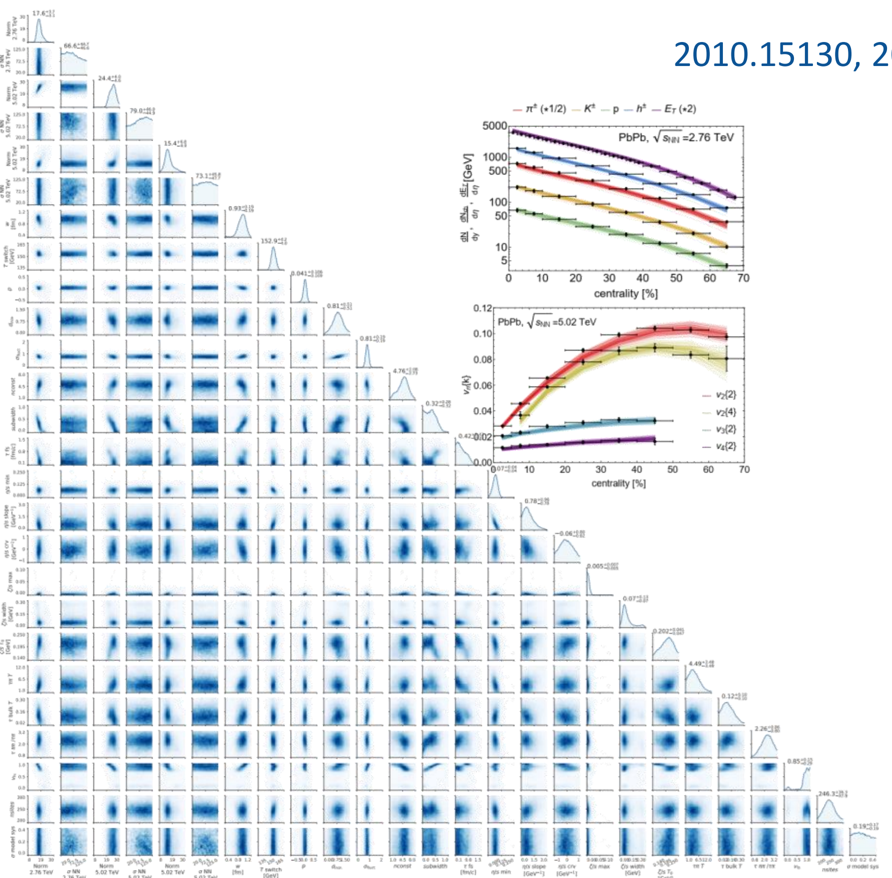
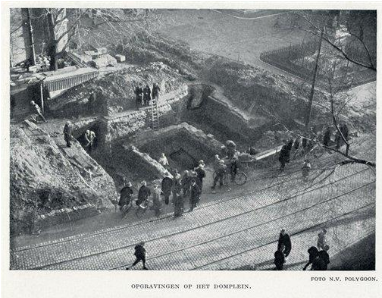
A global analysis of Heavy Ion Collisions with transverse momentum dependence

Towards a precision analysis of heavy ion collisions

Based on *Trajectum* with Govert Nijs
2010.15130, 2010.15134 with Govert Nijs, Umut Gursoy and Raimond Snellings





Roman excavations in Utrecht (from Trajectum, or bridge) in 1929

Wilke van der Schee

Vith Initial Stages 2021
Weizmann Institute of Science, 12 January 2021

What are the open questions?

1. Particle ratios: (sizeable) deviations from thermal equilibrium
   - Viscous corrections within hydrodynamics

Hydro at large Reynolds

2. Hydrodynamics for very peripheral PbPb and *p*Pb?
   - How fast? 0.1 fm/c or 1.5 fm/c?
   - T-dependent shear viscosity, bulk viscosity
   - Second order transport relevant?

Is QGP strongly coupled?
At which energy scale?
Non-conformal?

3. Initial shape: how to convert colliding nucleons to energy density
   - Not even settled if binary collisions are ruled out (!)
   - More profound in *p*Pb: spherical proton unlikely

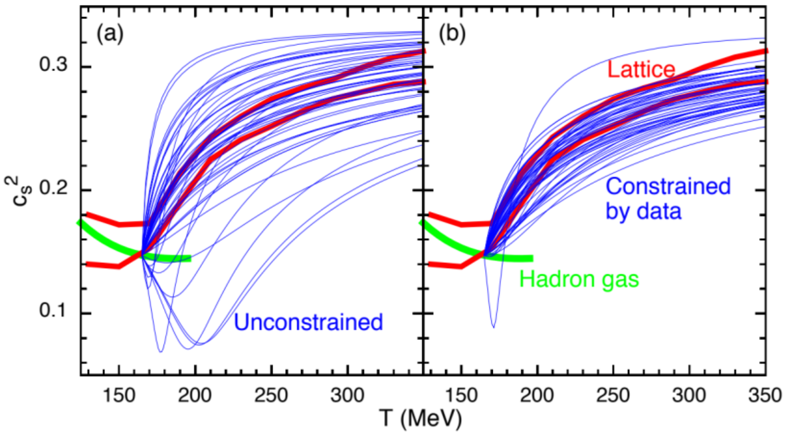
What are the d.o.f.?
Partons? Glasma?

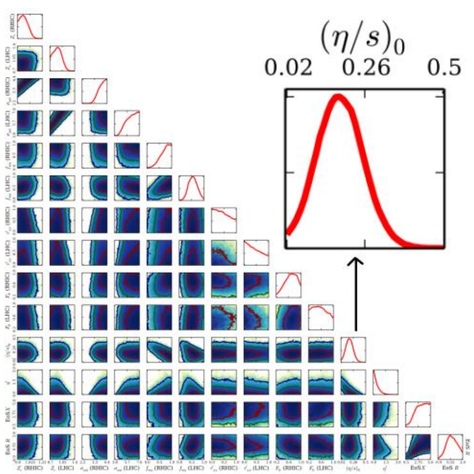
First global analyses

Precise questions require precise understanding of interplay of rich physics in heavy ion collisions

Constraining EOS (Jan 2015)

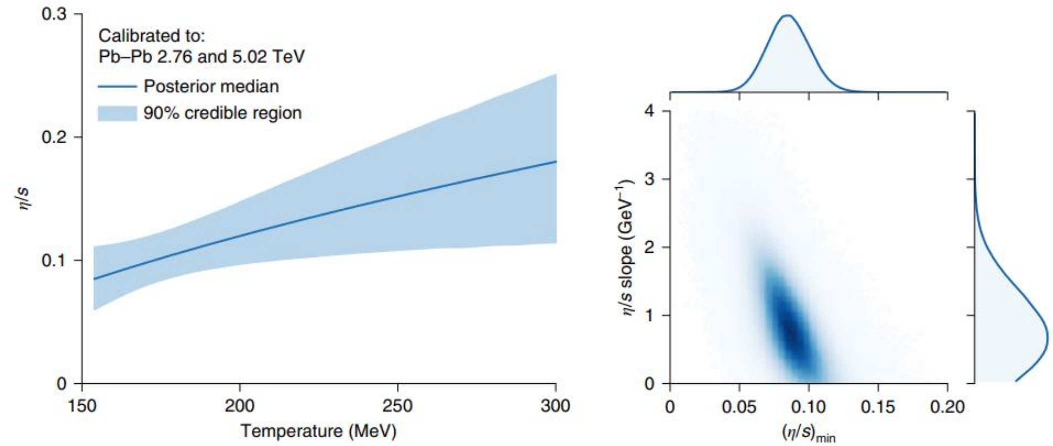
Prior Posterior (data)





Constraining η/s (2019, Nature Physics)

η/s versus temperature Posterior (**η/s+slope**)

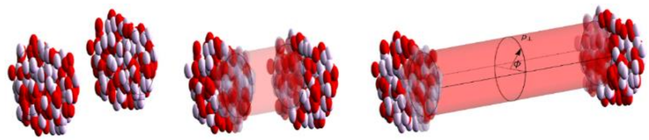


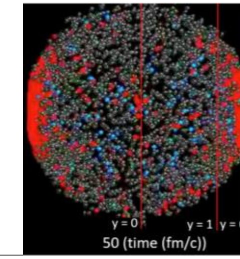
Jonah E. Bernhard, J. Scott Moreland and Steffen A. Bass
Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma

Important: `average' viscosity better constrained than T-dependent viscosity

S. Pratt, E. Sangaline, P. Sorensen and H. Wang, Constraining the Eq. of State of Super-Hadronic Matter from Heavy-Ion Collisions (2015)
Evan Sangaline and Scott Pratt, Towards a Deeper Understanding of How Experiments Constrain the Underlying Physics of Heavy-Ion Collisions (2015)

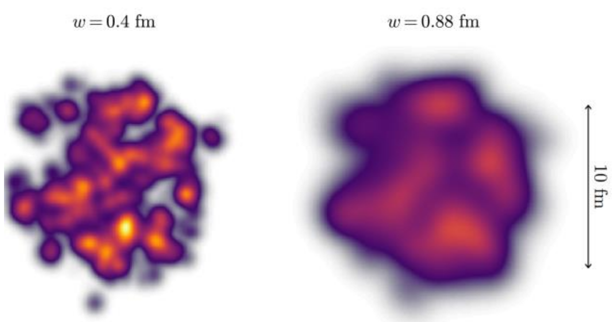
Standard model of heavy ion collisions





Initial stage (9)

Subnucleonic structure? (7)

