Color Glass Condensate at next-to-leading order meets HERA data

Henri Hänninen

University of Jyväskylä, Finland

In collaboration with
G. Beuf, T. Lappi, H. Mäntysaari
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DIS in the Dipole Picture at leading order

In Dipole Picture at Leading Order $\gamma^* p$ cross section using optical theorem:

$$
\sigma_{L,T}^{LO}(x_B, Q^2) \sim 4N_c\alpha_{em} \sum_f e_f^2 \int_0^1 dz_1 \int_{x_0, x_1} |\Psi_{\gamma^*, T \to q\bar{q}}|^2 N(x_{01}),
$$

$$
1 - N(x_{01}) \equiv S_{01} := \frac{1}{N_c} \langle \text{Tr} U(x_0) U^\dagger(x_1) \rangle_x
$$

$U = \text{Wilson line}$
Target evolution: BK equation

Target evolution is described approximatively\(^1\) by the Balitsky-Kovchegov (BK) equation:

\[
\partial_y \langle S_{01} \rangle_y = \frac{\bar{\alpha}_s}{2\pi} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} [\langle S_{02} \rangle_y \langle S_{21} \rangle_y - \langle S_{01} \rangle_y] .
\]

- Perturbative energy evolution
- Starts from a non-perturbative initial shape
- In a nutshell
  - Describe inclusive HERA data well
  - Simultaneous description of HERA heavy quark data not as good

\(^1\)Mean field (large \(N_c\)) approx. of B-JIMWLK
NLO DIS cross section in the Dipole Picture

Next-to-Leading Order $\gamma^* p$ cross section can be partitioned as

$$\sigma^{NLO}_{L,T} = \sigma^{IC}_{L,T} + \sigma^{qg}_{L,T} + \sigma^{dip}_{L,T},$$

where the NLO contributions are\(^2\)\(^3\):

$$\sigma^{qg}_{L,T} = 8 N_c \alpha_{em} \frac{\alpha_s C_F}{\pi} \sum_f e_f^2 \int_0^1 d z_1 \int_{z_{2,\text{min}}}^{1-z_1} \frac{d z_2}{z_2} \int_{x_0,x_1,x_2} K^{NLO}_{L,T}(z_1,z_2,x_0,x_1,x_2),$$

$$\sigma^{dip}_{L,T} = 4 N_c \alpha_{em} \frac{\alpha_s C_F}{\pi} \sum_f e_f^2 \int_0^1 d z_1 \int_{x_0,x_1} K^{LO}_{L,T}(z_1,x_0,x_1) \left[ \frac{1}{2} \ln^2 \left( \frac{z_1}{1-z_1} \right) - \frac{\pi^2}{6} + \frac{5}{2} \right],$$

$$z_{2,\text{min}} = e^{Y_0,\text{if } x_{Bj} Q_0^2} Q_0^2 Z_2$$

N.B. Evolution range is controlled by $z_{2,\text{min}}$ at NLO.

Initial condition and fit schemes

- Parametrize IC at $Y_{0,BK} = Y_{0,if}$ or at $Y_{0,BK} > 0$ and freeze dipoles when $Y < Y_{0,BK}$.
  - Use MV-$\gamma$ parametrization.

- Resolve the transient effect\(^4\) ($\sigma^{gg} \to 0$, $\sigma^{dip} \neq 0$ as $z_{2,\text{min}} \to 1$) by setting $Y_{0,if} = 0$.
  - Effective dipole prescription needed when $Y \in [Y_{0,if}, Y_{0,BK}]$.

- Evolution equations: approximations of the full NLOBK
  - Projectile momentum fraction: KCBK and ResumBK
  - Target momentum fraction: TBK

- Running coupling prescriptions
  - *Bal+SD*: Balitsky prescription in LOBK and smallest dipole elsewhere
    - Shortest length scale $\sim$ largest momentum scale dominates
  - *parent dipole*

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Beyond LO: Evolution in projectile rapidity $Y$

- Projectile rapidity $Y \sim \ln W^2$
- Higher order effects: resum large transverse log enhanced contributions

Collinear resummation of large transverse logs leads to "ResumBK" \(^5\)

$$
\partial_Y S(x_{01}, Y) = \int d^2x_2 K_{DLA} K_{STL} K_{BK} [S(x_{02}) S(x_{21}) - S(x_{01})].
$$

Another technique leads to a kinematic constraint (KCBK) and non-local equation \(^6\)

$$
\partial_Y S(x_{01}, Y) = \int d^2z K_{BK} \theta (Y - \Delta_{012} - Y_{0,if})
\times [S(x_{02}, Y - \Delta_{012}) S(x_{21}, Y - \Delta_{012}) - S(x_{01}, Y)].
$$

\(^6\) G. Beuf, Phys. Rev. D 89 (2014) no. 7 074039
Beyond LO: Evolution in target rapidity $\eta$

Recent study\(^7\) argues that evolution should be expressed in $\eta \sim \ln \frac{1}{x_{Bj}}$:

$$\partial_\eta \bar{S}(x_{01}, \eta) = \int d^2x_2 K_{BK} \theta(\eta - \eta_0 - \delta) [\bar{S}(x_{02}, \eta - \delta_{02}) \bar{S}(x_{21}, \eta - \delta_{21}) - \bar{S}(x_{01}, \eta)].$$

- Evolution in $\eta$, DIS impact factors in $Y$: need shift $\eta = Y - \rho$.
  - $\rho \equiv \ln \frac{1}{\min\{1, x_{ij}^2 Q_0^2\}}$
- LO DIS fits done to HERA data with good results\(^8\).

