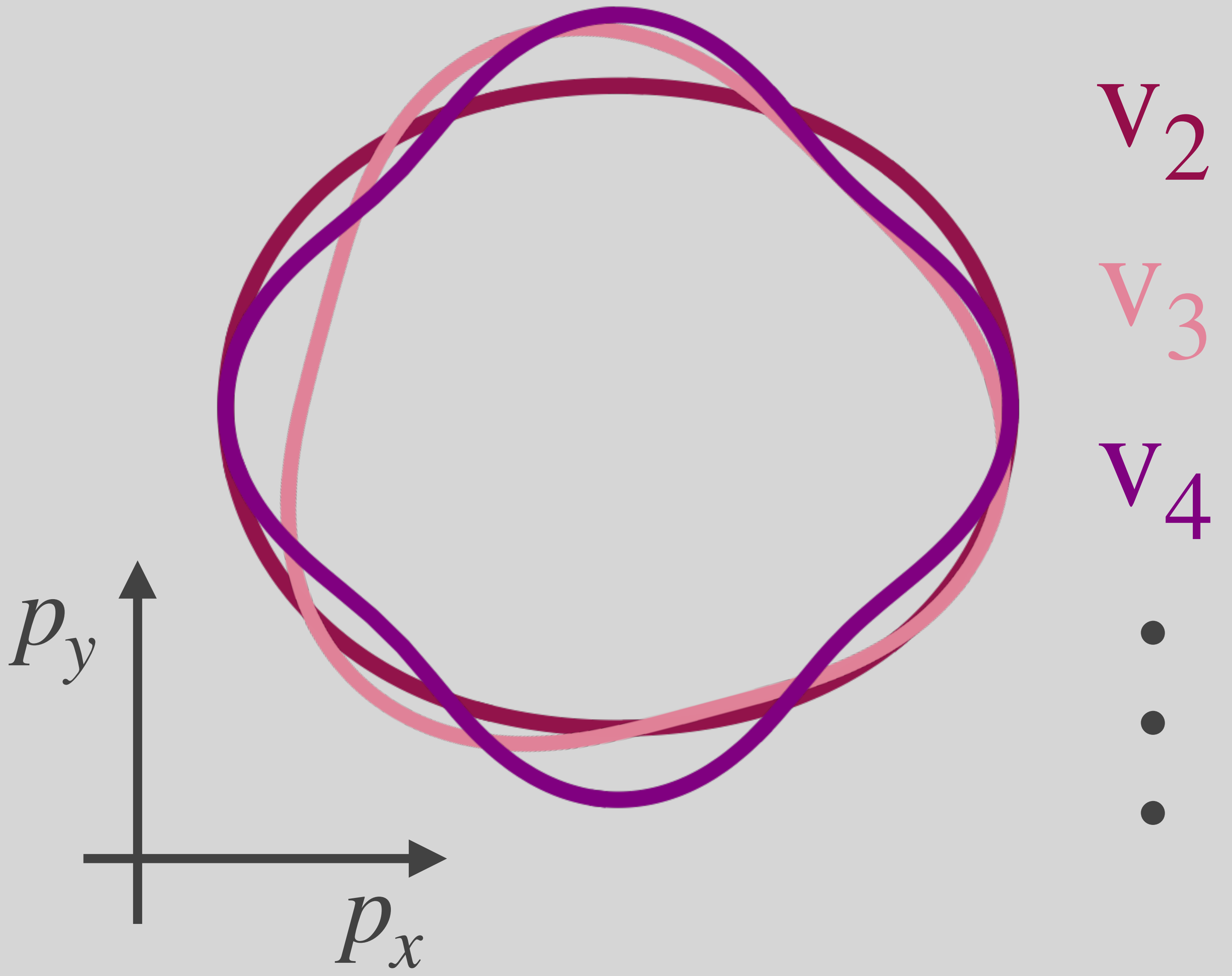


Theoretical description

~~**Hydrodynamics**~~
~~**Collective behaviour**~~
~~$N_{\mathrm{resc}} \gg 1$~~

Kinetic theory
Few collision regime
$N_{\mathrm{resc}} < 1$

Final state
Anisotropies
Momentum space



Reasons for kinetic theory approach

Small and dilute systems

Question to be answered

for our approaches

How do flow harmonics v_n evolve in time with dependence on the various initial eccentricities ϵ_m?


Methods

Analytical approach

to calculate time-dependent v_n

Analytical model

Calculate time-dependent anisotropic flow coefficients

$$v_n\left(t, p_\perp\right) = \frac{\iint \cos\left(n\left(\phi - \Psi_n\right)\right) f\left(t, \mathbf{x}, p_\perp, \phi\right)\, d^2\mathbf{x}\, d\phi}{\iint f\left(t, \mathbf{x}, p_\perp, \phi\right)\, d^2\mathbf{x}\, d\phi}$$

Analytical approach

to calculate time-dependent v_n

Analytical model

↓

Loss term in Boltzmann equation

- Calculate time-dependent AFC

$$v_n(t, p_\perp) = \frac{\iint \cos\left(n\left(\phi - \Psi_n\right)\right) f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}{\iint f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}$$

Use classical relativistic Boltzmann equation as equation of motion

$$\left[\partial_t + \mathbf{v} \cdot \nabla\right] f\left(t, \mathbf{x}, \mathbf{p}\right) = C_{coll}\left[f\left(t, \mathbf{x}, \mathbf{p}\right)\right]$$

with loss term of 2-to-2 elastic collision kernel

Analytical approach

to calculate time-dependent v_n

- Calculate time-dependent AFC

$$\mathrm{v}_n\left(t,p_\perp\right) = \frac{\iint \cos\left(n\left(\phi - \Psi_n\right)\right) f\left(t,\mathbf{x},p_\perp,\phi\right) d^2\mathbf{x}\, d\phi}{\iint f\left(t,\mathbf{x},p_\perp,\phi\right) d^2\mathbf{x}\, d\phi}$$

- Use

$\left[\partial_t\right.$

wit

Analytical model

Loss term in Boltzmann equation

$\mathcal{O}\left(N_{\mathrm{resc}}\right) = 1$

Few collision limit

Dependence on free-streaming distribution function

$$f^{(0)}\left(t,\mathbf{x},\mathbf{p}\right) = f^{(0)}\left(0,\mathbf{x}-t\mathbf{v},\mathbf{p}\right)$$

Analytical approach

to calculate time-dependent v_n

Analytical model

↓

Loss term in Boltzmann equation

↓

$\mathcal{O}\left(N_{\text{resc}}\right) = 1$

- Calculate time-dependent AFC

$$v_n\left(t, p_\perp\right) = \frac{\iint \cos\left(n\left(\phi - \Psi_n\right)\right) f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}{\iint f\left(t, \mathbf{x}, p_\perp, \phi\right) d^2\mathbf{x}\, d\phi}$$

- Use classical relativistic Boltzmann equation as equation of motion

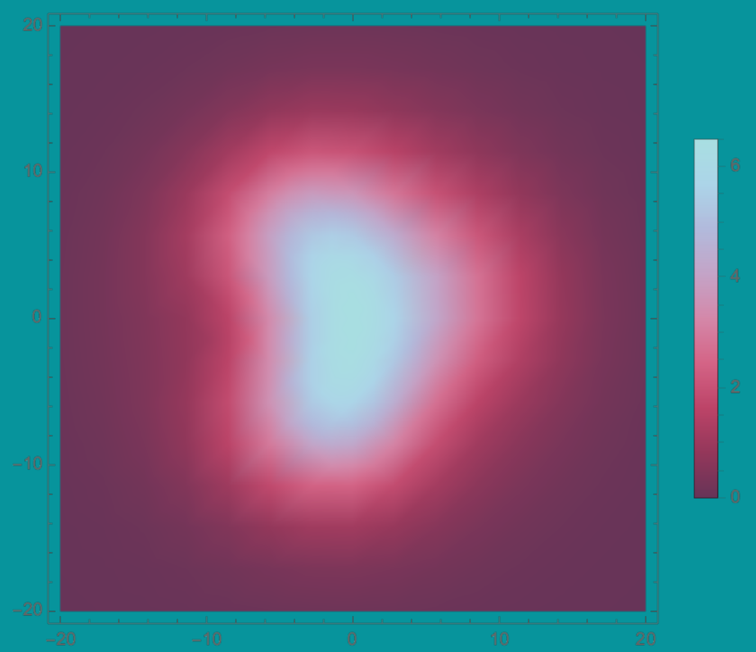
$$\left[\partial_t + \mathbf{v} \cdot \nabla\right] f\left(t, \mathbf{x}, \mathbf{p}\right) = C_{coll}\left[f\left(t, \mathbf{x}, \mathbf{p}\right)\right]$$

with lo

- Few co

- Depen

Initial distribution function (including initial eccentricities) as input for my calculation



Analytical approach

to calculate time-dependent v_n

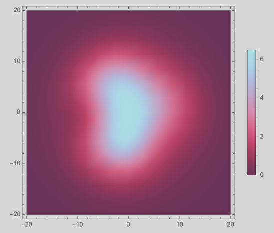
- Calculate time-dependent AFC

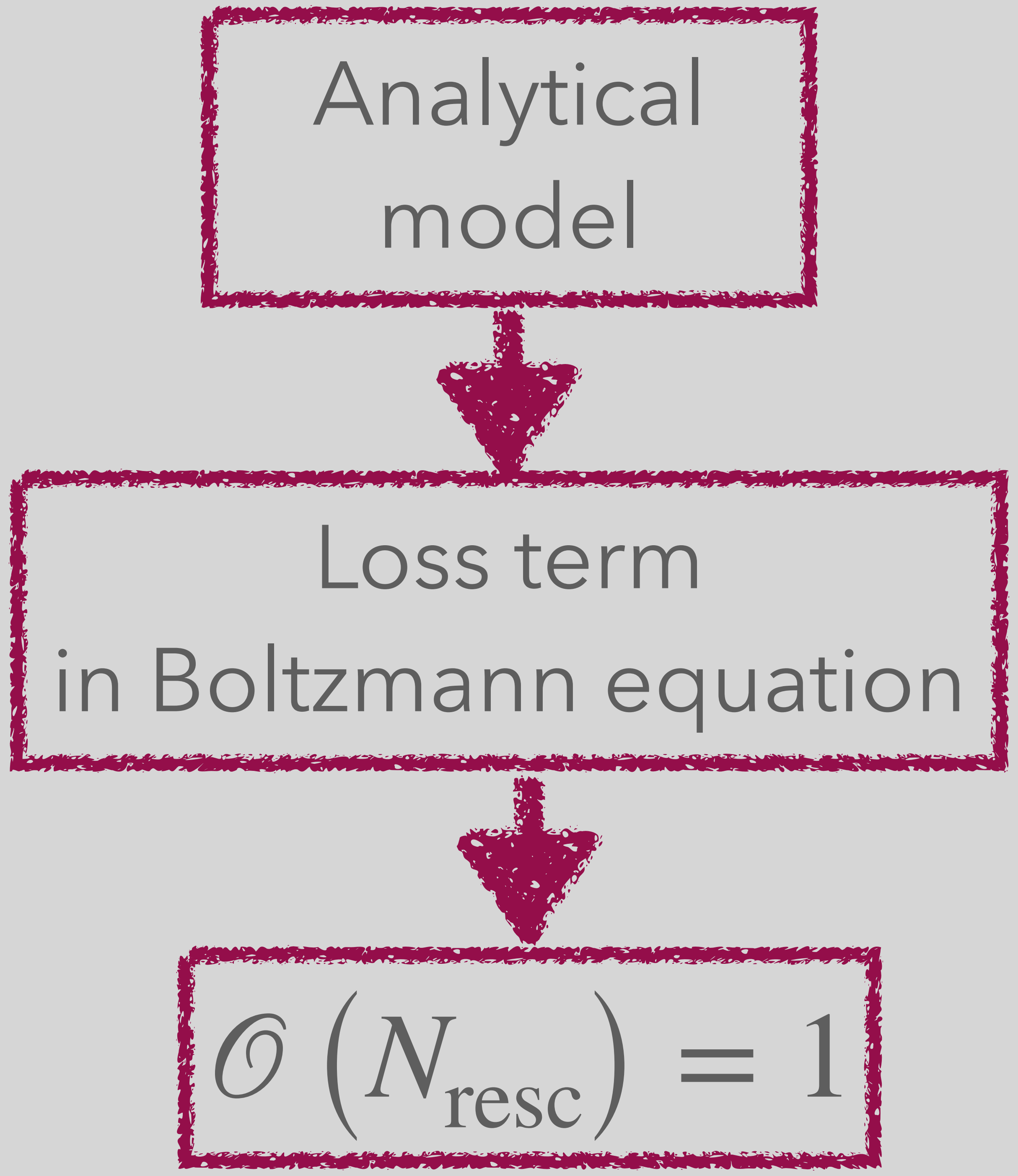
$$v_n(t, p_\perp) = \frac{\iint \cos\big(n(\phi - \Psi_n)\big) f(t, \mathbf{x}, p_\perp, \phi)\, d^2\mathbf{x}\, d\phi}{\iint f(t, \mathbf{x}, p_\perp, \phi)\, d^2\mathbf{x}\, d\phi}$$

