Far From Equilibrium Initial Conditions and the Search For the QCD Critical Point

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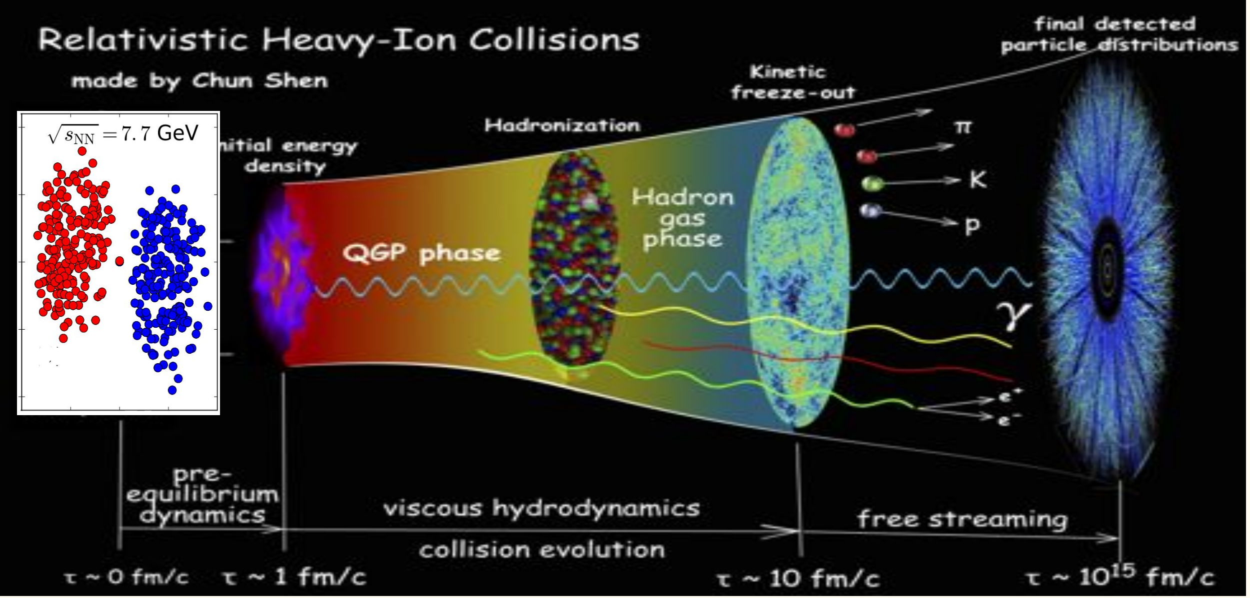
Overview

- What is needed for a full, dynamical model of the physics at the Beam Energy Scan?
  - Where does this work fit?

- Israel-Stewart vs DNMR
  - Most robust approaching criticality?
  - Far-from-equilibrium influence?

- Moving towards a (3+1)BSQ, SPH code
  - Where are we and what's left?



Initial State

- ➢ Baryon Stopping (some work has been done)
  C. Shen, B. Schenke *Phys.Rev.* C. **97**
- ➢ Initializing full $T^{\mu\nu}$
  - ○ Also problem at $\mu_B = 0$

Equation of State

- ➢ QCD EoS and Fermi Sign Problem

Freeze-out

- ➢ Need to conserve locally, not just on average D. Oliinychenko, et al., *Phys. Rev.C* 102 (2020) 3
- ➢ Out of equilibrium corrections
  - ○ Also problem at $\mu_B = 0$

Hydro Implementation

- ➢ (3+1)D with finite BSQ
- ➢ Transport Coefficients?
- ➢ Critical Fluctuations
- ➢ Correct Formulation?

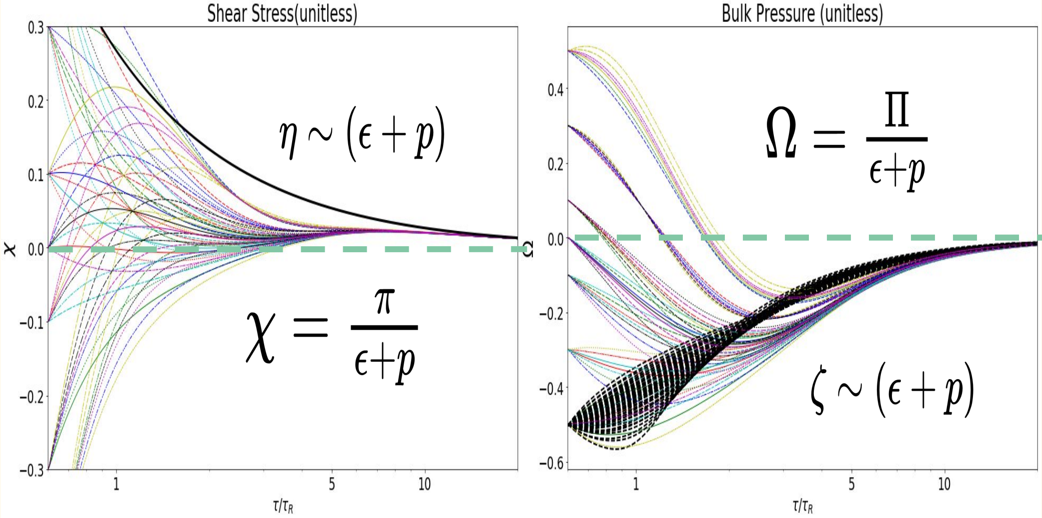
This talk touches on a few different areas of needed research

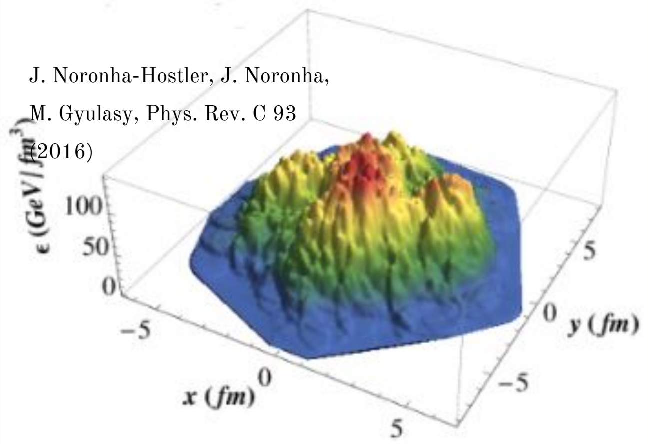
For more: poster: D. Almaalol (C), talk: J. Noronha (NT2), talks: L. Du (IS6), C. Shen (CD5)

