Emergence of prescaling in far-from-equilibrium quark-gluon plasma

Initial Stages
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Introduction

- The early stages of a weakly-coupled QGP after a heavy-ion collision constitutes a non-hydrodynamic system, far from thermal equilibrium.

- Yet, there is mounting evidence that even at this early stage the evolution of the plasma is governed by only a handful of degrees of freedom.

    —> “Hydrodynamization”

- However, these “slow” degrees of freedom elicit some questions:

    - Is there systematic way to identify them at all times?

    - In what sense can we extend hydrodynamics to earlier times?

The path to hydrodynamics in the "Bottom-up" thermalization scenario

[1] Baier, Mueller, Schiff and Son, PLB (2001)

In the BMSS scenario,

1. Over-occupied hard gluons $f_g \gg 1$ at very early times $1 \ll Q_s\tau \ll \alpha_s^{-3/2}$
2. Hard gluons become under-occupied $f_g \ll 1$, when $\alpha_s^{-3/2} \ll Q_s\tau \ll \alpha_s^{-5/2}$
3. Thermalization of the soft sector after $\alpha_s^{-5/2} \ll Q_s\tau$

We will be working inside this regime

—> Well before hydrodynamics & thermalization

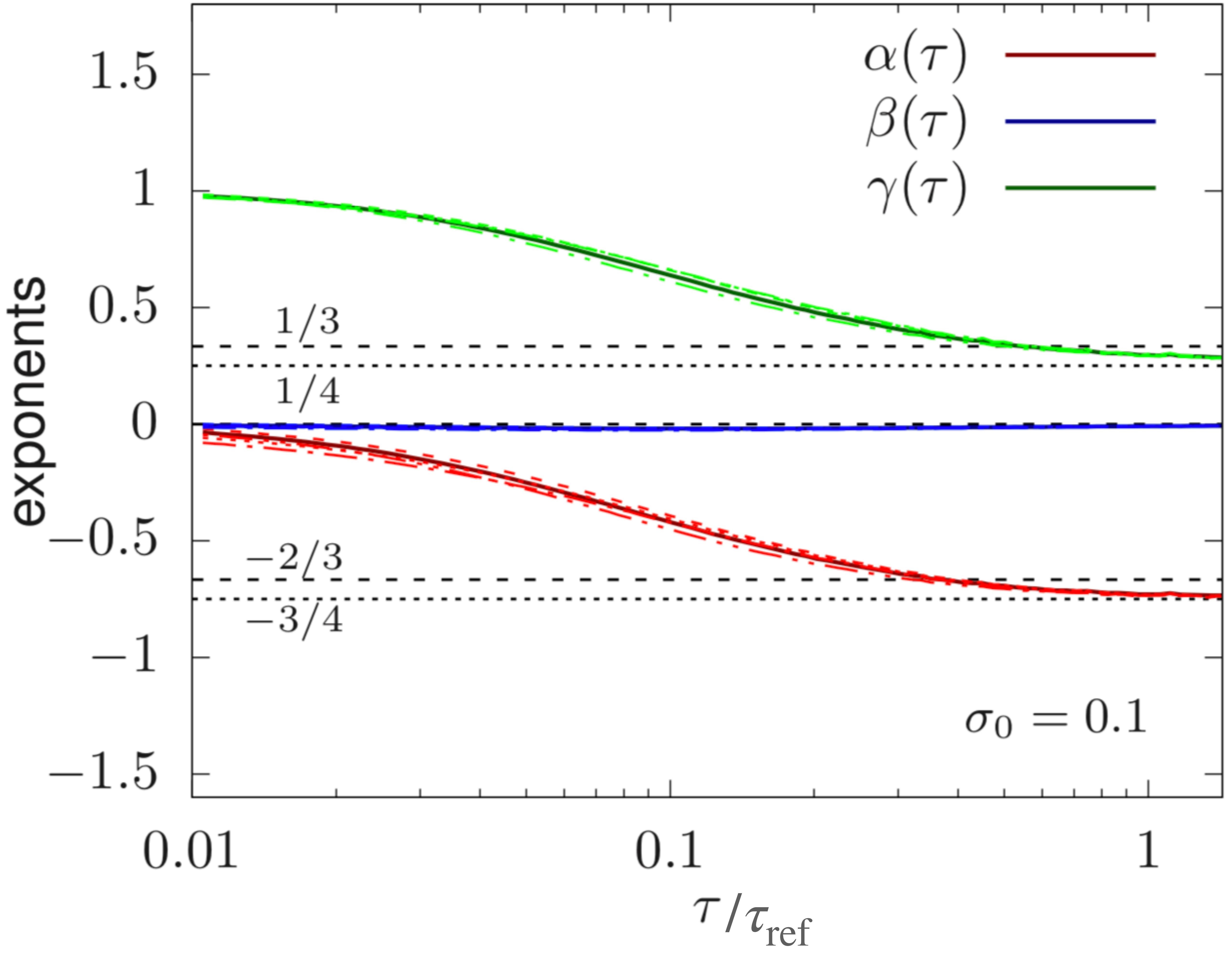
Motivation

Observation of prescaling in far-from-equilibrium QCD kinetic theory

- Prescaling: time-dependent scaling

$$f(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S\left(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z\right)$$

([2] A. Mazeliauskas, J. Berges (2019))



Motivation

$$-\frac{\partial}{\partial \tau} f(\mathbf{p},\tau) + \frac{p_z}{\tau}\frac{\partial}{\partial p_z} f(\mathbf{p},\tau) = \mathscr{C}[f(\mathbf{p})]$$

