Particle production beyond eikonal accuracy in the dilute-dense CGC framework

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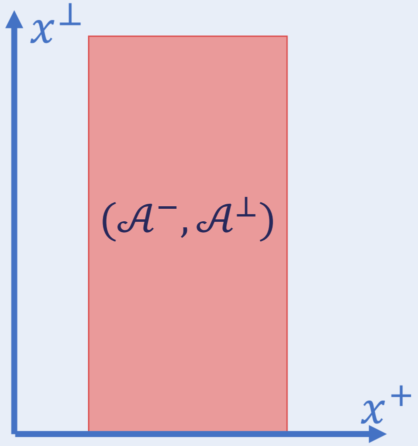
The eikonal approximation

Boosting the target

Dense and left moving: Strong classical field

$$\mathcal{A}^\mu(x) \to \begin{cases} \frac{1}{\gamma}\mathcal{A}^+\left(\gamma x^+, \frac{x^-}{\gamma}, \boldsymbol{x}\right) \\ \gamma \mathcal{A}^-\left(\gamma x^+, \frac{x^-}{\gamma}, \boldsymbol{x}\right) \\ \mathcal{A}^\perp\left(\gamma x^+, \frac{x^-}{\gamma}, \boldsymbol{x}\right) \end{cases}$$

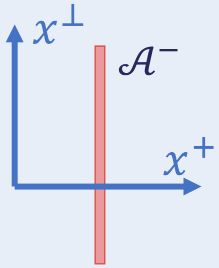


Eikonal approximation

Infinity Lorentz factor: $\gamma \to \infty$

$$\mathcal{A}^\mu(x) \to \begin{cases} 0 \\ \delta(x^+)\mathcal{A}^-(\boldsymbol{x}) \\ 0 \end{cases}$$

Shockwave

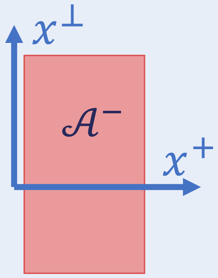


Our approach

We still neglect the transverse component of the field

$$\mathcal{A}^\mu(x) \to \begin{cases} 0 \\ \mathcal{A}^-(x^+, \boldsymbol{x}) \\ 0 \end{cases}$$

Finite length L^+



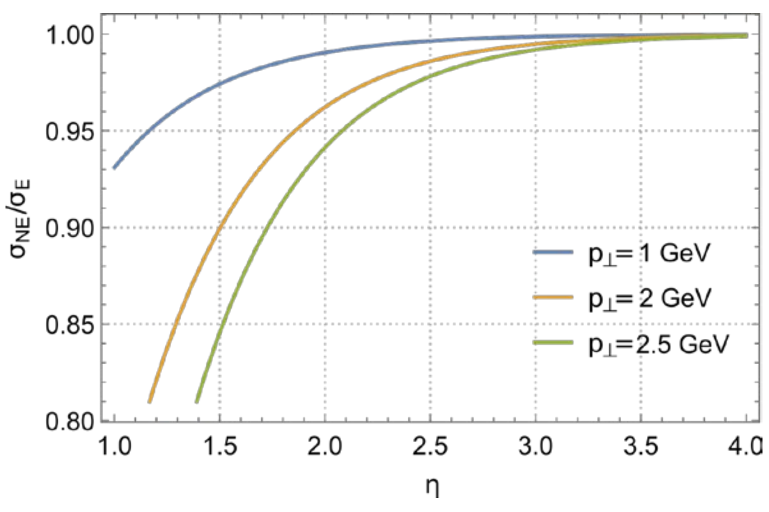
! The projectile is eikonal and dilute

$$j^\mu(x) = \delta^{\mu +}\delta(x^-)\rho^b(\boldsymbol{x})$$

Non-eikonal pp scattering

Gluon production in dilute-dilute (pp) collisions:

Non-eikonal corrections are sizeable at mid-rapidity



[**P. A.** ,T. Altinoluk, N. Armesto, arXiv:1902.04483]
[**P. A.** ,T. Altinoluk, N. Armesto, arXiv:1907.03668]