\(^7\)B. Ducloué et al., JHEP 04 (2019) 081
All three BK equations can fit the full HERA data well.

Even combined HERA data cannot differentiate between BK equations and running coupling scheme choices.

Bal+SD prescription overall performed slightly worse in $\chi^2/N$. 

Bal+SD rc, $Y_{0,BK} = 0$
Subtracting heavy quarks from HERA data

- NLO impact factors calculated only for massless quarks
- Fit "light quark reduced cross section" data made from HERA data by subtracting heavy quarks
- Charm, bottom contributions not measured in same bins as full cross sections
- Interpolate c, b data with LO IPsat fit
- Incorrect treatment of uncertainties

The solid and dashed lines show the calculated cross sections from the IPsat fit that are used to generate the pseudodata.

\[ \sigma_r(Q^2) = 2.7 \text{ GeV}^2 \]

\[ \sigma_r(Q^2) = 6.5 \text{ GeV}^2 \]

\[ \sigma_r(Q^2) = 18 \text{ GeV}^2 \]

\[ \sigma_r(Q^2) = 60 \text{ GeV}^2 \]

\[ \sigma_r(Q^2) = 120 \text{ GeV}^2 \]

\[ \sigma_r(Q^2) = 200 \text{ GeV}^2 \]

\[ x_B \]

\[ Q^2 = 120 \text{ GeV}^2 \]

\[ Q^2 = 200 \text{ GeV}^2 \]

\[ Q^2 = 2.7 \text{ GeV}^2 \]

\[ Q^2 = 6.5 \text{ GeV}^2 \]

\[ Q^2 = 18 \text{ GeV}^2 \]

\[ Q^2 = 60 \text{ GeV}^2 \]

\[ Q^2 = 120 \text{ GeV}^2 \]

\[ Q^2 = 200 \text{ GeV}^2 \]

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Fits to light quark data

NLO CGC can fit light quark data as well.

- Findings from fits:
  - Light quarks need larger $\sigma_0$ and $C^2$
  - $\alpha_s(k^2 \sim C^2/r^2) \Rightarrow$ large $C^2$
    means slow evolution

- Interpretation:
  - A large slowly evolving non-perturbative hadronic contribution
  - Fitted parameters effectively take into account non-perturbative effects

<table>
<thead>
<tr>
<th>Data</th>
<th>$\alpha_s$</th>
<th>$\chi^2/N$</th>
<th>$Q^2_{s,0}$</th>
<th>$C^2$</th>
<th>$\gamma$</th>
<th>$\sigma_0/2$ [mb]</th>
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<tbody>
<tr>
<td>HERA</td>
<td>Bal+SD</td>
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Henri Hänninen (JYU)
BK predictions at LHeC kinematics

- Anomalous dimension evolves differently in $Y$ and $\eta$ evolution, possible effect on $Q^2$ dependence
- Differences moderate even at LHeC kinematics

- Effect between $Y$ and $\eta$ evolution slightly enhanced in $F_L$
- $F_L$ is sensitive to smaller dipoles

Henri Hänninen (JYU)  NLO fits to HERA data  January 12, 2021  11/14
Fit comparison to H1 $F_L$ data

- $F_L$ computed with HERA $\sigma_{\text{red}}$ fits compared to H1 $F_L$ data
- Fits describe $F_L$ nicely
- KCBK, ResumBK and TBK equivalent
- Would start to see differences between evolutions at smaller $x_{Bj}$, moderately high $Q^2$
Conclusions

- NLO DIS cross section and small-x evolution: first NLO fits to HERA data
  - KCBK, ResumBK, and TBK all describe the combined HERA data well
- Important test for CGC at NLO
- The fits to the light-quark-only data imply the presence of a substantial non-perturbative contribution
- Would be preferable to fit precise $F_{2,c}$
  - Include massive quarks once NLO impact factors become available
- Precise $F_2$ and $F_L$ data over a wide kinematical range in $x$ and $Q^2$ can help to constrain the evolution equations
Thank you!
Fit results: KCBK

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Fits to HERA and light quark data with Kinematically Constrained BK.
## Fits: ResumBK

Fits to HERA and light quark data with Collinearly Resummed BK.

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## Fits: TBK

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Fits to HERA and light quark data with Target momentum fraction BK.
In $Y$, at $r \sim 1/Q_s$, parent dipole increases, smallest dipole decreases $\gamma$

- At asymptotically small dipoles $\gamma$ fixed
- Evolved $\gamma$ meet on a curve that fits the data

In $\eta$, evolution at $r \sim 1/Q_s$ decreasing with either coupling

- Evolves towards asymptotic $\gamma \sim 0.6$ at large $\eta$