

Full next-to-eikonal quark propagator in the CGC and applications

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Outline of the talk

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Eikonal approximation and beyond

High-energy scatterings:

projectile : dilute proton – target : dense nucleus (CGC)

(anti)quark parton – large gluon background field $\mathcal{A}^\mu(x)$

Eikonal approximation : Schockwave approximation for a x^- -going target

- * target gets Lorentz contracted to $x^+ = 0$ (zero width)
- * independence on x^- due to Lorentz time dilation
- * large hierarchy between components of the background field

Background field in the eikonal limit: $\mathcal{A}^\mu(x^-, x^+, \mathbf{x}) \approx \delta^{\mu-} \mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

Strong hierarchy of components of \mathcal{A}^μ with respect to the boost factor γ :

$$\mathcal{A}^- = O(\gamma) \gg \mathcal{A}_\perp = O(1) \gg \mathcal{A}^+ = O(1/\gamma)$$

Eikonal approximation \equiv leading power in γ or total energy of the collision

Subleading powers : potentially important corrections to CGC at moderate energies (EIC/RHIC)

Beyond eikonal approximation, at Next-to-Eikonal accuracy (NEik):

- * target with finite width \Rightarrow transverse motion of the parton within the medium
- * interactions with \mathcal{A}_\perp field cannot be neglected

High-energy expansion with small parameters $\sim \frac{L^+}{k^+} Q^2$ (L^+ : target width, k^+ : parton momentum, Q^2 : any transverse scale in the problem)

Quark propagator - basics

Full quark Feynman propagator in background field $\mathcal{A}^\mu(x)$

$$S_F(x, y)_{\alpha\beta} = S_{0,F}(x, y)_{\alpha\beta} + \delta S_F(x, y)_{\alpha\beta}$$

free propagator + corrections due to interactions
with the background field

Free quark Feynman propagator:

$$S_{0,F}(x, y)_{\alpha\beta} = (\mathbf{1})_{\alpha\beta} \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{i(\not{k} + m)}{[k^2 - m^2 + i\epsilon]}$$

Corrections:

- at the eikonal order

$$\delta S_F \Big|_{\text{Eik}} \equiv \delta S_F \Big|_{\text{pure } \mathcal{A}^-}$$

- at the next-to-eikonal order

$$\delta S_F \Big|_{\text{NEik}} \equiv \delta S_F \Big|_{\text{pure } \mathcal{A}^-}^{\text{NEik}} + \delta S_F \Big|_{\text{single } \mathcal{A}_\perp}^{\text{NEik}} + \delta S_F \Big|_{\text{double } \mathcal{A}_\perp}^{\text{NEik}}$$

Quark propagator in the eikonal limit

$$S_F(x, y)_{\alpha\beta} \Big|_{\text{Eik}} = S_{0,F}(x, y)_{\alpha\beta} + \delta S_F(x, y)_{\alpha\beta} \Big|_{\text{pure } \mathcal{A}^-}$$

In eikonal limit, the quark already interacts with arbitrarily many \mathcal{A}^- fields
For generic x and y , with notations $\underline{k} \equiv (k^+, \mathbf{k})$, and \bar{k} on-shell version of k :

$$\begin{aligned} S_F(x, y)_{\alpha\beta} \Big|_{\text{Eik}} &= \mathbf{1}_{\alpha\beta} \delta^{(3)}(\underline{x} - \underline{y}) \operatorname{sgn}(x^- - y^-) \frac{\gamma^+}{4} \\ &+ \int \frac{d^3 \underline{q}}{(2\pi)^3} \int \frac{d^3 \underline{k}}{(2\pi)^3} 2\pi \delta(q^+ - k^+) e^{-ix \cdot \underline{q} + iy \cdot \bar{k}} \frac{(\not{q} + m)\gamma^+(\not{k} + m)}{(2k^+)^2} \\ &\times \int d^2 \mathbf{z} e^{-iz \cdot (\mathbf{q} - \mathbf{k})} \left\{ \theta(k^+) \theta(x^+ - y^+) \mathcal{U}_F(x^+, y^+; \mathbf{z})_{\alpha\beta} \right. \\ &\quad \left. - \theta(-k^+) \theta(y^+ - x^+) \mathcal{U}_F^\dagger(y^+, x^+; \mathbf{z})_{\alpha\beta} \right\} \end{aligned}$$

