Linearized kinetic description of non-equilibrium dynamics in pp and pA collisions

Clemens Werthmann, Sören Schlichting

Bielefeld University

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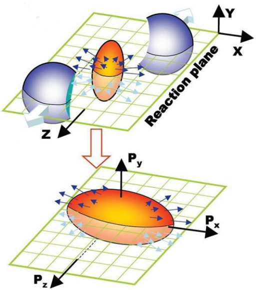
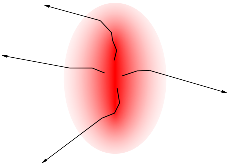
- describe QCD fireball created in a hadronic collision
- examine how spatial anisotropies in the initial state (ϵ_n) dynamically create momentum anisotropies in the final state (v_n) in small systems
- small densities, large gradients: hydro not necessarily applicable; alternative: microscopic description in terms of kinetic theory
- numerical transport codes simulate these dynamics quite well

  AMPT: He, Edmonds, Lin, Liu, Molnar, Wang PLB 753 (2016) 506

  BAMPS: Greif, Greiner, Schenke, Schlichting, Xu PRD 96 (2017) 091504
- want analytical treatment for better understanding and to find parametric dependencies



Hiroshi Masui (2008)



Modelling the fireball

- microscopic description in terms of on-shell phase-space distribution of gluons:

$$f(\tau, \vec{x}_T, \eta, \vec{p}_T, y) = \frac{(2\pi)^3}{\nu_g} \frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3p}(\tau, \vec{x}_T, \eta, \vec{p}_T, y)$$

  - boost invariance: dependence only on $y - \eta \Rightarrow \underbrace{2}_{\vec{x}_T} + \underbrace{3}_{(\vec{p}_T, "p_\parallel")} + \underbrace{1}_{\tau}$ D description
  - observables expressed as constant densities $\mathrm{d}X/\mathrm{d}\eta$ (or ratios)
- time evolution: Boltzmann equation

$$p^\mu \partial_\mu f = C[f]$$

- will solve this both analytically and numerically!

Analytical Description

relativistic Boltzmann equation

$$p^\mu \partial_\mu f = C[f]$$

- integro-differential equation, need simplifications for analytical solution
  - small system $\Rightarrow$ "opacity expansion" in number of scatterings

$$\text{0th order}: \quad p^\mu \partial_\mu f^{(0)} = 0$$
$$\text{1st order}: \quad p^\mu \partial_\mu f^{(1)} = C[f^{(0)}]$$
$$\text{etc...}$$

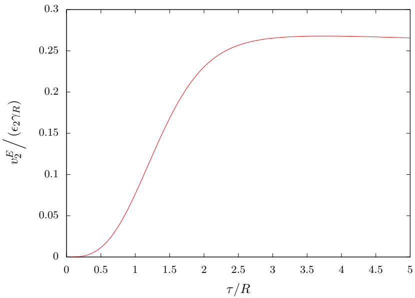
Heiselberg, Levy PRC 59 (1999) 2716
Borghini, Gombeaud EPJC 71 (2011) 1612
Romatschke EPJC 78 (2018) 636
Kurkela, Wiedemann, Wu PLB 783 (2018) 274

  - initial condition: isotropic gaussian energy density $\epsilon(x_T)$ and linearization in small anisotropic $\delta\epsilon(\vec{x}_T)$ on top
  - for now: $C_{RTA}[f] = \frac{p_\mu u^\mu}{\tau_R}(f_{eq} - f)$, $\tau_R = 5\frac{\eta}{s}T^{-1}$

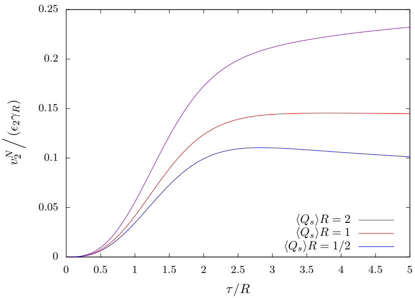
    time evolution depends on opacity $\gamma_R = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{30}{\nu_g \pi^2}\frac{1}{2\pi}\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}R\right)^{1/4}$

$$\gamma_R \approx 0.74\,\frac{0.2}{\eta/s}\left(\frac{R}{0.4\,\mathrm{fm}}\right)^{1/4}\left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{5\,\mathrm{GeV}}\right)^{1/4}$$

E_T-weighted



N-weighted



- energy weighted response fully determined, particle number weighted response needs parametric input
- most of the buildup happens around $\tau/R = 0.5 - 2$
- depending on $\langle Q_s \rangle R$: increasing or decreasing trend at late τ
- realistic prediction of final v_2: hadronization?

