Nonperturbative properties of overoccupied gluonic plasmas

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Initial Stages 2021,
January 13, 2021

Talk mainly based on: arXiv:2101.02715

PRD 100, 094022 (2019), [arXiv:1907.05892]
PRD 98, 014006 (2018), [arXiv:1804.01966]

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2. Highly occupied gauge theories in 2+1D: self-similar attractor, PRD 100, 094022 (2019)
3. Broad excitations in a 2+1D overoccupied gluon plasma, arXiv:2101.02715
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Introduction

Goal

Learn about nonperturbative properties of nonequilibrium QCD

- Initial stages in heavy-ion collisions suitable playground
  - ✓ For high energy / weak coupling limit: $g^2 \equiv 4\pi\alpha_s \ll 1$
  - ✓ Gauge fields initially large $A \sim 1/g$, overoccupied
  - ✓ Nonperturbative and perturbative methods available!
- Classical-statistical lattice simulations vs. kinetic theory, HTL

Introduction

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- Here focus on **excitation spectra of 2+1D** gauge theories
  - ✓ Initially color fields approx. boost invariant (Glasma) $\Rightarrow$ eff. 2+1D

    McLerran, Venugopalan (1999); Krasnitz, Venugopalan (1999, 2000, 2001); Lappi (2003, 2006, 2011); ...
  - ✓ Later: Bottom-up scenario with kinetic theory in anisotropic 3+1D

    Baier, Mueller, Schiff, Son (2001); Berges, KB, Schlichting, Venugopalan (2013); Kurkela, Zhu (2015); Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney (2019) ..., see Talks by J.-F. Paquet, M. Strickland
- Quasiparticles? When is kinetic theory applicable?

Considered models and initial conditions

- Model 1, 2+1D: $\mathrm{SU}(N_c)$ pure gauge theory ($N_c = 2$) in 2+1D
- Model 2, Glasma-like 2+1D: add adjoint scalar ϕ to model 1
  - Dimensionally reduced 3+1D, like 'Glasma'
  - Differences: Minkowski metric, small long. pressure $P_L > 0$
  - Model of expanding Glasma for later times $\tau \gtrsim 1/Q_s$

- Initial conditions (with $E = \partial_0 A$): highly occupied

$$f(t=0,p) = \frac{Q}{g^2}\, n_0\, e^{-\frac{p^2}{2p_0^2}} \quad \text{with} \quad f(t,p) \propto \frac{\langle |E_T(t,p)|^2 \rangle}{p}$$

with $Q \propto \sqrt[4]{T^{00}}$ and subscript T for 'transverse' polarization

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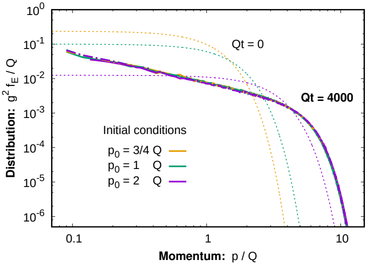
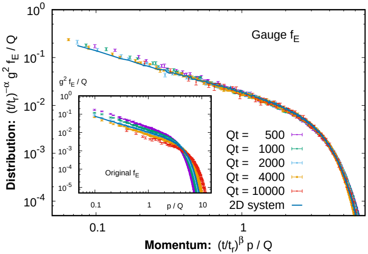
Self-similar attractor





- Both 2+1D theories approach a classical self-similar attractor

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal scaling exponents insensitive to details of initial conditions

$$\beta = -\frac{1}{5}, \quad \alpha = 3\beta \quad \text{(energy conserv.)}$$

- Hard scale $\Lambda(t) = \langle p \rangle \sim Q(Qt)^{1/5}$, occupancy $f(t,\Lambda) \sim (Qt)^{-3/5}$

Perturbative explanation of scaling exponents

- Soft scale (Debye mass) from HTL

$$m_D^2 \approx d_{\mathrm{pol}} N_{\mathrm{c}} \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \frac{g^2 f(t,p)}{p}$$