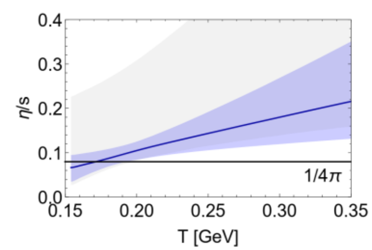


Non-thermal flow? (2)
for time τ with varying speed (***new***)

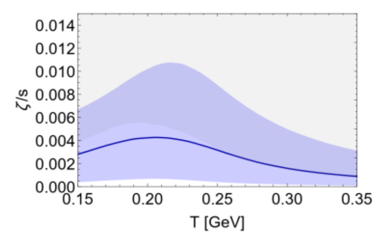
Fluctuations? (1)

Viscous hydrodynamics (9)

Shear viscosity (3)



Bulk viscosity (3)



Second order transports: 3 (***new***)

Cascade of hadrons (1)

Convert quark-gluon plasma at T_{switch} to particles following Boltzmann distribution (particlization, 1)

Subtle: viscous corrections

Evolve particles with hadronic code: SMASH

Jonah Bernhard, Scott Moreland and Steffen Bass, Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma (2019)
Govert Nijs, WS, Umut Gursoy and Raimond Snellings, A Bayesian analysis of Heavy Ion Collisions with Trajectum (2020)

Hydrodynamics: first and second order

1. Constitutive relations for the stress tensor, with p(ρ) EOS from HotQCD

$$T^{\mu\nu} = \rho u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

With shear and bulk tensors:

$$D\Pi = -\frac{1}{\tau_\Pi}\left[\Pi + \zeta\nabla\cdot u + \delta_{\Pi\Pi}\nabla\cdot u\Pi - \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right],$$

$$\Delta^\mu_\alpha\Delta^\nu_\beta D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\Big[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \delta_{\pi\pi}\pi^{\mu\nu}\nabla\cdot u - \phi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} + \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} - \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}\Big].$$

2. We vary the green coefficients, η and ζ as a function of temperature, 2nd order according to $\frac{\tau_\Pi s T \delta^2}{\zeta}$, $\frac{\tau_\pi s T}{\eta}$ and $\frac{\tau_{\pi\pi}}{\tau_\pi}$

Performing a global analysis

Bayes theorem:

$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{\mathrm{exp}}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det\left(\Sigma(\boldsymbol{x})\right)}}\mathcal{P}(\boldsymbol{x})$$

$$\text{with} \quad \Delta^2 = (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\mathrm{exp}}) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\mathrm{exp}})$$

We have a 20-dimensional parameter space and 514 datapoints

- Run model on 1000 `design' points, spaced on a latin hypercube
- `Interpolate' results by training a Gaussian Process Emulator

Markov Chain Monte Carlo (emcee2.2)

- Obtain sample of 10^6 likely values

Compare posterior with data

- From emulator (emulator has its own uncertainty estimate)
- A high statistics run at the optimal value (MAP, maximum a posteriori)

Experimental observables: *a wealth of data*

1. Yields, spectra, identified $v_n\{2\}$ versus p_T, *p*Pb and PbPb (514 datapoints)

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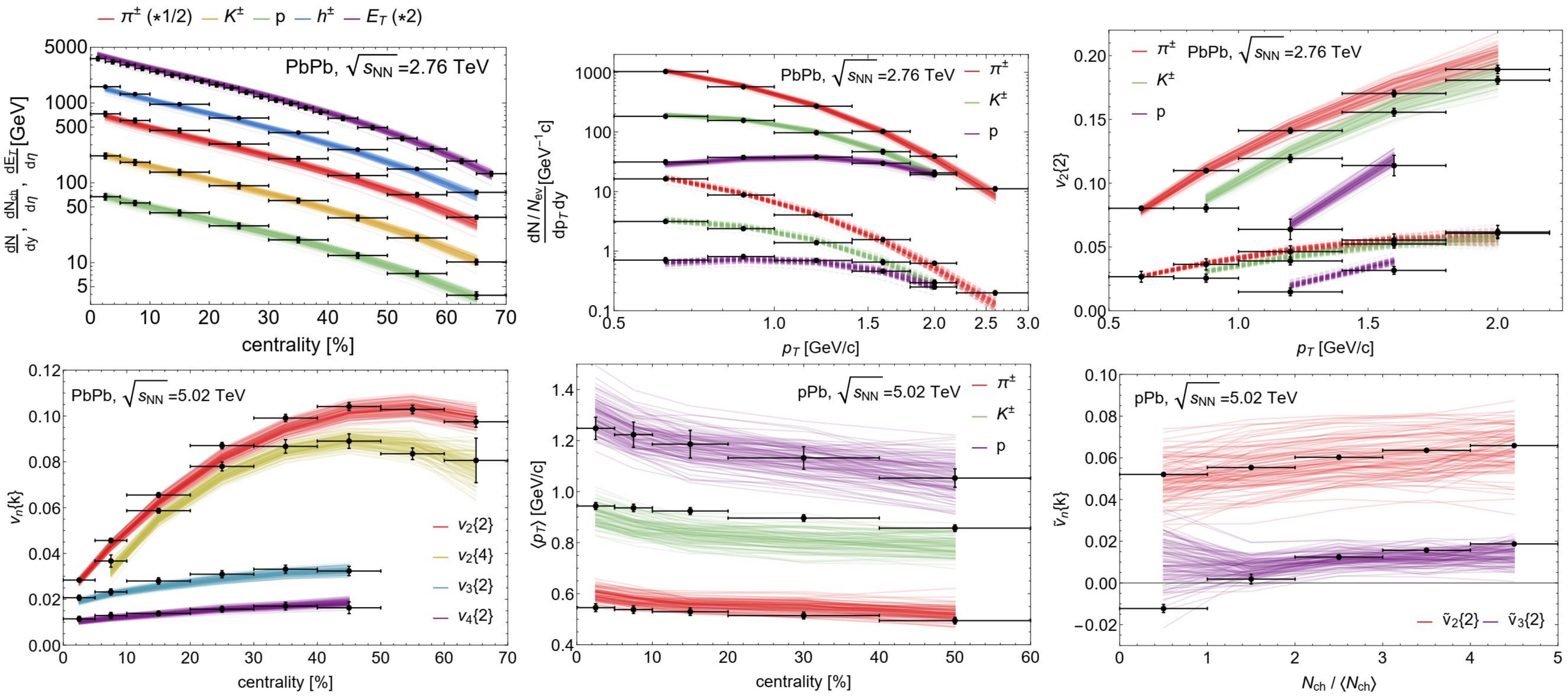
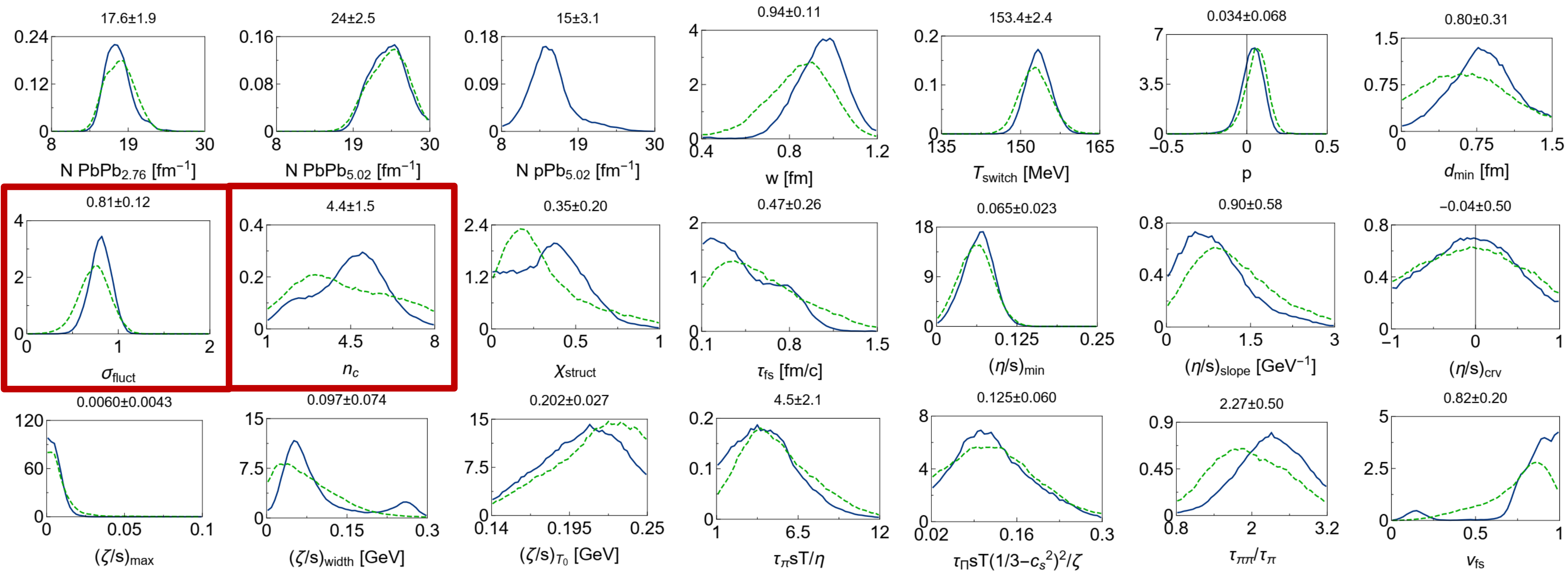
2. Note: points are highly correlated, without p_T dependence effectively only 6 to 8 principal components (PCs), for 20 parameters. With p_T-dependence roughly 12 PCs.