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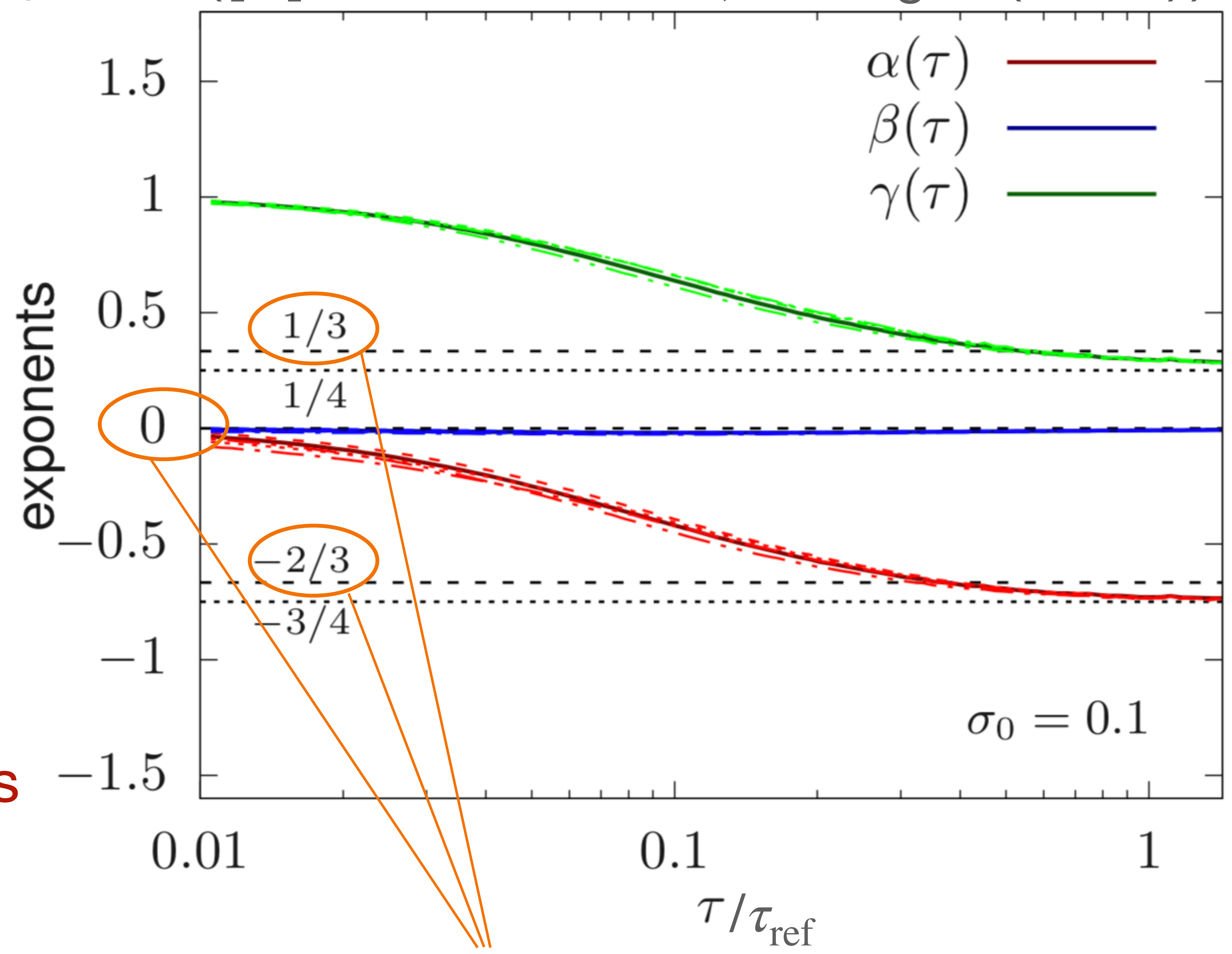
$$f(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S\left(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z\right)$$

Universal distribution function of the scaling regime [3]

Time-dependent scaling exponents

([2] A. Mazeliauskas, J. Berges (2019))



Universal scaling exponents, BMSS scenario [1]

In [2], the setup of the simulation featured

$$1 < 70 = \tau_0 Q_s \ll g^{-3} = 10^9$$

$$1 \ll 7000 = \tau_{\rm ref} Q_s \ll g^{-3} = 10^9,$$

consistent with the first stage of the bottom-up scenario [1].