- Momentum diffusion with kinetic estimates (Backup)

$$\Lambda^2 \sim \hat{q}\, t \quad \text{with} \quad \hat{q} \sim \int \mathrm{d}q_\perp \frac{\mathrm{d}\Gamma}{\mathrm{d}q_\perp} q_\perp^2$$

leads to observed $\Lambda \sim Q(Qt)^{1/5}$ and $m_D \sim Q(Qt)^{-1/5}$

- Kinetic estimates work $\Rightarrow$ eff. kinetic descr. may exist for $p \gg m_D$
- *Remark:* in 2+1D soft momentum transfer $q_\perp \sim m_D$ crucial

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Spectral and statistical correlation functions

- Kinetic theory requires dominance of quasiparticles $\Rightarrow$
  What are the relevant excitations out of equilibrium / at initial stages?
- Knowledge of spectral function needed ($\dot{\rho} = \partial_t \rho$, $E = \partial_t A$)

$$\dot{\rho}(x, x') = \frac{i}{N_c^2 - 1} \left\langle \left[\hat{E}(x), \hat{A}(x') \right] \right\rangle$$

- Statistical correlator in general independent of $\dot{\rho}$

$$\langle EE \rangle(x, x') = \frac{1}{2(N_c^2 - 1)} \left\langle \left\{ \hat{E}(x), \hat{E}(x') \right\} \right\rangle$$

- Distinguish polarizations (transverse T, longitudinal L, scalar ϕ)
- Fourier transf. in $t - t'$ and $\vec{x} - \vec{x}'$ to frequency ω and momentum $\vec{p}$

Perturbative computation: HTL results

- Hard loop (HTL) framework applicable for $m_D/\Lambda \ll 1$; in thermal equ. for $g \sim m_D/T \ll 1$. Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...
- In 3+1D $m_D^2 = 4N_c \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{p} \sim g^2 f\, \Lambda^2 \Rightarrow$ HTL applicable
- In 2+1D soft-soft interactions important

$$m_D^2 \approx d_{\rm pol} N_{\rm c} \int \frac{\mathrm{d}^2p}{(2\pi)^2} \frac{g^2 f(t,p)}{\sqrt{m_D^2 + p^2}} \sim g^2 f\, \Lambda \ln\left(\frac{\Lambda}{m_D}\right)$$

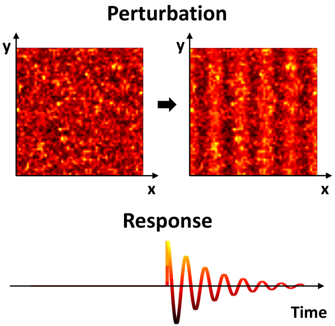
$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$

- Comparison to HTL still useful to extract nonperturbative features
- Quasiparticles in $\rho^{\rm HTL}(\omega, p)$ as $\sim \delta\big(\omega - \omega_\alpha^{\rm HTL}(p)\big)$
- All expressions depend only on m_D, computed consistently in HTL

Nonperturbative computation of spectral function ρ

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, A. Kurkela, T. Lappi, J. Peuron, *PRD 98, 014006 (2018),* Editors' suggestion

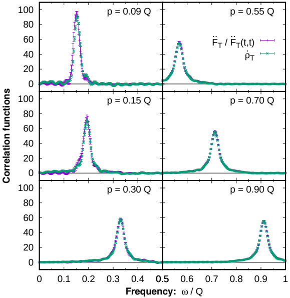


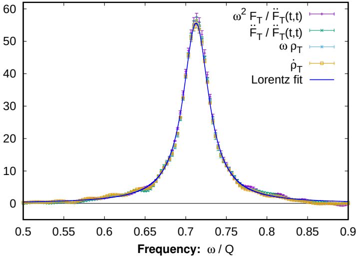
- Perturb $A(t,\vec{x}) \mapsto A(t,\vec{x}) + a(t,\vec{x})$
- Use temporal gauge $A_0 = 0$
- Class. EOM for A: $D_\mu F^{\mu\nu}[A] = 0$ (in gauge-covariant formulation)
- Linearized EOM for $a(t,\vec{x})$ (also in gauge-cov. formulation)
  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $G_R(t,t',p) = \theta(t-t')\ \boxed{\rho(t,t',p)}$ response related to $a(t,\vec{p})$

Similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020)

Application: isotropic 3+1D (PRD 98, 014006 (2018))





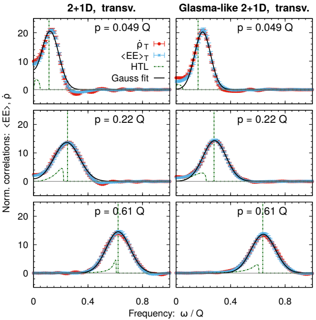
- Narrow Lorentzian quasiparticle peaks for all momenta, even $p \lesssim m_D$
- Generalized fluctuation dissipation relation (FDR) for $\alpha = T, L$

$$\frac{\langle EE \rangle_\alpha(t, \omega, p)}{\langle EE \rangle_\alpha(t, \Delta t{=}0, p)} \approx \frac{\dot{\rho}_\alpha(t, \omega, p)}{\dot{\rho}_\alpha(t, \Delta t{=}0, p)}$$

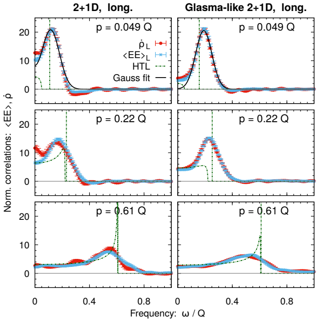
- HTL at LO describes main features well (Backup)
- Width $\gamma_\alpha(p) \ll \omega_\alpha(p)$ (Backup), decreases $\gamma_\alpha(t) \sim (Qt)^{-2/7} m_D(t)$

Now: correlations in 2+1D theories

Transverse



Longitudinal



- Generalized FDR observed
- Broad peaks $\gamma_\alpha \sim \omega_{\rm pl} \equiv \omega_T(p{=}0) \propto m_D$ [in HTL $2\omega_{\rm pl}^2 = m_D^2$]

$\Rightarrow$ **no quasiparticles for $p \lesssim m_D$!**

Now: correlations in 2+1D theories

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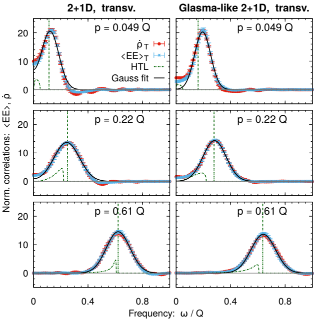

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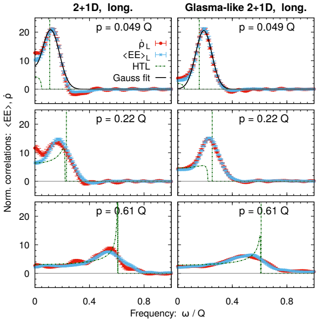