Posterior distributions

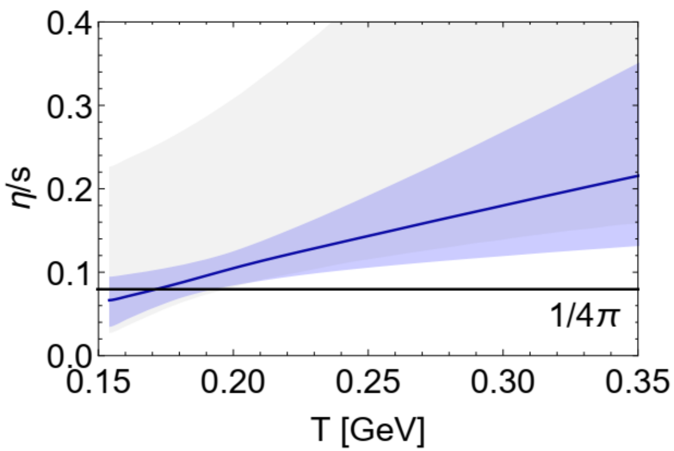
1. Dashed: without *p*Pb: indeed much flatter for e.g. n_c
2. Strong constraint on nucleon-nucleon fluctuations (also found at Duke)



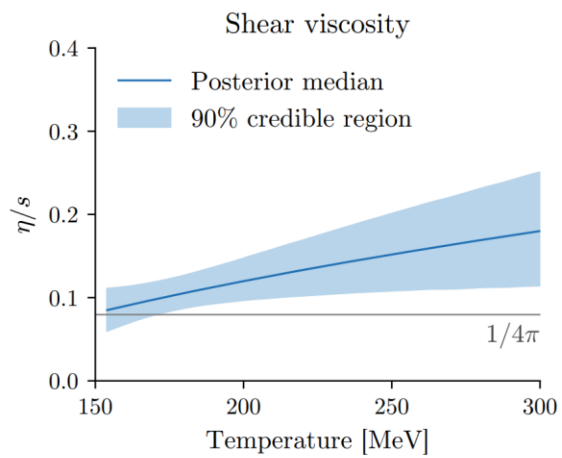
See also talk by
Derek Everett (Wed 16:10)

Posterior distributions – shear viscosity

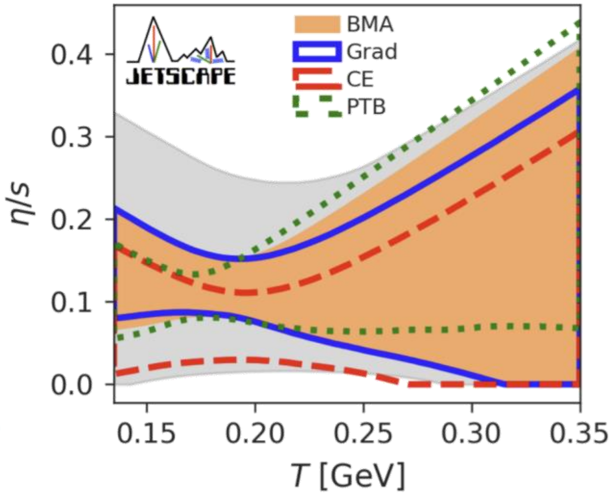
1. Shear viscosity consistent with previous work
   - More data, but also enlarged model → similar constraint on η/s
   - New JETSCAPE slightly broader band (larger priors, single PbPb energy but with RHIC)



Current work (2020)



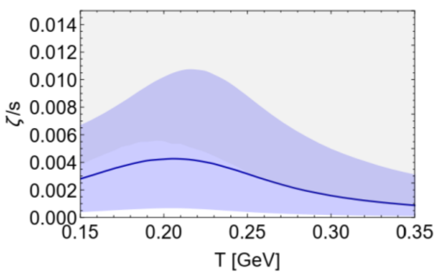
J. Bernhard, S. Moreland and S. Bass, Nature Physics (2019)

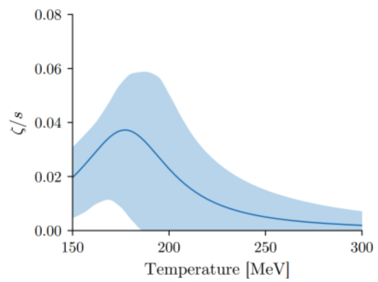


JETSCAPE (2020)

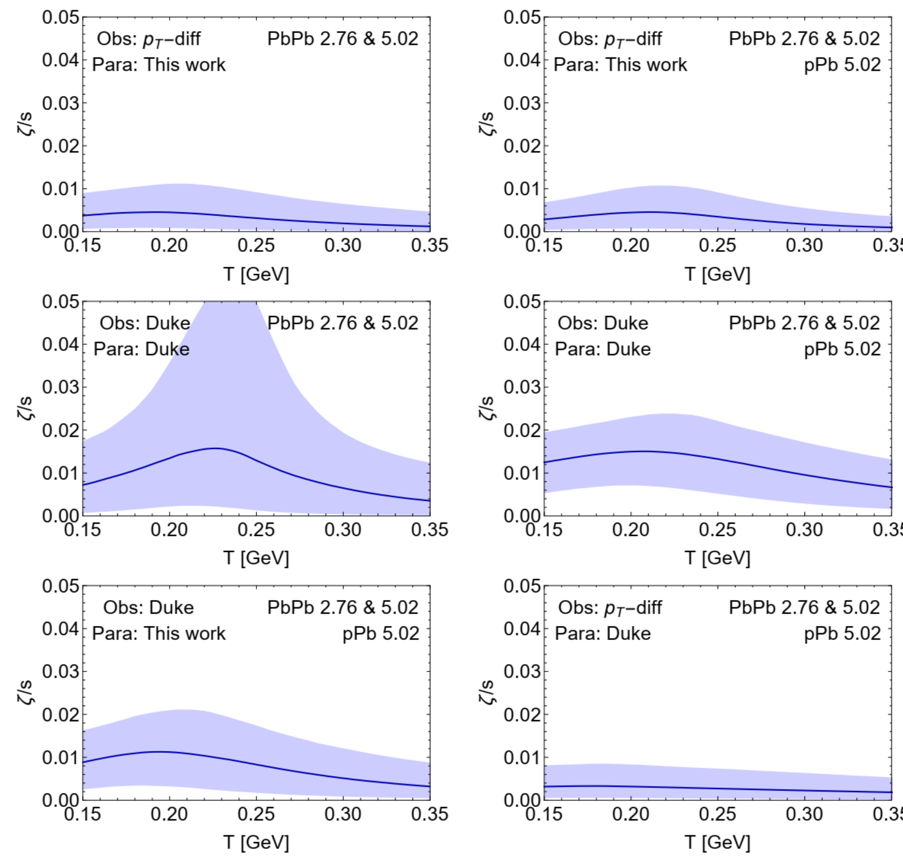
Posterior distributions – bulk viscosity:

Much smaller, even consistent with zero





J. Bernhard, S. Moreland and S. Bass, Nature Physics (2019)

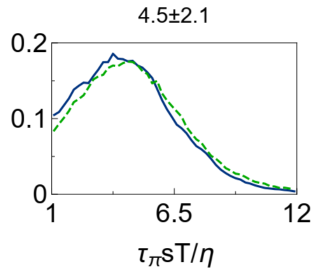
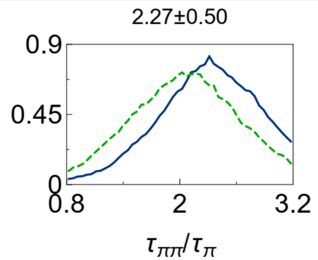


Bulk viscosity, varied several aspects:

- More limited parameter set
  - All versus only `Duke'
- Include or not include p-Pb collisions
- **Include p_T-differential observables**

Posterior distributions – 2nd order transport

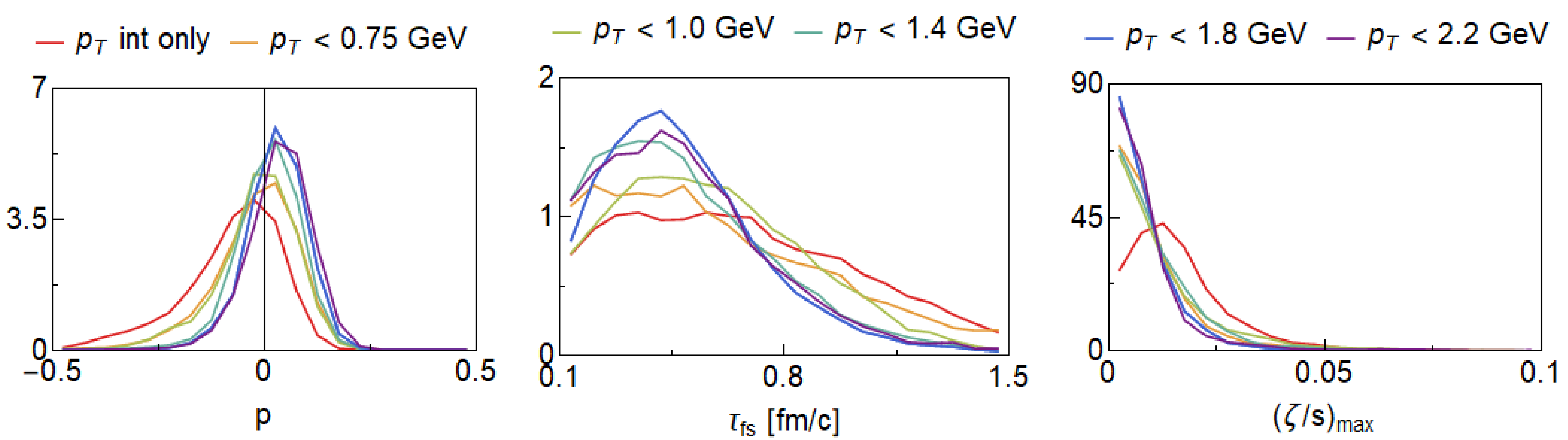
1. τ_π and $\tau_{\pi\pi}$ can be compared to strong and weak coupling values
   - Both consistent, AdS/CFT slightly favoured for $\tau_{\pi\pi}$

	this work	AdS/CFT	kinetic theory
$\frac{\tau_\pi s T}{\eta}$		$4 - \log(4) \approx 2.61$	5
$\frac{\tau_{\pi\pi}}{\tau_\pi}$		$\frac{88}{35(2-\log 2)} \approx 1.92$	$\frac{10}{7} \approx 1.43$