Attractors and Far-From-Equilibrium Hydro

Seems to be *robust* feature of kinetic theory and different hydrodynamic formulations:

Heller and Spalinski, PRL 115 (2015); Gabriel S. Denicol and Jorge Noronha, Phys. Rev. D 97; Romatschke P. J., High Energ. Phys. (2017); F. Bemfica, M. Disconzi, J. Noronha Phys. Rev. D 98 (2018); M. Strickland, J. Noronha, G. Denicol, *Phys.Rev.D 97 (2018) 3*





J. Noronha-Hostler, J. Noronha, M. Gyulasy, Phys. Rev. C 93 (2016)

The existence of attractors makes far-from-equilibrium Hydro plausible, and even likely in small systems.

A. Bzdak, et al., *Phys.Rev.*C 87 (2013); H. Niemi, G. Denicol, arXiv:1404.7327

Israel-Stewart vs DNMR

- ★ Independent and dynamic viscous currents that *relax* to Navier-Stokes values before equilibrium
- ★ 'Second order theory' in Knudsen and inverse Reynold's numbers

Relaxation equations with boost invariance and polar symmetry:

$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \frac{\eta T \pi^\eta_\eta}{2}\left(\beta_\pi + \tau\dot{\beta}_\pi\right)\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left[\zeta + \frac{\zeta T \Pi}{2}\left(\beta_\Pi + \tau\dot{\beta}_\Pi\right)\right]$$

W. Israel, J.M. Stewart, *Annals Phys.* 118 (1979)

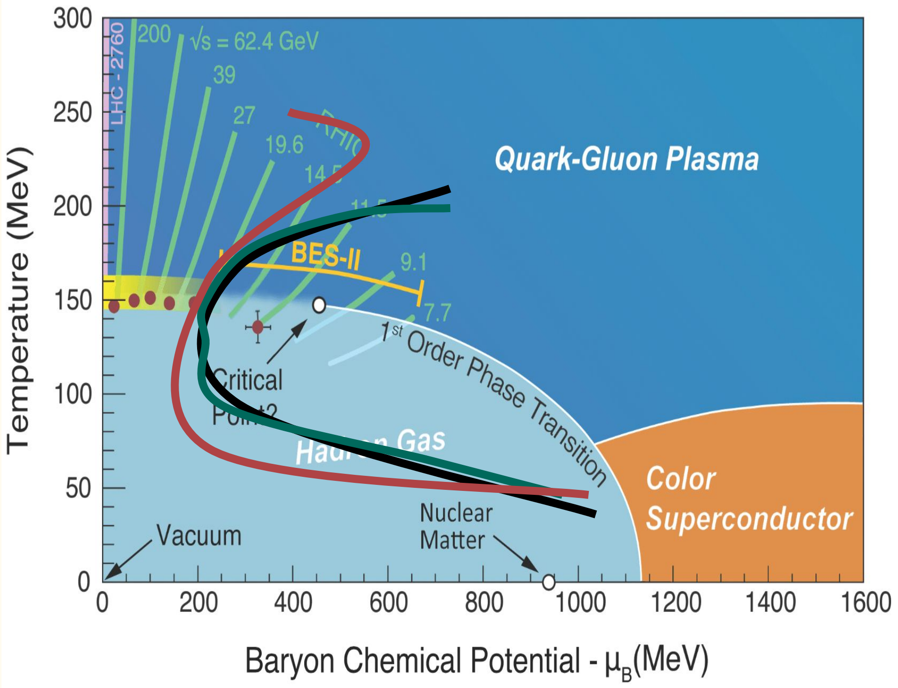
$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \pi^\eta_\eta\left(\delta_{\pi\pi} + \tau_{\pi\pi}\right) + \lambda_{\pi\Pi}\Pi\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left(\zeta + \delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi^\eta_\eta\right)$$