Eikonal interactions with the medium resummed into the Wilson lines:

$$\mathcal{U}_F(x^+, y^+; \mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!} \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+ t \cdot \mathcal{A}^-(z^+, \mathbf{z}) \right]^N$$

- background field $\mathcal{A}_a^-(z^+, \mathbf{z})$ has a finite support $[-L^+/2, L^+/2]$ - this is where the non-trivial medium contributions come from in the interval $[y^+, x^+]$
- if there is no support the propagator reduces to the Feynman propagator in vacuum

Subeikonal corrections: Brownian motion in a pure \mathcal{A}^- background

Full next-to-eikonal quark propagator:

$$S_F \Big|_{\text{NEik}} = \underbrace{S_F \Big|_{\text{Eik}} + \delta S_F \Big|_{\text{pure } \mathcal{A}^-}}_{S_F \Big|_{\text{pure } \mathcal{A}^-}} + \delta S_F \Big|_{\text{single } \mathcal{A}_\perp} + \delta S_F \Big|_{\text{double } \mathcal{A}_\perp}^{\text{NEik}}$$

From now on, always $x^+ > L^+/2$ and $y^+ < -L^+/2$: quark propagating through the whole target

Quark propagator in pure \mathcal{A}^- background field up to next-to-eikonal order for positive energy:

$$\begin{aligned} S_F(x, y)_{\alpha\beta} \Big|_{\text{pure } \mathcal{A}^-} &= \int \frac{d^3 \underline{q}}{(2\pi)^3} \int \frac{d^3 \underline{k}}{(2\pi)^3} 2\pi \delta(q^+ - k^+) \frac{\theta(k^+)}{(2k^+)^2} e^{-ix \cdot \underline{q} + iy \cdot \underline{k}} (\not{q} + m) \gamma^+ (\not{k} + m) \\ &\times \int d^2 \mathbf{z} e^{-iz \cdot (\mathbf{q} - \mathbf{k})} \left\{ \mathcal{U}_F \left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{z} \right) \right. \\ &\quad - \frac{(\mathbf{q}^j + \mathbf{k}^j)}{4k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{z} \right) \overleftrightarrow{\partial}_{z^j} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{z} \right) \right] \\ &\quad \left. - \frac{i}{2k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{z} \right) \overleftrightarrow{\partial}_{z^j} \overleftrightarrow{\partial}_{z^j} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{z} \right) \right] \right\} \end{aligned}$$

NEik corrections: transverse drift term + transverse Brownian motion term

Analog to earlier results on the gluon propagator with subeikonal corrections:

Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP **1407**, 068 (2014)

Altinoluk, Armesto, Beuf, Moscoso, JHEP **1601**, 114 (2016)

Subeikonal corrections: single \mathcal{A}_\perp insertion

Full next-to-eikonal quark propagator:

$$S_F \Big|_{\text{NEik}} = S_F \Big|_{\text{Eik}} + \delta S_F \Big|_{\text{pure } \mathcal{A}_\perp} + \delta S_F \Big|_{\text{single } \mathcal{A}_\perp} + \delta S_F \Big|_{\text{double } \mathcal{A}_\perp}$$

Replace one $\gamma^+ \mathcal{A}_a^-$ interaction by $\gamma^j \mathcal{A}_j^a$

$$\delta S_F(x, y) \Big|_{\text{single } \mathcal{A}_\perp}^{\text{NEik}} = \int d^4 z S_F(x, z) \Big|_{\text{Eik}} [-ig \gamma^j t^a] \mathcal{A}_j^a(z) S_F(z, y) \Big|_{\text{Eik}}$$

Subeikonal correction due to an interaction with \mathcal{A}_\perp :

$$\begin{aligned} \delta S_F(x, y) \Big|_{\text{single } \mathcal{A}_\perp}^{\text{NEik}} &= \int \frac{d^3 \underline{q}}{(2\pi)^3} \int \frac{d^3 \underline{k}}{(2\pi)^3} 2\pi \delta(q^+ - k^+) \frac{\theta(k^+)}{(2k^+)^3} e^{-ix \cdot \underline{q}} e^{iy \cdot \underline{k}} \\ &\times (\underline{q} + m) \gamma^j \gamma^+ \gamma^i (\underline{k} + m) \int d^3 \underline{z} \left[e^{-iz \cdot \underline{q}} \mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{z} \right) \right] \\ &\times \left[\overleftarrow{\partial}_{\mathbf{z}^j} [gt \cdot \mathcal{A}_i(\underline{z})] - [gt \cdot \mathcal{A}_j(\underline{z})] \overrightarrow{\partial}_{\mathbf{z}^i} \right] \left[\mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{z} \right) e^{iz \cdot \mathbf{k}} \right] \end{aligned}$$