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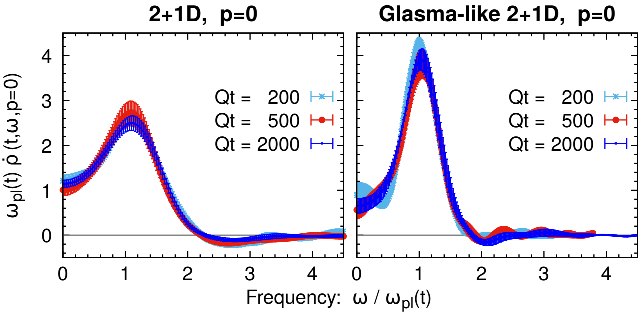
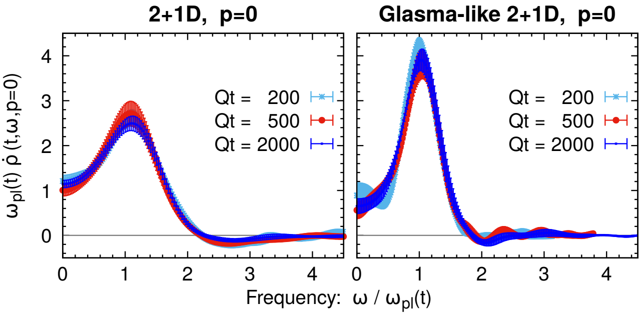
  $\Rightarrow$ **no quasiparticles for $p \lesssim m_D$!**
- Non-Lorentzian shape of the peaks (Backup)
- HTL curves (green) agree poorly (except for $\omega \ll p$ for long.)
- At low p: for $\omega \to 0$, $\dot\rho_T = \omega\rho_T$ finite, $\rho_T \sim 1/\omega$

Time dependence



- $\omega_{\rm pl}\,\dot\rho(t,\omega/\omega_{\rm pl},p/\omega_{\rm pl})$ is time independent (for all p, Backup)
- This implies $\boxed{\gamma_\alpha(t,p)\sim\omega_{\rm pl}(t)}\sim m_D\sim Q(Qt)^{-1/5}$
- Estimates as for 3+1D lead to $Q(Qt)^{-2/5}\Rightarrow$ different mechanism

Time dependence



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- Estimates as for 3+1D lead to $Q(Qt)^{-2/5}\Rightarrow$ different mechanism
- Also in classical thermal equilibrium $\gamma\sim\omega_{\rm pl}$ (Backup)
- No quasiparticles at low p seems to be quite general in 2+1D

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Conclusion

- In 2+1D, classical attractor observed and ρ extracted
- Much broader peaks in 2+1D than in 3+1D, with $\gamma(t) \sim m_D(t)$
  $\Rightarrow$ excitations too short-lived to form quasiparticles for $p \lesssim m_D$
- However, an effective kinetic description possible for $p \gg m_D$
  - ✓ explains scaling exponents of self-similar attractor
  - ✓ but requires nonperturbatively determined collision kernel
- *Outlook*
  - ρ in Bjorken expanding systems
  - How does an eff. kinetic theory in 2+1D systems look like?

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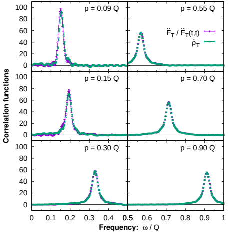
- *Outlook*
  - ρ in Bjorken expanding systems
  - How does an eff. kinetic theory in 2+1D systems look like?

Other nonperturbative properties out of equilibrium (selection)

- *Gauge-invariant condensation*, Berges, KB, Mace, Pawlowski, PRD 102, 034014 (2020)
- *Heavy quark diffusion in an overoccupied gluon plasma*,
  Talk by J. Peuron on Tue, at 19:20
- *Jet momentum broadening in real-time lattice simulations of the glasma*, Posters by D. Müller on We, and D. Schuh on Tue



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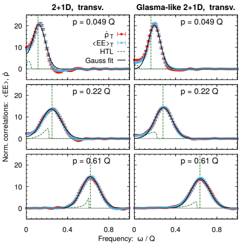
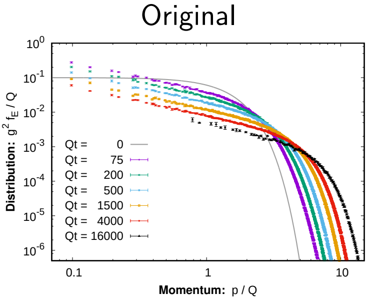


