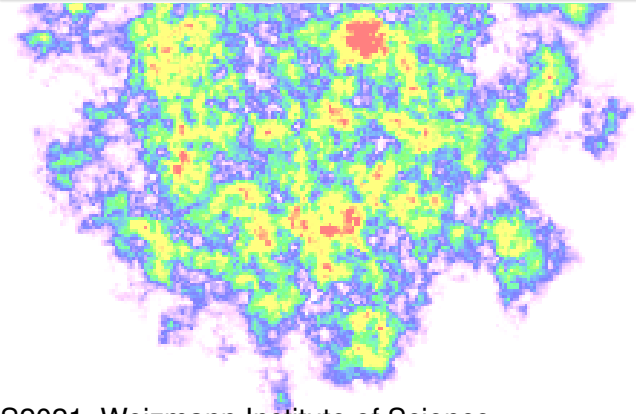
Towards fully 3-D simulations of heavy-ion collisons in the IP-Glasma Initial State



Sangyong Jeon

w/ Scott McDonald

and Charles Gale

Department of Physics
McGill University
Montréal, QC, CANADA

IS2021, Weizmann Institute of Science

McGill Team

- Charles Gale
- Sangyong Jeon
- Shuzhe Shi

- Chanwook Park → WSU
- Mayank Singh
- *Scott McDonald* → Brainbox AI: *Thesis work*
- Siggi Hauksson
- Rouzbeh Modarresi-Yazdi
- Jessica Churchil
- Matthew Heffernan
- Melissa Mendes
- Nicolas Fortier

Part of JETSCAPE and BEST. Many alumni.

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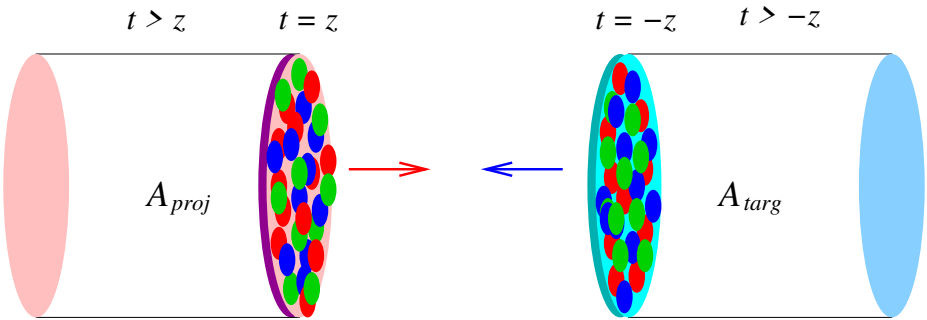
Why 3D?

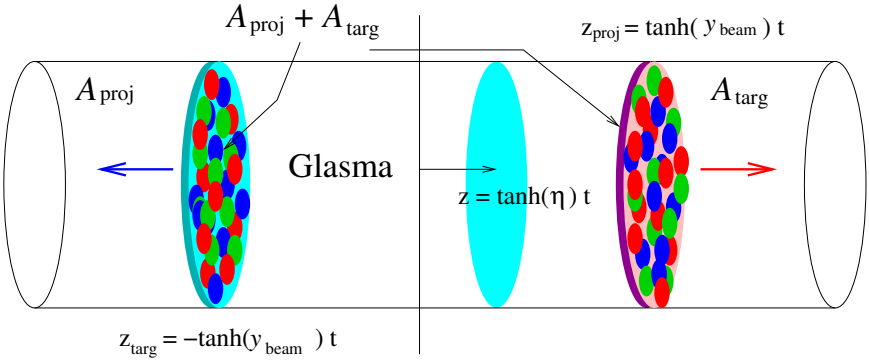
- Midrapidity observables are exhaustively studied
- More handles on characterizing QGP
- A lot of important physics in longitudinal dynamics (e.g. JIMWLK evolution)

Before the collision (2D limit)



- MV model picture
- Two nuclei approach each other accompanied by trailing gluon fields

After the collision (2D limit)



- Middle: Glasma - Result of interaction between A_{proj} and A_{targ}
- At the end-cap surfaces: *Glasma initial conditions at* $\tau = 0^+$ *in the* $y_{\text{beam}} \to \infty$ *limit*
  - $\mathcal{A}_i = \mathcal{A}_i^1 + \mathcal{A}_i^2,\ \mathcal{E}^\eta = ig[\mathcal{A}_i^1, \mathcal{A}_i^2],\ \mathcal{E}^i = 0,\ \mathcal{B}^i = 0,\ \mathcal{A}_\eta = 0$