- Use classical relativistic Boltzmann equation as equation of motion

$$\left[\partial_t + \mathbf{v} \cdot \nabla\right] f(t, \mathbf{x}, \mathbf{p}) = C_{coll}\left[f(t, \mathbf{x}, \mathbf{p})\right]$$

  with loss term of 2-to-2 elastic collision kernel

- Few collision limit
- Dependence on free-streaming distribution function $f^{(0)}(t, \mathbf{x}, \mathbf{p}) = f^{(0)}(0, \mathbf{x} - t\mathbf{v}, \mathbf{p})$
- Initial distribution function (including initial eccentricities) as input for my calculation 



Our approaches to calculate v_n

- Kinetic theory
- 2 dimensional
- Massless particles

Analytical model

↓

Loss term in Boltzmann equation

↓

$\mathcal{O}\left(N_{\mathrm{resc}}\right) = 1$

Numerical simulations

↓ ↓

$2 \to 0$ scatterings | $2 \to 2$ scatterings

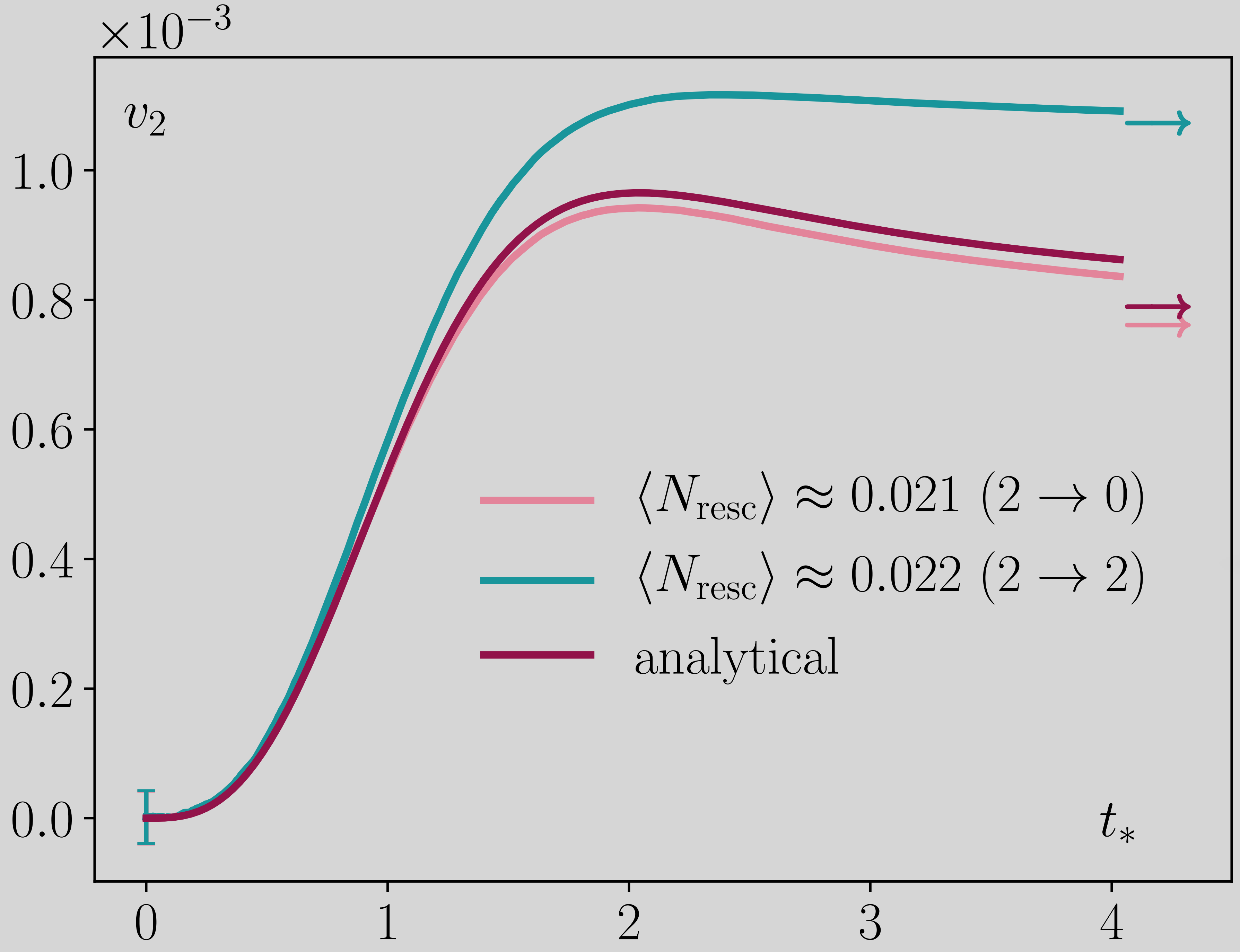
↓ ↓

$\mathcal{O}\left(N_{\mathrm{resc}}\right) \to \infty$

Hendrik Roch
Poster in session T

Results

v_2 from our approaches



Loss term dominates signal for $N_{resc} \lesssim 0.35$

Higher orders in N_{resc} can be neglected in few collision regime

Deviation increases with growing N_{resc}

Good agreement for small N_{resc}

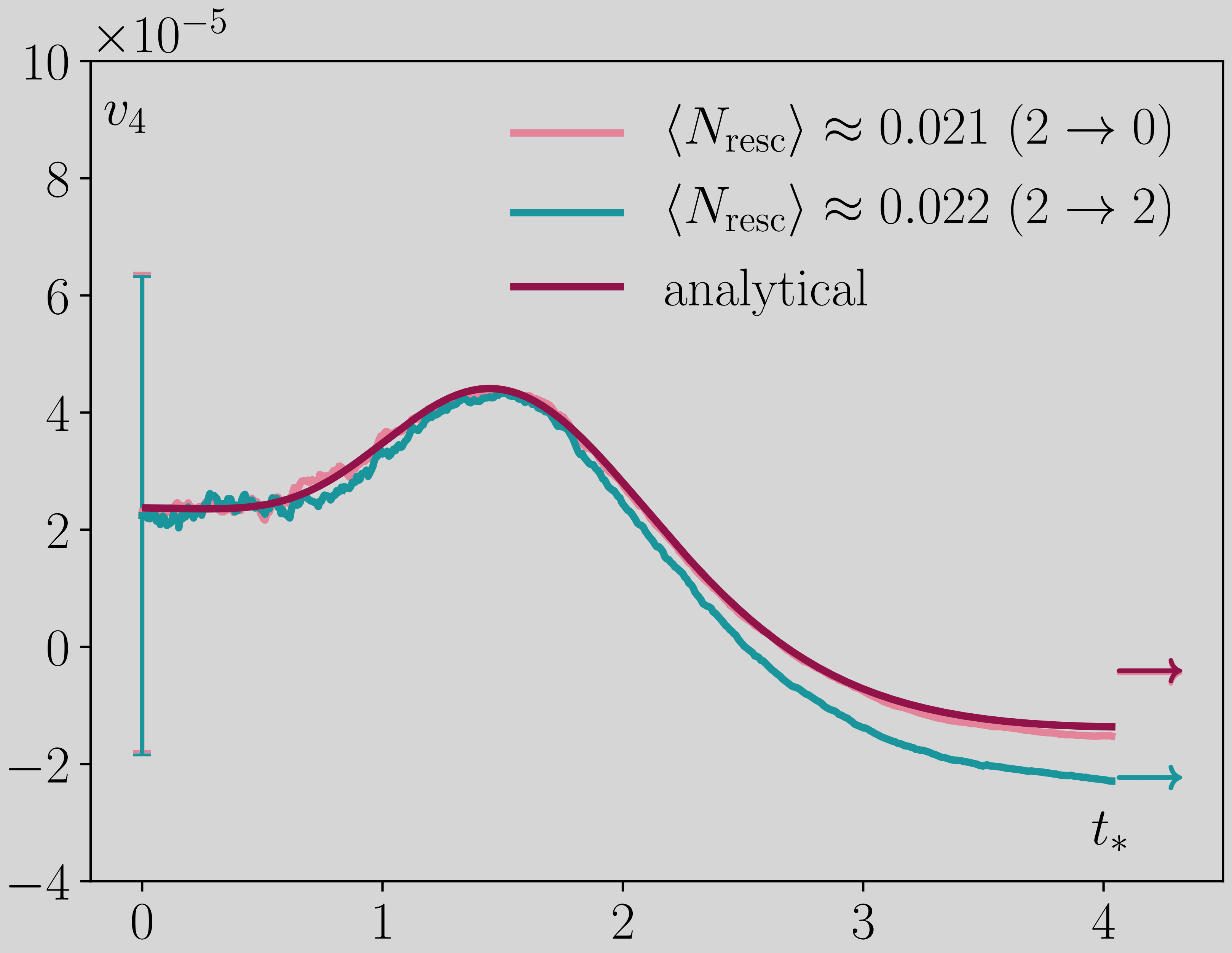
Expansion around small times $t \Rightarrow v_2 \propto \epsilon_2 \, t^3$

- Analytical formula (with $t_* \equiv t/R$)

$$v_2(t_*) = \frac{64\sqrt{\pi}\, N_{resc}\, \epsilon_2}{27\left(8 + 3\sqrt{2}\, \epsilon_2^2\right)} e^{-\frac{2\, t_*^2}{3}} \left[-t_*\, I_0\left(\frac{2\, t_*^2}{3}\right) + \left(2\, t_* + \frac{3}{t_*}\right)\, I_1\left(\frac{2\, t_*^2}{3}\right) \right]$$

v_4 from our approaches

with dependence on ϵ_4



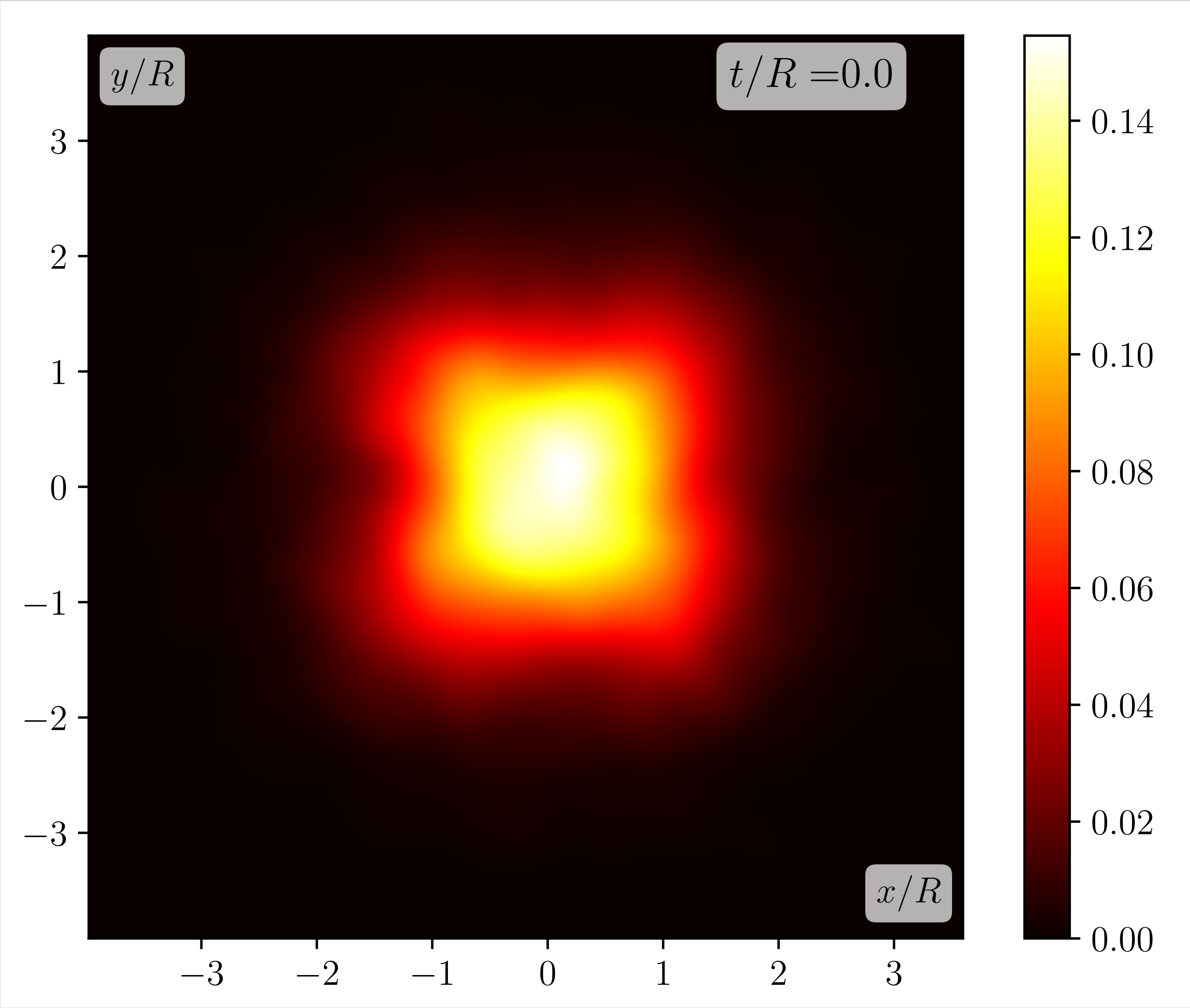
Good agreement for small $N_{\rm resc}$

Expansion around small times t
$\Rightarrow \mathrm{v}_4 \propto \epsilon_4\, t^5$