Reminder: $x^+ > L^+/2$ and $y^+ < -L^+/2$:
quark propagating through the whole medium

Subeikonal corrections: double \mathcal{A}_\perp insertion

Full next-to-eikonal quark propagator:

$$S_F \Big|_{\text{NEik}} = S_F \Big|_{\text{Eik}} + \delta S_F \Big|_{\text{pure } \mathcal{A}^-}^{\text{NEik}} + \delta S_F \Big|_{\text{single } \mathcal{A}_\perp}^{\text{NEik}} + \delta S_F \Big|_{\text{double } \mathcal{A}_\perp}^{\text{NEik}}$$

Replace two $\gamma^+ \mathcal{A}_a^-$ interactions by $\gamma^j \mathcal{A}_j^b$ and $\gamma^i \mathcal{A}_i^a$

$$\begin{aligned} \delta S_F(x, y) \Big|_{\text{double } \mathcal{A}_\perp}^{\text{NEik}} &= \int d^4 z \int d^4 z' S_F(x, z') \Big|_{\text{Eik}} [-ig \gamma^j t^b] \mathcal{A}_j^b(z') \\ &\quad \times S_F(z', z) \Big|_{\text{Eik}} [-ig \gamma^i t^a] \mathcal{A}_i^a(z) S_F(z, y) \Big|_{\text{Eik}} \end{aligned}$$

Contribution at NEik accuracy due to the instantaneous term in the middle eikonal propagator:

$$\begin{aligned} \delta S_F(x, y) \Big|_{\text{double } \mathcal{A}_\perp}^{\text{NEik}} &= \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} 2\pi \delta(q^+ - k^+) \frac{\theta(k^+)}{(2k^+)^3} e^{-ix \cdot \bar{\mathbf{q}}} e^{iy \cdot \bar{\mathbf{k}}} \\ &\quad \times (\not{\bar{\mathbf{q}}} + m) \gamma^j \gamma^+ \gamma^i (\not{\bar{\mathbf{k}}} + m) \int d^3 \underline{z} e^{-iz \cdot (\mathbf{q} - \mathbf{k})} \\ &\quad \times (-i) \mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{z} \right) [gt \cdot \mathcal{A}_j(z)] [gt \cdot \mathcal{A}_i(z)] \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{z} \right) \end{aligned}$$

Next-to-eikonal quark propagator - spinor structure

Spinor structure:

- \mathcal{A}^- field associated with $(\not{q} + m)\gamma^+(\not{k} + m)$
- \mathcal{A}_\perp field associated with $(\not{q} + m)\gamma^j\gamma^+\gamma^i(\not{k} + m)$

- separate symmetric and anti-symmetric parts:

$$\gamma^j\gamma^+\gamma^i = \delta^{ij}\gamma^+ + \gamma^+\frac{[\gamma^i, \gamma^j]}{2}$$

(helicity independent + helicity dependent)

Helicity dependence:

$$[\gamma^i, \gamma^j] = -4i\epsilon^{ij}S^3$$

S^3 - helicity operator, acting on spinors as:

$$S^3 u(\check{k}, h) = hu(\check{k}, h)$$

$$S^3 v(\check{k}, h) = -hv(\check{k}, h)$$

Quark propagator

$$S_F(x, y) = S_F(x, y)\Big|_{\text{unpol.}} + S_F(x, y)\Big|_{\text{h. dep.}}$$

Next-to-eikonal quark propagator - full result

Unpolarized part

$$\begin{aligned}
 S_F(x, y) \Big|_{\text{unpol.}} &= \int \frac{d^3 q}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} 2\pi \delta(q^+ - k^+) \frac{\theta(k^+)}{(2k^+)^2} e^{-ix \cdot \bar{q}} e^{iy \cdot \bar{k}} (\not{q} + m) \gamma^+ (\not{k} + m) \\
 &\times \int d^2 \mathbf{z} e^{-iz \cdot (\mathbf{q} - \mathbf{k})} \left\{ \mathcal{U}_F \left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{z} \right) \right. \\
 &- \frac{(\mathbf{q}^j + \mathbf{k}^j)}{4k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{z} \right) \overleftarrow{\mathcal{D}}_{\mathbf{z}^j} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{z} \right) \right] \\
 &\left. - \frac{i}{2k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{z} \right) \overleftarrow{\mathcal{D}}_{\mathbf{z}^j} \overrightarrow{\mathcal{D}}_{\mathbf{z}^j} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{z} \right) \right] \right\}
 \end{aligned}$$

