Towards fully 3-D simulations of heavy-ion collisions in the IP-Glasma Initial State

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Part of JETSCAPE and BEST. Many alumni.
Why 3D?

- Midrapidity observables are exhaustively studied
- More handles on characterizing QGP
- A lot of important physics in longitudinal dynamics (e.g. JIMWLK evolution)
Before the collision (2D limit)

- **MV model picture**
- **Two nuclei approach each other accompanied by trailing gluon fields**
After the collision (2D limit)

Middle: Glasma - Result of interaction between $A_{\text{proj}}$ and $A_{\text{targ}}$

At the end-cap surfaces: *Glasma initial conditions at $\tau = 0^+$ in the $y_{\text{beam}} \to \infty$ limit*

- $A_i = A_i^1 + A_i^2$, $\mathcal{E}_\eta = ig[A_i^1, A_i^2]$, $\mathcal{E}^i = 0$, $B^i = 0$, $A_\gamma = 0$
2D IP-Glasma has been successful


See also [Phys. Rev. Lett. 110, 012303, Gale, Jeon, Schenke, Tribedy, and Venugopalan]
Going to 3+1D

- Add the rapidity direction to the IP-Glasma model – JIMWLK evolution
- Retain good features of the 2D IP-Glasma model
- Complications: $\gamma_{\text{beam}} < \infty$
  - Exact classical solutions not available $\implies$ Boundary still at $\tau = 0^+$
  - Validity limited within the plateau
  - Need to deal with

$$E = \int d\eta d^2x_\perp \left( \frac{\tau}{2} ((E^\eta)^2 + (B^\eta)^2) + \frac{1}{2\tau} (E^2_\perp + B^2_\perp) \right)$$
Setting up the coordinate system

- Where to put $t = 0, z = 0$ is arbitrary when the nuclei are not infinitely thin.
- Can choose $t = 0, z = 0$ so that there are no color charges within the forward lightcone.

EoM:

$$[\mathcal{D}_\mu, G^{\mu\nu}] = 0 \text{ in the } A_\tau = 0 \text{ gauge}$$

- Evolution starts at $\tau_0 \ll 1 / Q_s$. Typically set to $\tau_0 = 0.01 \text{ fm}$. 
Finding the 3D Initial conditions

**Goals**

- Stay as close to the 2D initial conditions as possible
- Energy deposition only when there is overlap

**2D**

\[ A_i = A(A)_i \]

\[ A_\eta = 0 \]

\[ \text{J}_A(x) = \delta^\mu\nu(\delta(x^+)\rho(x)) \]

\[ \text{J}_B(x) = \delta^\mu\nu(\delta(x^+)\rho(x)) \]

**3D**

\[ A_i = A(A)_i \]

\[ A_\eta = 0 \]

\[ \text{J}_A(x) = (-i/g)V\partial_\eta V^\dagger \]

\[ \text{J}_B(x) = (-i/g)V\partial_\eta V^\dagger \]

**Gauss' Law**

\[ [D^\mu, \phi] = E^\mu \]

\[ [D^\mu, [D^\mu, \phi]] = -[D_\eta, E_\eta] \]

\[ A(B)_\eta = 0 \]

\[ A(A)_\eta = 0 \]

\[ A_\eta = A(A)_\eta + A(B)_\eta \]
Rapidity dependence


- Interpretation: How the gluon field appears in a moving frame

- Finite $\eta > 0 \implies$ Moving frame with $v^z = \tanh \eta$

- The projectile has $\gamma_p = \cosh(y_{beam} - \eta)$ and the target has $\gamma_t = \cosh(y_{beam} + \eta)$

- The target appears much denser than the projectile $\implies \partial_\eta V \neq 0$

CGC & JIMWLK: Work by Venugopalan, McLerran, Jalilian-Marian, Iancu, Weigert, Leonidov, Kovner, Kovchegov, Dumitru, Gelis, Blaizot, Kharzeev, Nardi, Levin, Krasnitz, Nara, Lappi, Mäntysaari and many others
Initial conditions in other approaches