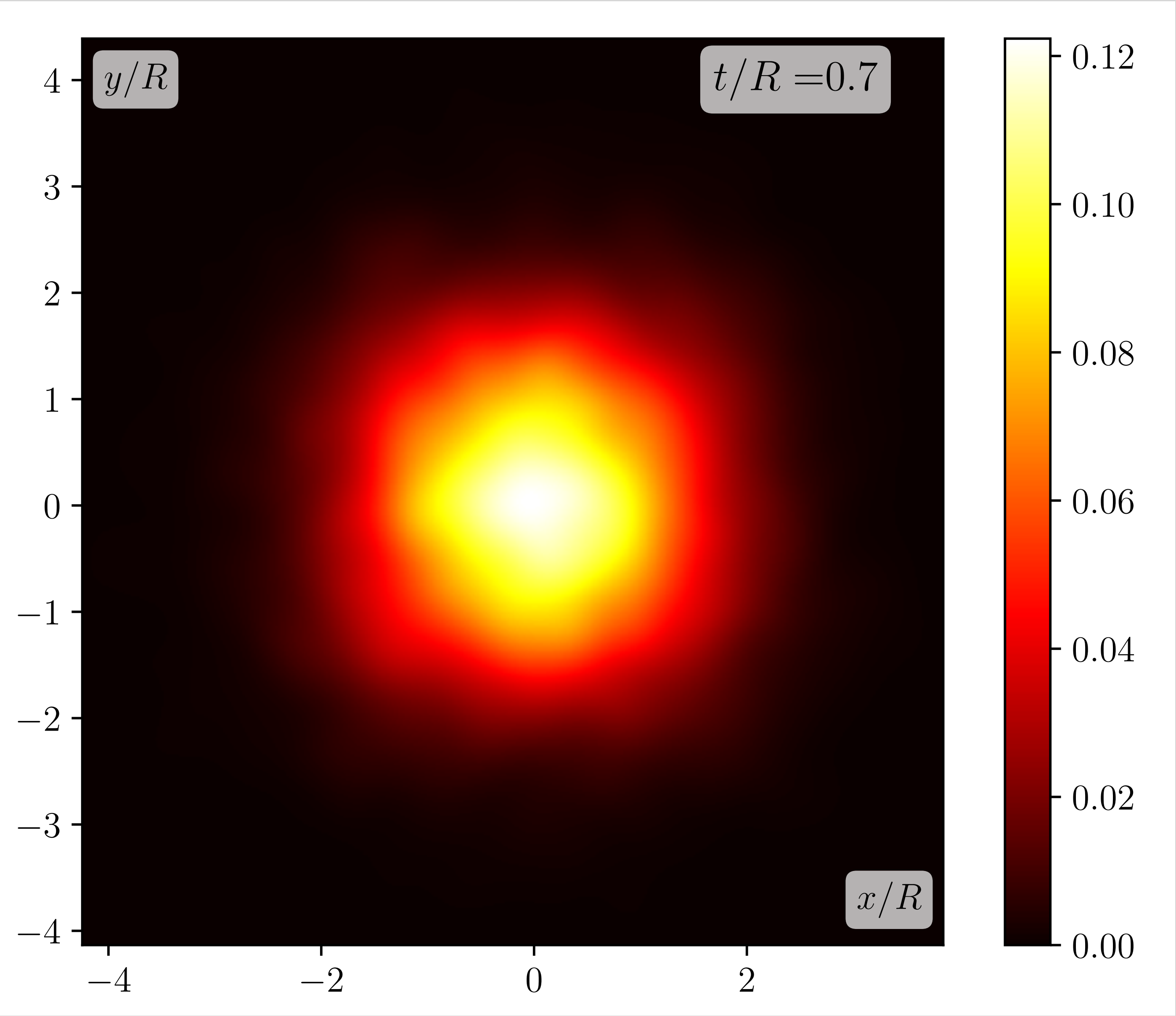
- Analytical formula (with $t_* \equiv t/R$)

$$\mathrm{v}_4(t_*) = \frac{2048\sqrt{\pi}\, N_{\rm resc}\,\epsilon_4}{405\left(384 + 35\sqrt{2}\,\epsilon_4^2\right)} e^{-\frac{2t_*^2}{3}} \left[-\left(5\,t_*^3 + 21\,t_* + \frac{54}{t_*}\right) I_0\!\left(\frac{2\,t_*^2}{3}\right) + \left(5\,t_*^3 + 24\,t_* + \frac{63}{t_*} + \frac{162}{t_*^3}\right) I_1\!\left(\frac{2\,t_*^2}{3}\right)\right]$$

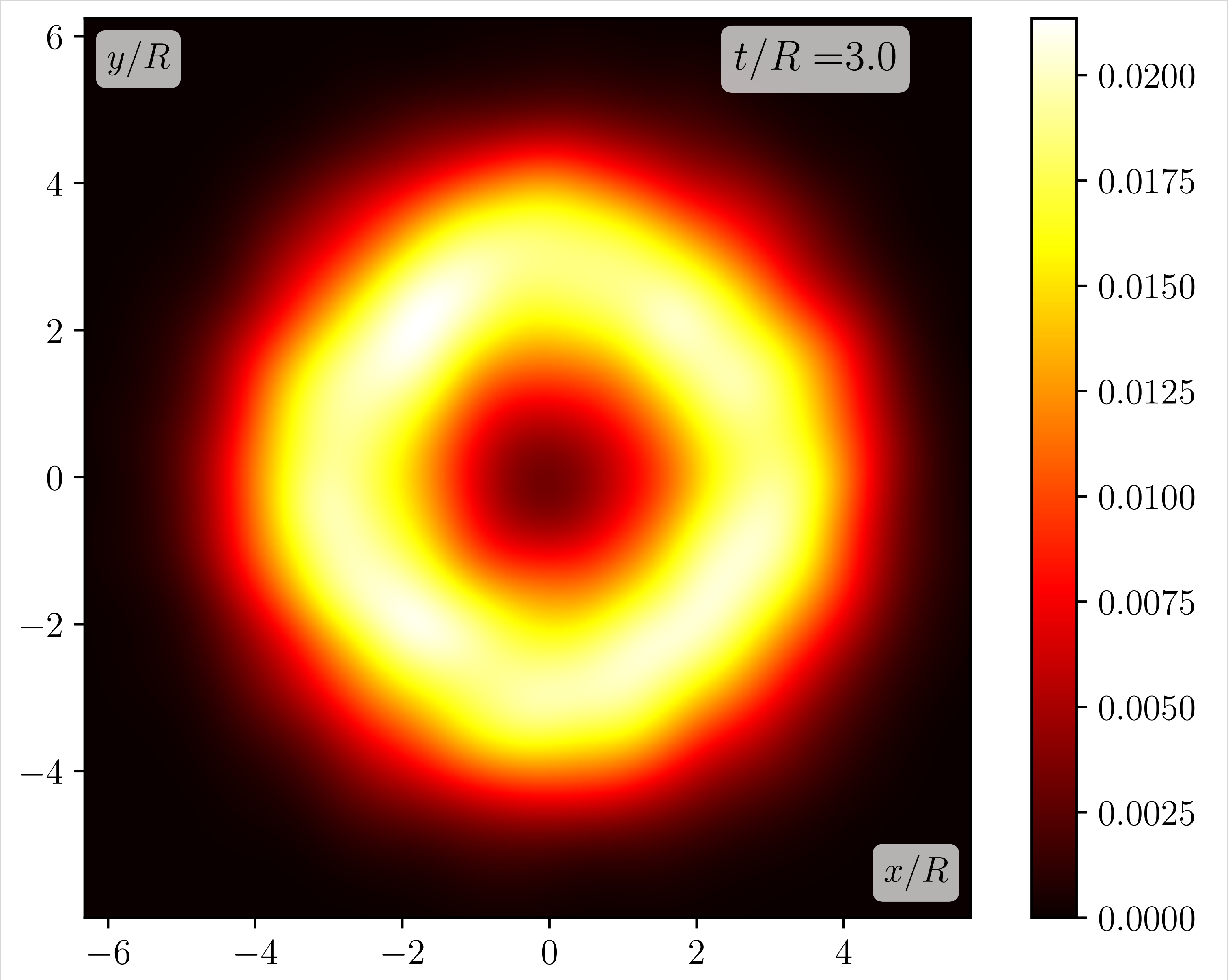
Density plots from numerical simulations