Motivation

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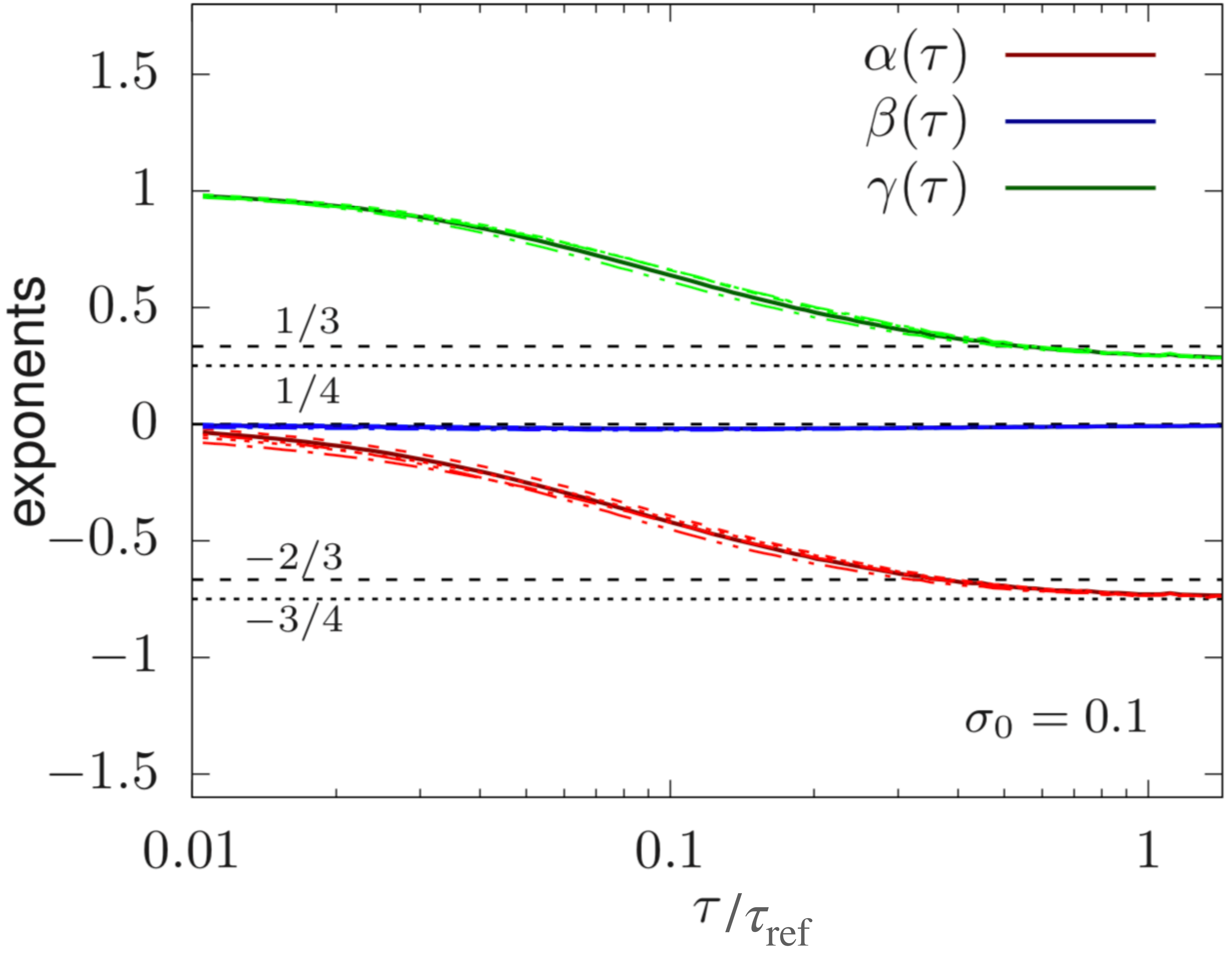
- Prescaling: time-dependent scaling

$$f(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S\big(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z\big)$$

- In this phase, three "slow" apparent degrees of freedom, α, β, γ govern the evolution.

- How do these exponents emerge? Are they actually degrees of freedom?

([2] A. Mazeliauskas, J. Berges (2019))



Adiabatic Hydrodynamization

—> Plenary talk by Jasmine Brewer [4], Friday 17:45

- Look at the kinetic equation as a Hamiltonian system:

$$\partial_y \vec{\psi} = -\mathscr{H}[y; \{F_i[\vec{\psi}]\}]\vec{\psi}.$$

—> Each eigenstate $\vec{\psi}$ of $\mathscr{H}$ constitutes an effective degree of freedom. ($\mathscr{H}\vec{\psi} = E\vec{\psi}$)

—> If the rate of change of $\mathscr{H}$ is smaller than the gap between its eigenvalues, then the lowest energy eigenstates should dominate the "slow" evolution of the system.

—> These modes should describe the "slow" properties of the plasma.

- Since α, β, γ are "slow" quantities, is it possible to understand prescaling from this point of view?

Summary of results

(what we will discuss today)

- We show that at early times the time-dependent scaling exponents are determined by the evolution of only <u>one</u> slowly varying parameter.
- We establish the nature of the slow modes in kinetic theory at early times in the small-angle scattering approximation from the adiabatic hydrodynamization perspective.

Early-time prescaling

Solution scheme

- At early times, we can use the small-angle scattering approximation, and that the typical gluon momenta satisfy $p_z \ll p_\perp \approx p$. It follows that

momentum diffusion constant [5]

$$\frac{\partial}{\partial \tau} f - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \approx \hat{q}(y) \frac{\partial^2 f}{\partial p_z^2} \approx \hat{q}(y)\, \nabla^2_{\mathbf{p}} f,$$

where $\hat{q} = 4\pi N_c^2 \alpha_s^2 l_{Cb} \int_p (1+f) f \approx 4\pi N_c^2 \alpha_s^2 l_{Cb} \int_p f^2$.

over-occupied distribution $f \gg 1$

- To solve this, we will treat $\hat{q}$ as a time-dependent parameter and look for time-dependent scaling solutions $\langle p_\perp^m p_z^n \rangle = D_{n,m} A(\tau) B(\tau)^m C(\tau)^n$.

$$\implies \alpha = \alpha[\tau; \hat{q}, \partial_\tau \hat{q}; f_0], \quad \beta = \beta[\tau; \hat{q}, \partial_\tau \hat{q}; f_0], \quad \gamma = \gamma[\tau; \hat{q}, \partial_\tau \hat{q}; f_0].$$

- The only approximation we make is that $\partial \log \hat{q} / \partial \log \tau$ is varying slowly.