Sensitivity to p_T-differential observables

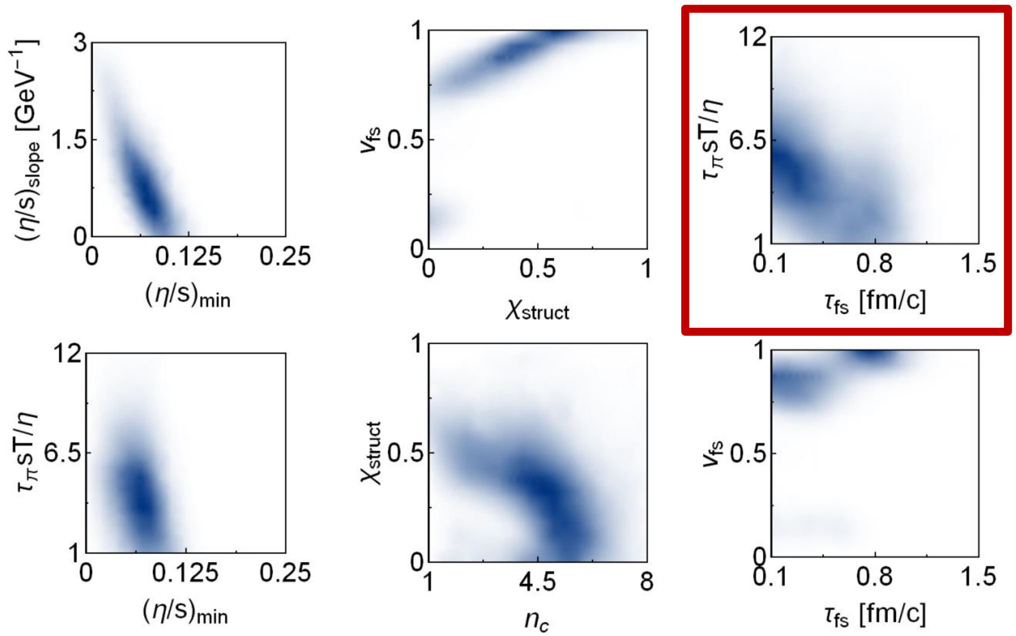
Vary maximum p_T of observables to identify their constraining power

- First two bins till 1 GeV give strongest constraints (if at all, selection shown)
  - Also due to tougher statistics at higher p_T: emulator error
- Viscous corrections at freeze-out very uncertain at large p_T
  - Encouraging that results are quite insensitive to high p_T bins



Correlations among the parameters

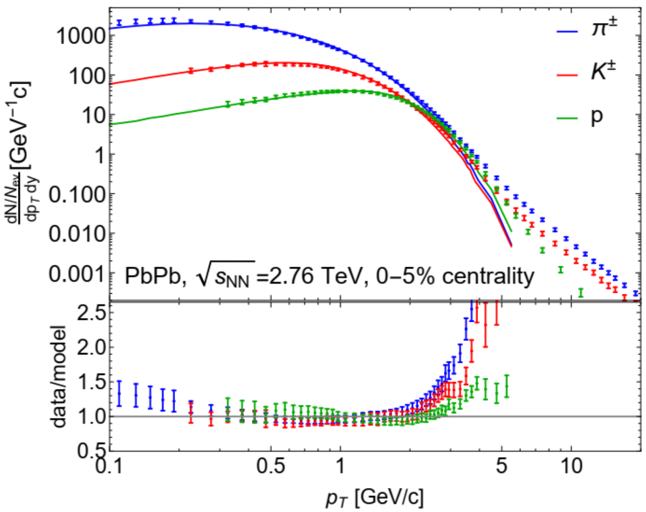
1. Interesting correlation between free streaming time and τ_π