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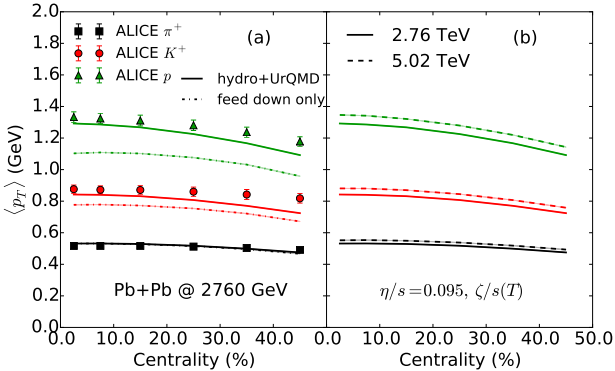
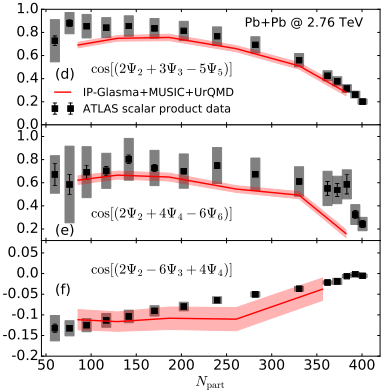
2D IP-Glasma has been successful

[Phys. Rev. C 95, 064913 (2017), McDonald, Shen, Fillion-Gourdeau, Jeon, and Gale]



PID'ed mean p_T



v_n correlation results

See also [Phys. Rev. Lett. 110, 012303, Gale, Jeon, Schenke, Tribedy, and Venugopalan]

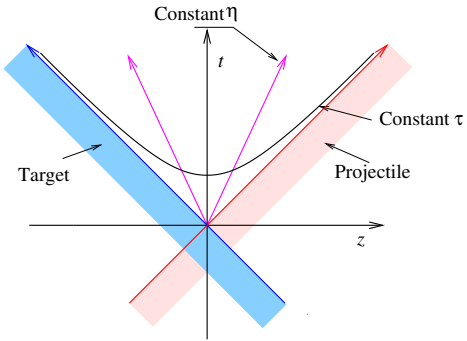
Going to 3+1D

- Add the rapidity direction to the IP-Glasma model – JIMWLK evolution
- Retain good features of the 2D IP-Glasma model
- Complications: $y_{\rm beam} < \infty$
  - Exact classical solutions not avaliable $\Longrightarrow$ Boundary still at $\tau = 0^+$
  - Validity limited within the plateau
  - Need to deal with

$$E = \int d\eta d^2 x_\perp \left(\frac{\tau}{2} \left((\mathcal{E}^\eta)^2 + (\mathcal{B}^\eta)^2 \right) + \frac{1}{2\tau} \left(\mathbf{E}_\perp^2 + \mathbf{B}_\perp^2 \right) \right)$$

Evolution in the forward light cone

Setting up the coordinate system



- Where to put $t = 0, z = 0$ is arbitrary when the nuclei are not infinitely thin
- Can choose $t = 0, z = 0$ so that there are no color charges within the forward lightcone $\Longrightarrow$ EoM:

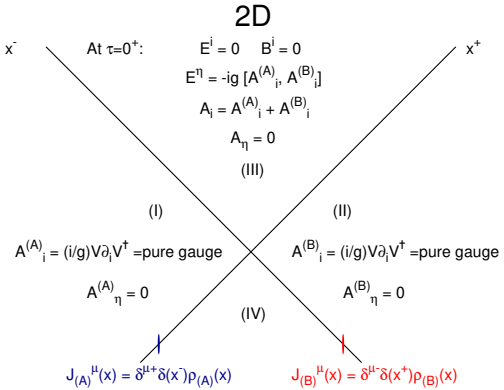
$$[\mathcal{D}_\mu, \mathcal{G}^{\mu\nu}] = 0 \text{ in the } \mathcal{A}_\tau = 0 \text{ gauge}$$

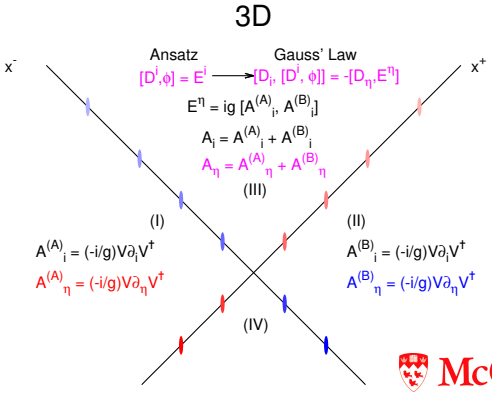
- Evolution starts at $\tau_0 \ll 1/Q_s$. Typically set to $\tau_0 = 0.01$ fm.