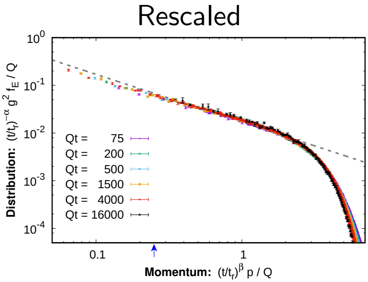
3+1D

2+1D

Backup slides

Self-similarity of 2+1D theory (PRD 100, 094022 (2019))





- Self-similar evolution

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal scaling exponents

$$\beta = -\frac{1}{5}, \quad \alpha = 3\beta \quad \text{(energy conserv.)}$$

Perturbative explanation of scaling exponents (PRD 100, 094022 (2019))

- Soft scale (Debye mass) from HTL

$$m_D^2 \approx d_{\rm pol} N_{\rm c} \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \frac{g^2 f(t,p)}{p} \sim g^2 f \, \Lambda \, \ln(\Lambda/m_D)$$

(Log from soft-soft interactions $\Rightarrow$ breakdown of HTL at m_D in 2+1D)

- Scaling exponents from kinetic arguments:

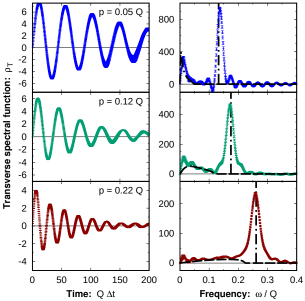
Elastic scattering rate: $$\frac{\mathrm{d}\Gamma}{\mathrm{d}q_\perp} \sim \frac{g^4}{(q_\perp^2 + m_D^2)^2} \int \mathrm{d}^2 p \, f(1+f)$$

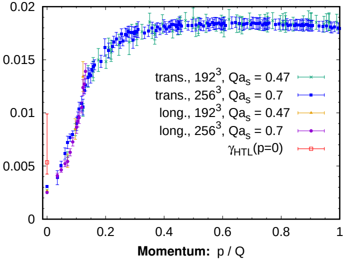
Momentum diffusion: $$\hat{q} \sim \int \mathrm{d}q_\perp \frac{\mathrm{d}\Gamma}{\mathrm{d}q_\perp} q_\perp^2 \sim \frac{\Lambda^2 \, (g^2 f)^2}{m_D}$$

From broadening $\Lambda^2 \sim \hat{q}\, t$ follows $\Lambda \sim Q(Qt)^{1/5}$

- In 2+1D $q_\perp \sim m_D$ crucial! If $q_\perp \sim \Lambda$, then $\Lambda \sim Q(Qt)^{1/7}$ instead!
- Kinetic estimates work $\Rightarrow$ eff. kinetic descr. may exist for $p \gg m_D$

Spectral function in isotr. 3+1D (PRD 98, 014006 (2018))



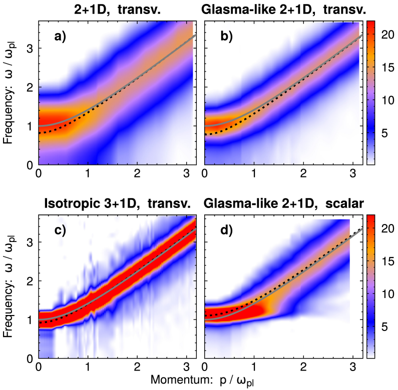


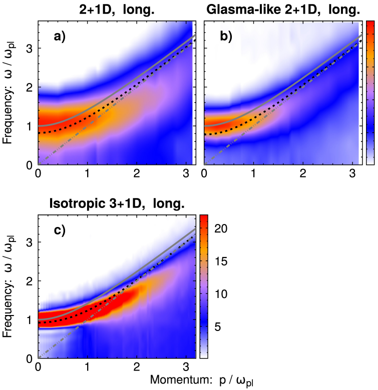
- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