- Initial eccentricity ϵ_4 only

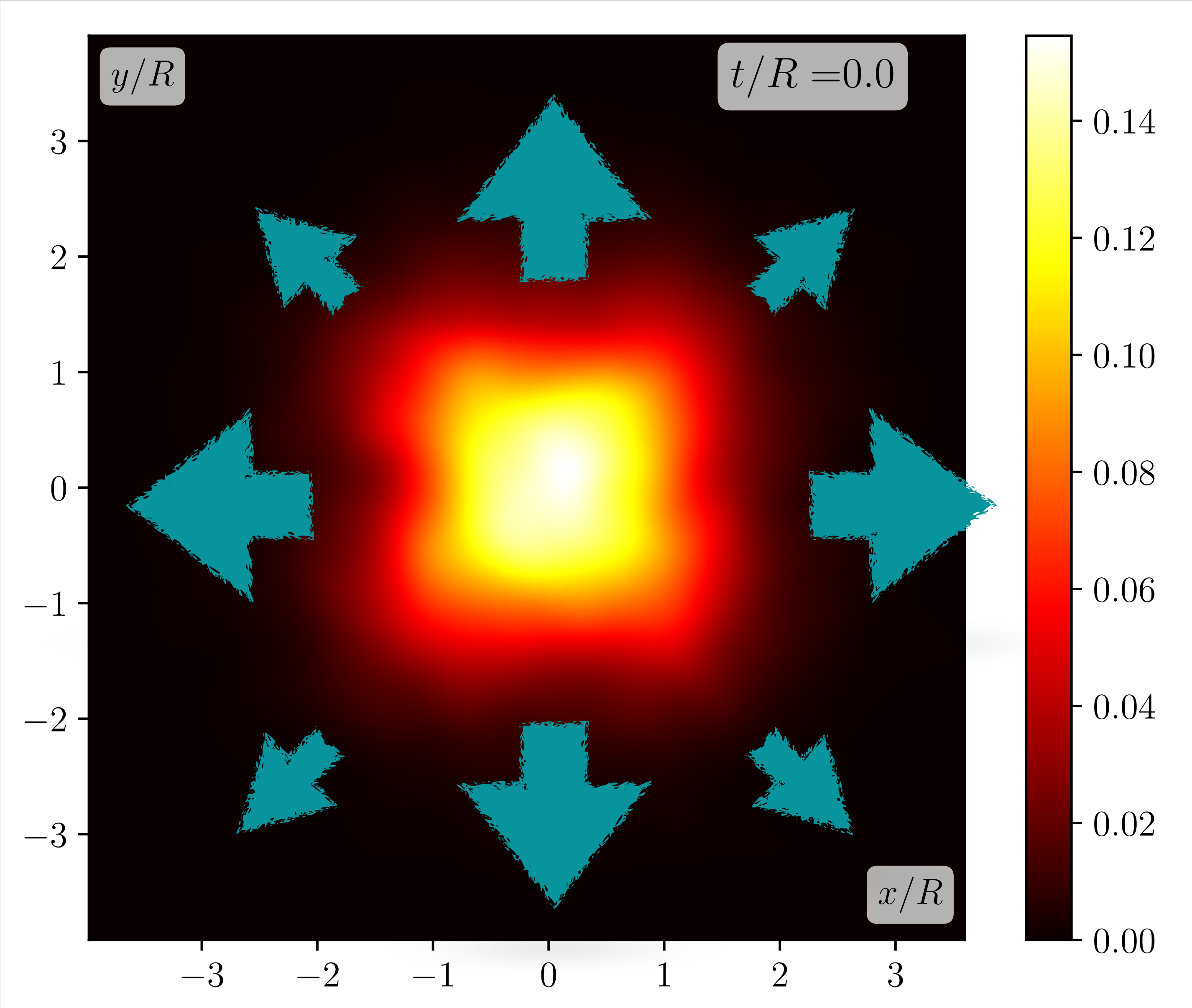


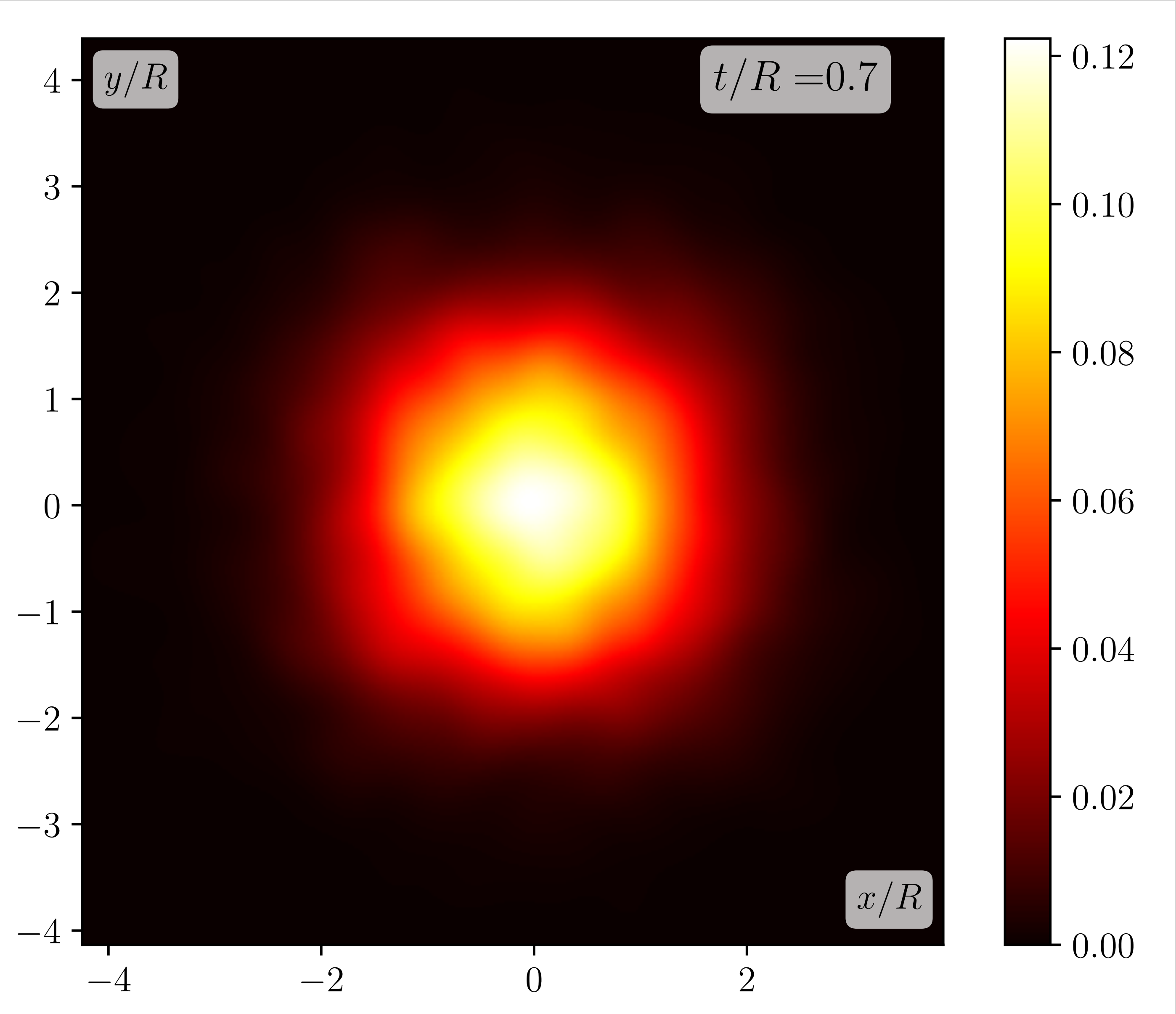
- $\pi/4$-rotated high density region ⇒ locally $\epsilon_4 < 0$
- Dilute regions ⇒ small contribution to v_4