Finding the 3D Initial conditions

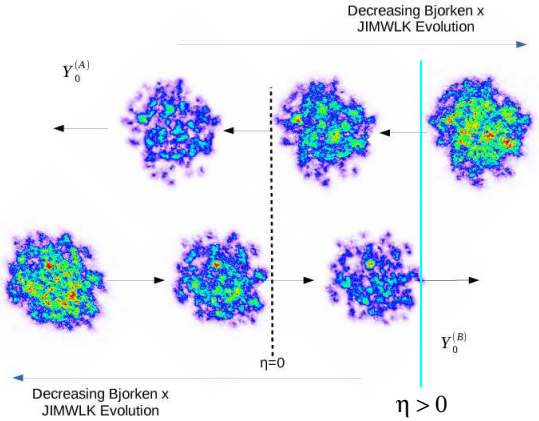
Goals

- Stay as close to the 2D initial conditions as possible
- Energy deposition only when there is overlap





Rapidity dependence



- JIMWLK evolution following Lappi and Mäntysaari [Eur. Phys. J. C73, 2307 (2013)]
- Interpretation: How the gluon field appears in a moving frame
- Finite $\eta > 0 \Longrightarrow$ Moving frame with $v^z = \tanh \eta$
- The projectile has $\gamma_p = \cosh(y_{\text{beam}} - \eta)$ and the target has $\gamma_t = \cosh(y_{\text{beam}} + \eta)$
- The target appears much denser than the projectile $\Longrightarrow \partial_\eta V \neq 0$

CGC & JIMWLK: Work by Venugopalan, McLerran, Jalilian-Marian, Iancu, Weigert, Leonidov, Kovner, Kovchegov, Dumitru, Gelis, Blaizot, Kharzeev, Nardi, Levin, Krasnitz, Nara, Lappi, Mäntysaari and many others

Initial conditions in other approaches

- Phys. Rev. D 74, 045011 (2006), Romatschke and Venugopalan: 2D initial condition plus η dependent factorized random noise
- Phys. Rev. Lett. 111, 232301 (2013), Epelbaum and Gelis: 2D initial condition plus random initial field for the quantum fluctuations
- Phys. Rev. C 89, 034902 (2014), Ozonder and Fries based on Lam and Mahlon: 2D-like initial condition with boosted Coulomb field for η dependence
- Phys. Rev. D 94, 014020 (2016), Gelfand, Ipp and Müller: 2D MV model IC. 3D Evolution in (t, z). The sources move with $v = \pm c$
- Phys. Rev. C 94, 044907 (2016), Schenke and Schlichting: 2D IC & 2D evolution for each η slice
- ArXiv:2010.11172, Schlichting and Singh: 2D MV model performed in (t, z). The sources move with $v = \pm c$

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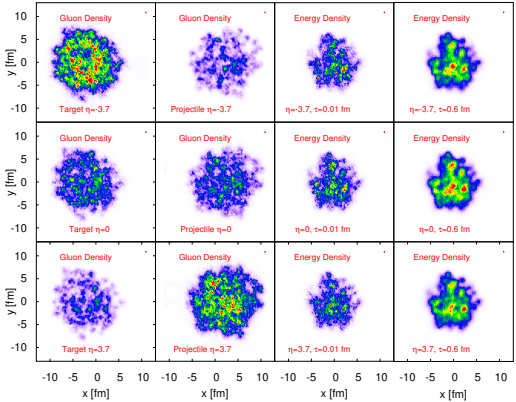
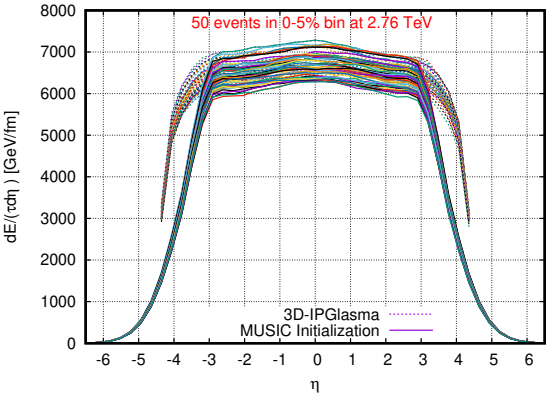
3D-Glasma Results