Helicity-dependent part

$$\begin{aligned}
 S_F(x, y) \Big|_{\text{h. dep.}} &= \int \frac{d^3 q}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} 2\pi \delta(q^+ - k^+) \frac{\theta(k^+)}{(2k^+)^3} e^{-ix \cdot \bar{q}} e^{iy \cdot \bar{k}} \\
 &\times (\not{q} + m) \gamma^+ \frac{[\gamma^i, \gamma^j]}{4} (\not{k} + m) \int d^2 \mathbf{z} e^{-iz \cdot (\mathbf{q} - \mathbf{k})} \\
 &\times \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{z} \right) g t \cdot \mathcal{F}_{ij}(\underline{z}) \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{z} \right)
 \end{aligned}$$

The final result fully gauge invariant (due to covariant derivatives):

$$\overrightarrow{\mathcal{D}}_{z^\mu} \equiv \partial_{z^\mu} + igt \cdot \mathcal{A}_\mu(\underline{z}); \quad \overleftarrow{\mathcal{D}}_{z^\mu} \equiv \overrightarrow{\mathcal{D}}_{z^\mu}^\dagger; \quad \overleftarrow{\mathcal{D}}_{z^\mu} \equiv \overrightarrow{\mathcal{D}}_{z^\mu} - \overleftarrow{\mathcal{D}}_{z^\mu}$$

Longitudinal chromo-magnetic field of the target associated with helicity:

$$\mathcal{F}_{ij}^a(\underline{z}) \equiv \partial_{z^i} \mathcal{A}_j^a(\underline{z}) - \partial_{z^j} \mathcal{A}_i^a(\underline{z}) - gf^{abc} \mathcal{A}_i^b(\underline{z}) \mathcal{A}_j^c(\underline{z})$$

Scattering amplitude from quark propagator

- Simplest observable where NEik correction matter: quark-target cross section
- Need for the relevant scattering amplitude

S-matrix element

Formal definition: $S_{q(\bar{q}, h', \beta) \leftarrow q(\bar{k}, h, \alpha)} = \langle 0 | \hat{b}_{\text{out}}(\bar{q}, h, \beta) \hat{b}_{\text{in}}^\dagger(\bar{k}, h, \alpha) | 0 \rangle$

$$= (2k^+) 2\pi \delta(q^+ - k^+) i \mathcal{M}_{\alpha\beta}^{hh'}(\underline{k}, \mathbf{q})$$

LSZ reduction: $S_{q(\bar{q}, h', \beta) \leftarrow q(\bar{k}, h, \alpha)} = \lim_{x^+ \rightarrow \infty} \lim_{y^+ \rightarrow -\infty} \int d^2\mathbf{x} \int dx^- \int d^2\mathbf{y} \int dy^-$

$$\times e^{i\mathbf{x} \cdot \bar{\mathbf{q}} - iy \cdot \bar{\mathbf{k}}} \bar{u}(\bar{q}, h') \gamma^+ S_F(x, y)_{\alpha\beta} \gamma^+ u(\bar{k}, h)$$

Quark-target scattering amplitude

$$i \mathcal{M}_{\alpha\beta}^{hh'}(\underline{k}, \mathbf{q}) = \delta_{hh'} \int d^2\mathbf{z} e^{-i\mathbf{z} \cdot (\mathbf{q} - \mathbf{k})} \left\{ \mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{z}\right) \right.$$

$$+ \frac{\epsilon^{ij} h}{2k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) (-igt \cdot \mathcal{F}_{ij}(z)) \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right) \right]$$

$$- \frac{i}{2k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) \overleftarrow{\mathcal{D}}_{z^+ j} \overrightarrow{\mathcal{D}}_{z^+ j} \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right) \right]$$

$$\left. - \frac{(\mathbf{q}^j + \mathbf{k}^j)}{4k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}\right) \overleftarrow{\mathcal{D}}_{z^+ j} \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}\right) \right] \right\}_{\alpha\beta}$$