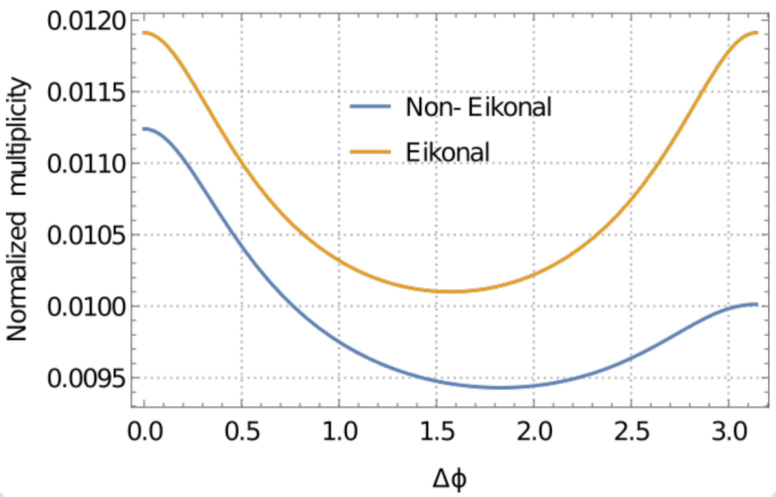
Double gluon production in the CGC is symmetric in transverse momentum:

$$\sigma(\boldsymbol{k}_1, \boldsymbol{k}_2) = \sigma(\boldsymbol{k}_1, -\boldsymbol{k}_2)$$

Non-eikonal corrections break the symmetry:

$$\sigma(\boldsymbol{k}_1, \boldsymbol{k}_2) \neq \sigma(\boldsymbol{k}_1, -\boldsymbol{k}_2)$$

Odd azimuthal harmonics



Dilute-dense scattering (pA) in the CGC

Single gluon spectrum

$$(2\pi)^3(2k^+)\frac{\mathrm{dN}}{dk^+d^2\boldsymbol{k}} = \langle|\mathcal{M}_\lambda^a(k)|^2\rangle_{p,T}$$

LSZ reduction formula

$$\mathcal{M}_\lambda^a(k) = \epsilon_\lambda^{\mu*}\int d^4x\, e^{ik\cdot x}\, \Box_x\, A_\mu^a(x)$$

Looking for the retarted gluon field

Target: dense and left moving

$\mathcal{A}^\mu(x) = \delta^{\mu-}\mathcal{A}^-(x^+,\boldsymbol{x})$

Projectile: dilute and right moving

$j^\mu(x) = \delta^{\mu+}j^+(x^-,\boldsymbol{x}) = \delta^{\mu+}\delta(x^-)\rho^b(\boldsymbol{x})$

Linear response: $A_a^\mu = \mathcal{A}_a^\mu + a_a^\mu + \mathcal{O}(g^3)$ → $a_a^\mu(x) = -i\int d^4y\, G_R^{\mu\nu}(x,y)_{ab}\, j_{\nu b}(x)$

➡ LC Gauge: $A^+ = 0\,;\ \epsilon_\lambda^+ = 0$ ➡ $\mathcal{M}_\lambda^a(k) = \epsilon_\lambda^{i*}\int d^4x\, e^{ik\cdot x}\, \Box_x\, a_a^i(x)$

⬇

We are only interested in the transverse part of $a^\mu(x)$

Gluon propagator through a dense medium

Scalar retarded propagator

Solve the 2+1 dimensional Schrödinger equation

$$\left[\delta^{ab}\left(i\,\partial_{x^+}+\frac{\partial_{\boldsymbol{x}}^2}{2k^+}\right)+g\mathcal{A}^-(x^+,\boldsymbol{x})^{ab}\right]\mathcal{G}_{k^+}^{bc}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})=i\delta^{ac}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta(x^+-y^+)$$

The solution is a Brownian motion around the path $\boldsymbol{z}(z^+)$

$$\mathcal{G}_{k^+}^{ab}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})=\Theta(x^+-y^+)\int_{\boldsymbol{y}}^{\boldsymbol{x}}\mathcal{D}\boldsymbol{z}(z^+)\exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+}dz^+\,\dot{\boldsymbol{z}}^2(z^+)\right]\mathcal{U}^{ab}\big(x^+,y^+;\boldsymbol{z}(z^+)\big)$$

↳ Propagator of a gluon moving through a dense medium (extensively used in jet quenching)

Wilson Line

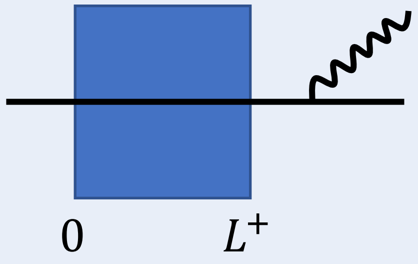
$$\mathcal{U}^{ab}\big(x^+,y^+;\boldsymbol{z}(z^+)\big)=\mathcal{P}^+\exp\left\{ig\int_{y^+}^{x^+}dz^+\,\mathcal{A}^-\big(z^+,\boldsymbol{z}(z^+)\big)\right\}^{ab}$$