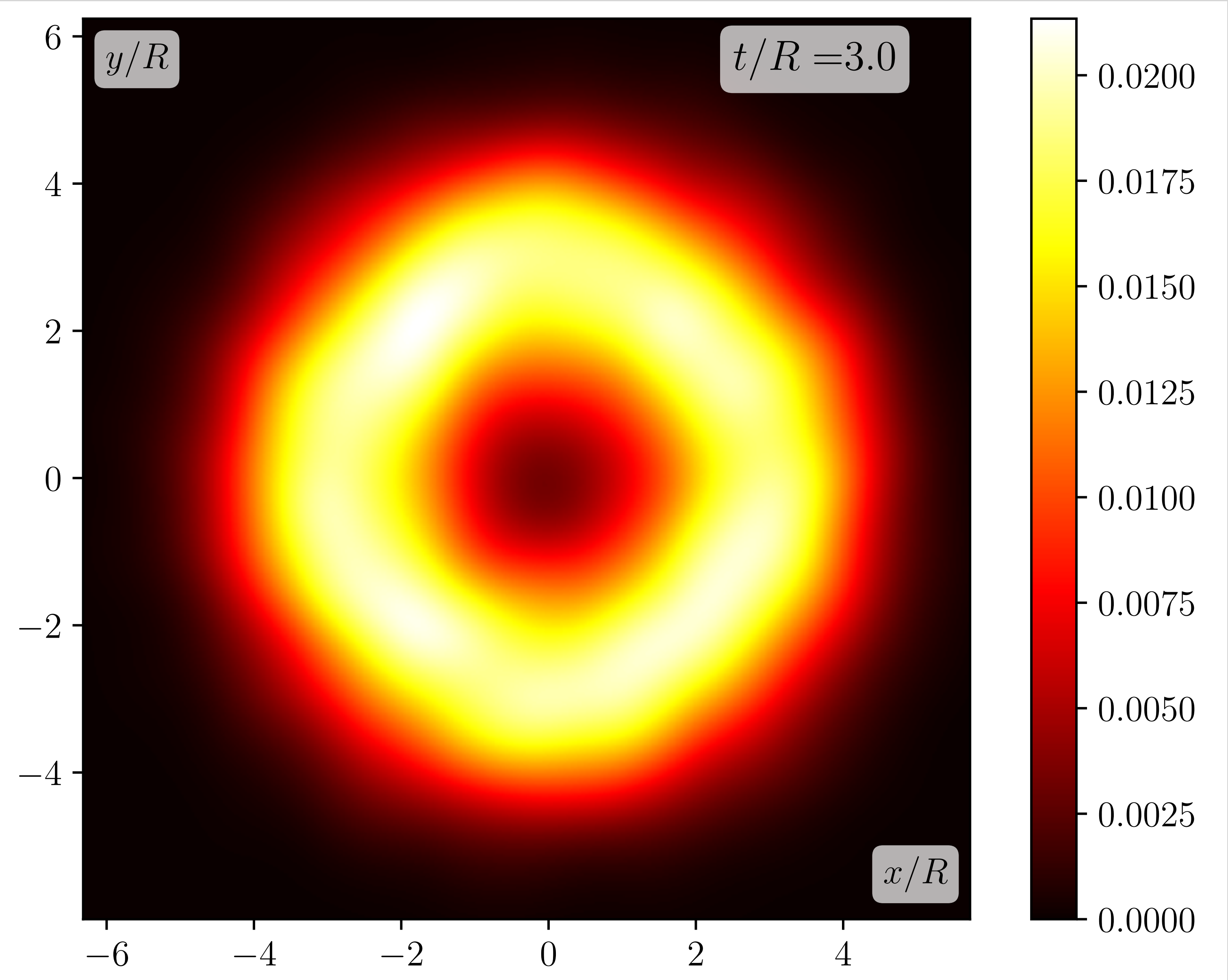


- More diluted system
- Denser regions still with $\epsilon_4 < 0$

Density plots from numerical simulations





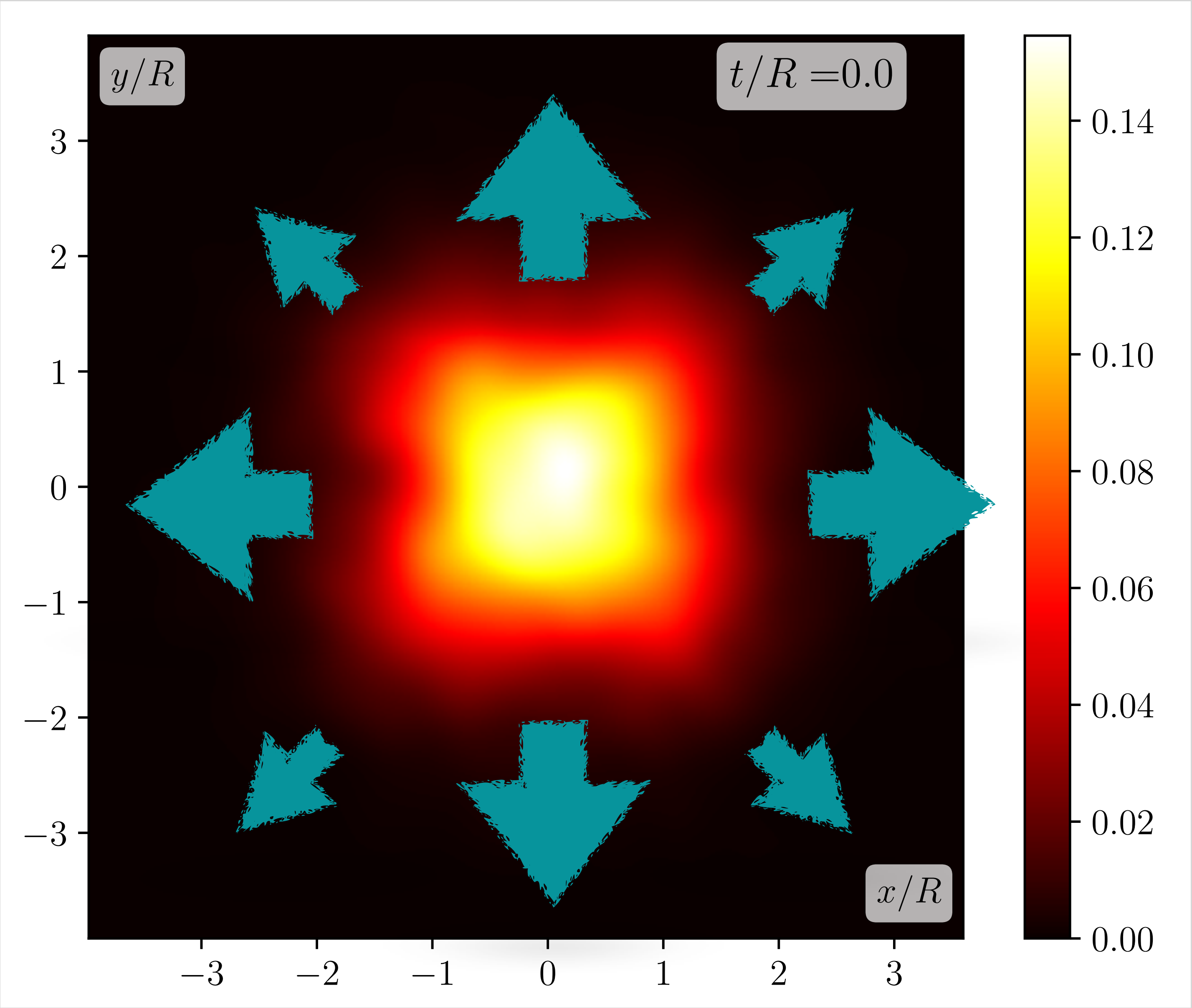


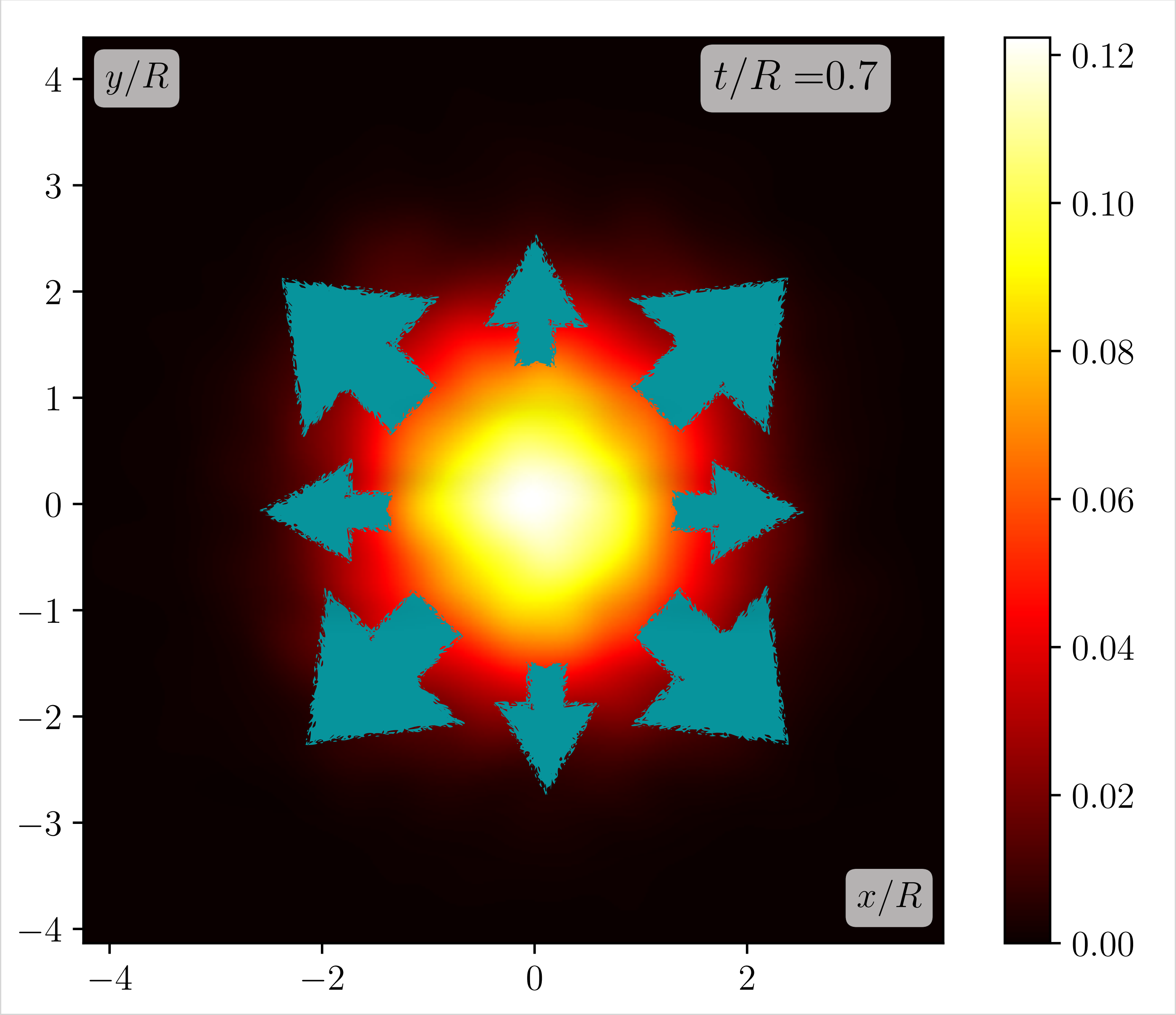
- Initial eccentricity ϵ_4 only

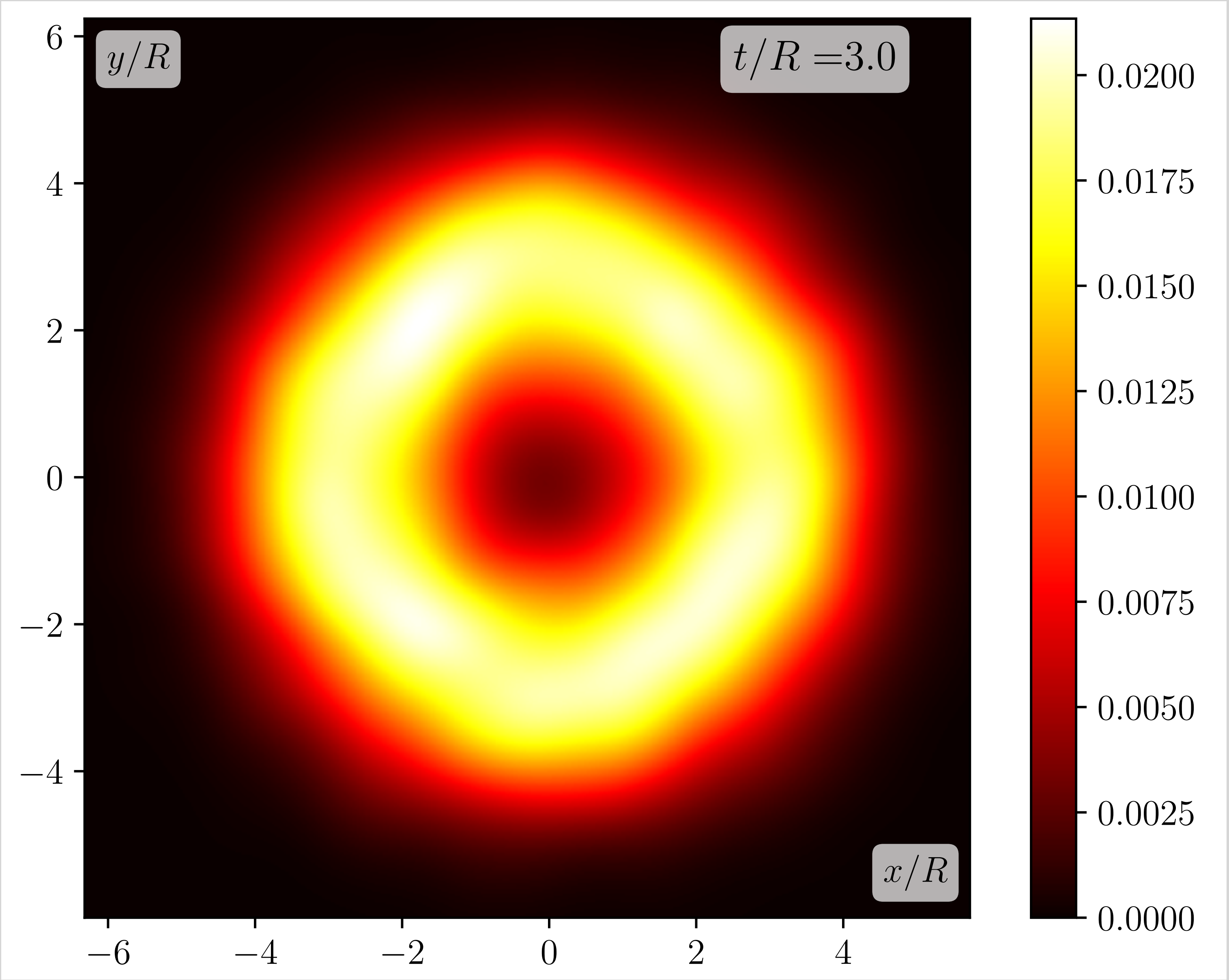
- $\pi/4$-rotated high density region ⇒ locally $\epsilon_4 < 0$
- Dilute regions ⇒ small contribution to v_4