- 1200 3D-IP-Glasma events within $0 - 50\,\%$
- 100 UrQMD events per 3D-IPG+MUSIC event

3D Evolutions

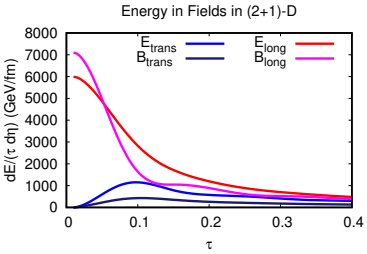


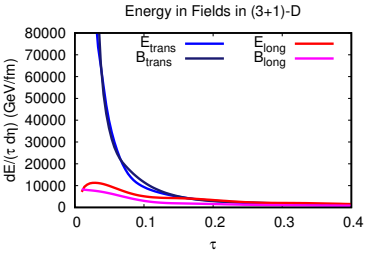
Initial state gluon densities ($\mathrm{Tr}(1-V)/N_c$) and final state enegy densities

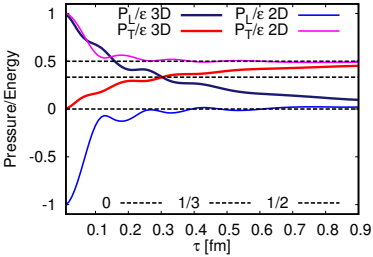


- Initial JIMWLK evolution between $\eta = 4.25$ and $\eta = -4.25$
- Hydro initial condition with a stretched Gaussian

2D vs 3D Evolutions







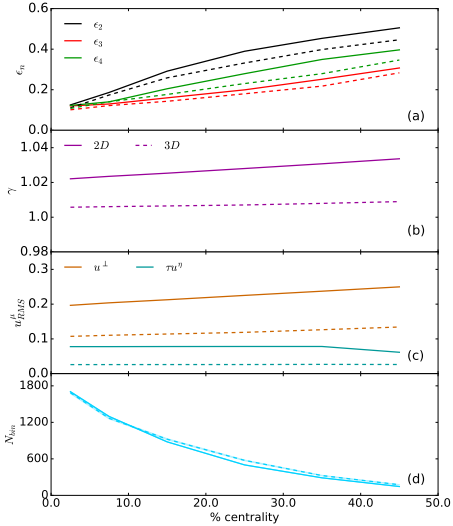
- Note the scale – 3D initial energy is *much* higher because of

$$E = \int d\eta d^2x_\perp \, \tau \left(\frac{1}{2} \left((\mathcal{E}^\eta)^2 + (\mathcal{B}^\eta)^2 \right) + \frac{1}{2\tau^2} \left(\mathbf{E}_\perp^2 + \mathbf{B}_\perp^2 \right) \right)$$

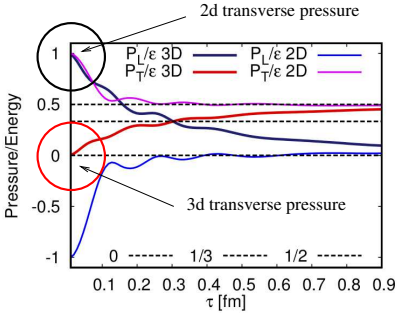
- In 3D, one *cannot* set $\mathbf{E}_\perp = 0$ and $\mathbf{B}_\perp = 0$
- Large τ behaviours are similar

Similar behaviours of $P_{T,L}/\varepsilon$ seen by Gelfand, Ipp and Müller [Phys. Rev. D 94, 014020 (2016)]

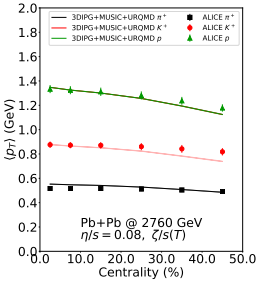
2D vs 3D Evolutions

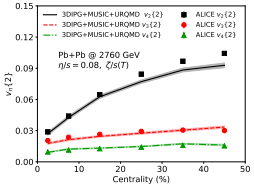


- 3D evolution results in less developed initial state eccentricity and flows
- Why: Pressure