Numerical nonlinear simulation

Setup of numerical nonlinear simulation

- comparing with numerical treatment:
  - find range of validity of linearizations in γ_R, ϵ_n
  - check for interesting observables
- Setup: switch to describing momentum moments of f

Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting PRD 102 (2020) 056003

$$C_l^m = \int \frac{\mathrm{d}^2 p_T \, \mathrm{d}p_\eta}{(2\pi)^3} p^\tau Y_l^m(\theta_p, \phi_p) f$$

  - can extract only energy weighted observables
- discretization: cutoff at finite l_{max}, lattice in $\vec{x}_T$

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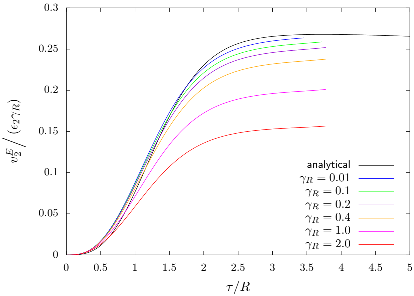
- discretization: cutoff at finite l_{max}, lattice in $\vec{x}_T$
- taking moments of Boltzmann equation: different momentum components have different angular dependence $\Rightarrow$ non-diagonal

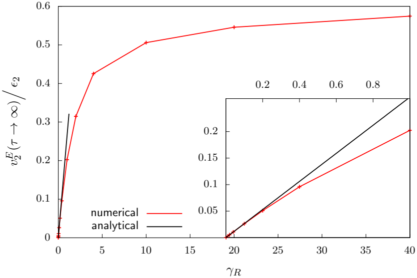
$$\partial_\tau (p^\tau f) = f \partial_\tau p^\tau - p^i \partial_i f + \frac{p^\mu u_\mu}{\tau_R}(f_{eq} - f)$$

$$\Rightarrow \partial_\tau C_l^m = \sum_{l',m'} (b_{ll'}^{mm'} + c_{ll'}^{mm'} \partial_1 + d_{ll'}^{mm'} \partial_2 + e_{ll'}^{mm'}(u^\mu)) C_{l'}^{m'} + E_l^m(u^\mu, T)$$

buildup of v_2^E



γ_R-dep. of final v_2^E



- smaller γ_R: closer to analytical result
- no significant deviation from $v_2^E \propto \epsilon_2$ for $\epsilon_2 \leq 0.2$, $\gamma_R \leq 4$
- in γ_R-dependence of final values:
  - saturation for large γ_R
  - tangential to analytical result for small γ_R
  - similar to previous numerical analysis: Kurkela, Wiedemann, Wu EPJC 79 (2019) 759, 965

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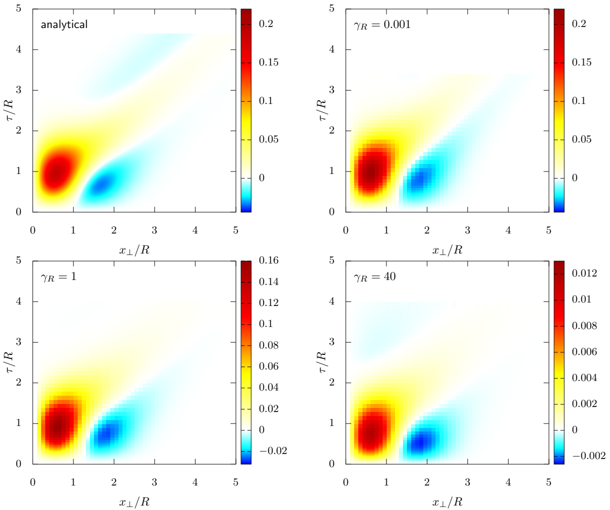

- flipped sign in outskirts of the system: particles in the peaks of the almond shape?
- larger γ_R: scaled down, relevant area shrinks towards lower τ

Summary and Outlook

So far

- successfull analytical computation of v_2 to linear order in opacity and eccentricity in RTA
- numerical results match for small opacities, are linear in eccentricity
- v_2 buildup: flipped sign in outer regions

Next steps

- use numerical code to check more observables (e.g. v_3, v_4), test initial conditions
- switch to more realistic QCD collision kernel

Backup

PDE Setup

coordinates:

$$\tau = \sqrt{t^2 - z^2} \qquad \eta = \operatorname{artanh}(z/t) \qquad y = \operatorname{artanh}(p_z/E)$$

Boltzmann equation:

$$[\underbrace{p_T \cosh(y-\eta)}_{p^\tau}\partial_\tau + p_\perp^i \partial_i + \underbrace{\frac{p_T}{\tau}\sinh(y-\eta)}_{p^\eta}\partial_\eta]f = C[f]$$

initial condition:

$$f^{(0)}(\tau_0, \vec{x}_\perp, \vec{p}_\perp, y-\eta) = \frac{(2\pi)^3}{\nu_g}\frac{\delta(y-\eta)}{\tau_0 p_\perp} F\left(\frac{Q_s(\vec{x}_\perp)}{p_\perp}\right)$$

position dependent momentum scale $Q_s(\vec{x}_\perp)$ chosen such that

$$\epsilon(\tau_0, \vec{x}_\perp) = \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\frac{1}{2\pi R^2 \tau_0}\exp\left(-\frac{\vec{x}_\perp^2}{2R^2}\right)\left\{1 + \delta_n\left(\frac{x_\perp}{R}\right)^n \exp\left(-\frac{x_\perp^2}{2R^2}\right)\cos[n(\varphi_x - \psi_n)]\right\}$$