- $\gamma_{T/L}(p)$ beyond HTL at LO
- Here first determination!
- 'isotropic' $\gamma_T(p) \approx \gamma_L(p)$
- HTL prediction $\gamma(p = 0)$

  Braaten, Pisarski, PRD 42, 2156 (1990)

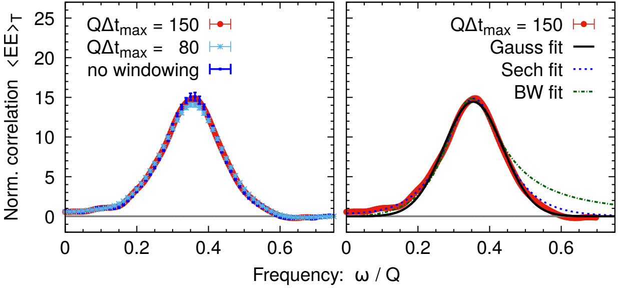
Correlation functions in 2+1D (arXiv:2101.02715)





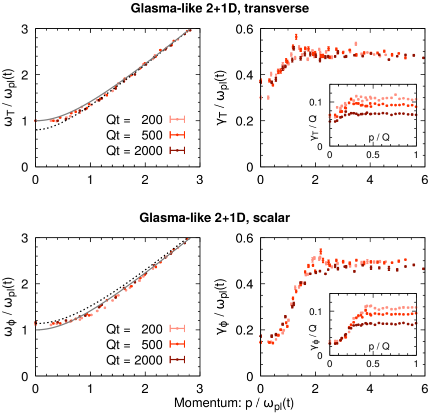
- Black dashed: $\omega_\alpha^{\mathrm{HTL}}(p)$, gray: $\sqrt{\omega_{\mathrm{pl}}^2 + p^2}$, with $\omega_{\mathrm{pl}} \propto m_D$
- Both dispersions agree sufficiently well with the results
- Scalar excitations narrow for $p \lesssim m_D$, agree with transv. for $p \gg m_D$

Shape of the excitation peaks in 2+1D (arXiv:2101.02715)



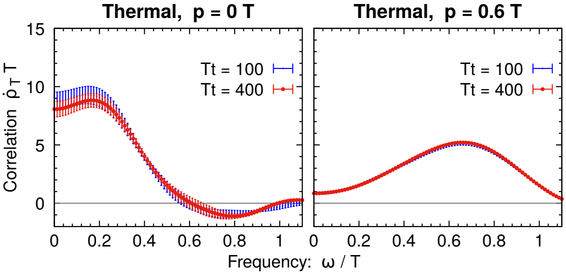
- *Left:* Different ways of computing the Fourier transform are consistent
- *Right:* Peak has non-Lorentzian shape (not Breit-Wigner)

Dispersion relations, damping rates in Glasma-like 2+1D (arXiv:2101.02715)



- *Left:* Dispersions $\omega_\alpha(t,p)/\omega_{\mathrm{pl}}(t)$
- *Right:* Peak width $\gamma_\alpha(t,p)/\omega_{\mathrm{pl}}(t)$
- As functions of $p/\omega_{\mathrm{pl}}(t)$ time independent $\Rightarrow \gamma(t,p) \sim \omega_{\mathrm{pl}}(t)$
- Scalar excitation narrow for $p \lesssim m_D$, but same t dependence

ρ in 2+1D classical thermal equilibrium (arXiv:2101.02715)



- Qualitatively similar behavior as far from equilibrium
  - ✓ Broad gluonic excitations with $\gamma(p) \sim \omega_{\rm pl}$
  - ✓ HTL provides poor description
  - ✓ For $\omega \to 0$, $\dot{\rho}_T = \omega\rho_T$ finite at low p
- Interpret.: qualitative features may be generic to 2+1D gauge theories