Results

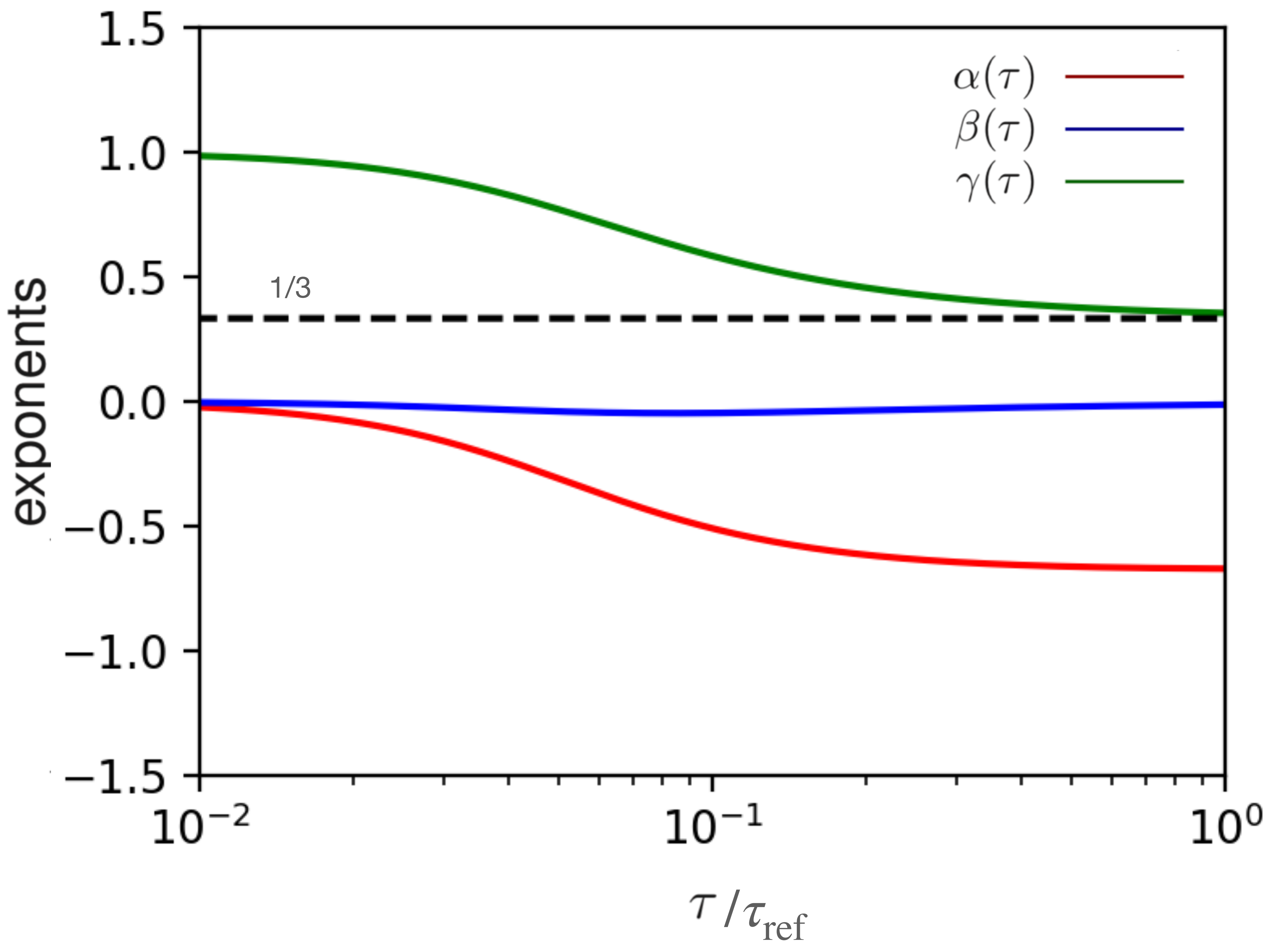
Note that (pre)scaling implies that $\frac{\partial \log \hat{q}}{\partial \log \tau} = 2\alpha - 2\beta - \gamma$.

—> Then, replacing the expressions

$$\alpha = \alpha[\tau; \hat{q}, \partial_\tau \hat{q}]$$
$$\beta = \beta[\tau; \hat{q}, \partial_\tau \hat{q}]$$
$$\gamma = \gamma[\tau; \hat{q}, \partial_\tau \hat{q}]$$

one gets a 1st order ODE for $\hat{q}$.

Solving it, $\hat{q}$ fully determines α, β, γ.



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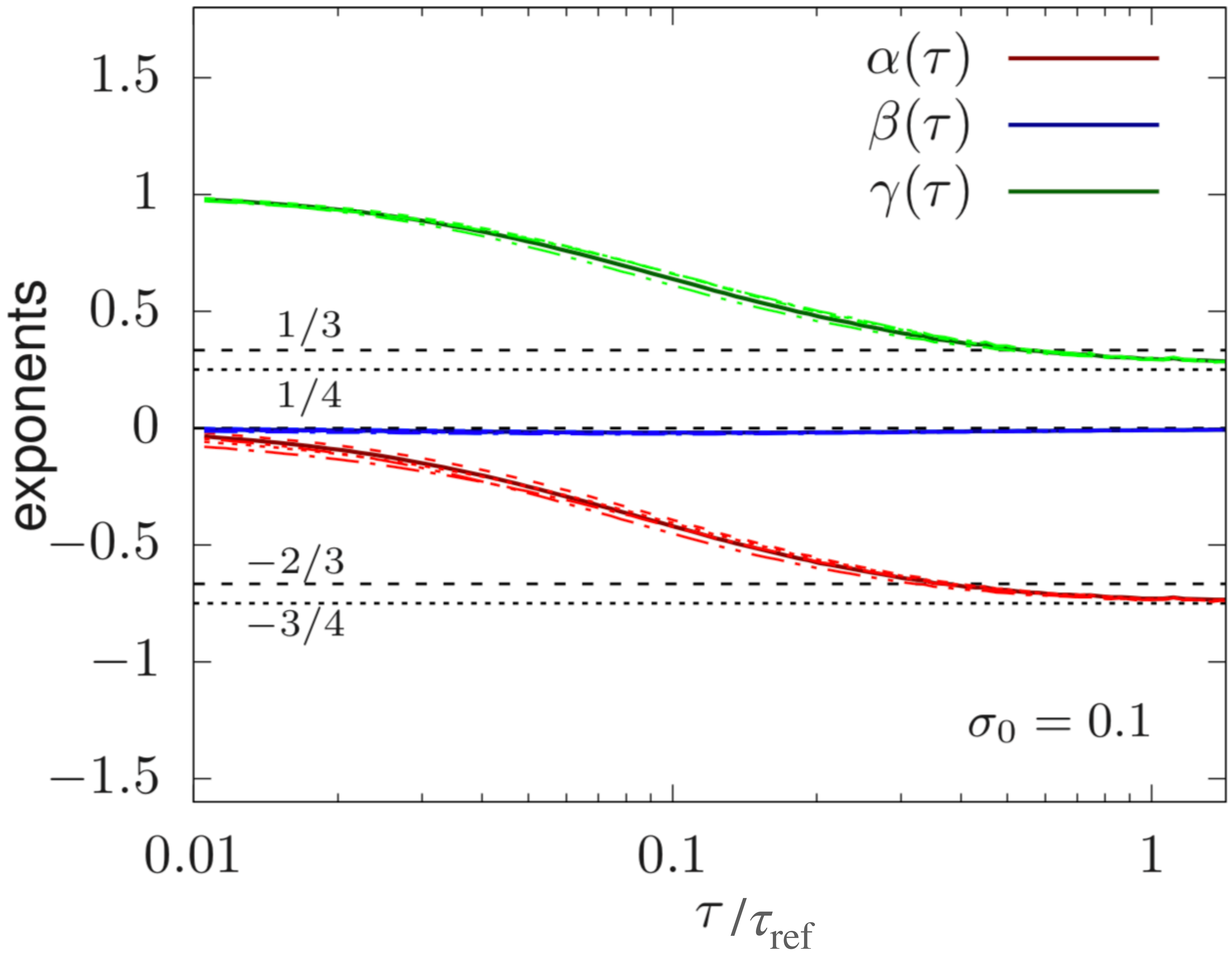