G. Denicol, et al, *Eur.Phys.* J.A 48 (2012)

Biggest difference: Transport Coefficients

An Evolving Hydrodynamic Picture



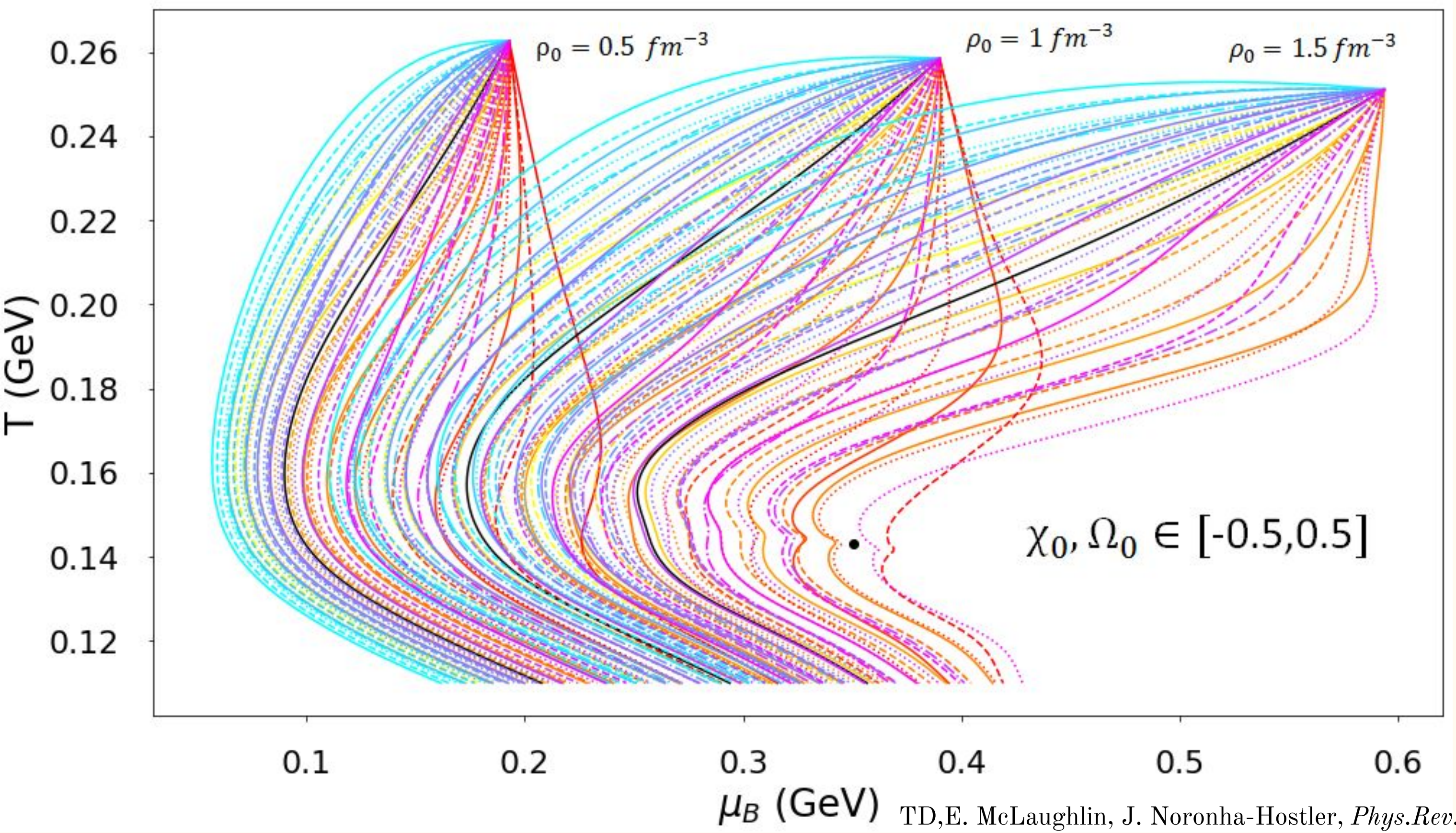
- ➢ Without viscous effects there is no entropy production. System evolves isentropically

- ➢ Close to equilibrium scenario may not alter trajectories dramatically

- ➢ Far from equilibrium effects may significantly affect trajectories.

  In fact..

Out Of Equilibrium Effects Are Important



TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev*.D 102 (2020) 7

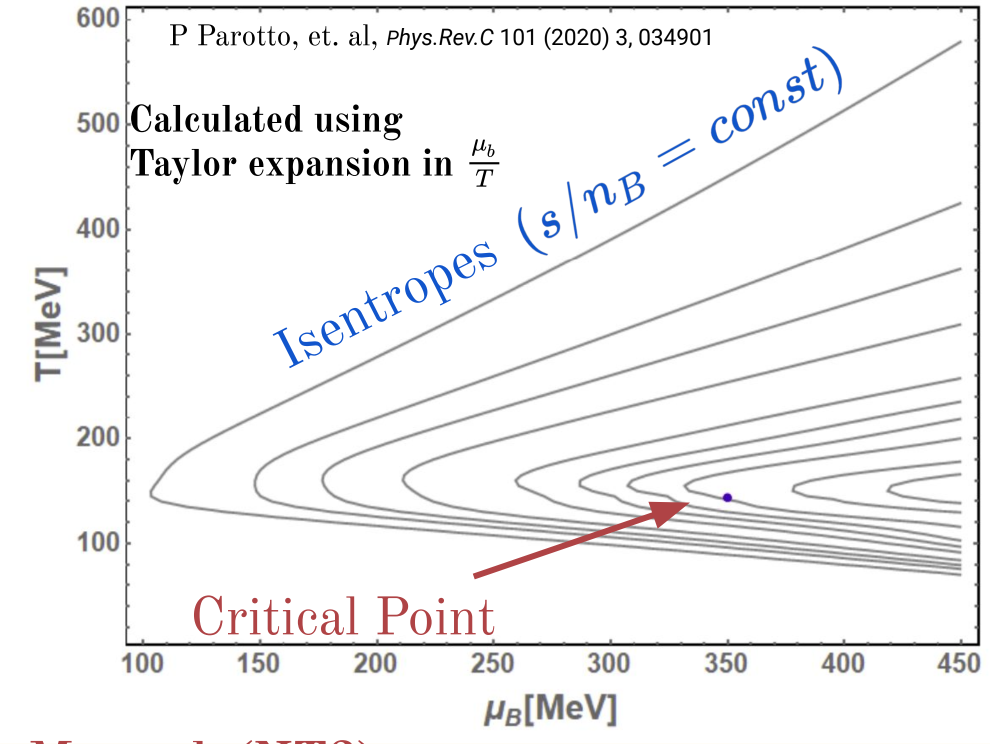
Phenomenological Tool:

Lattice QCD EoS with Parameterized CP (3D Ising University Class)