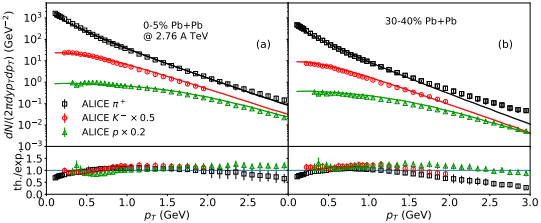


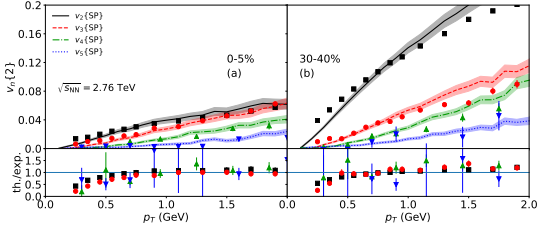
Results – Transverse dynamics





Mean p_T and Integrated v_n

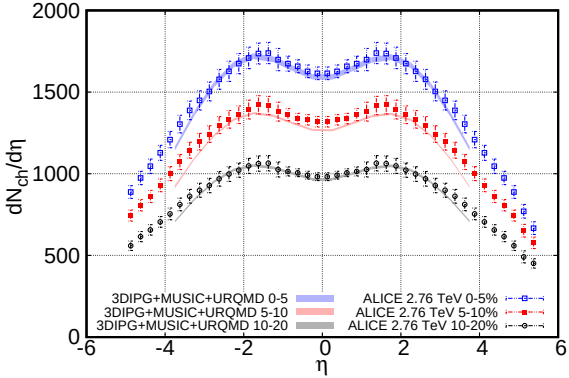




p_T spectra and differential v_n

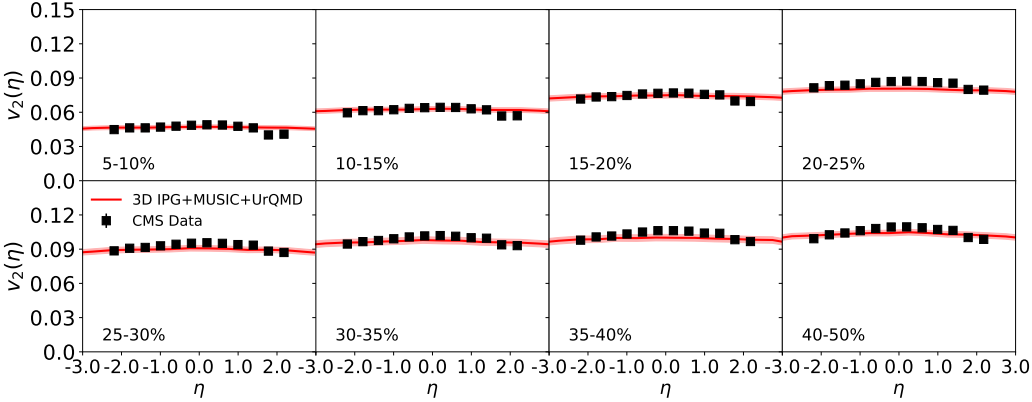
- Reasonable description mid-rapidity dynamics.
- $\eta/s = 0.08$
- ζ/s: The same as the 2D calculations in McDonald et al.'s PRC95, 064913 (2017).

Results – Longitudinal dynamics



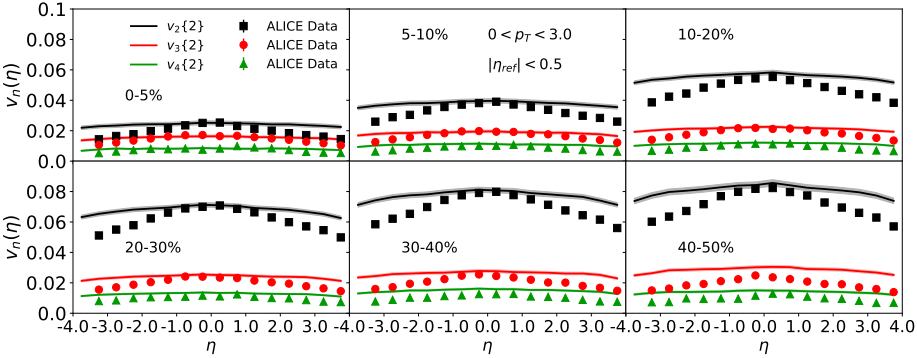
- Can capture global longitudinal dynamics
- Not yet applicable to $|\eta| \gtrsim 4$