- Phys. Rev. D 74, 045011 (2006), Romatschke and Venugopalan: 2D initial condition plus $\eta$ dependent factorized random noise
- Phys. Rev. Lett. 111, 232301 (2013), Epelbaum and Gelis: 2D initial condition plus random initial field for the quantum fluctuations
- Phys. Rev. C 89, 034902 (2014), Ozonder and Fries based on Lam and Mahlon: 2D-like initial condition with boosted Coulomb field for $\eta$ dependence
- Phys. Rev. D 94, 014020 (2016), Gelfand, Ipp and Müller: 2D MV model IC. 3D Evolution in $(t, z)$. The sources move with $\nu = \pm c$
- Phys. Rev. C 94, 044907 (2016), Schenke and Schlichting: 2D IC & 2D evolution for each $\eta$ slice
- ArXiv:2010.11172, Schlichting and Singh: 2D MV model performed in $(t, z)$. The sources move with $\nu = \pm c$
3D-Glasma Results

- 1200 3D-IP-Glasma events within 0 – 50%
- 100 UrQMD events per 3D-IPG+MUSIC event
3D Evolutions

Initial state gluon densities \((\text{Tr}(1 - V)/N_c)\) and final state energy densities

- Initial JIMWLK evolution between \(\eta = 4.25\) and \(\eta = -4.25\)
- Hydro initial condition with a stretched Gaussian

50 events in 0-5% bin at 2.76 TeV

\(\frac{dE}{d\eta} [\text{GeV/fm}]\)
Note the scale – 3D initial energy is much higher because of 
\[ E = \int \frac{1}{\tau} \left( \frac{1}{2} ((E^\eta)^2 + (B^\eta)^2) + \frac{1}{2\tau^2} (E_\perp^2 + B_\perp^2) \right) \]

In 3D, one cannot set \( E_\perp = 0 \) and \( B_\perp = 0 \)

Large \( \tau \) behaviours are similar

Similar behaviours of \( P_{T,L}/\varepsilon \) seen by Gelfand, Ipp and Müller [Phys. Rev. D 94, 014020 (2016)]
2D vs 3D Evolutions

- 3D evolution results in less developed initial state eccentricity and flows
- Why: Pressure

Jeon (McGill)
Results – Transverse dynamics

- Reasonable description mid-rapidity dynamics.

- $\eta/s = 0.08$

- $\zeta/s$: The same as the 2D calculations in McDonald et al.'s PRC95, 064913 (2017).

Mean $p_T$ and Integrated $v_n$

$p_T$ spectra and differential $v_n$
Can capture global longitudinal dynamics

Not yet applicable to $|\eta| \gtrsim 4$
Results – Longitudinal dynamics

- Reference: $|\eta| < 2.4$
- $0.3\text{ GeV} < p_T < 3\text{ GeV}$
Results – Longitudinal dynamics

- Reference: $|\eta| < 0.5$
- $p_T > 0$
- Need more decorrelation – Thermal fluctuations?
Results – Longitudinal dynamics

Lower centrality: Fluctuation driven
Higher centrality: Geometry driven

Caveat: $\eta_b$ ranges vs a fixed $\eta_b$
More statistics needed
Summary & Perspectives

- Saturation physics provides good picture of initial interactions
- Going 3D is non-trivial but doable
- Good description of 3D physics possible
- A lot of physics to learn: Saturation physics, JIMWLK evolution, ...
- To do 1: Introduce more fluctuations in $\eta$
- To do 2: More events!
Backup Slides
Before the collision

**Single nucleus**

**Venugopalan-McLerran picture**

- Large $x$ partons $\approx$ Lorentz contracted colour source on the light cone $t = \pm z$
- Small $x$ partons $\approx$ Classical gluon field
- Most of entropy is in the gluon field $\Rightarrow$ Creates the QGP medium
Initial energy distribution

- $\sqrt{s_{NN}} = 2.76$ TeV
- This is within the “plateau”
Energy distribution after YM evolution

\[ \sqrt{s_{NN}} = 2.76 \text{ TeV} \]