- ➢ Ideal hydrodynamics evolves along isentropic trajectories

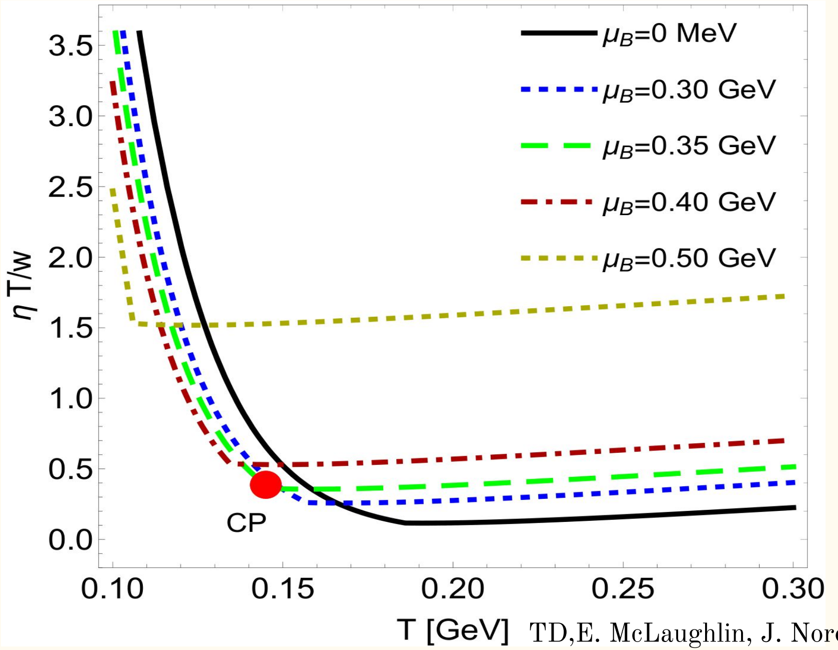
- ➢ How do out of equilibrium effects influence trajectories?

- ➢ Effect of criticality?

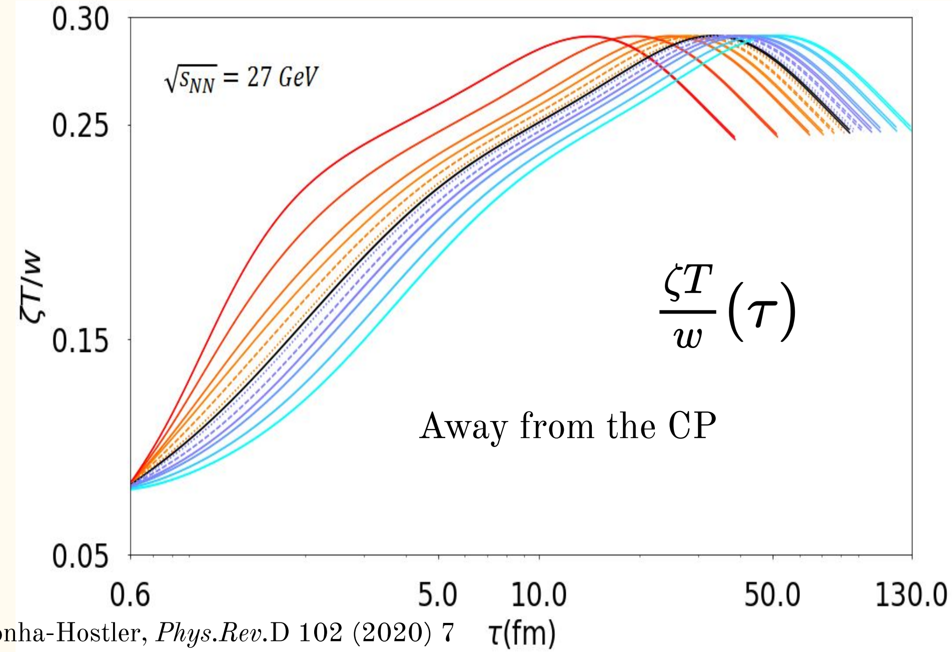


For more on this EoS, see talk: Mroczek (NT2)

Transport Coefficient Behavior



TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

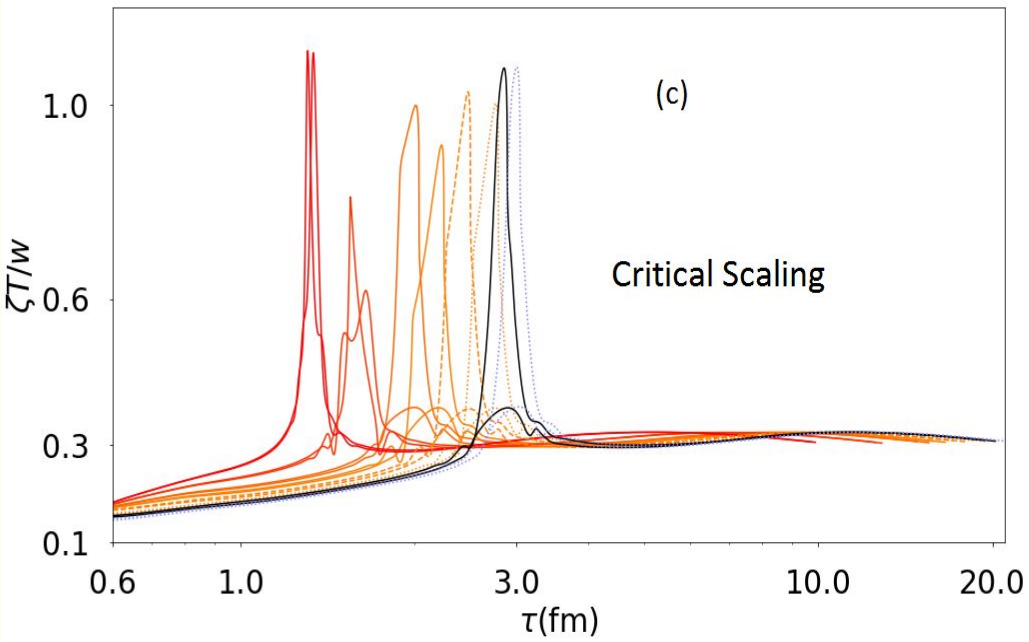


Shear viscosity not sensitive to criticality explicitly

Bulk viscosity away from CP shows similar behavior

Influence of Criticality on Bulk Viscosity

TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7



Monnai, Akihiko et al, Nucl. Phys. ,A967,2017

Critically Scaled Bulk:

$$\left(\frac{\zeta T}{w}\right)_{CS} = \frac{\zeta T}{w}\left[1 + \left(\frac{\xi}{\xi_0}\right)^3\right]$$