Gluon production matrix

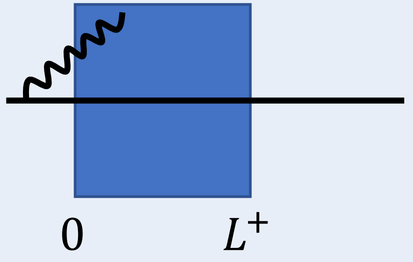
Single gluon production amplitude

$$\mathcal{M}^a_\lambda(k) \propto$$
$$2\,\frac{\boldsymbol{k}^i}{\boldsymbol{k}^2}\int_{\boldsymbol{y}} e^{i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{y}}\,\mathcal{U}_{\boldsymbol{y}}(L^+,0)^{ab}$$
$$-2\,\frac{\boldsymbol{q}^i}{\boldsymbol{q}^2}\int_{\boldsymbol{y},\boldsymbol{x}} e^{i\boldsymbol{q}\cdot\boldsymbol{y}-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\,\mathcal{G}^{ab}_{k^+}(L^+,\boldsymbol{x};0,\boldsymbol{y})$$
$$+\,\frac{1}{k^+}\int_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{q}\cdot\boldsymbol{y}-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\int_0^{L^+} dy^+\left[\partial_{\boldsymbol{y}^i}\mathcal{G}^{ac}_{k^+}(L^+,\boldsymbol{x};y^+,\boldsymbol{y})\right]\mathcal{U}_{\boldsymbol{y}}(y^+,0)^{cb}$$

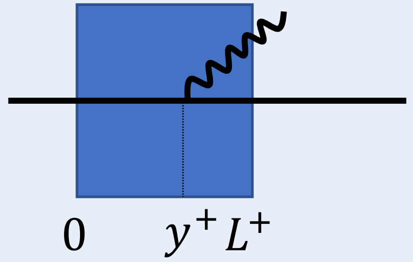
After contribution



Before contribution (doesn't appear in jet quenching)



Inside contribution (completely non eikonal, $L^+ \neq 0$)



Generalizing the dipole operator

Single gluon spectrum

$$\frac{\mathrm{dN}}{dk^+d^2\boldsymbol{k}} \propto \left\langle \mathcal{M}_\lambda^a(k)\mathcal{M}_\lambda^a(k)^\dagger \right\rangle_{p,T}$$

3 ingredients

$$d_0 = \frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{U}_{\boldsymbol{y}}(x^+,y^+)\mathcal{U}_{\bar{\boldsymbol{y}}}^\dagger(x^+,y^+)\right]\right\rangle_T$$

$$d_1 = \frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{U}_{\bar{\boldsymbol{y}}}^\dagger(x^+,y^+)\right]\right\rangle_T$$

$$d_2 = \frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{G}_{k^+}^\dagger(x^+,\bar{\boldsymbol{x}};y^+,\bar{\boldsymbol{y}})\right]\right\rangle_T$$

… and also a fourth one

$\left\langle \rho^a(\boldsymbol{x})\rho^{b*}(\boldsymbol{y})\right\rangle_p \propto \delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})$ (MV model)

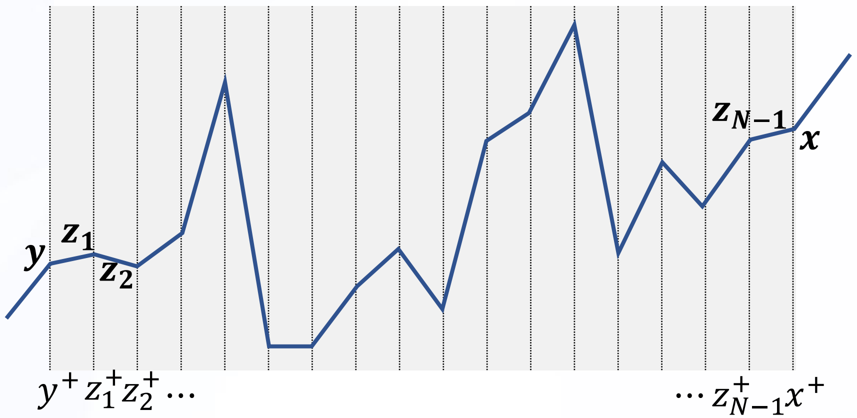
GBW model

$$\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{U}_y\mathcal{U}_{\bar{y}}^\dagger\right]\right\rangle_T = \exp\left\{-\frac{Q_S^2}{4}(\boldsymbol{y}-\bar{\boldsymbol{y}})^2\right\}$$