Unpolarized cross section

Differential cross section for quark scattering on the target is

$$\frac{d^2\sigma^{qA \rightarrow q+X}}{d^2\mathbf{q}} = \frac{1}{(2\pi)^2} \frac{1}{2N_c} \sum_{h,h'} \sum_{\alpha,\beta} \mathcal{M}_{\alpha\beta}^{hh'}(\underline{k}, \mathbf{q})^\dagger \mathcal{M}_{\alpha\beta}^{hh'}(\underline{k}, \mathbf{q}) \Big|_{q^+ = k^+}$$

Cross section averaged over the target

$(z - z') \equiv \mathbf{r}$ and $(z + z') \equiv 2b$

$$\left\langle \frac{d^2\sigma^{qA \rightarrow q+X}}{d^2\mathbf{q}} \right\rangle_A = \frac{1}{(2\pi)^2} \int d^2\mathbf{r} e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}} \left\{ 1 - \bar{P}(\mathbf{r}) \right. \\ \left. + \left(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j + \mathbf{k}^j)}{4k^+} [\mathcal{O}_{(1)}^j(\mathbf{r}) - \mathcal{O}_{(1)}^{\dagger j}(\mathbf{r})] - \frac{i}{2k^+} [\mathcal{O}_{(2)}(\mathbf{r}) - \mathcal{O}_{(2)}^\dagger(\mathbf{r})] \right) \right\}$$

Dipole operator:

$$d_F(\mathbf{r}) = 1 - \bar{P}(\mathbf{r}) + \bar{O}(\mathbf{r})$$

$$d_F(\mathbf{r}) = \frac{1}{N_c} \int d^2\mathbf{b} \left\langle \text{tr} \left[\mathcal{U}_F^\dagger \left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{b} - \frac{\mathbf{r}}{2} \right) \mathcal{U}_F \left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{b} + \frac{\mathbf{r}}{2} \right) \right] \right\rangle_A$$

Decorated dipole operators:

$$\mathcal{O}_{(1)}^j(\mathbf{r}) = \frac{1}{N_c} \int d^2\mathbf{b} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left\langle \text{tr} \left[\mathcal{U}_F^\dagger \left(\mathbf{b} - \frac{\mathbf{r}}{2} \right) \right. \right. \\ \left. \left. \times \mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{b} + \frac{\mathbf{r}}{2} \right) \overleftrightarrow{\mathcal{D}}_{\mathbf{b}^j + \frac{\mathbf{r}^j}{2}} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{b} + \frac{\mathbf{r}}{2} \right) \right] \right\rangle_A$$

$$\mathcal{O}_{(2)}(\mathbf{r}) = \frac{1}{N_c} \int d^2\mathbf{b} \int_{-L^+/2}^{L^+/2} dz^+ \left\langle \text{tr} \left[\mathcal{U}_F^\dagger \left(\mathbf{b} - \frac{\mathbf{r}}{2} \right) \right. \right. \\ \left. \left. \times \mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{b} + \frac{\mathbf{r}}{2} \right) \overleftrightarrow{\mathcal{D}}_{\mathbf{b}^j + \frac{\mathbf{r}^j}{2}} \overleftrightarrow{\mathcal{D}}_{\mathbf{b}^j + \frac{\mathbf{r}^j}{2}} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{b} + \frac{\mathbf{r}}{2} \right) \right] \right\rangle_A$$

Unpolarized cross section : antiquark and symmetries

Anti-quark–target differential cross section:

$$\left\langle \frac{d^2 \sigma_{\bar{q}A \rightarrow \bar{q}+X}}{d^2 \mathbf{q}} \right\rangle_A = \frac{1}{(2\pi)^2} \int d^2 \mathbf{r} e^{-i(\mathbf{q}-\mathbf{k}) \cdot \mathbf{r}} \left\{ 1 - \bar{P}(\mathbf{r}) - \left(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j + \mathbf{k}^j)}{4k^+} [\mathcal{O}_{(1)}^j(\mathbf{r}) - \mathcal{O}_{(1)}^{\dagger j}(\mathbf{r})] - \frac{i}{2k^+} [\mathcal{O}_{(2)}(\mathbf{r}) - \mathcal{O}_{(2)}^{\dagger}(\mathbf{r})] \right) \right\}$$