Away from the critical point, only has effect from speed of sound

Correlation length calculated in linear parametrization model

Systematic Formulation Comparison

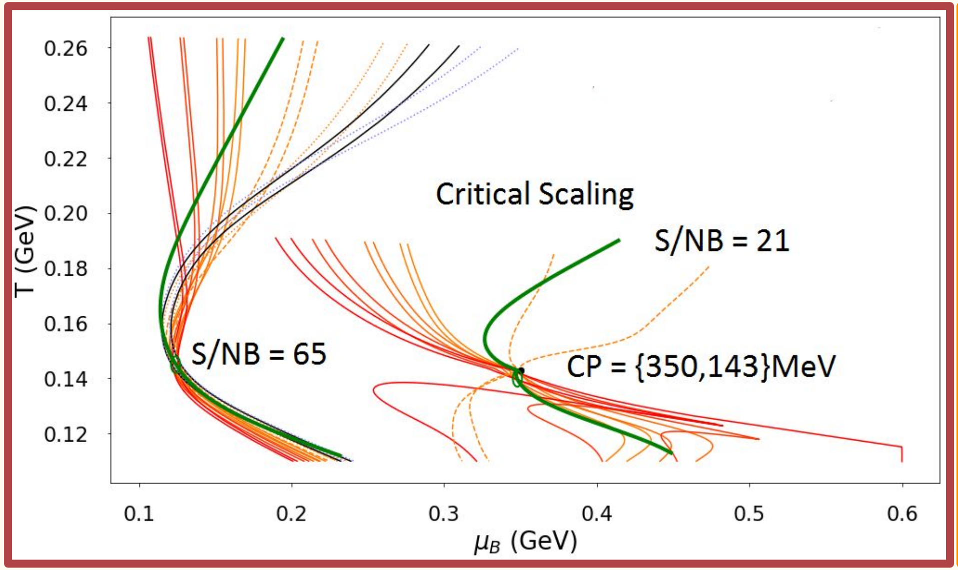
- Run *many* hydro events, systematically scan through initial conditions for $\{\chi, \Omega\}$
  - That is, the full $T^{\mu\nu}$

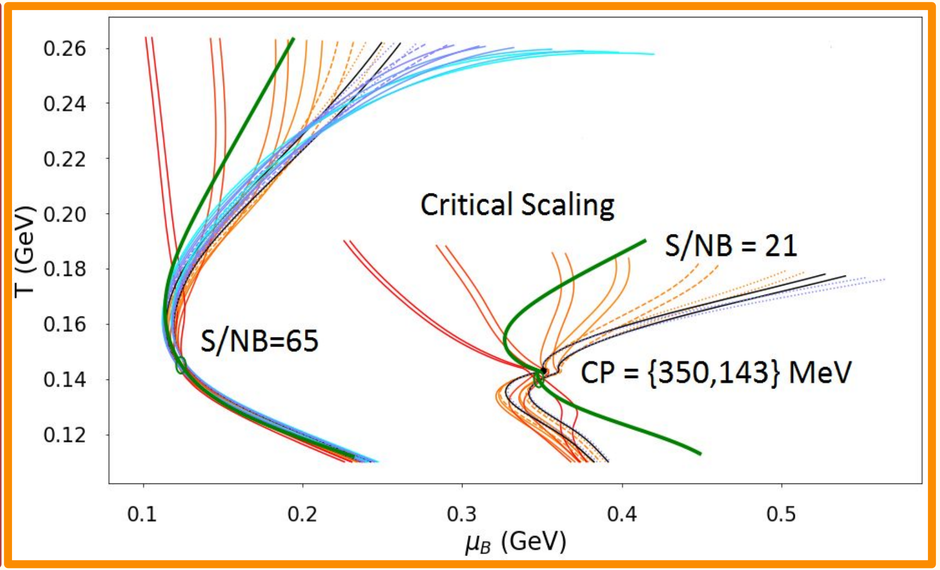
- Only select on events that pass through the same freeze-out point
  - Taken from:
    P. Alba, et al. *Phys.Rev.C* 101 (2020) 5
  - Ideal hydro base of comparison

Israel-Stewart

$\{T(\tau), \mu_B(\tau)\}$

DNMR





TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

Green Line: Equilibrium Hydro trajectory

Takeaways:

1. Pushed to or away from CP on EbE basis
2. Degeneracy of final state mapping to initial
3. DNMR seems more robust than Israel-Stewart