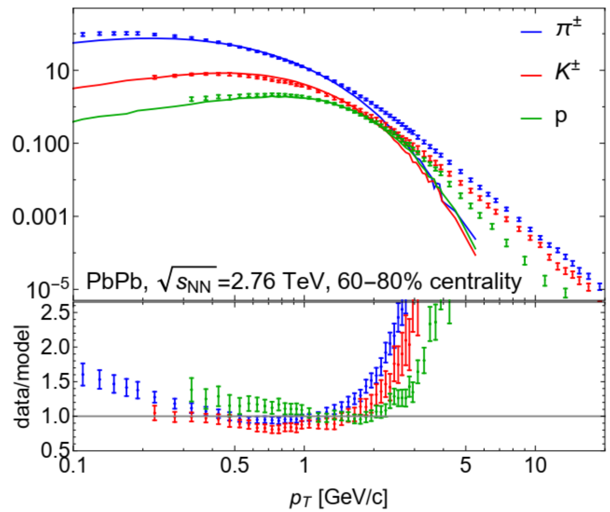
MAP: maximum a posteriori: spectra

1. High statistics run at (almost) optimal parameters, compared with ALICE data

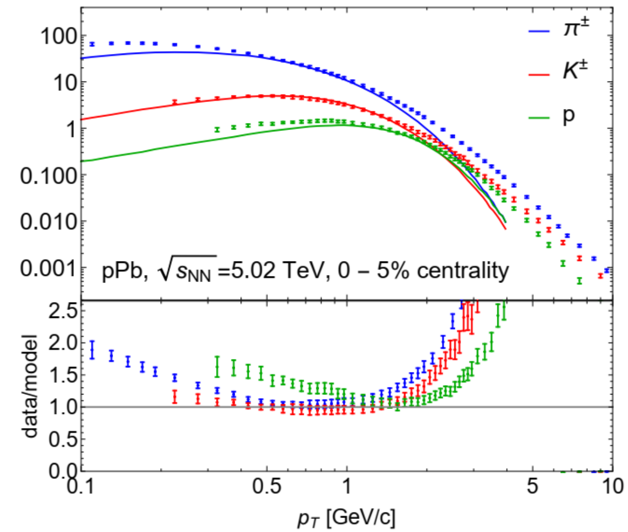
Central PbPb



Very peripheral PbPb

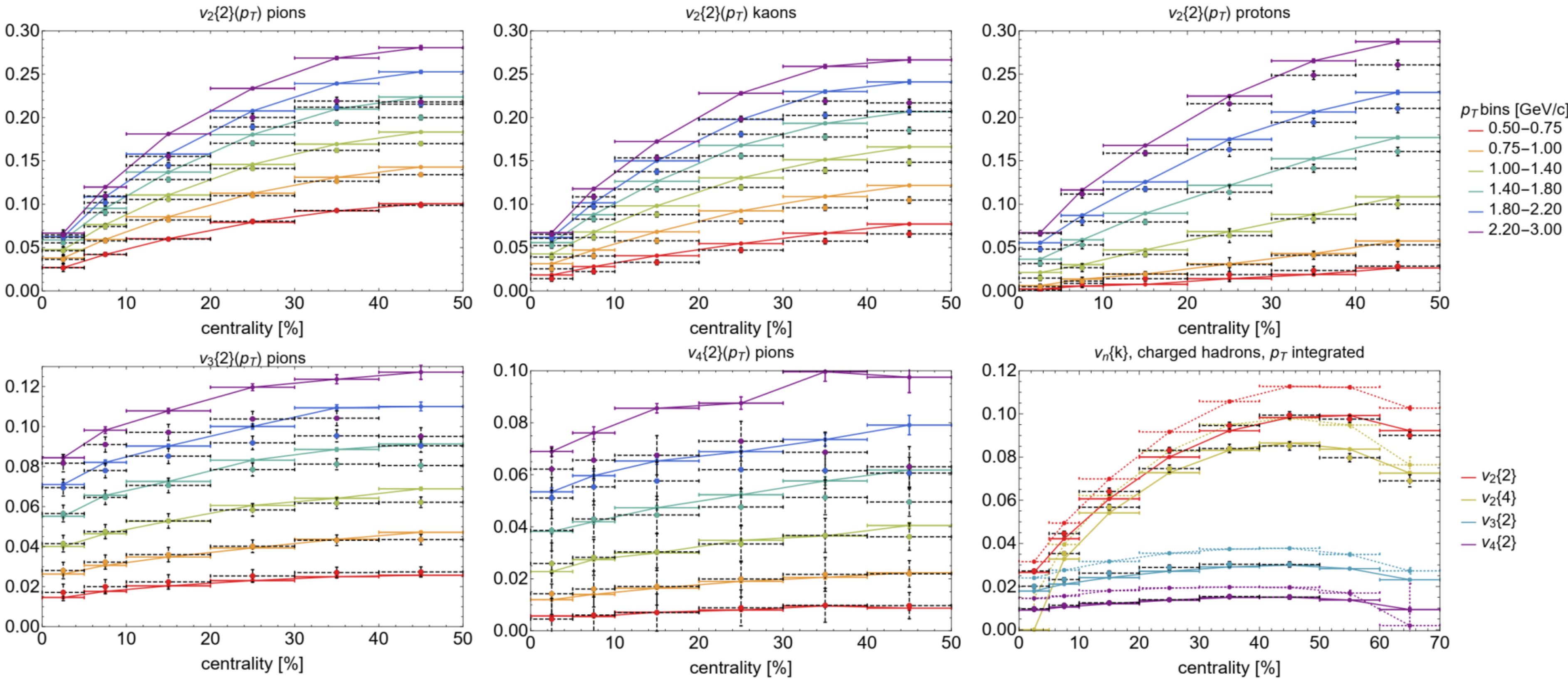


Central *p*Pb



MAP: PbPb anisotropic flow

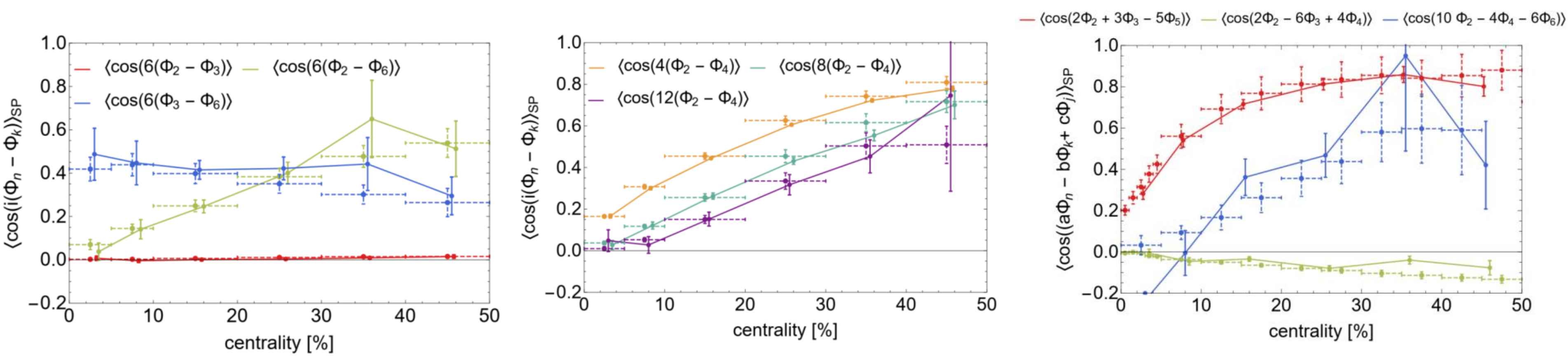
1. Anisotropic flow matches well, except for a few high p_T bins



See also talk by
Matthew Heffernan (Mon 18:40)

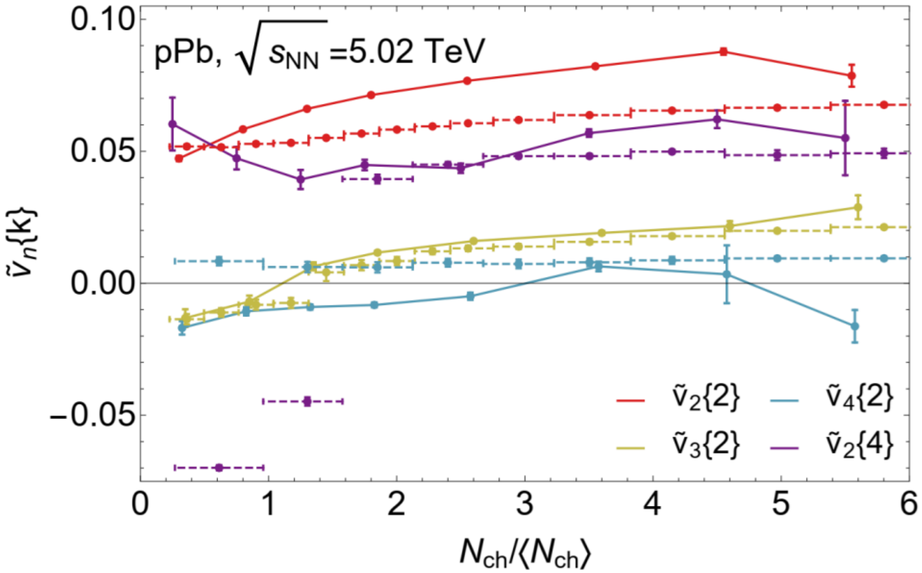
MAP: PbPb event plane angles

1. A non-trivial check: the event-plane angle correlations (not fitted)
2. Non-trivial to fit: needs a very specific η/s(T)



MAP: *p*Pb anisotropic flow

1. Emulator and MCMC are less precise for *p*Pb: uncertainty is statistical only
2. Shows potential to obtain imaginary $v_n\{2\}$ (= negative $\tilde{v}_n\{k\}$), in agreement with ATLAS low multiplicity result
3. Sheds new light on discussion of hydro versus sign of $\langle\langle 2\rangle\rangle_n$



$$Q_n = \sum_{i=1}^{M} e^{in\phi_i}$$

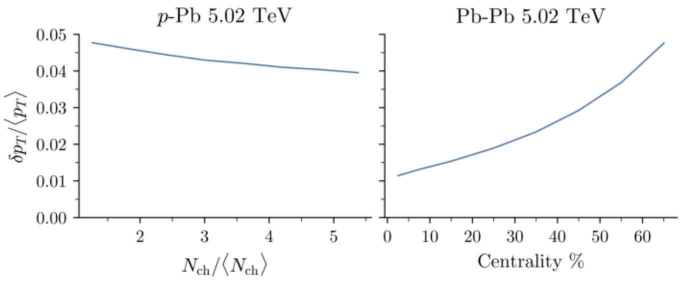
$$\langle 2\rangle_n = \frac{|Q_n|^2 - M}{M(M-1)}$$

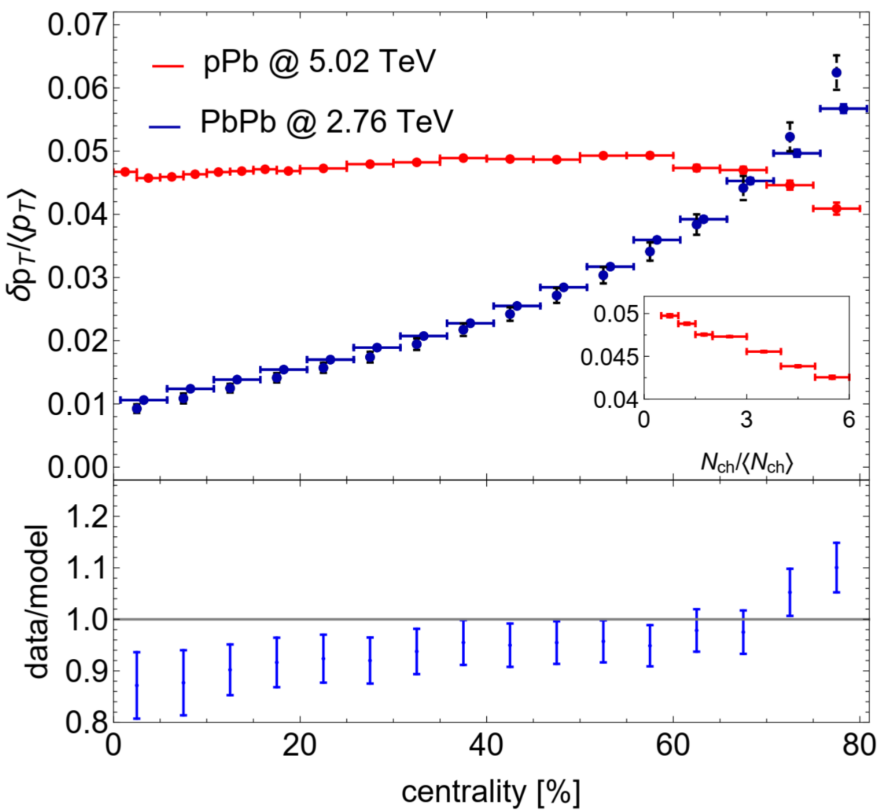
$$v_n\{2\} = \sqrt{\langle\langle 2\rangle\rangle_n}$$

$$\delta p_T^2 = \left\langle\left\langle (p_{T,i} - \langle p_T \rangle)(p_{T,j} - \langle p_T \rangle) \right\rangle\right\rangle$$

A prediction for *p*Pb: momentum fluctuations

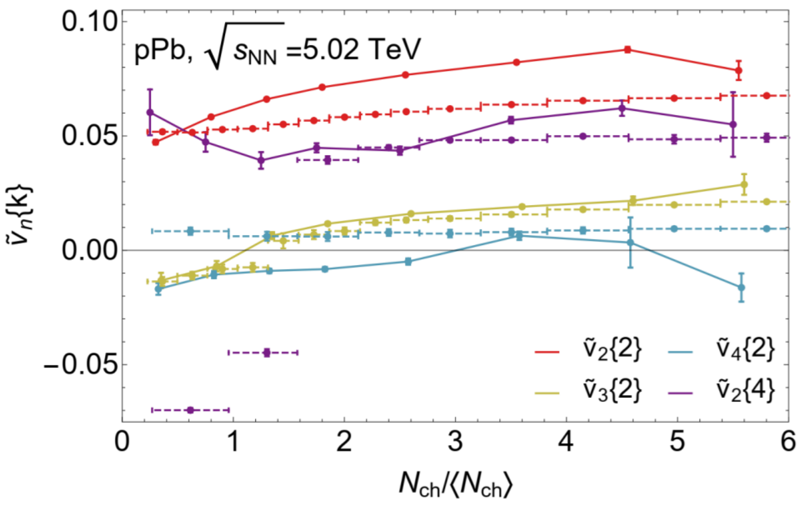
1. Rather hard to find observables that have not been measured
2. Typically hard to match to data
3. Roughly comparable to results by Scott:

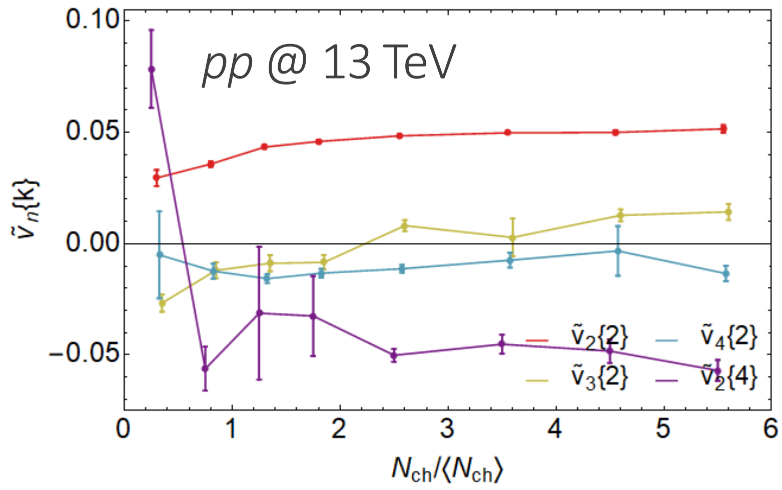




MAP: *pp* anisotropic flow

1. Preliminary results for *pp*; different sign for $v_2\{4\}^2$ (?)





Discussion

1. A road to precision analysis of the quark-gluon plasma
   - Measuring transport and initial stage `beyond η/s', revisiting bulk viscosity
   - Hints on constraints for second order transport

2. Encouraging results
   - p_T-differential anisotropic flow sheds new light on global analyses
   - Surprisingly small bulk viscosity, consistent with zero
   - An excellent fit of event-plane angles (MAP results)
   - Interesting MAP results on flow in *p*Pb (imaginary $v_3\{2\}$)

3. Study is still limited:
   - Still significant uncertainties in ***initial stage*** and ***particlisation***
   - Data set still fairly small, should still include RHIC results

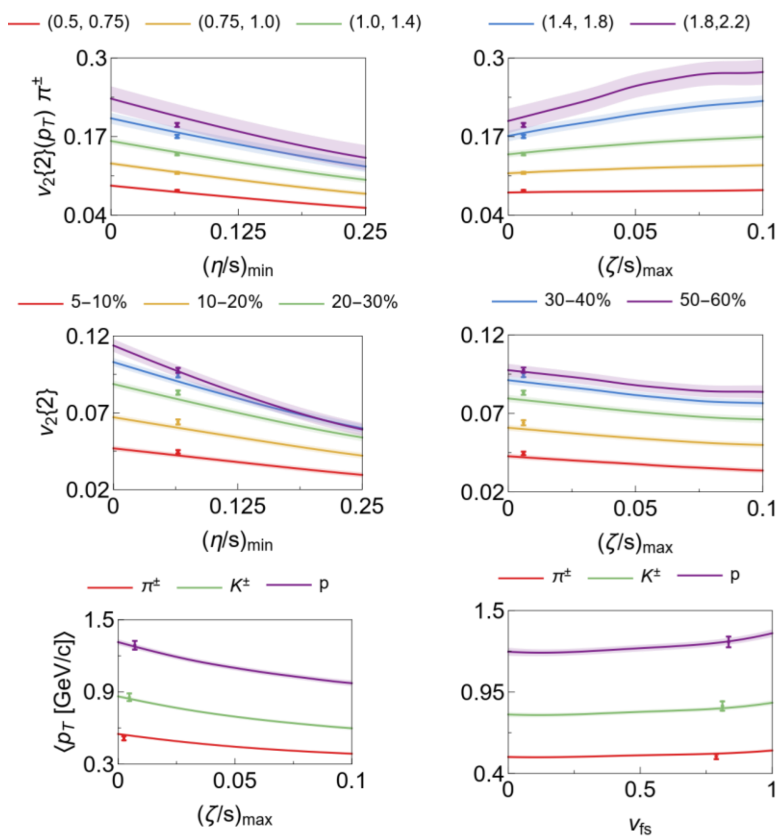
Jonah E. Bernhard, Bayesian parameter estimation for relativistic heavy-ion collisions (PhD thesis, 2018)
Scott Moreland, Initial conditions of bulk matter in ultrarelativistic nuclear collisions (PhD thesis, 2019)

Back up

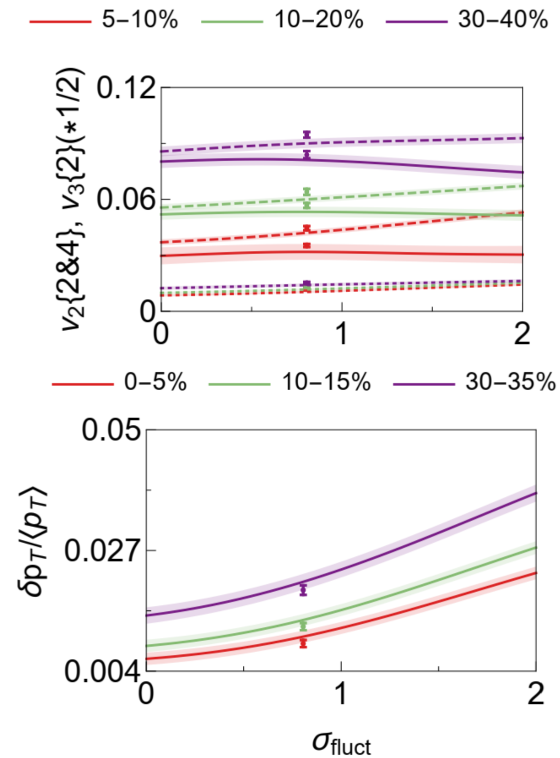
The emulator: Viscosities and fluctuations

also note: emulator uncertainty (50-60%, or $v_2\{4\}$)

v_2 increases in every p_T bin, but decreases on average for ζ.



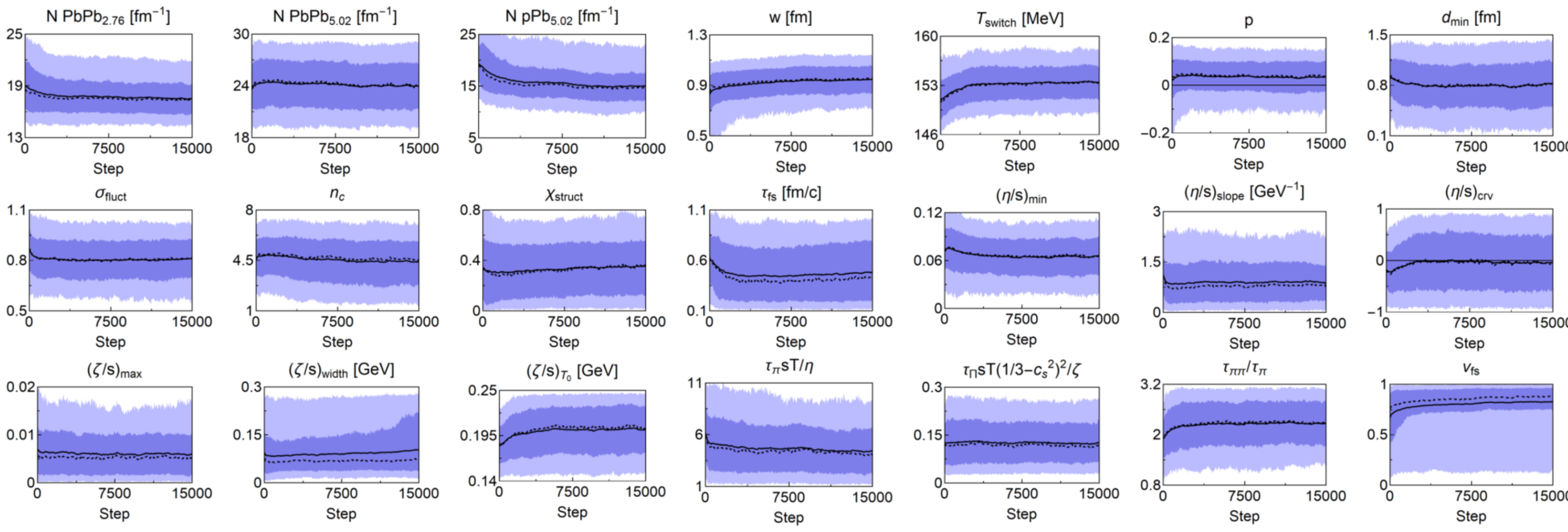
$v_2\{2\}$ - $v_2\{4\}$ increases when increasing fluctuations



Making the chain: (pt)emcee

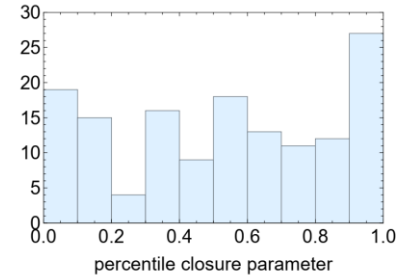
1. Constructing a chain of posteriors (MCMC); we use emcee