- By definition : $d_F(\mathbf{r})^\dagger = d_F(-\mathbf{r})$
⇒ Decomposition $d_F(\mathbf{r}) = 1 - \bar{P}(\mathbf{r}) + \bar{O}(\mathbf{r})$ with
 - Real part $1 - \bar{P}(\mathbf{r})$ even in \mathbf{r}
 - Imaginary term $\bar{O}(\mathbf{r})$ odd in \mathbf{r}
- Signature transformation ($\mathcal{U} \rightarrow \mathcal{U}^\dagger$) and charge conjugation ($q \rightarrow \bar{q}$):

$\bar{P}(\mathbf{r})$: Pomeron is **even** under both transformations

$\bar{O}(\mathbf{r})$: Odderon is **odd** under both transformations

- * Eikonal terms contain both Pomeron and Odderon
- * Next-to-eikonal corrections are of Odderon-type

Quark helicity asymmetry

The difference between the cross sections for a quark of positive and negative helicity scattering on the nucleus target is:

$$\frac{d^2 \Delta \sigma^{qA \rightarrow q+X}}{d^2 \mathbf{q}} \equiv \frac{1}{(2\pi)^2} \frac{1}{2N_c} \sum_{h, h'} \sum_{\alpha, \beta} (2h) \mathcal{M}_{\alpha\beta}^{hh'}(\underline{k}, \mathbf{q})^\dagger \mathcal{M}_{\alpha\beta}^{hh'}(\underline{k}, \mathbf{q}) \Big|_{q^+ = k^+}$$

Quark helicity asymmetry averaged over the target

$$\left\langle \frac{d^2 \Delta \sigma^{qA \rightarrow q+X}}{d^2 \mathbf{q}} \right\rangle_A = \frac{1}{(2\pi)^2} \int d^2 \mathbf{r} e^{-i(\mathbf{q}-\mathbf{k}) \cdot \mathbf{r}} \frac{(-i)}{4k^+} \left[O_{(3)}(\mathbf{r}) - O_{(3)}^\dagger(\mathbf{r}) \right]$$

New decorated dipole operator:

$$O_{(3)}(\mathbf{r}) = \frac{1}{N_c} \int d^2 \mathbf{b} \int_{-L^+/2}^{L^+/2} dz^+ \left\langle \text{Tr} \left[\mathcal{U}_F^\dagger \left(\mathbf{b} - \frac{\mathbf{r}}{2} \right) \right. \right. \\ \left. \left. \times \mathcal{U}_F \left(\frac{L^+}{2}, z^+; \mathbf{b} + \frac{\mathbf{r}}{2} \right) \left\{ \epsilon^{ij} \left[g t \cdot \mathcal{F}_{ij} \left(z^+, \mathbf{b} + \frac{\mathbf{r}}{2} \right) \right] \right\} \mathcal{U}_F \left(z^+, -\frac{L^+}{2}; \mathbf{b} + \frac{\mathbf{r}}{2} \right) \right] \right\rangle_A$$

The anti-quark helicity asymmetry is identical!

Remarks:

- $O_{(3)}(\mathbf{r})$ behaves neither like Pomeron nor like Odderon (odd under signature transformation but even under charge conjugation)
- \mathcal{F}_{ij} and $O_{(3)}(\mathbf{r})$ vanish for unpolarized target, but not for longitudinally polarized target \Rightarrow Double longitudinal spin asymmetry A_{LL}

Summary and final remarks

- Full next-to-eikonal (NEik) expression for the quark propagator through the background field derived
 - Corrections due to transverse motion (drift and Brownian) of the quark while crossing the Lorentz contracted target
 - Corrections due to interaction with the \mathcal{A}_\perp components of the background field
 - However, effects of x^- dependence of the \mathcal{A}_μ and of the \mathcal{A}^+ component would appear only at NNEik accuracy
- Gauge covariant expression: covariant derivatives and field strength insertions in Wilson lines
- Eikonal limit blind to spin, whereas a NEik term is proportional to helicity
- Detailed comparison with results of [Chirilli \(2019\)](#) to be done, but helicity piece consistent with results of [Kovchegov et al. \(2016-2020\)](#)
- qA (and $\bar{q}A$) cross section (unpolarized and helicity asymmetry) studied at NEik
- **Derived quark propagator is of a general form and therefore of a general use: applicable for different scattering processes**
(calculation of DIS di-jet production in progress, and other observables in project)