Results – Longitudinal dynamics



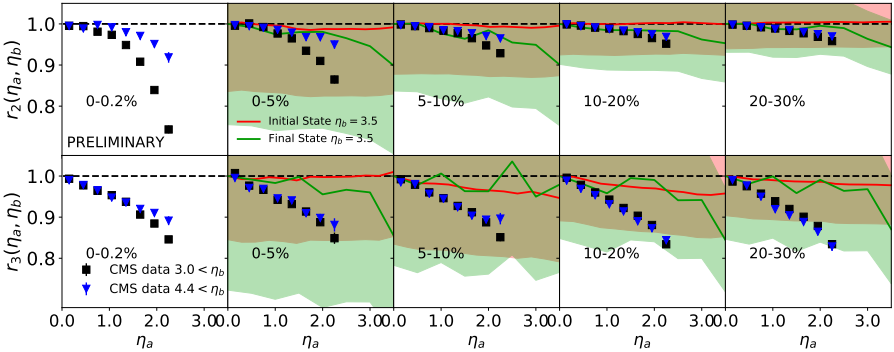
- Reference: $|\eta| < 2.4$
- $0.3\,\text{GeV} < p_T < 3\,\text{GeV}$

Results – Longitudinal dynamics



- Reference: $|\eta| < 0.5$
- $p_T > 0$
- Need more decorrelation – Thermal fluctuations?

Results – Longitudinal dynamics



- Lower centrality: Fluctuation driven
- Higher centrality: Geometry driven

- Caveat: η_b ranges vs a fixed η_b
- More statistics needed

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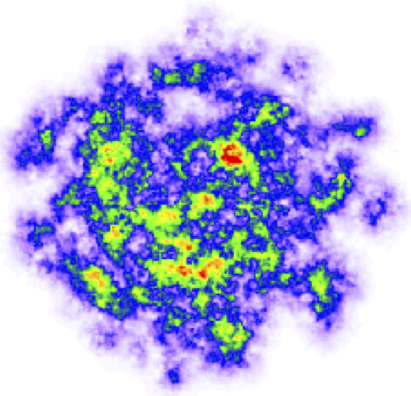
Summary & Perspectives

- Saturation physics provides good picture of initial interactions
- Going 3D is non-trivial but doable
- Good description of 3D physics possible
- A lot of physics to learn: Saturation physics, JIMWLK evolution, ...
- To do 1: Introduce more fluctuations in η
- To do 2: More events!

Backup Slides

Before the collision

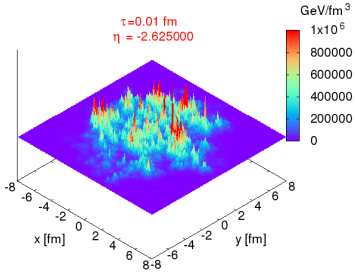
Single nucleus

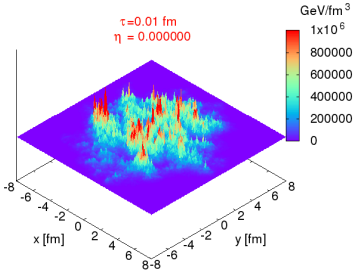


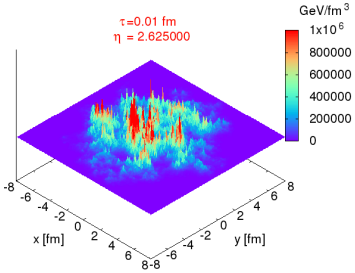
Venugopalan-McLerran picture

- Large x partons $\approx$ Lorentz contracted colour source on the light cone $t = \pm z$
- Small x partons $\approx$ Classical gluon field
- Most of entropy is in the gluon field $\Longrightarrow$ Creates the QGP medium