2. Necessary to verify convergence:



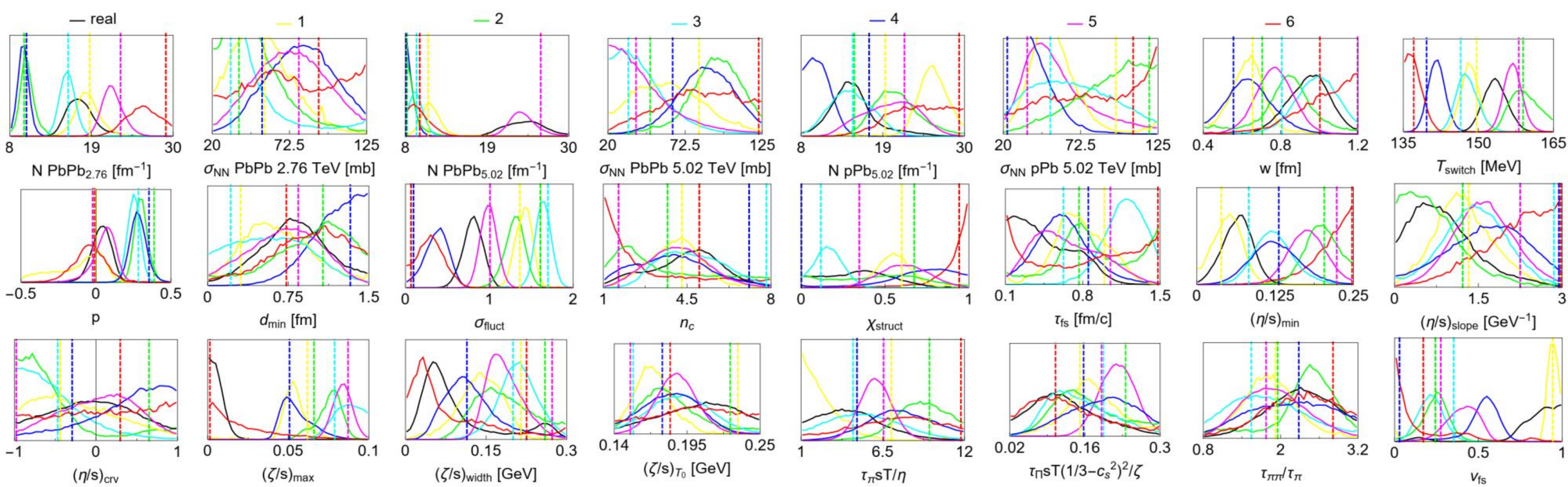
3. Still quite expensive to emulate:
   600 walkers * 15000 steps * 10% acceptance with 2000 points = 3 weeks

Wilke van der Schee, CERN



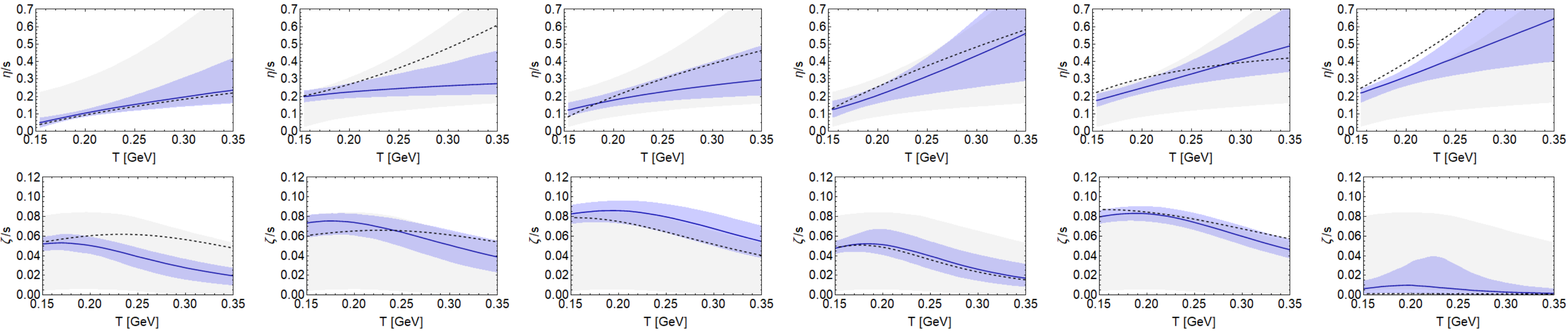
Selected results: closure test

1. We chose six random parameter points (sometimes at edge of prior)
   - Try to extract parameters from model-generated `experimental' data
2. Verifies model + shows sensitivity data on parameter
   - Output indeed consistent with input
   - Sensitive to viscosities, less so for second order

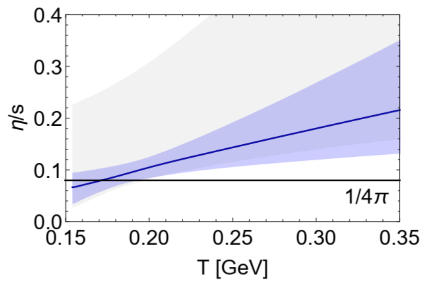


Closure test: viscosities

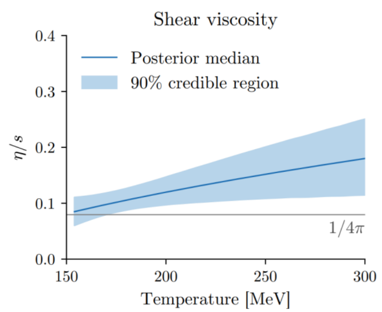
1. Closure test works well for both viscosities
2. Most sensitive to low-T region



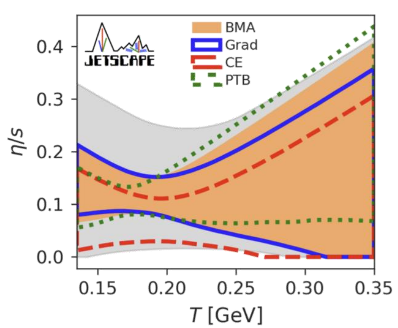
Posterior distributions – shear viscosity



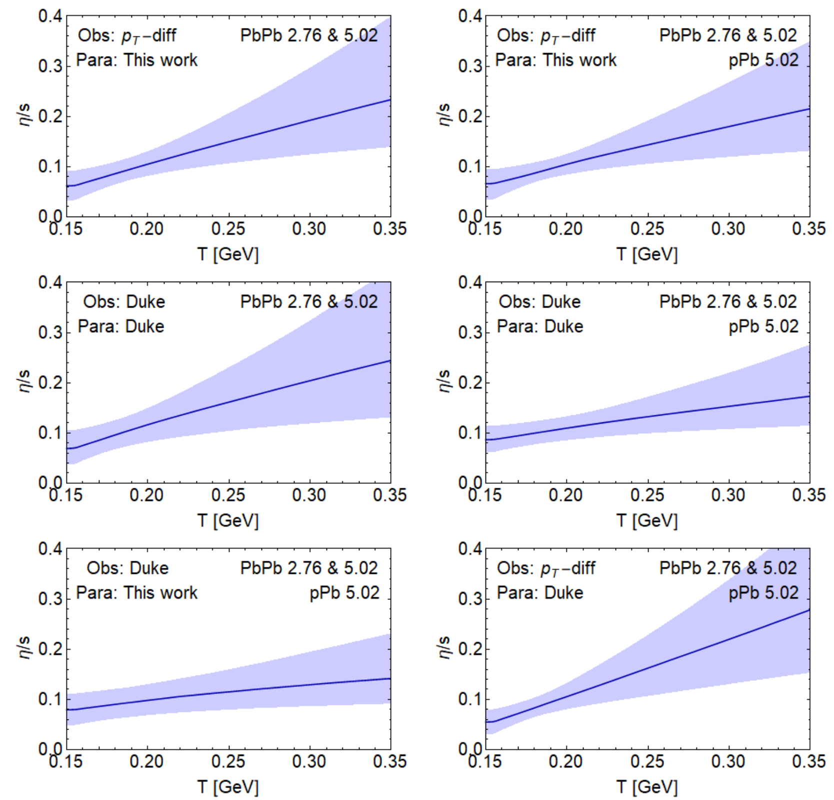
Current work (2020)



J. Bernhard, S. Moreland and S. Bass, Nature Physics (2019)



JETSCAPE (2020)



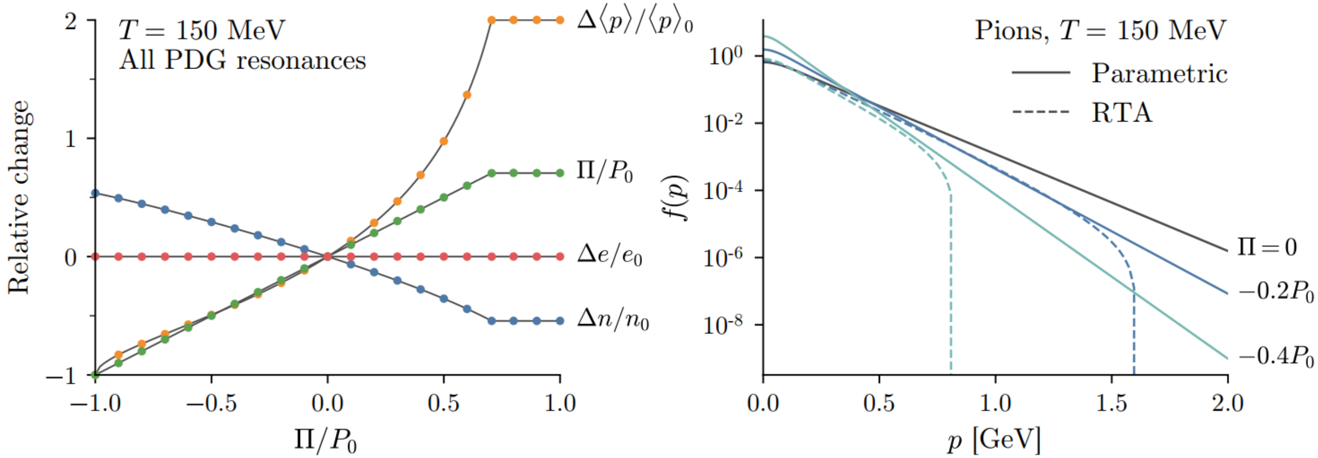
Particlization: viscous corrections

$$T^{\mu\nu} = \sum_{\text{sp}} g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(p),$$