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zeroth order $p^\mu \partial_\mu f^{(0)} = 0$:

$$f^{(0)}(\tau, \vec{x}_\perp, \vec{p}_\perp, y-\eta) = f^{(0)}\left(\tau_0, \vec{x}_\perp - \vec{v}_\perp t(\tau, \tau_0, y-\eta), \vec{p}_\perp, \mathrm{arsinh}\left(\frac{\tau}{\tau_0}\sinh(y-\eta)\right)\right)$$

first order $p^\mu \partial_\mu f^{(1)} = C[f^{(0)}]$:

$$f^{(1)}(\tau, \vec{x}_T, \vec{p}_T, y-\eta) = \int_{\tau_0}^{\tau} \mathrm{d}\tau' \left(\frac{C[f^{(0)}]}{p^\tau}\right)\left(\tau', \vec{x}'_T, \vec{p}_T, y-\eta'\right)$$

collision kernel: find local rest frame and temperature using Landau matching to compute $C_{RTA}[f^{(0)}] = \frac{p_\mu u^\mu T}{5\,\eta/s}(f_{eq} - f)$ where $f_{eq} = \frac{1}{\exp(p_\mu u^\mu/T)-1}$

$$T^{\mu\nu} = \int \frac{\mathrm{d}^3p}{(2\pi)^3 p^\tau} p^\mu p^\nu f^{(0)} \qquad \epsilon u^\mu = u_\nu T^{\nu\mu} \qquad \epsilon = \frac{\nu_g \pi^2}{30} T^4$$

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extract momentum distribution and compute its moments

- jacobian from milne coordinates

$$\frac{\mathrm{d}N}{\mathrm{d}^2 p_T \mathrm{d}y}(\tau) = \nu_g \int \mathrm{d}\sigma_\mu p^\mu f = \int \mathrm{d}^2 x_T \mathrm{d}\eta \; p_T \tau \cosh(y-\eta) \nu_g f(\tau, \vec{x}_T, \eta, \vec{p}_T, y)$$

- extract relevant moments; flow harmonics $v_n(p_T)$ in terms of weighted p_T-averages:

$$V_{m,n} = \int \mathrm{d}^2 p_T \; p_T^m e^{in\phi_p} \frac{\mathrm{d}N}{\mathrm{d}^2 p_T \mathrm{d}y} \qquad v_n^{(m)} = \frac{V_{m,n}}{V_{m,0}}$$

- in total: 6d integral over $\tau', \vec{x}_T, \eta, \vec{p}_T$. 4 computed analytically, 2 numerically

Numerical Setup

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$$C_l^m = \int \frac{\mathrm{d}^2 p_T \, \mathrm{d}p_\eta}{(2\pi)^3} p^\tau Y_l^m(\theta_p, \phi_p) f$$

- $Y_l^m(\theta_p, \phi_p)$: spherical harmonics
- ϕ_p: azimuthal momentum angle
- θ_p defined by $\cos\theta_p = \frac{p_\eta}{\tau p^\tau}$; dispersion relation $p^\tau = \sqrt{p_\perp^2 + p_\eta^2/\tau^2}$

$$\partial_\tau (p^\tau f) = f \partial_\tau p^\tau - p^i \partial_i f + \frac{p^\mu u_\mu}{\tau_R}(f_{eq} - f)$$

- taking moments: $p^1 = p_T \cos\phi_p$, $p^2 = p_T \sin\phi_p$, $p_T = p^\tau \sin\theta_p$, $p_\eta = \tau p^\tau \cos\theta_p$ together with $Y_l^m(\theta_p, \phi_p)$ result in linear combination of $Y_{l'}^{m'}(\theta_p, \phi_p)$

$$\Rightarrow \partial_\tau C_l^m = \sum_{l',m'} (b_{ll'}^{mm'} + c_{ll'}^{mm'} \partial_1 + d_{ll'}^{mm'} \partial_2 + e_{ll'}^{mm'}(u^\mu)) C_{l'}^{m'} + E_l^m(u^\mu, T)$$

- E_l^m are the moments of the equilibrium term $\frac{p_\mu U^\mu}{\tau_R} f_{eq}$

$$\partial_\tau C_l^m = \sum_{l',m'} (b_{ll'}^{mm'} + c_{ll'}^{mm'}\partial_1 + d_{ll'}^{mm'}\partial_2 + e_{ll'}^{mm'}(u^\mu))C_{l'}^{m'} + E_l^m(u^\mu, T)$$

Computation:

- $C[f]$ local $\Rightarrow$ change of C_l^m due to blue and purple terms can be computed for each $\vec{x}_T$-site individually
- red term needs nonlocal discretization of derivative $\Rightarrow$ treated separately in Fourier space
- time evolution via Runge-Kutta-algorithm with time step $0.002\,\min(\tau, \mathrm{R})$
- parameters
  - $\vec{x}_T$ lattice size 128x128
  - Lattice spacing chosen such that the total lattice has size $12R$
  - $l_{max} = 64$
  - initial time $\tau_0 = 0.01R$