- More diluted system
- Denser regions still with $\epsilon_4 < 0$

Density plots from numerical simulations





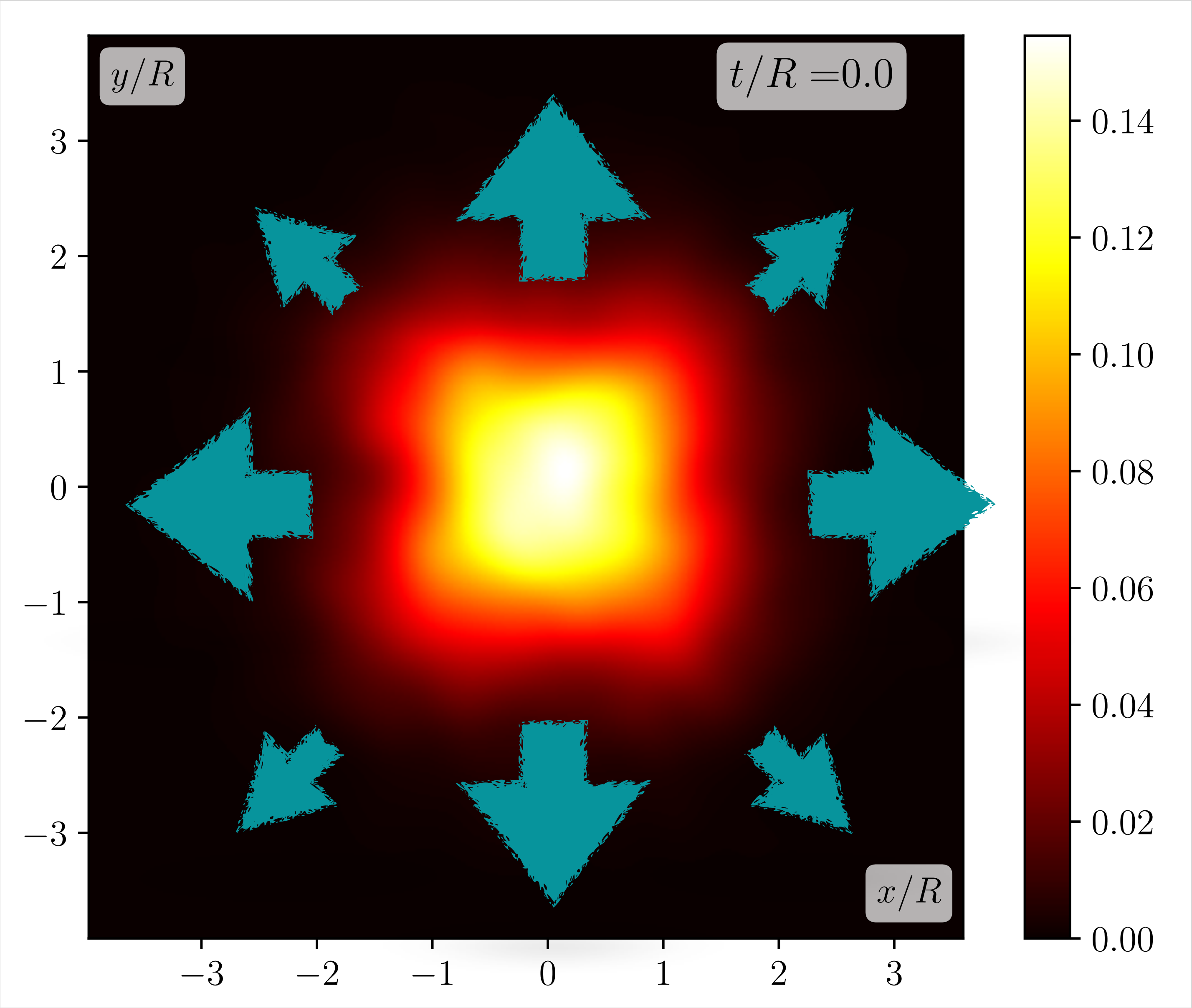


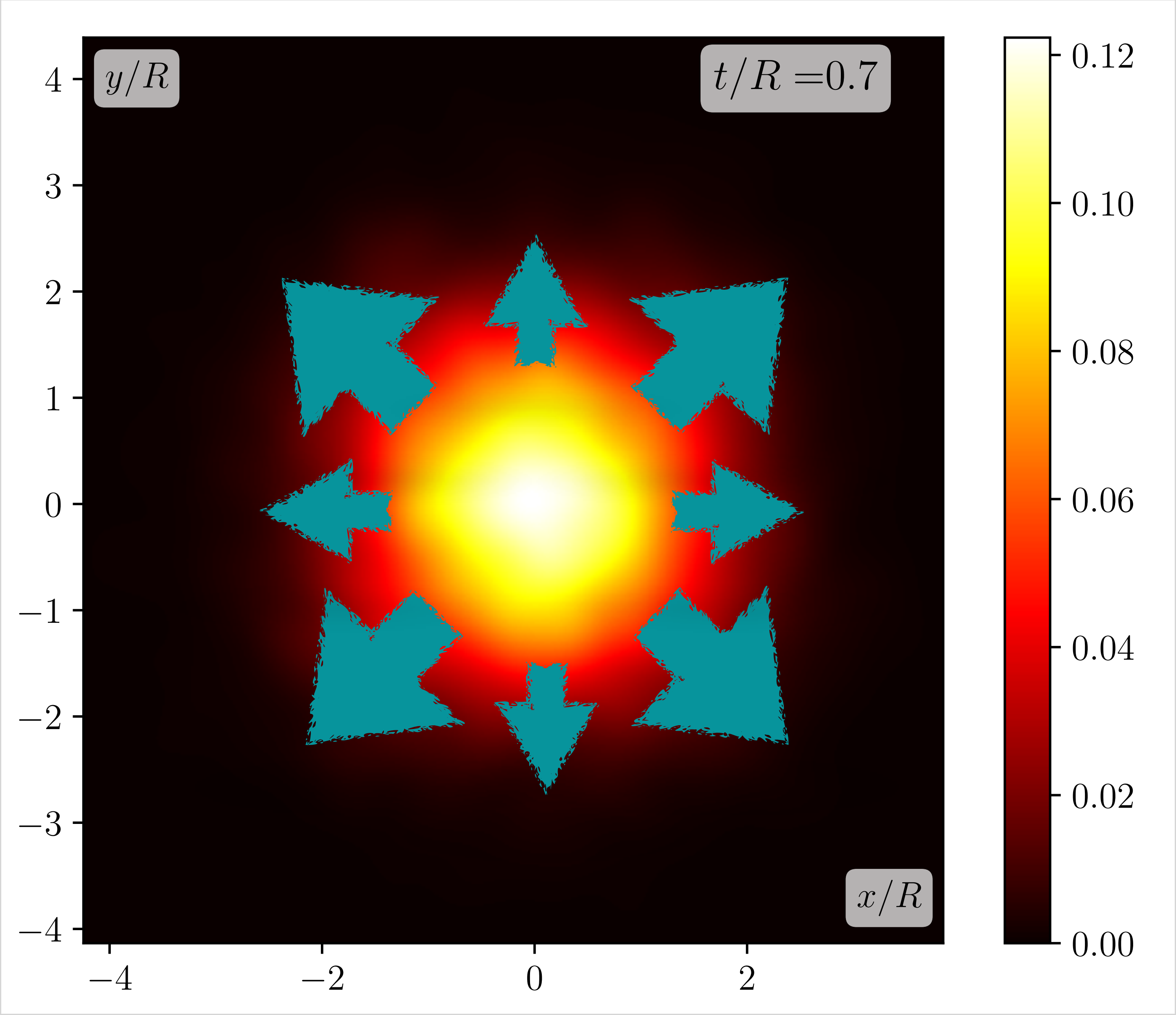
- Initial eccentricity ϵ_4 only

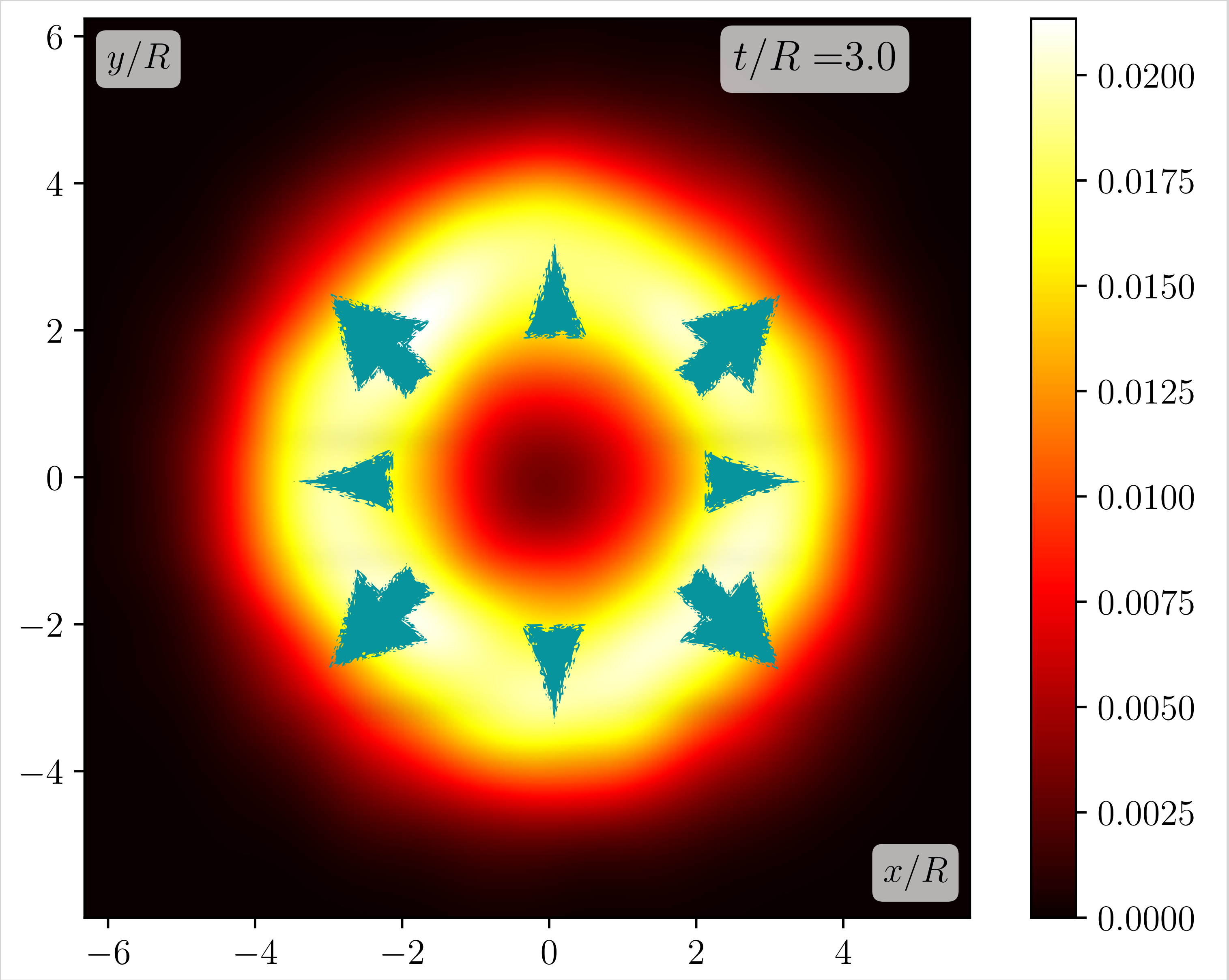
- $\pi/4$-rotated high density region $\Rightarrow$ locally $\epsilon_4 < 0$
- Dilute regions $\Rightarrow$ small contribution to v_4

- More diluted system
- Denser regions still with $\epsilon_4 < 0$

Density plots from numerical simulations





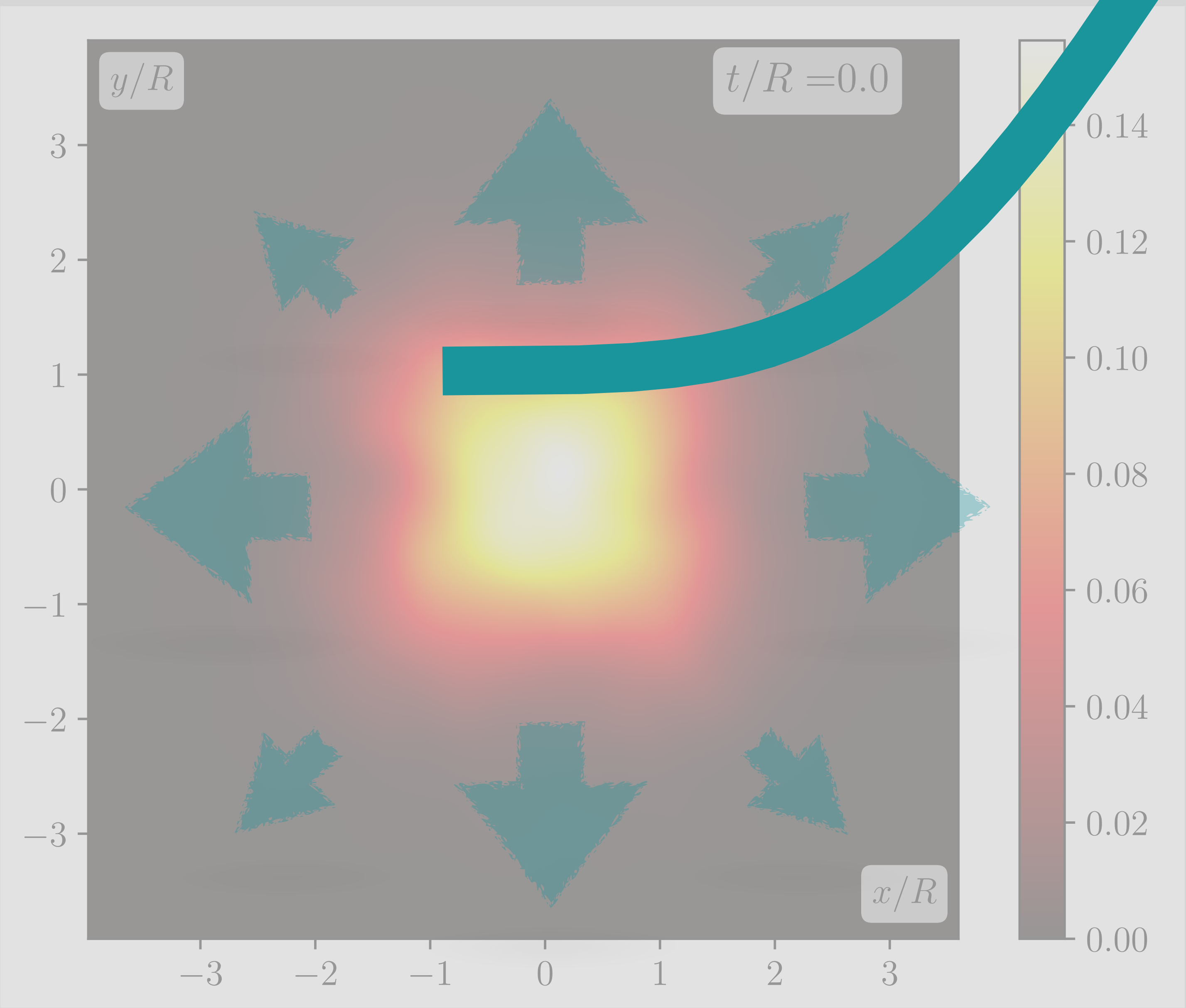


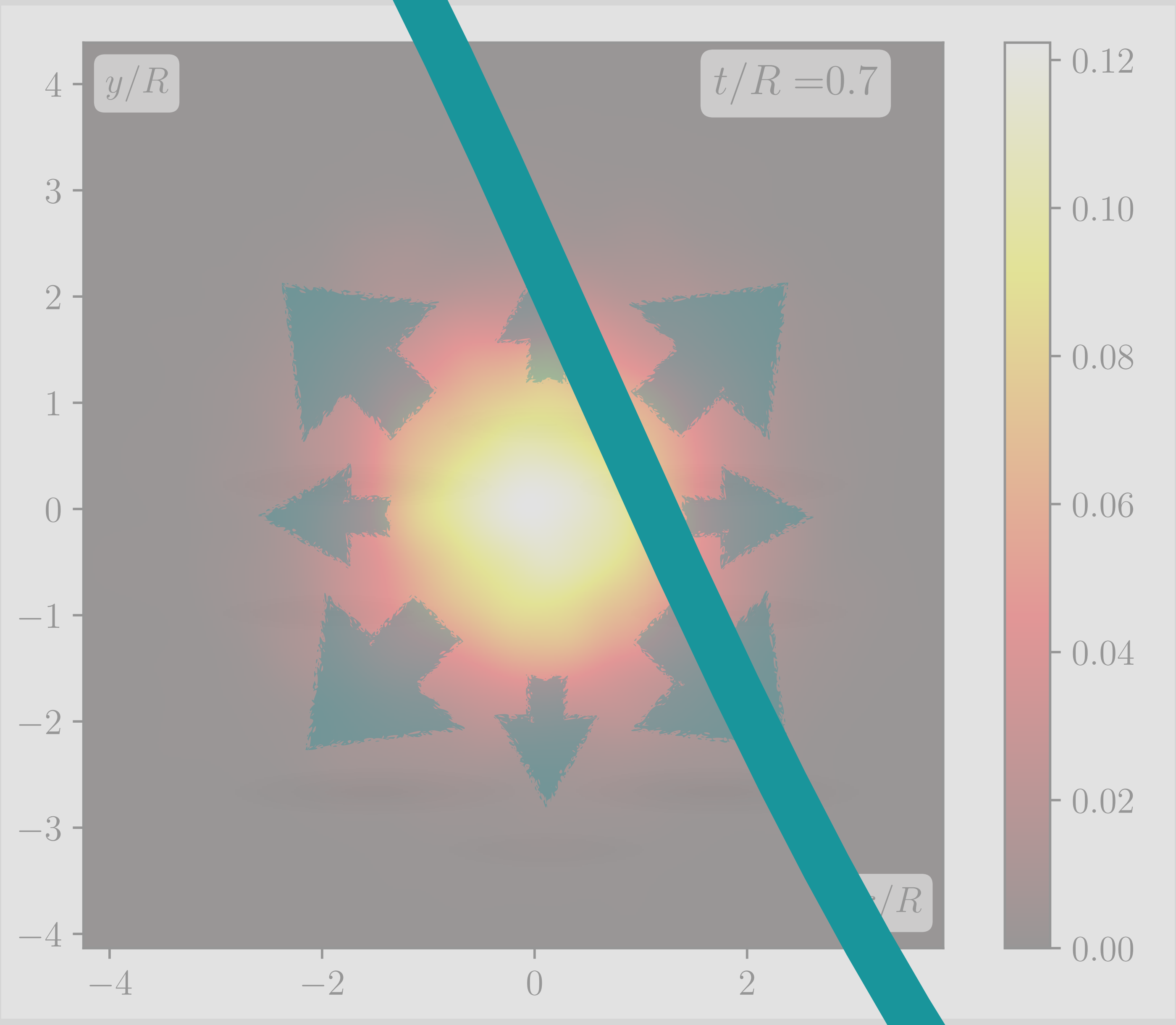
- Initial eccentricity ϵ_4 only

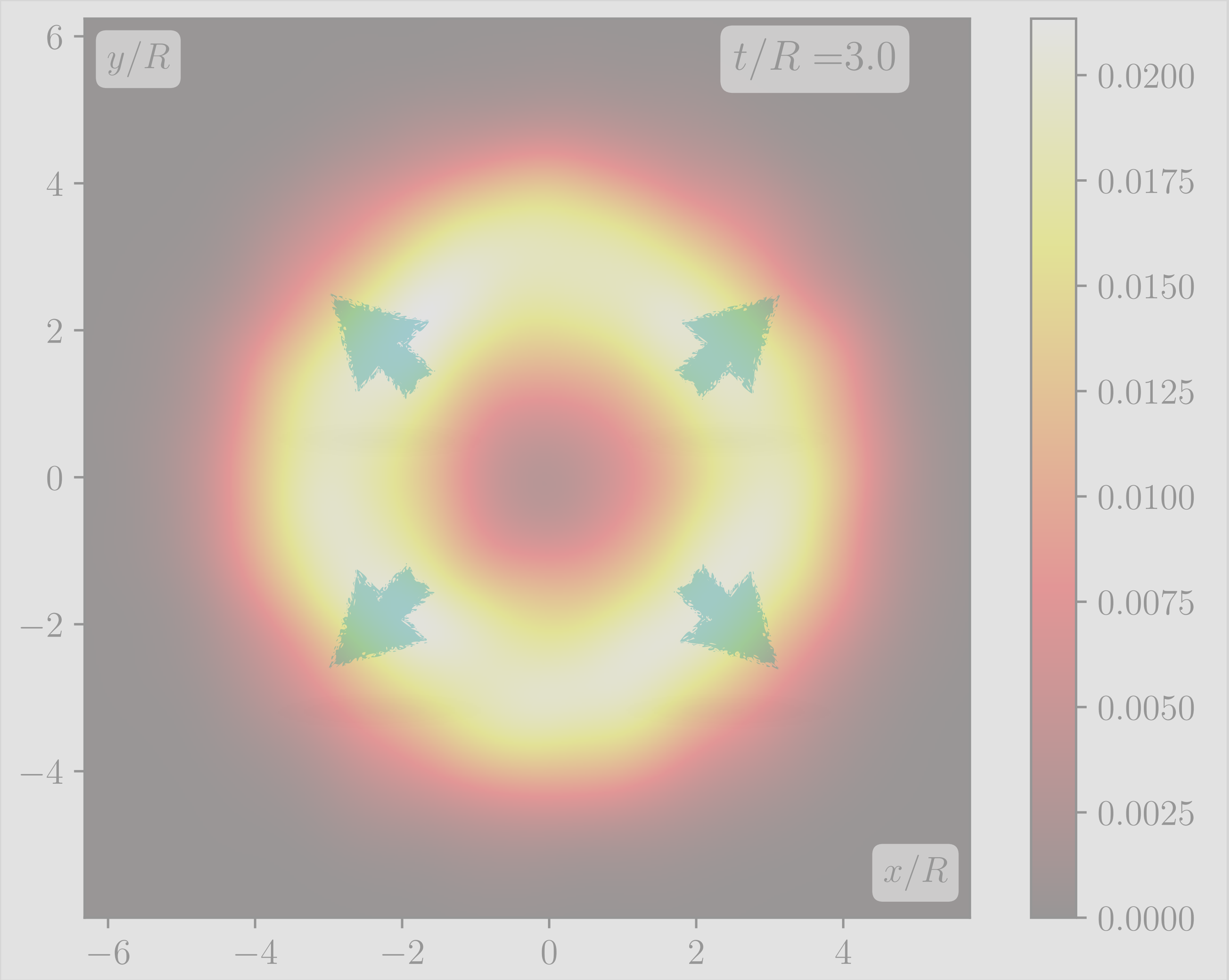
- $\pi/4$-rotated high density region $\Rightarrow$ locally $\epsilon_4 < 0$
- Dilute regions $\Rightarrow$ small contribution to v_4