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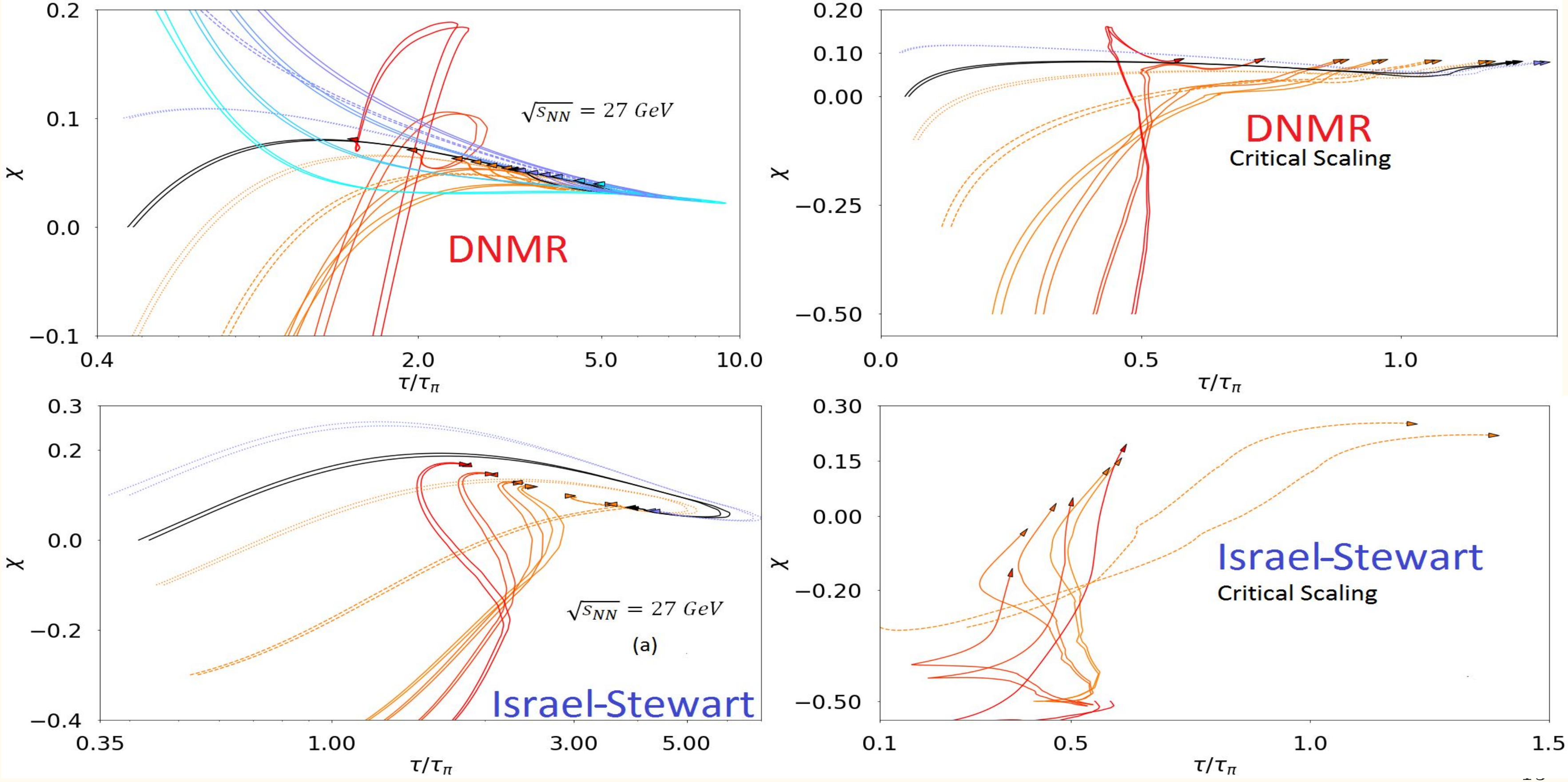
Far From CP

$$\chi = \frac{\pi}{\epsilon + p}$$

Close to CP

DNMR

Israel-Stewart



Far From CP

$$\Omega = \frac{\Pi}{\epsilon + p}$$

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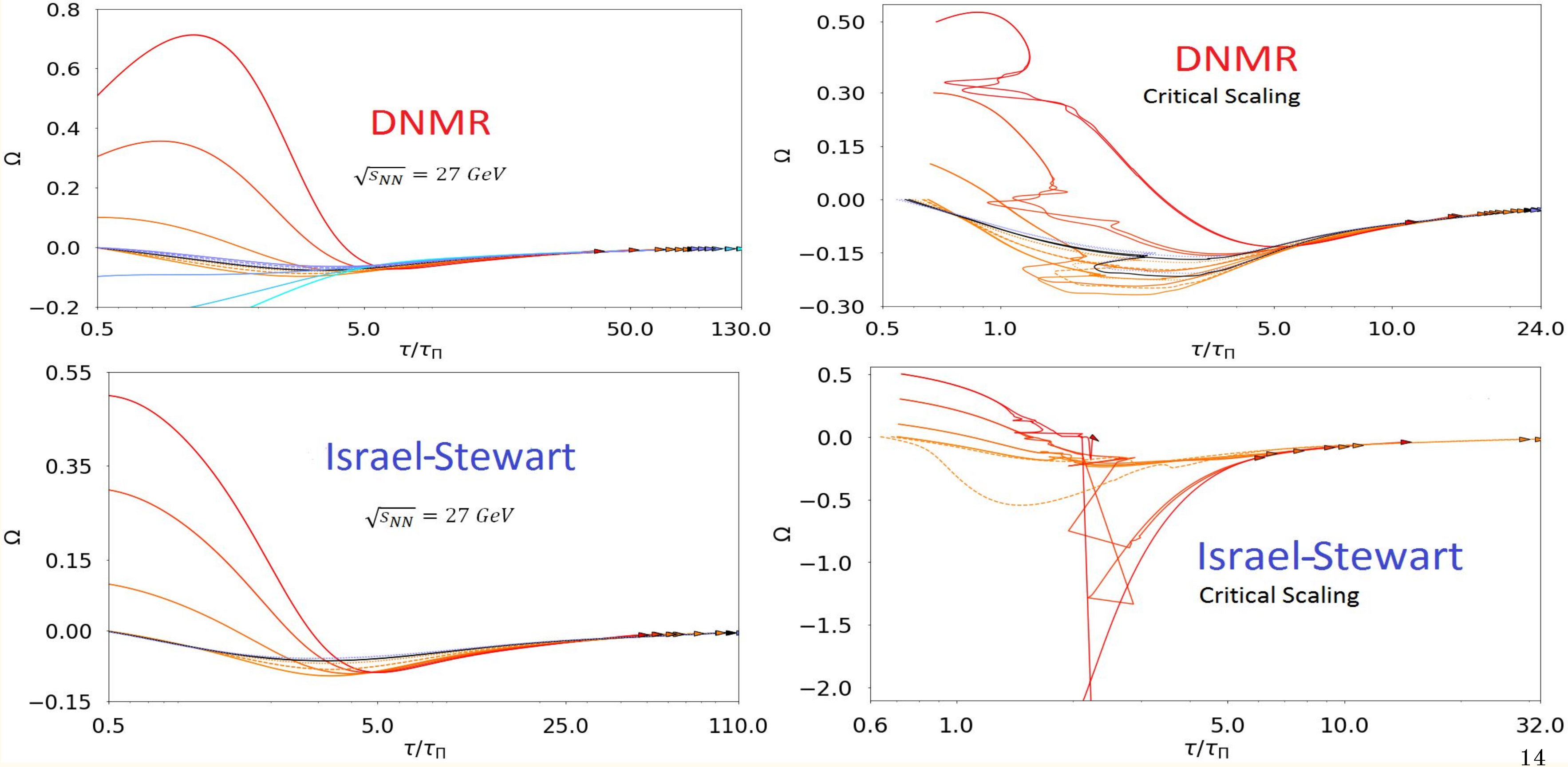
Close to CP