Initial energy distribution

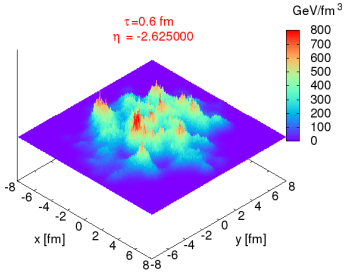


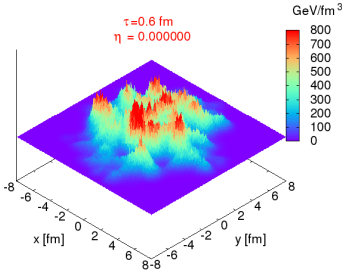


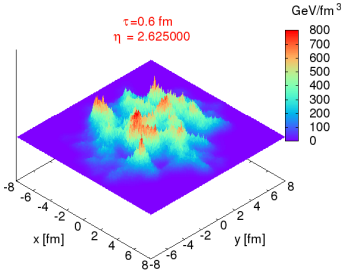


- $\sqrt{s_{NN}} = 2.76\,\text{TeV}$
- This is within the “plateau”

Energy distribution after YM evolution







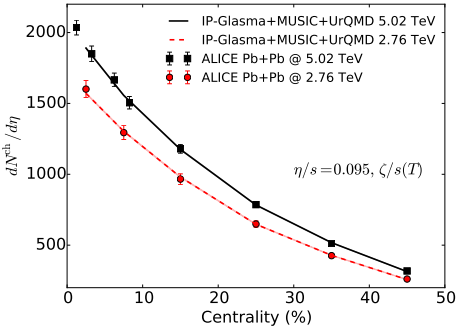
- $\sqrt{s_{NN}} = 2.76\,\text{TeV}$
- This is within the “plateau”

A bit of technical detail

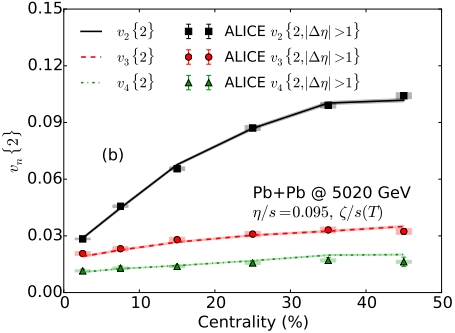
- New implementation of 3D SU(3) real-time CYM in $\tau, \eta, \mathbf{x}_\perp$
- Fully in-house code
- Time-evolution method: Leap-frog
- Gauss law solver: non-Abelian Jacobi method
- Running coupling JIMWLK following Lappi and Mäntysaari
- Initial y for JIMWLK: ± 4.25
- Hydro: MUSIC in 3+1D mode
- Hadronic afterburner: UrQMD

2D IP-Glasma has been successful

[Phys. Rev. C 95, 064913 (2017), McDonald, Shen, Fillion-Gourdeau, Jeon, Gale]



Centrality selection



Integrated v_n

- Centrality selections done by generating sample min-bias events and binning them – Turned out to be crucial
- Shear viscosity fixed by fitting the integrated v_2

Going 3D – Early attempts

[Phys.Lett.B424:15-24,1998, McLerran & Venugopalan]

- MV's definition of space-time rapidity

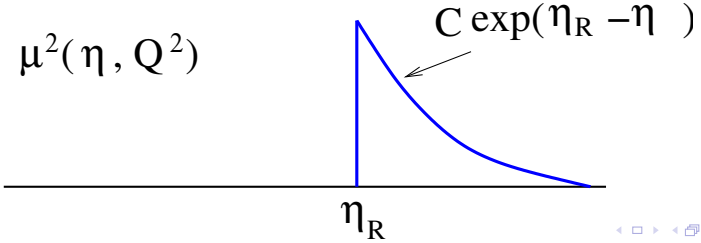
$$\eta = \eta_R + \ln(x_R^- / x^-)$$

with $x_R^- = \sqrt{2}R/\gamma$. The Gauss law

$$[\mathcal{D}_i, \mathcal{G}^{i+}] = g\rho(x^-, \mathbf{x}_\perp) \text{ becomes } [\mathcal{D}_i, \partial_\eta \mathcal{A}_i] = g\rho(\eta, \mathbf{x}_\perp)$$

with $\rho(\eta, \mathbf{x}_\perp) = x^- \rho(x^-, \mathbf{x}_\perp)$

- Color fluctuation scale per unit rapidity



Source locations

[Phys.Rev.D78:054019,2008, Gelis, Lappi, Venugopalan.