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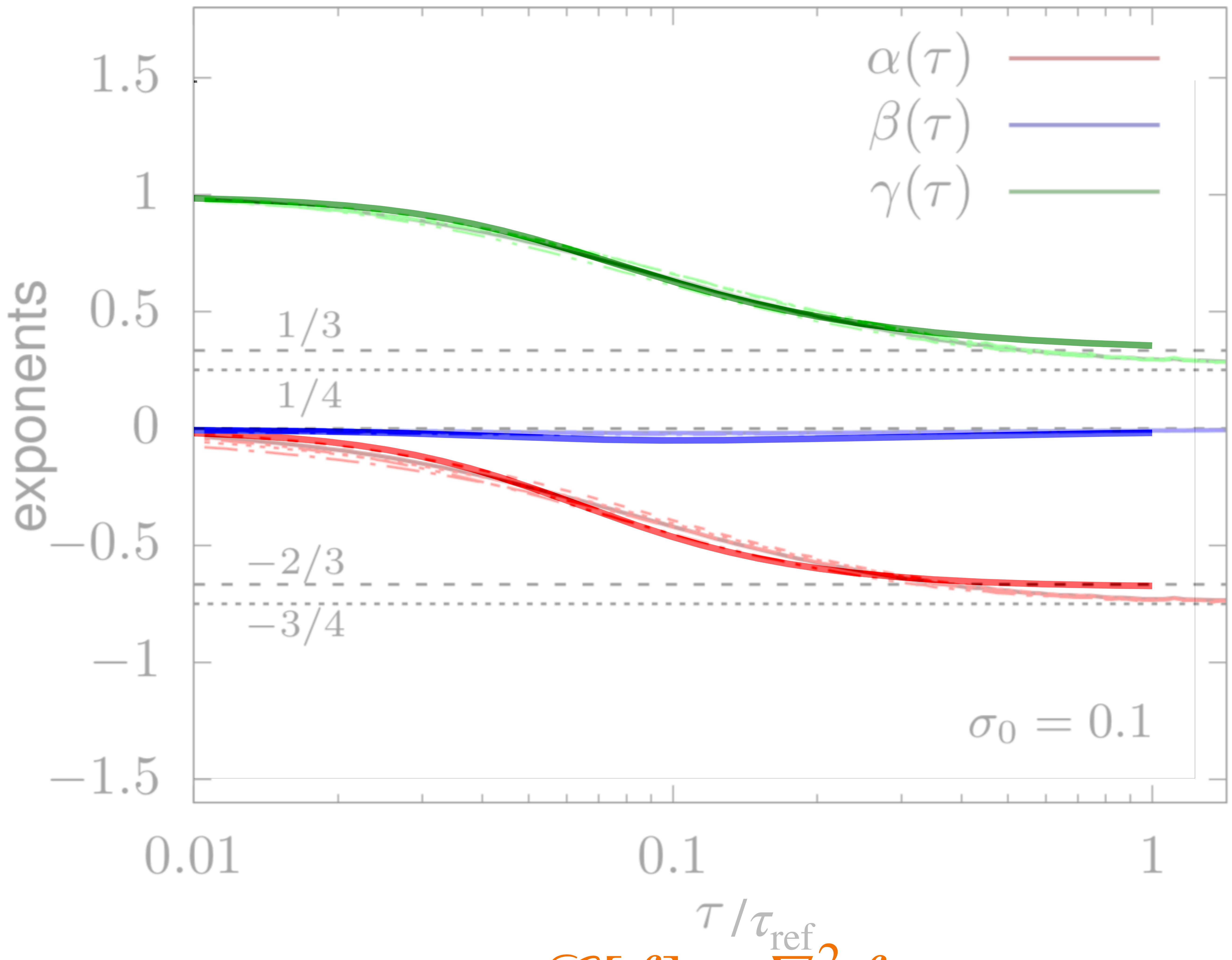