DNMR

Israel-Stewart



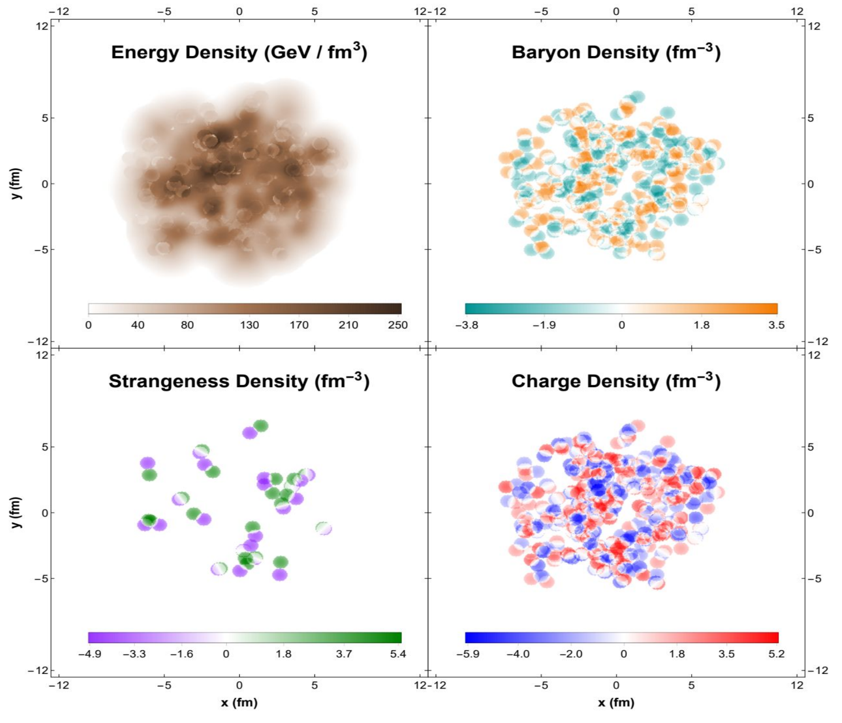
Next Steps towards BES Hydrodynamics

Start with what we know: LHC energies and ICCING

Initializing
Conserved
Charges
in Nuclear
Geometries

M. Martinez, et al., arXiv:1911.12454

M. Martinez, et al., arXiv:1911.10272



Initializing charges at 0 ***net*** density allows study of diffusion in a better controlled environment

Requirements:

- (2+1) dimensional Hydro
  - V-USPhydro
- 4D EoS Noronha-Hostler, et al. *Phys.Rev.C 100 (2019)*
  - $\{\epsilon, \rho_B, \rho_Q, \rho_S\} \rightarrow \{T, \mu_B, \mu_Q, \mu_S\}$

For more on ICCING, see poster: Carzon (session O)

Smooth Particle Hydrodynamics In a Nutshell

Two Main Approximations:

1. Coarse grain value of local quantities with kernel function

$$\int d\mathbf{r}' W(|\mathbf{r}-\mathbf{r}'|, h) = 1$$

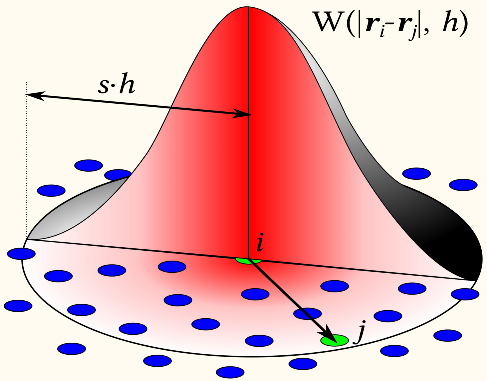
$$with\ W(|\mathbf{r}-\mathbf{r}'|, h) \to 0\ for\ |\mathbf{r}-\mathbf{r}'| \sim \mathcal{O}(h)$$

$$\sigma = \frac{1}{V} = \frac{\gamma}{V^*}\ Local\ conserved\ fluid\ cell\ volume$$

2. Represent system with finite number of SPH "*particles*", introduces notion of "*reference density*"

Any local quantity can then be expressed as:

$$a_{SPH}(\mathbf{r}, t) = \sum_{\alpha}^{N_{SPH}} \nu_\alpha \frac{a(\mathbf{r}_\alpha, t)}{\sigma^*(\mathbf{r}_\alpha, t)} W(|\mathbf{r}-\mathbf{r}'|), h)$$



Full Israel-Stewart With BSQ Diffusion in SPH

$$\tau_\Pi \dot{\Pi} + \Pi = -(\zeta + \tau_\Pi \Pi)\theta - \frac{\tau_\Pi}{2\beta_\Pi}\dot{\beta}_\Pi \Pi - \frac{\zeta}{\beta}\sum_{q,q'}\left(\gamma_0^{qq'} D_\mu n_q^\mu + \frac{1}{2} n_q^\mu (\nabla_\mu \gamma_0^{qq'} - \gamma_0^{qq'} \frac{1}{\sigma}\nabla_\mu \sigma)\right)$$

$$\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = (2\eta\sigma^{\mu\nu} + \tau_\pi \pi^{\mu\nu})\theta - \frac{\tau_\pi}{2\beta_\pi}\dot{\beta}_\pi \pi^{\mu\nu} - \frac{2\eta}{\beta}\sum_{q,q'}\left(\gamma_1^{qq'} \nabla^{\langle\mu} n_q^{\nu\rangle} + \frac{1}{2}(n_q^{\langle\mu} \nabla^{\nu\rangle} \gamma_1^{qq'} - \gamma_1^{qq'} n_q^{\langle\mu} \frac{1}{\sigma}\nabla^{\nu\rangle}\sigma)\right)$$

$$\tau_{qq'} \dot{n}_{q'}^\mu + n_q^\mu = -(\kappa_{qq'} \nabla^\mu \alpha_{q'} + \tau_{qq'} n_{q'}^\mu)\theta - \frac{\tau_{qq'}}{2\beta_{qq'}}\dot{\beta}_{qq'} n_{qq'}^\mu - \frac{\kappa_{qq'}}{\beta}\left(\gamma_0^{qq'} \nabla^\mu \Pi - \frac{\Pi}{2}(\nabla^\mu \gamma_0^{qq'} + 3\gamma_0^{qq'} \frac{1}{\sigma}\nabla^\mu \sigma)\right) - \frac{\kappa_{qq'}}{\beta}\left(\gamma_1^{qq'} \nabla_\nu \pi^{\mu\nu} + \frac{\pi^{\mu\nu}}{2}(\nabla_\nu \gamma_1^{qq'} - \gamma_1^{qq'} \frac{1}{\sigma}\nabla_\nu \sigma)\right)$$