Localized GBW model

$$\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{U}_{\boldsymbol{y}}(x^+,y^+)\mathcal{U}_{\bar{\boldsymbol{y}}}^\dagger(x^+,y^+)\right]\right\rangle_T = \exp\left\{-\frac{Q_S^2}{4}(\boldsymbol{y}-\bar{\boldsymbol{y}})^2\frac{x^+-y^+}{L^+}\right\}$$

Discretizing the longitudinal space



generalized dipole operator

$$\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{G}_{k^+}(x^+,\bar{\boldsymbol{x}};y^+,\bar{\boldsymbol{y}})^\dagger\right]\right\rangle_T$$

Discretizing the longitudinal space

$$\mathcal{G}^{ab}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y}) = \Theta(x^+-y^+)\int_{\boldsymbol{y}}^{\boldsymbol{x}}\mathcal{D}\boldsymbol{z}(z^+)\exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+}dz^+\,\dot{\boldsymbol{z}}^2(z^+)\right]\mathcal{U}^{ab}\left(x^+,y^+;\boldsymbol{z}(z^+)\right)$$

$$\mathcal{G}^{ab}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})$$

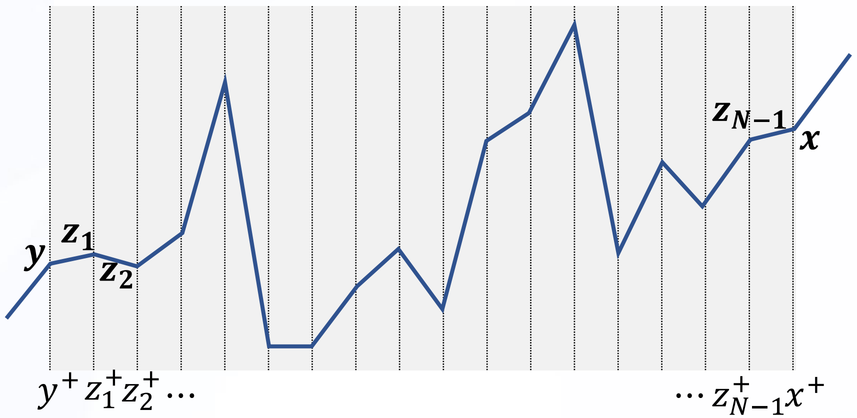
$$= \lim_{N\to\infty}\Theta(x^+-y^+)\int\left(\prod_{n=1}^{N-1}d^2\boldsymbol{z}_n\right)\left(\frac{-ik^+N}{2(x^+-y^+)}\right)^N\exp\left[\frac{i\,k^+N}{2(x^+-y^+)}\sum_{n=0}^{N-1}(\boldsymbol{z}_{n+1}-\boldsymbol{z}_n)^2\right]\mathcal{P}^+\left\{\prod_{n=1}^{N-1}\exp\left[ig\frac{x^+-y^+}{N}\mathcal{A}^-(z_n^+,\boldsymbol{z}_n)\right]\right\}^{ab}$$

$z_0 = y$
$z_N = x$

Discretizing the space

Non-eikonal dipole operator

$$\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{G}_{k^+}(x^+,\overline{\boldsymbol{x}};y^+,\overline{\boldsymbol{y}})^\dagger\right]\right\rangle_T$$



Discretizing the longitudinal space

$$\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{G}_{k^+}(x^+,\overline{\boldsymbol{x}};y^+,\overline{\boldsymbol{y}})^\dagger\right]\right\rangle_T$$

$$= \lim_{N\to\infty}\Theta(x^+-y^+)\int\left(\prod_{n=1}^{N-1}d^2\boldsymbol{z}_n d^2\overline{\boldsymbol{z}}_n\right)\left(\frac{-ik^+N}{2(x^+-y^+)}\right)^{2N}\exp\left[\frac{i\,k^+N}{2(x^+-y^+)}\sum_{n=0}^{N-1}\{(\boldsymbol{z}_{n+1}-\boldsymbol{z}_n)^2-(\overline{\boldsymbol{z}}_{n+1}-\overline{\boldsymbol{z}}_n)^2\}\right]$$