$\mathscr{C}[f] \propto \nabla^2_{\mathbf{p}} f$

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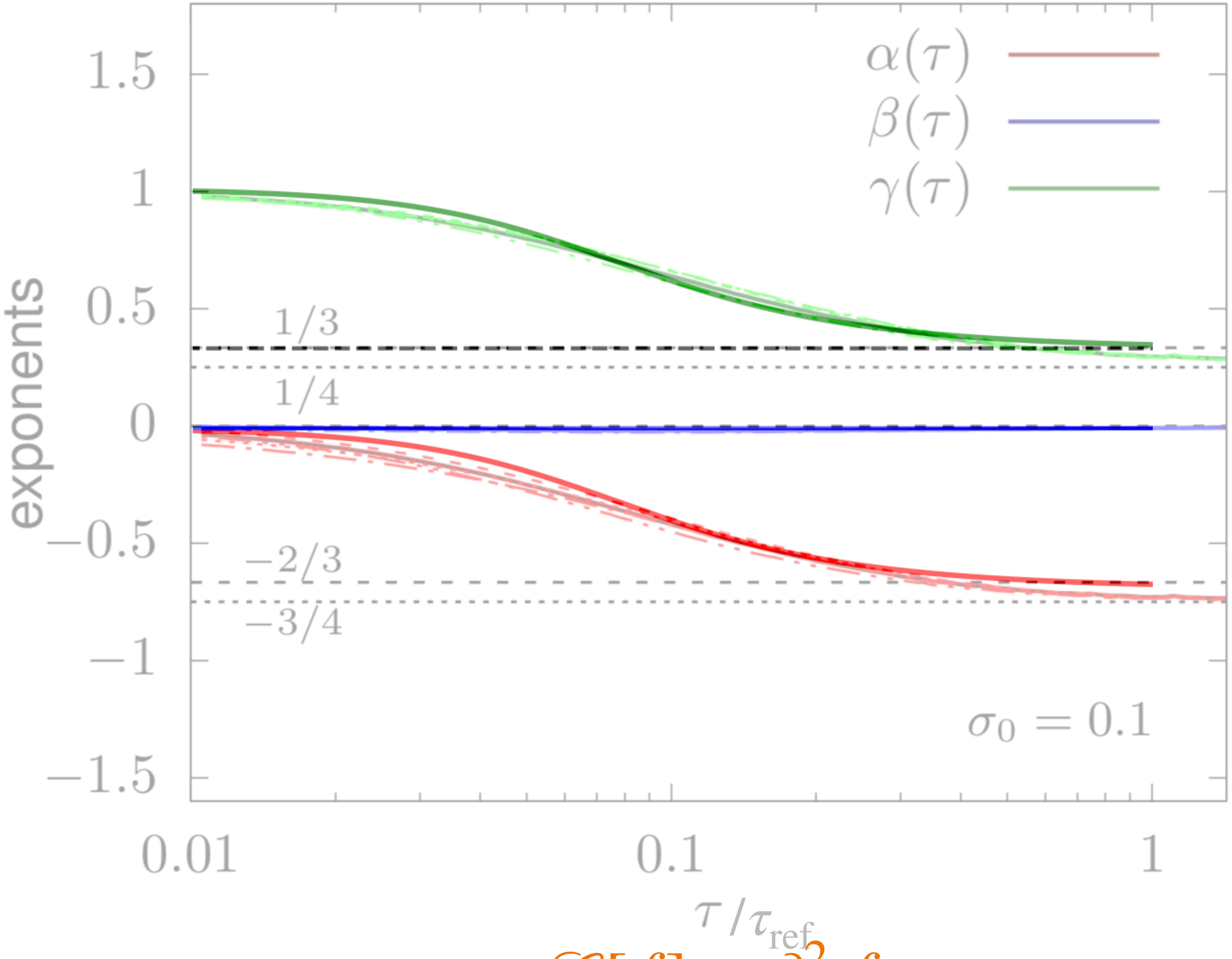

$\mathscr{C}[f] \propto \partial_{p_z}^2 f$

The adiabatic perspective: scaling

Why is this time-dependent scaling solution preferred?

—> Kinetic equation as a Hamiltonian system: $\partial_y \overrightarrow{\psi} = -\mathscr{H}[y; \{F_i[\overrightarrow{\psi}]\}]\overrightarrow{\psi}$.

—> Lowest energy eigenstate (for simplicity $\mathscr{C}[f] \propto \partial_{p_z}^2 f$) is given by

$$|\psi_0\rangle \longleftrightarrow \langle p_z^{2n} p_\perp^m \rangle \propto \frac{(2n)!}{n!} \left(\frac{\tau \hat{q}}{2}\right)^n, \; E_0 = 1.$$

—> This state exhibits time-dependent scaling. It follows that

$$\alpha = -\frac{1}{2}\frac{\partial \log \hat{q}}{\partial \log \tau} - \frac{3}{2}, \quad \gamma = -\frac{1}{2} - \frac{1}{2}\frac{\partial \log \hat{q}}{\partial \log \tau}, \quad \beta = 0, \quad \text{and} \quad \frac{\partial \log \hat{q}}{\partial \log \tau} = 2\alpha - 2\beta - \gamma.$$

—> Putting these together, $\alpha = -2/3, \quad \gamma = 1/3, \quad \beta = 0.$

The adiabatic perspective: prescaling

Eigenvalues of $\mathscr{H}$: $E_n = 2n + 1 \implies$ Energy gap.