Conclusions and Future Work

- There is still **much** theoretical work and modeling to be done for the upcoming BES 2
  - Hydro Formulation comparison is one area

- These results show that DNMR may offer more robust solutions when the system exhibits criticality

- Studying charge diffusion at LHC energies offers a means of controlled study before moving on to (3+1) BSQ Hydro

Backup Slides

$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{4\eta}{3\tau} - \frac{\eta T \pi^\eta_\eta}{2}\left(\frac{\beta_\pi}{\tau} + \dot{\beta}_\pi\right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{\zeta T \Pi}{2}\left(\frac{\beta_\Pi}{\tau} + \dot{\beta}_\Pi\right)$$

$$\tau_\pi = \frac{5\ \eta}{\epsilon + p}$$

$$\tau_\Pi = \frac{\zeta}{15(\epsilon + p)\left(\frac{1}{3} - c_s^2\right)^2}$$

$$\lambda_{\pi\Pi} = \frac{6}{5}\tau_\pi$$

$$\delta_{\pi\pi} = \frac{4}{3}\tau_\pi$$

$$\tau_{\pi\pi} = \frac{10}{7}\tau_\pi$$

$$\lambda_{\Pi\pi} = \tau_\Pi \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\tau_\Pi$$

$$\delta_{\Pi\Pi} = \frac{2}{3}\tau_\Pi$$

$$\beta_\pi = \frac{\tau_\pi}{2\eta T}$$

$$\beta_\Pi = \frac{\tau_\Pi}{\zeta T}.$$

$$\dot{\epsilon} = -\frac{1}{\tau}\left[\epsilon + p + \Pi - \pi^\eta_\eta\right]$$

$$\tau_\pi \dot{\pi}^\eta_\eta + \pi^\eta_\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \pi^\eta_\eta\left(\delta_{\pi\pi} + \tau_{\pi\pi}\right) + \lambda_{\pi\Pi}\Pi\right]$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left(\zeta + \delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi^\eta_\eta\right)$$

$$\dot{\rho} = -\frac{\rho}{\tau}$$

Keeping Track of Reference Density: Example

$$\Pi + \tau_\Pi \frac{d}{d\tau}\Pi + = -\zeta\theta \implies \frac{\Pi}{\sigma} + \tau_\Pi \frac{d}{d\tau}\left(\frac{\Pi}{\sigma}\right) = -\frac{\zeta}{\sigma}\theta$$

$$\frac{\Pi}{\sigma} + \tau_\Pi \frac{1}{\sigma}\frac{d}{d\tau}\Pi + \tau_\Pi \Pi \frac{d}{d\tau}\frac{1}{\sigma} = -\frac{\zeta}{\sigma}\theta$$

$$\Pi + \tau_\Pi \frac{d}{d\tau}\Pi + \tau_\Pi \Pi \sigma \frac{d}{d\tau}\frac{1}{\sigma} = -\zeta\theta$$

Use relation

$$\sigma\frac{d}{d\tau}\frac{1}{\sigma} = \theta$$

$$\Pi + \tau_\Pi \frac{d}{d\tau}\Pi + = -(\zeta + \tau_\Pi \Pi)\theta$$

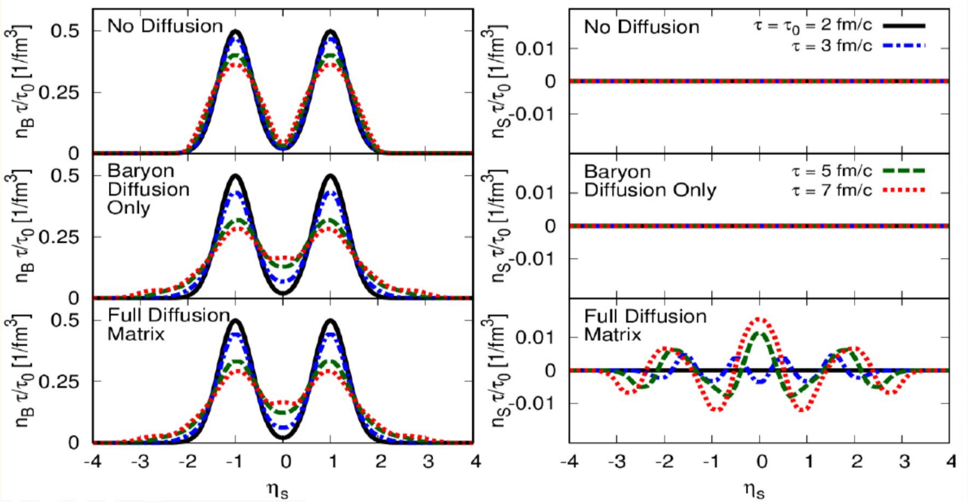
Extra term unique to SPH

Relativistic Coupled Diffusion

Generalization: $for\ q, l \in \{B, S, Q\}$

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)

$$\tau_q \dot{n}_q^\mu + n_q^\mu = \kappa_q \nabla^\mu \alpha_q \Rightarrow \sum_l \tau_{ql} \dot{n}_l^\mu + n_q^\mu = \sum_l \kappa_{ql} \nabla^\mu \alpha_l$$



(2+1) SPH equations for Israel-Stewart currently being derived and implemented into existing v-USPhydro code