- This is within the “plateau”
A bit of technical detail

- New implementation of 3D SU(3) real-time CYM in $\tau, \eta, x_\perp$
- Fully in-house code
- Time-evolution method: Leap-frog
- Gauss law solver: non-Abelian Jacobi method
- Running coupling JIMWLK following Lappi and Mäntysaari
- Initial $y$ for JIMWLK: $\pm 4.25$
- Hydro: MUSIC in 3+1D mode
- Hadronic afterburner: UrQMD
2D IP-Glasma has been successful

[Phys. Rev. C 95, 064913 (2017), McDonald, Shen, Fillion-Gourdeau, Jeon, Gale]

- Centrality selections done by generating sample min-bias events and binning them – Turned out to be crucial
- Shear viscosity fixed by fitting the integrated $v_2$
Going 3D – Early attempts


- MV’s definition of space-time rapidity

\[ \eta = \eta_R + \ln\left(\frac{x_R^-}{x^-}\right) \]

with \( x_R^- = \sqrt{2R}/\gamma \). The Gauss law

\[ [D_i, G^{i+}] = g\rho(x^-, x_\perp) \text{ becomes } [D_i, \partial_\eta A_i] = g\rho(\eta, x_\perp) \]

with \( \rho(\eta, x_\perp) = x^- \rho(x^-, x_\perp) \)

- Color fluctuation scale per unit rapidity

\[ \mu^2(\eta, Q^2) = C \exp(\eta_R - \eta) \]

\( \eta_R \)
Within the JIMWLK approach

- The JIMWLK color sources are spatially located at or below $x^- = \epsilon$.
- Gluon field at the rapidity $y$ given by
  $$\nabla^2_{\perp} \alpha(y, \mathbf{x}_{\perp}) = -\rho(y, \mathbf{x}_{\perp})$$
- The source exists only within $0 < x^- < \epsilon$
More refined approach – JIMWLK

- The JIMWLK RG evolution equation

\[
\frac{\partial \mathcal{P}[\alpha]}{\partial y} = \int_{\mathbf{u}_\perp, \mathbf{v}_\perp} \frac{\delta}{\delta \alpha_a(\mathbf{u}_\perp)} \eta^{ab}(\mathbf{u}_\perp | \mathbf{v}_\perp) \frac{\delta}{\delta \alpha_b(\mathbf{v}_\perp)} \mathcal{P}[\alpha]
\]

evolves the gluon field \( \alpha \) from one rapidity to another.


\[
V_x(y + dy) = \exp \left\{ -i \frac{\sqrt{\alpha_s} dy}{\pi} \int_z \mathbf{K}_{x-z} \cdot \tilde{\xi}_z \right\} V_x \exp \left\{ i \frac{\sqrt{\alpha_s} dy}{\pi} \int_z \mathbf{K}_{x-z} \cdot (V_z^\dagger \tilde{\xi}_z V_z) \right\}
\]

- Gluon field: \( A^i = (i/g) V \partial^i V^\dagger \)

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**Rapidity Dependence**

- **Comes from JIMWLK**

\[
V_{\perp}(y + dy) = \exp \left(-i \frac{\sqrt{dy}}{\pi} \int_{z_{\perp}} \sqrt{\alpha_S} K_{x_{\perp} - z_{\perp}} \cdot \tilde{\xi}_{z_{\perp}} \right)
\]

\[
\times V_{\perp}(y) \exp \left(i \frac{\sqrt{dy}}{\pi} \int_{z_{\perp}} \sqrt{\alpha_S} K_{x_{\perp} - z_{\perp}} \cdot (V_{z_{\perp}}^\dagger \tilde{\xi}_{z_{\perp}} V_{z_{\perp}}) \right)
\]

with

\[
\langle \tilde{\xi}_{a,i} \tilde{\xi}_{b,j} \rangle = \delta^{ab} \delta^{ij} \delta(x_{\perp} - y_{\perp})
\]

- **Running coupling:**

\[
\alpha_S(k_{\perp}) = \frac{4\pi}{\beta \ln \left[ \left( \frac{\mu^2_0}{\Lambda^2_{QCD}} \right)^{1/c} + \left( \frac{k^2_{\perp}}{\Lambda^2_{QCD}} \right)^{1/c} \right]^c}
\]