—> After a sufficiently long time the state will be governed by the lowest modes.

$$\implies \text{initial condition}: |\psi\rangle = A_0 |\psi_0\rangle + A_1 |\psi_1\rangle.$$

—> Solving for the scaling exponents (perturbatively in A_1/A_0) gives

$$\gamma = -\frac{1}{2}\left(1 + \frac{\partial \log \hat{q}}{\partial \log \tau}\right) + \frac{A_1}{A_0}\frac{(\tau_I/\tau)^2}{4\tau\hat{q}}\left(3 + \frac{\partial \log \hat{q}}{\partial \log \tau}\right)^2, \quad \beta = 0, \quad \alpha = \gamma - 1.$$

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"0th order" BMSS exponent

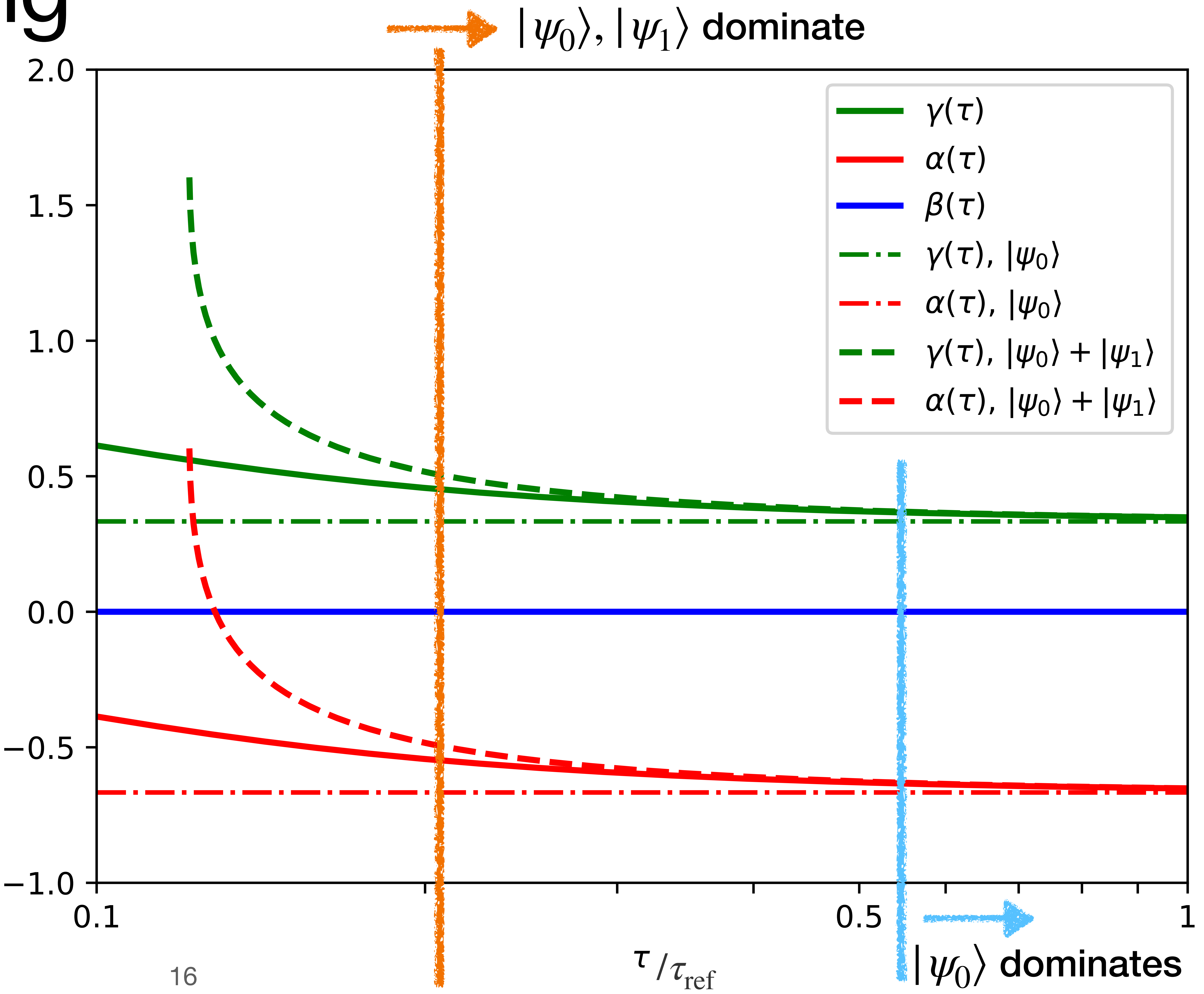
"Perturbative parameter"

"1st order" correction —> Prescaling

Now we solve the ODE $\dfrac{\partial \log \hat{q}}{\partial \log \tau} = 2\alpha - 2\beta - \gamma$

Adiabatic prescaling

- Prescaling emerges as the lowest excited states decay.
- Appears before reaching the time-independent scaling regime, for any initial condition.
- For specific choices of initial conditions (which requires $f_0 \sim f_S$), prescaling can be extended to arbitrarily early times.



Summary

- Scaling and prescaling at early times after a heavy-ion collision can be explained by following the instantaneous eigenstates of lowest energy in the kinetic equation.

—>The use of such states can greatly simplify the analysis of the QGP, even at very early times.

- This analysis extends that of [4] to an earlier stage in the QGP hydrodynamization.

Outlook

- To do: follow the evolution of a “lowest energy” eigenstate from early times until hydrodynamics.

—> Also: study other setups with the adiabatic framework.

- How to probe different scaling regimes: exponent-independent ratios of moments.

—> In particular: cumulants that vanish for specific forms of scaling distributions f_S.

Thanks!

References

- [1] R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B 502, 51-58 (2001), [arXiv:hep-ph/0009237 [hep-ph]]
- [2] A. Mazeliauskas and J. Berges, Phys. Rev. Lett. 122, no.12, 122301 (2019), [arXiv:1810.10554 [hep-ph]]
- [3] J. Berges, K. Boguslavski, S. Schlichting and R. Venugopalan:
  Phys. Rev. D 89, no.7, 074011 (2014), [arXiv:1303.5650 [hep-ph]],
  Phys. Rev. D 89, no.11, 114007 (2014), [arXiv:1311.3005 [hep-ph]]
- [4] J. Brewer, L. Yan and Y. Yin, [arXiv:1910.00021 [nucl-th]]
- [5] A. Kurkela and G. D. Moore, JHEP 12, 044 (2011), [arXiv:1107.5050 [hep-ph]]

Extra slides

How to write the kinetic equation as in the adiabatic theorem of quantum mechanics

Consider a collision integral of the form

$$\mathscr{C}[f] = -\sum_i \lambda_i(\tau; f)\big(\mathscr{O}_i f\big),$$

where λ_i are numbers that may depend non-linearly on f, and $\mathscr{O}_i$ are linear differential operators acting on f.