Also see Annals of Physics 340, No. 1, 119-170, S. Jeon]

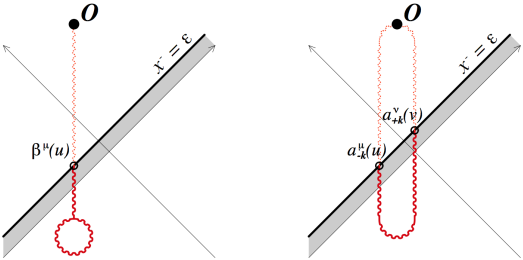


Figure 3: NLO corrections in the single nucleus case, seen as an initial value problem on the surface $x^- = \epsilon$. The shaded area represents the domain where the nuclear color sources live ($0 \leq x^- \leq \epsilon$). The field fluctuations represented in red continue to evolve in the region $x^- > \epsilon$ until they hit the operator we want to evaluate. However, this evolution is entirely hidden in the dependence of the classical field upon its initial value at $x^- = \epsilon$, and we do not need to consider it explicitly.

Within the JIMWLK approach

- The JIMWLK color sources are spatially located at or below $x^- = \epsilon$.
- Gluon field at the rapidity y given by

$$\nabla_\perp^2 \alpha(y, \mathbf{x}_\perp) = -\rho(y, \mathbf{x}_\perp)$$

- The source exists only within $0 < x^- < \epsilon$

More refined approach – JIMWLK

- The JIMWLK RG evolution equation

$$\frac{\partial \mathcal{P}[\alpha]}{\partial y} = \int_{\mathbf{u}_\perp,\mathbf{v}_\perp} \frac{\delta}{\delta \alpha_a(\mathbf{u}_\perp)} \eta^{ab}(\mathbf{u}_\perp|\mathbf{v}_\perp) \frac{\delta}{\delta \alpha_b(\mathbf{v}_\perp)} \mathcal{P}[\alpha]$$

  evolves the gluon field α from one rapidity to another

- We use the solution by Lappi and Mäntysaari [Eur. Phys. J. C73 (2013) 2307]

$$V_{\mathbf{x}}(y+dy) = \exp\left\{-i\frac{\sqrt{\alpha_s dy}}{\pi}\int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}} \cdot \tilde{\xi}_{\mathbf{z}}\right\} V_{\mathbf{x}} \exp\left\{i\frac{\sqrt{\alpha_s dy}}{\pi}\int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}} \cdot \left(V_{\mathbf{z}}^\dagger \tilde{\xi}_{\mathbf{z}} V_{\mathbf{z}}\right)\right\}$$

- Gluon field: $A^i = (i/g) V \partial^i V^\dagger$

CGC & JIMWLK: Work by Venugopalan, McLerran, Jalilian-Marian, Iancu, Weigert, Leonidov, Kovner, Kovchegov, Dumitru, Gelis, Blaizot, Kharzeev, Nardi, Levin, Krasnitz, Nara, Lappi, Mäntysaari and many others

Rapidity Dependence

- Comes from JIMWLK

$$V_{\mathbf{x}_\perp}(y+dy) = \exp\left(-i\frac{\sqrt{dy}}{\pi}\int_{\mathbf{z}_\perp}\sqrt{\alpha_S}\,\mathbf{K}_{\mathbf{x}_\perp-\mathbf{z}_\perp}\cdot\tilde{\boldsymbol{\xi}}_{\mathbf{z}_\perp}\right)$$
$$\times\, V_{\mathbf{x}_\perp}(y)\exp\left(i\frac{\sqrt{dy}}{\pi}\int_{\mathbf{z}_\perp}\sqrt{\alpha_S}\,\mathbf{K}_{\mathbf{x}_\perp-\mathbf{z}_\perp}\cdot\left(V^\dagger_{\mathbf{z}_\perp}\tilde{\boldsymbol{\xi}}_{\mathbf{z}_\perp}V_{\mathbf{z}_\perp}\right)\right)$$

with

$$\langle\tilde{\xi}^{a,i}_{\mathbf{x}_\perp}\tilde{\xi}^{b,j}_{\mathbf{y}_\perp}\rangle = \delta^{ab}\delta^{ij}\delta(\mathbf{x}_\perp-\mathbf{y}_\perp)$$

- Running coupling:

$$\alpha_S(\mathbf{k}_\perp) = \frac{4\pi}{\beta\ln\left[\left(\left(\frac{\mu_0^2}{\Lambda^2_{\rm QCD}}\right)^{1/c}+\left(\frac{\mathbf{k}_\perp^2}{\Lambda^2_{\rm QCD}}\right)^{1/c}\right)^c\right]}$$