1. Particles in fluid restframe cannot be in thermal equilibrium
2. Several methods that (only/mostly) agree for small deviations

$f(p) \to z_{\text{bulk}} f(p + \lambda_{\text{bulk}} p)$ parametric, rescale p: fix z and λ such that e and P match

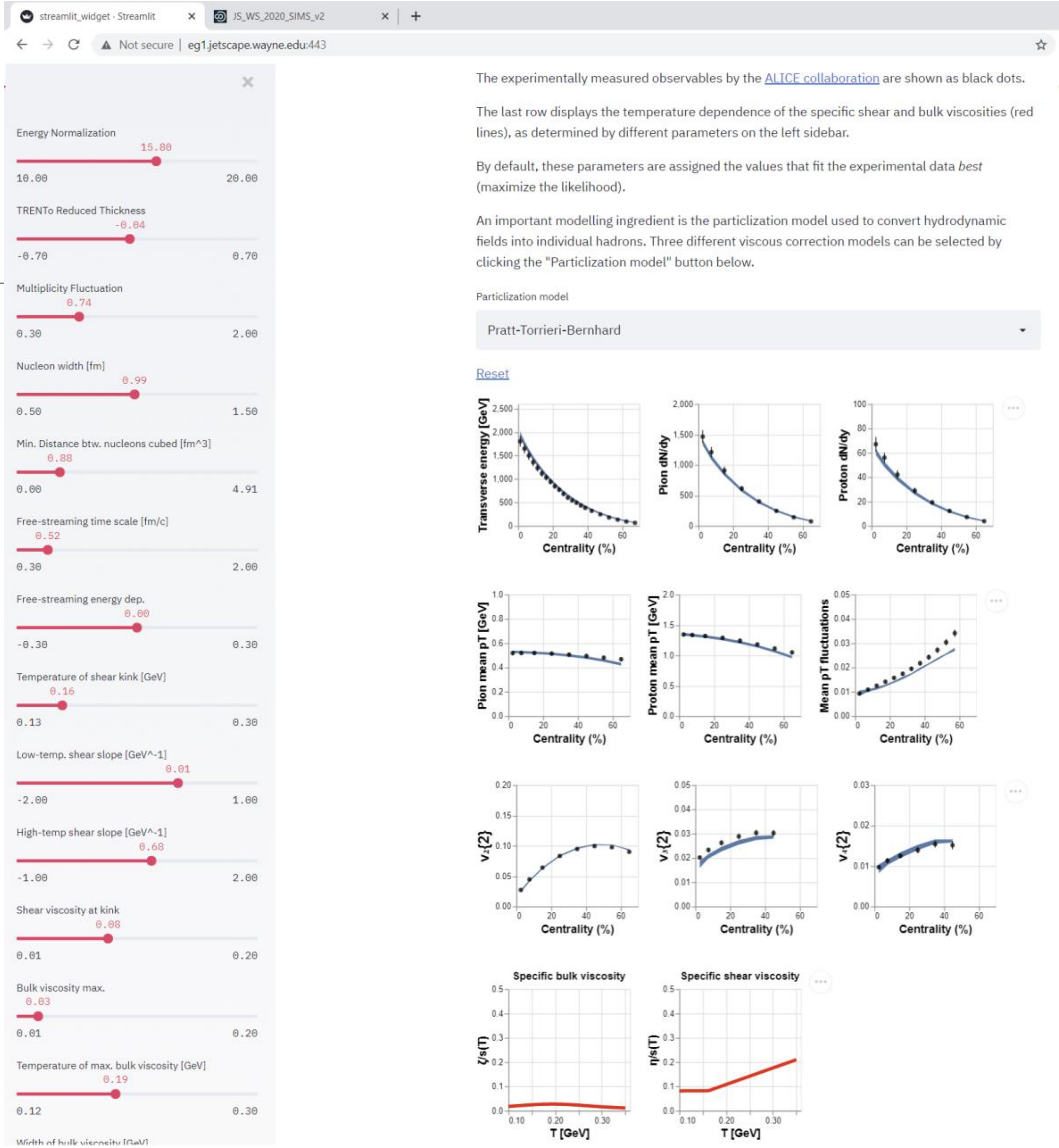
$\delta f = f_0(1 \pm f_0)\frac{\tau}{ET}\left[\frac{1}{2\eta}p^i p^j \pi_{ij} + \frac{1}{\zeta}\left(\frac{p^2}{3} - c_s^2 E^2\right)\Pi\right]$ change $f(p)$ directly, motivated by RTA



\`Parametric' clearly better at high p_T, but somewhat ad-hoc and species independent

Comparison with JETSCAPE

Results seem to be in relatively good agreement. Data is quite consistent without a sizeable bulk viscosity.

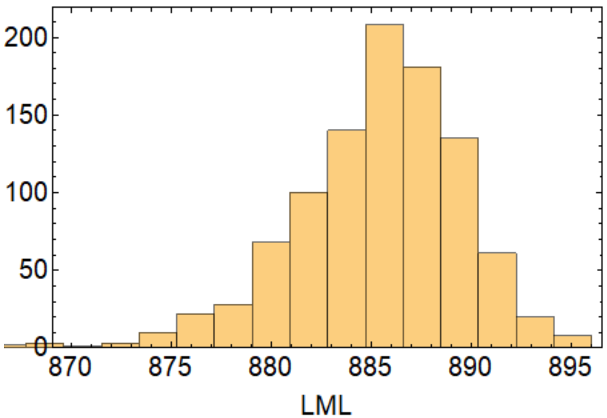


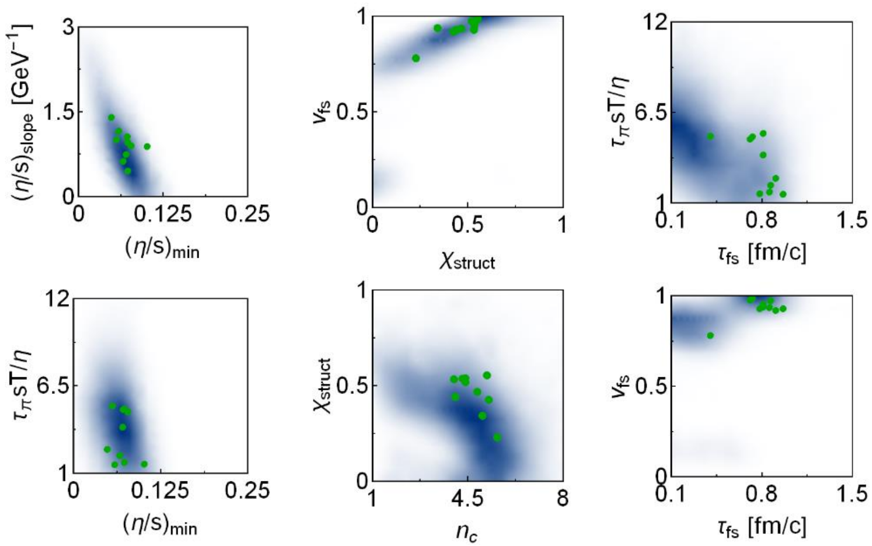
MAP: maximum a posteriori

$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{\text{exp}}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det\left(\Sigma(\boldsymbol{x})\right)}}\mathcal{P}(\boldsymbol{x})$$

$$\text{with} \quad \Delta^2 = \left(\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}}\right) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot \left(\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}}\right),$$

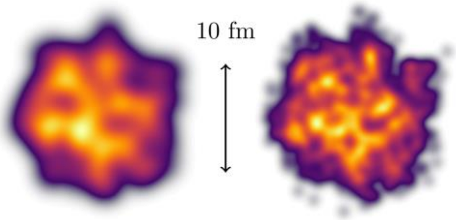
1. Subtle to find the `true' maximum of the LML:
2. take 3000 points from the posterior chain, plot highest 10 LMLs (LML > 893):





3. We decided to use expectation value of each param as `MAP' (LML = 884)

Initial geometry: two (three?) uncertainties



1. The structure of nucleons
   - n_c constituents of Gaussian subwidth *v* within a nucleon of width *w*
   - Nucleons placed according to MC Glauber

2. How do colliding (sub)nucleons deposit their energy? $\mathcal{T} = \left(\frac{\mathcal{T}_A^p + \mathcal{T}_B^p}{2}\right)^{1/p}$
   - For $p = 0$ we get $\mathcal{T} = \sqrt{\mathcal{T}_A \mathcal{T}_B}$: close to EKRT or Holography ($\mathcal{T} \approx (\mathcal{T}_A \mathcal{T}_B)^{4/9}$)
   - Does not quite allow binary scaling?

3. (Quantum) fluctuations in the above: Gamma-distribution:
   - Goes beyond MC Glauber fluctuations $p(T) = \frac{1}{\Gamma(1/\sigma)\sigma^{1/\sigma}} T^{(1-\sigma)/\sigma} e^{-T/\sigma}$

Scott Moreland, Jonah Bernhard and Steffen Bass, Estimating nucleon substructure properties in a unified model of p-Pb and Pb-Pb collisions (2018)
WS and Bjoern Schenke, Rapidity dependence in holographic heavy ion collisions (2015)