Then, by taking moments $\left(\mathrm{e.g.}\quad \mathrm{n}_{n,m} = \int_p p_z^{2n} p_\perp^m f\right)$, one arrives at

$$\partial_{\log\tau}\mathrm{n}_{n,m} = -(2n+1)\mathrm{n}_{n,m} - \sum_i \lambda_i M^{\mathscr{O}_i}_{n,m;n',m'}\mathrm{n}_{n',m'},$$

which is of the form $\partial_y \vec{\psi} = -\mathscr{H}[y; \{F_i[\vec{\psi}]\}]\vec{\psi}$.

Explicit form of the Hamiltonians

- If $\mathscr{C}[f] = -\hat{q}\,\nabla^2_{\mathbf{p}} f$, we have (in the $\mathrm{n}_{n,m} = \langle p_z^{2n} p_\perp^m \rangle$ basis)

$$\partial_{\log\tau}\mathrm{n}_{n,m} = -(2n+1)\mathrm{n}_{n,m} + \tau\hat{q}(\tau)\left[2n(2n-1)\mathrm{n}_{n-1,m} + m^2\mathrm{n}_{n,m-2}\right],$$

which means that

$$\mathscr{H}_{n,m;n',m'} = (2n+1)\delta_{n,n'}\delta_{m,m'} - \tau\hat{q}\left[(2n)(2n-1)\delta_{n-1,n'}\delta_{m,m'} + m^2\delta_{n,n'}\delta_{m-1,m'}\right].$$

- If we only keep the longitudinal momentum derivatives, $\mathscr{C}[f] = -\hat{q}\partial^2_{p_z} f$,

$$\partial_{\log\tau}\mathrm{n}_{n,m} = -(2n+1)\mathrm{n}_{n,m} + \tau\hat{q}(\tau)2n(2n-1)\mathrm{n}_{n-1,m},$$

$$\mathscr{H}_{n,m;n',m'} = (2n+1)\delta_{n,n'}\delta_{m,m'} - \tau\hat{q}(2n)(2n-1)\delta_{n-1,n'}\delta_{m,m'}.$$

Scaling around the late-time attractor:

- Recently, Almalool, Kurkela, Strickland (2020) showed that an aHydro ansatz

$$f(\mathbf{p};\tau) = f_{\text{Bose}}\left(\frac{\sqrt{\mathbf{p}^2 + \xi^2(\tau)p_z^2}}{\Lambda(\tau)}\right),$$

fixing ξ, Λ such that the energy-momentum tensor matches that of a full kinetic theory simulation.

—> This is also a time-dependent scaling distribution. How do we understand this?

Relaxation Time Approximation (RTA)

(near Hydro)

- As an illustrative example, consider the RTA approximation to the kinetic equation:

$$\partial_\tau f - \frac{p_z}{\tau}\partial_{p_z} f = -\frac{1}{\tau_R(\tau)}\left(f - f_{\text{eq}}\right).$$

- After the transients have died out, most of the moments behave as

$$\mathrm{n}_{n,m} \sim T^{(2n+m+3)},$$

where T is determined by the energy density of the system.

$$\implies \alpha = 0, \quad \beta = \gamma = -\frac{\partial_y T}{T}.$$

- This is also a time-dependent scaling regime, but the shape of the distribution function is different.

A way to distinguish the two regimes: a "phase transition" of the distribution function

- (Time-dependent) Scaling greatly simplifies the dynamics of a system. However,

  —> α, β, γ do not give information on the shape of the distribution function.

  —> Moreover, for scaling to take place, that shape must remain fixed, and it must be independent of α, β, γ.

- One can use this fact to find quantities independent of α, β, γ that remain constant.

  —> For instance, under (time-dependent) scaling, the ratio

$$\frac{\left\langle p_z^2 p_\perp^2 \right\rangle^2}{\left\langle p_z^4 \right\rangle \left\langle p_\perp^4 \right\rangle} \quad \text{is constant.}$$

$\Longrightarrow$ If this ratio changes, then scaling must be broken, signaling a "phase transition" out of that regime. One can use it as an order parameter to distinguish different "phases."

Explicit solutions to the scaling exponents' ODE

Motivated by [2], if the shape of the initial distribution is Gaussian, with $\langle p_z^2 \rangle = \sigma_z^2$ and $\langle p_\perp^2 \rangle = \sigma_\perp^2$, we find $(y \equiv \log(\tau/\tau_I), \quad q \equiv \tau\hat{q}, \quad g_q \equiv -\partial_y q/q)$

$$\gamma = 1 - \frac{qe^{2y}(1 - g_q/2)}{qe^{2y} - q_0 + \sigma_z^2(1 - g_q/2)}$$

$$\beta = -\frac{qg_q/2}{q_0 - q + \sigma_\perp^2 g_q(0)/2}$$

$$\alpha = -\frac{qe^{2y}(1 - g_q/2)}{qe^{2y} - q_0 + \sigma_z^2(1 - g_q/2)} - \frac{qg_q}{q_0 - q + \sigma_\perp^2 g_q(0)/2} = \gamma - 1 + 2\beta$$

Scaling exponents for higher initial occupancy

- In [2], it was also considered an initial occupancy 6 times higher.
- Prescaling starts later than in the case considered in the main section.

—> The comparison to our results starts later.

Compare with [2]:
$\mathscr{C}[f] \propto \nabla_{\mathbf{p}}^2 f$