- More diluted system
- Denser regions still with $\epsilon_4 < 0$

Density plots from numerical simulations







- Initial eccentricity ϵ_4 only

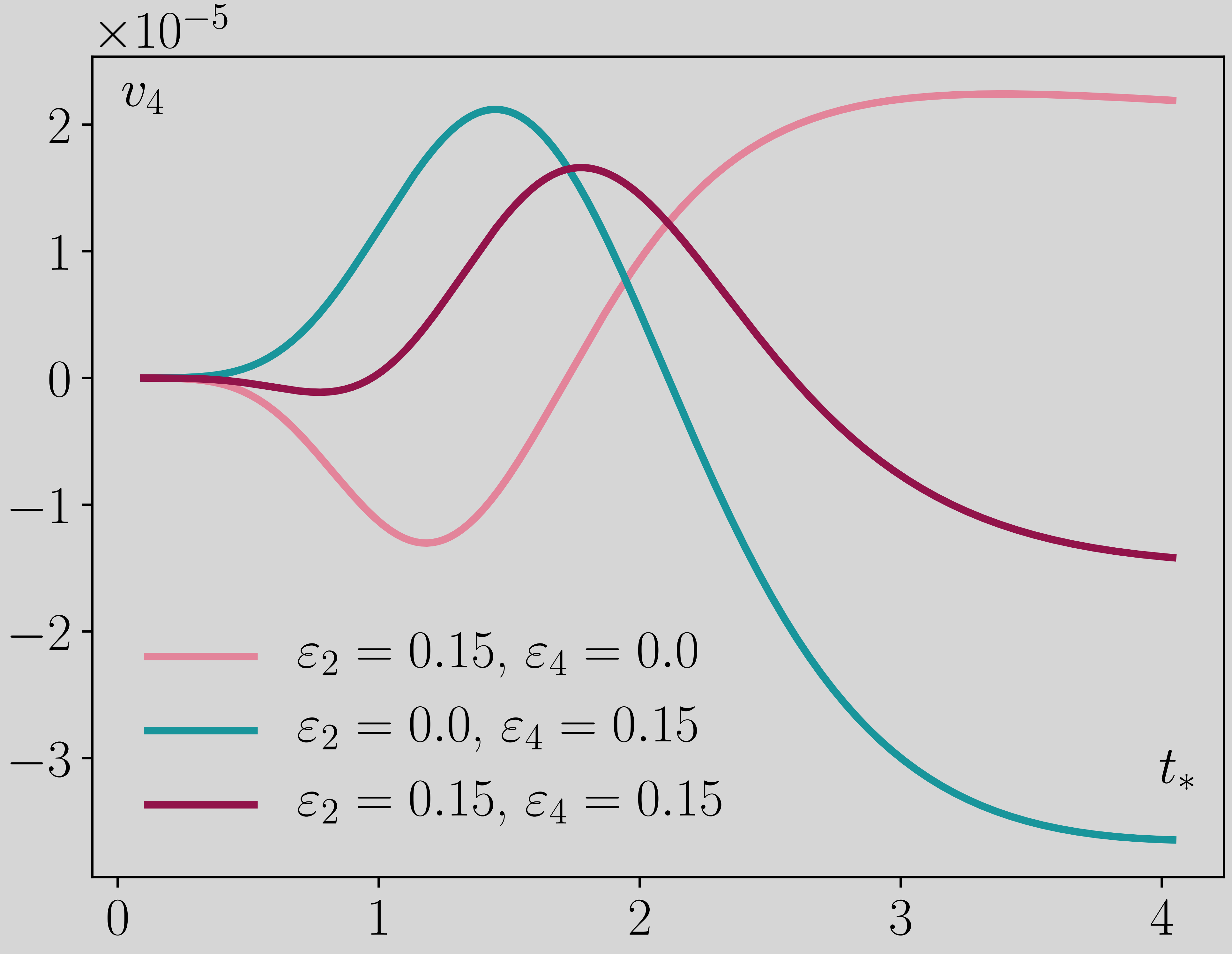
- $\pi/4$-rotated high density region $\Rightarrow$ locally $\epsilon_4 < 0$
- Dilute regions $\Rightarrow$ small contribution to v_4

- More diluted system
- Denser regions still with $\epsilon_4 < 0$

$v_4(t_*)$

v_4 from our analytical approach

with dependence on ϵ_2 and ϵ_4



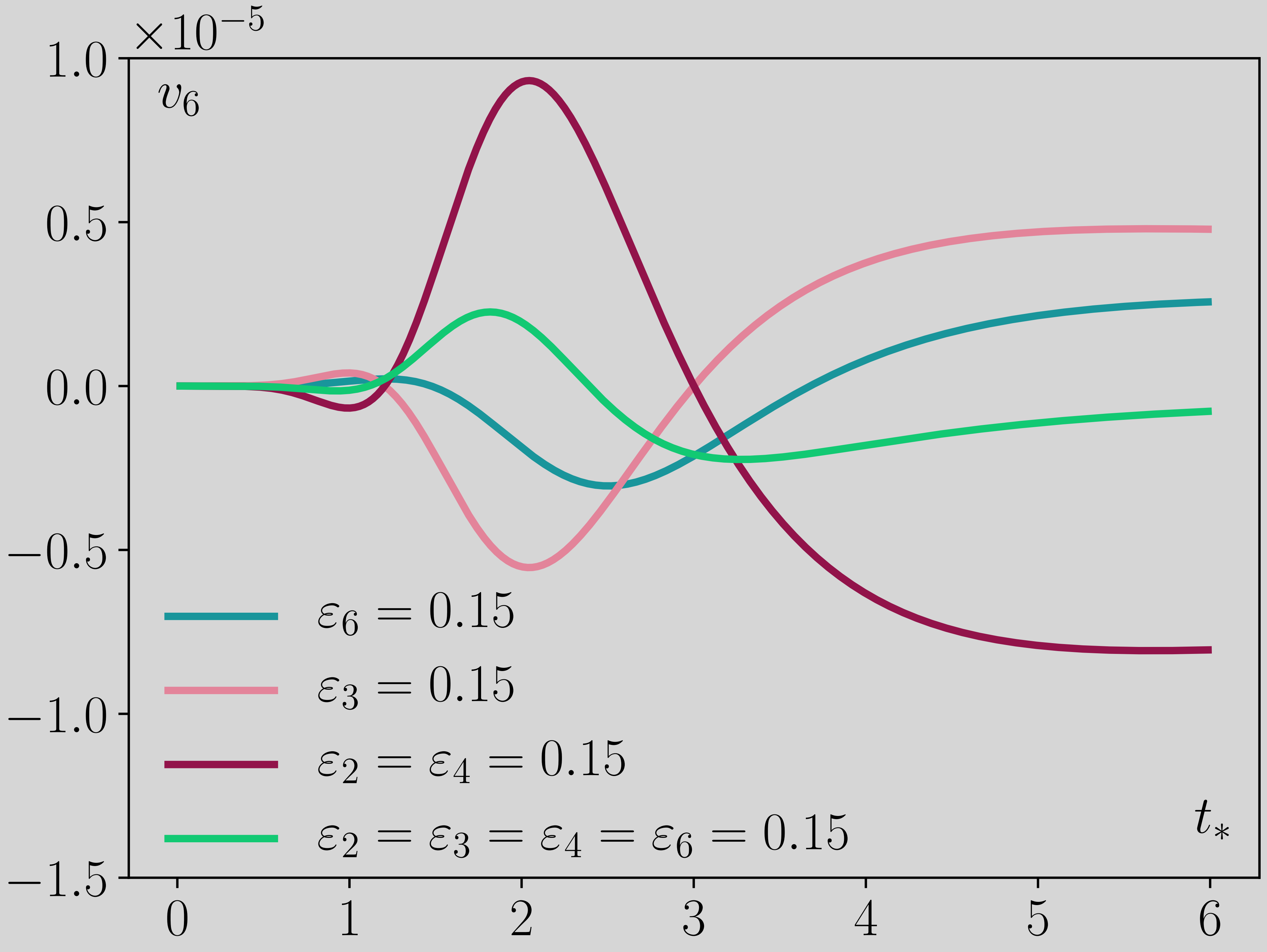
Two contributions from ϵ_4 and ϵ_2^2 to v_4

Oscillating v_4 with dependence on time t

Contributions compensate each other

v_6 from our analytical approach

with dependence on ϵ_2, ϵ_3, ϵ_4 and ϵ_6



- Three contributions from ϵ_6, ϵ_3^2 and $\epsilon_2\epsilon_4$ to v_6
- Oscillating v_6 with dependence on time t
- Additional zero compared to v_4 signal
- Contributions compensate each other

Summary

of our results on time-dependent anisotropic flow coefficients v_{2n} in few collision regime

- Loss term dominates the signal of anisotropic flow coefficients v_{2n}
- Rediscovery of $v_{2n} \propto t^{2n+1}$ for kinetic theory (expansion around small times t)
- Oscillation of v_{2n} with dependence on time
- Compensation of linear and non-linear eccentricity-contributions to higher flow harmonics v_{2n}

Outlook

- Further comparison with RTA-results obtained by our colleagues
- Extension to 3-dimensions and description of massiv particles

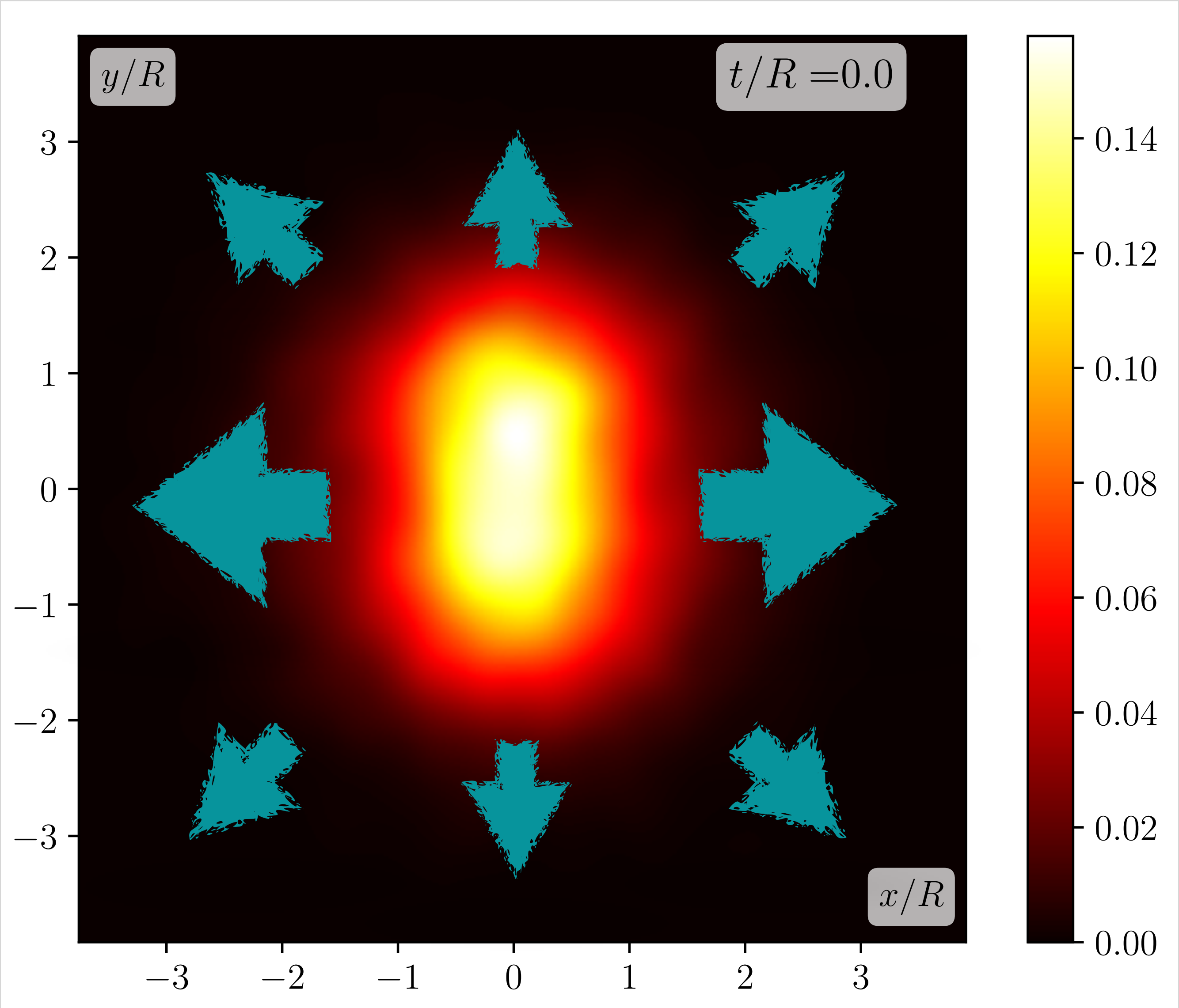
Outlook

- Further comparison with RTA-results obtained by our colleagues
- Extension to 3-dimensions and description of massiv particles

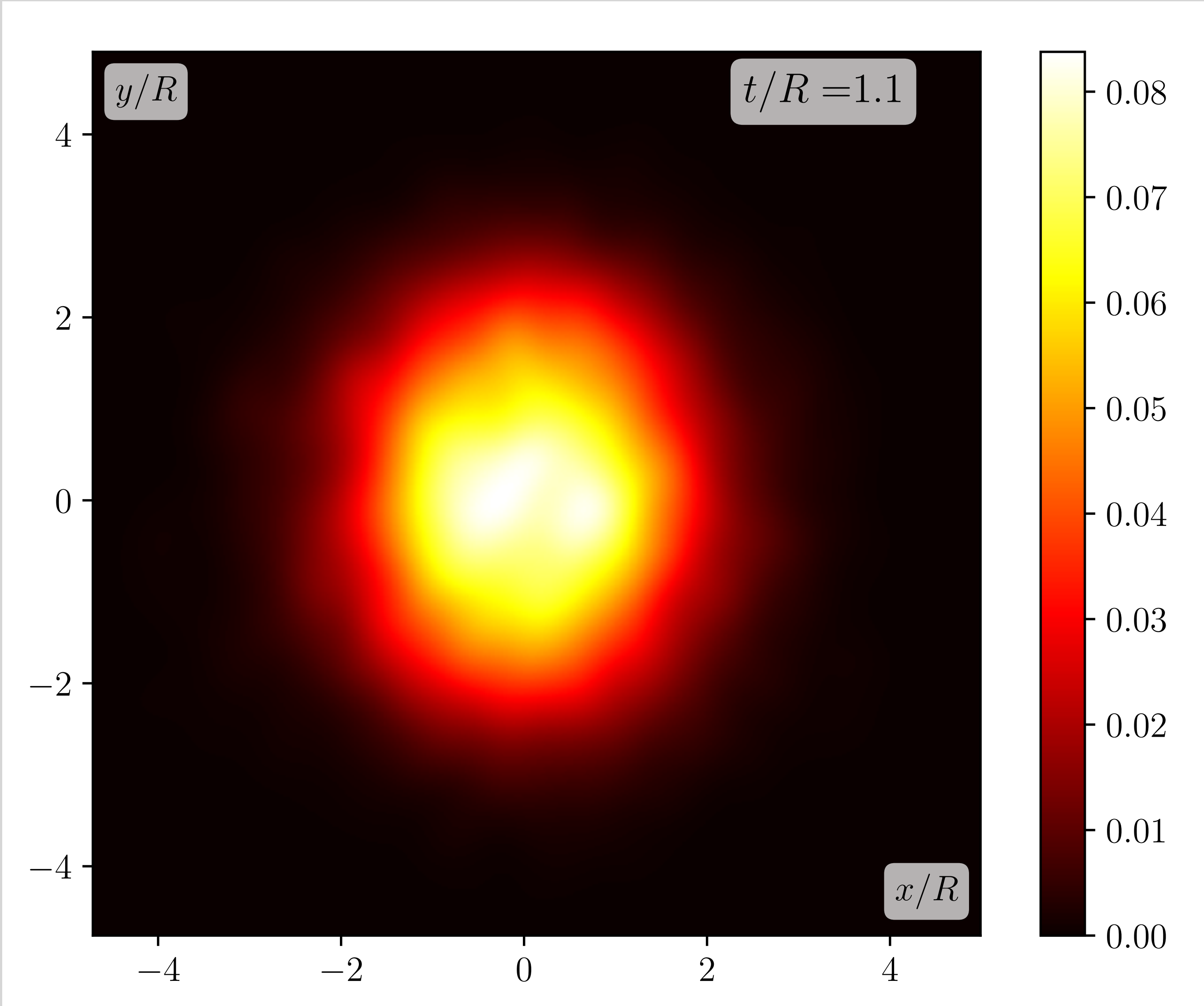
Thanks for your attention

Backup

Density plots from numerical simulations



$v_4(t_*)$



- Initial eccentricity ϵ_2 only
- Negative v_4 created

- Eccentricity $\epsilon_4 > 0$ created
- Leads to increase of v_4

Note: The v_2 signal is much stronger than the v_4 signal.
It is not easy to draw conclusions from this density plot.

AFC v_4 for our approaches

with dependence on ϵ_2 and ϵ_4

- Analytical formula

$$v_4(t_*) = \frac{64\sqrt{\pi}\, N_{\text{resc}}}{405\left(384 + 144\sqrt{2}\,\epsilon_2^2 + 35\sqrt{2}\,\epsilon_4^2\right)} e^{-t_*^2}$$

$$\left[243\,\epsilon_2^2\left(\left(5t_*^3 + 14t_* + \frac{24}{t_*}\right) I_0\left(t_*^2\right) - \left(5t_*^3 + 16t_* + \frac{28}{t_*} + \frac{48}{t_*^3}\right)\right) I_1\left(t_*^2\right)\right.$$

$$\left. + 32\epsilon_4 e^{\frac{t_*^2}{3}}\left(-\left(5t_*^3 + 21t_* + \frac{54}{t_*}\right) I_0\left(\frac{2\,t_*^2}{3}\right) + \left(5t_*^3 + 24t_* + \frac{63}{t_*} + \frac{162}{t_*^3}\right) I_1\left(\frac{2\,t_*^2}{3}\right)\right)\right]$$

AFC v_4 for our approaches

with dependence on ϵ_2 and ϵ_4

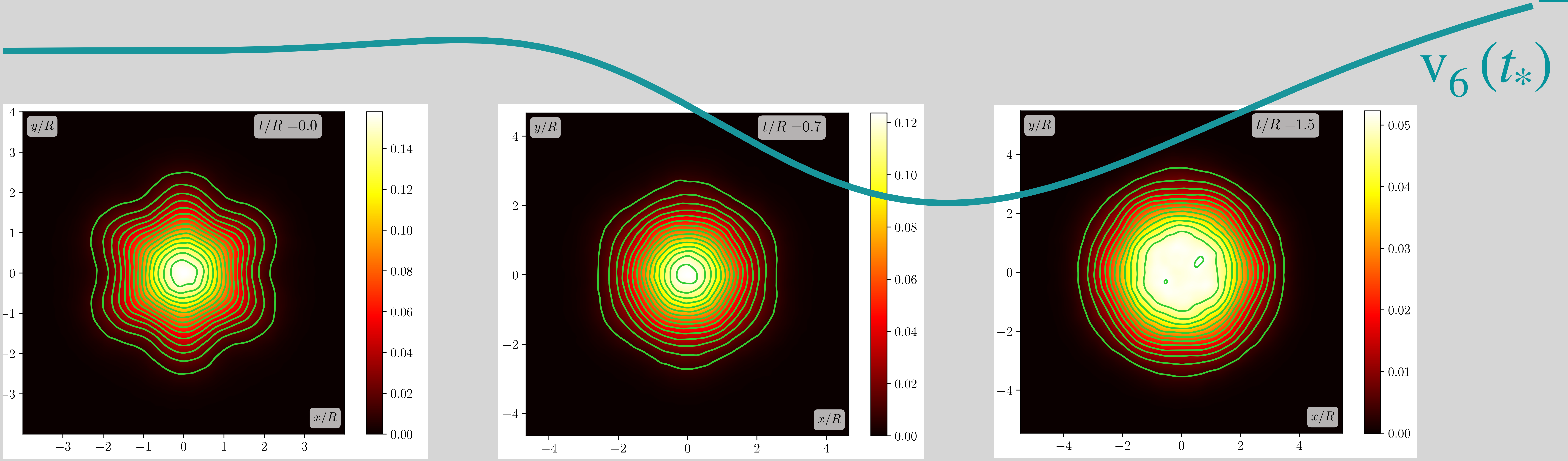
- Analytical formula

$$v_4(t_*) = \frac{64\sqrt{\pi}\,N_{\rm resc}}{405\left(384 + 144\sqrt{2}\,\epsilon_2^2 + 35\sqrt{2}\,\epsilon_4^2\right)} e^{-t_*^2}$$

$$+32\epsilon_4 e^{\frac{t_*^2}{3}}\left[-\left(5t_*^3 + 21t_* + \frac{54}{t_*}\right) I_0\left(\frac{2\,t_*^2}{3}\right) + \left(5t_*^3 + 24t_* + \frac{63}{t_*} + \frac{162}{t_*^3}\right) I_1\left(\frac{2\,t_*^2}{3}\right)\right]\Bigg]$$

$$v_4 \propto N_{\rm resc}\left[\kappa_{4,4}\,\epsilon_4 + \kappa_{4,22}\,\epsilon_2^2\right] t_*^5$$

Density plots from numerical simulations



- Initial eccentricity ϵ_6 only

- $\pi/6$-rotated high density region ⇒ locally $\epsilon_6 < 0$
- Dilute regions ⇒ small contribution to v_6

- Original orientation in denser region regained
- Dilute regions ⇒ small contribution to v_6

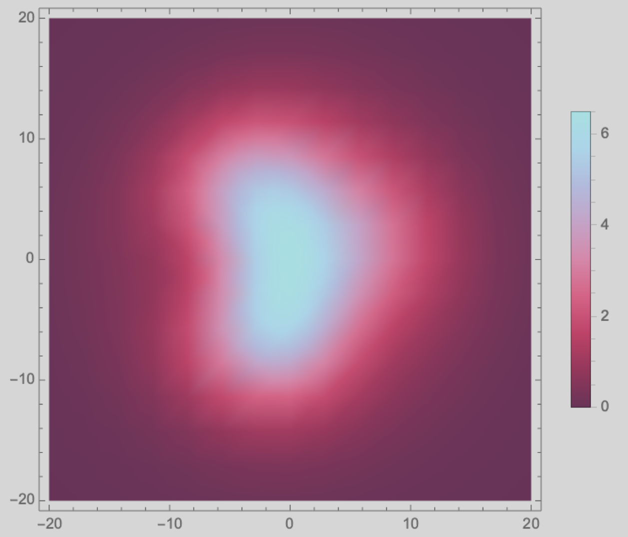
Initial distribution function

- Initially factorized in position and momentum space

$$f^{(0)}\left(0,\mathbf{r},\mathbf{p}\right) = G\left(\mathbf{r}\right) F\left(\mathbf{p}\right)$$

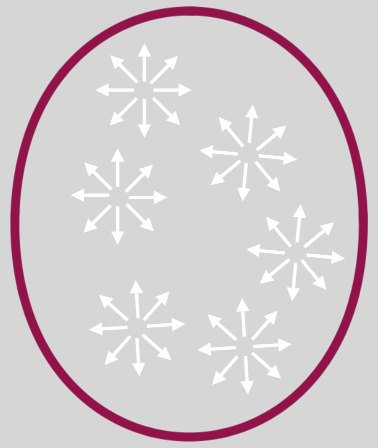
- Initial distribution function in position space

$$G\left(\mathbf{r}\right) \propto \exp\left(-\frac{r^2}{2R^2}\right)\left[1 + \sum_{n=2}^{\infty} \tilde{\epsilon}_n \left(\frac{r}{R}\right)^n \cos\left(n\left(\theta - \Psi_n\right)\right)\exp\left(-\frac{r^2}{2R^2}\right)\right]$$



- Initially isotropy in momentum space

$$\Rightarrow F\left(\mathbf{p}\right) = F\left(p_T\right)$$



OR

- Anisotropic initial momentum distribution

$$F\left(\mathbf{p}\right) = \tilde{F}\left(p_\perp\right)\left[1 + 2\sum_{k=2}^{\infty}\left(w_{k,c}\cos\left(k\,\phi\right) + w_{k,s}\sin\left(k\,\phi\right)\right)\right]$$

Eccentricity

- Formula $\epsilon_n = \dfrac{\int G(r,\theta)\ \cos\left(n\left(\theta - \Psi_n\right)\right)\ r^{n+1}\ \mathrm{d}\theta\ \mathrm{d}r}{\int G(r,\theta)\ r^{n+1}\ \mathrm{d}\theta\ \mathrm{d}r}$

  weighted with r^n

  $\Rightarrow$ Outer regions contribute stronger to ϵ_n