$$\times\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{U}_{\boldsymbol{z}_0}(z_0^+,z_1^+)\mathcal{U}_{\boldsymbol{z}_1}(z_1^+,z_2^+)\cdots\mathcal{U}_{\boldsymbol{z}_{N-1}}(z_{N-1}^+,z_N^+)\mathcal{U}_{\overline{\boldsymbol{z}}_{N-1}}(z_{N-1}^+,z_N^+)^\dagger\cdots\mathcal{U}_{\overline{\boldsymbol{z}}_1}(z_1^+,z_2^+)^\dagger\mathcal{U}_{\overline{\boldsymbol{z}}_0}(z_0^+,z_1^+)^\dagger\right]\right\rangle_T$$

Localization argument

The MV model is local

For a generic functional $F[\rho_T]$ of the target charge density:

$$\langle F(x^+,z^+)F(z^+,x^+)\rangle_T = \langle F(x^+,z^+)\rangle_T \langle F(z^+,x^+)\rangle_T \quad ; \quad y^+ < z^+ < x^+$$

The target ensemble is color neutral

$$\left\langle \mathcal{U}^{ab}\mathcal{U}^{bc\dagger}\right\rangle_T = \frac{\delta^{ac}}{N_c^2-1}\left\langle Tr[\mathcal{U}\mathcal{U}^\dagger]\right\rangle_T$$

Localized GBW model

$$\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{U}_{\mathbf{z}_n}(z_n^+,z_{n+1}^+)\mathcal{U}_{\bar{\mathbf{z}}_n}(z_n^+,z_{n+1}^+)^\dagger\right]\right\rangle_T = \exp\left\{-\frac{Q_s^2}{4}(\mathbf{z}_n-\bar{\mathbf{z}}_n)^2\frac{z_{n+1}^+-z_n^+}{L^+}\right\} = \exp\left\{-\frac{Q_s^2}{4N}(\mathbf{z}_n-\bar{\mathbf{z}}_n)^2\right\}$$

Therefore

$$\frac{1}{N_c^2-1}\left\langle Tr\left[\mathcal{U}_{\mathbf{z}_0}(z_0^+,z_1^+)\mathcal{U}_{\mathbf{z}_1}(z_1^+,z_2^+)\cdots\mathcal{U}_{\mathbf{z}_{N-1}}(z_{N-1}^+,z_N^+)\mathcal{U}_{\bar{\mathbf{z}}_{N-1}}(z_{N-1}^+,z_N^+)^\dagger\cdots\mathcal{U}_{\bar{\mathbf{z}}_1}(z_1^+,z_2^+)^\dagger\mathcal{U}_{\bar{\mathbf{z}}_0}(z_0^+,z_1^+)^\dagger\right]\right\rangle_T$$

$$= \frac{1}{(N_c^2-1)^N}\left\langle Tr\left[\mathcal{U}_{\mathbf{z}_0}(z_0^+,z_1^+)\mathcal{U}_{\bar{\mathbf{z}}_0}(z_0^+,z_1^+)^\dagger\right]\right\rangle_T\cdots\left\langle Tr\left[\mathcal{U}_{\mathbf{z}_{N-1}}(z_{N-1}^+,z_N^+)\mathcal{U}_{\bar{\mathbf{z}}_{N-1}}(z_{N-1}^+,z_N^+)^\dagger\right]\right\rangle_T$$

$$= \exp\left\{-\frac{Q_s^2}{4N}\sum_{n=0}^{N-1}(\mathbf{z}_n-\bar{\mathbf{z}}_n)^2\right\}$$

Discretizing the space

Gaussians are doable

Performing the integral

$$\frac{1}{N_c^2-1}\langle Tr[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{G}^\dagger_{k^+}(x^+,\bar{\boldsymbol{x}};y^+,\bar{\boldsymbol{y}})]\rangle_T$$

$$= \lim_{N\to\infty}\Theta(x^+-y^+)\int\left(\prod_{n=1}^{N-1} d^2\boldsymbol{z}_n d^2\bar{\boldsymbol{z}}_n\right)\left(\frac{-ik^+N}{2(x^+-y^+)}\right)^{2N}\exp\left[\frac{i\,k^+N}{2(x^+-y^+)}\sum_{n=0}^{N-1}\{(\boldsymbol{z}_{n+1}-\boldsymbol{z}_n)^2-(\bar{\boldsymbol{z}}_{n+1}-\bar{\boldsymbol{z}}_n)^2\}\right]\exp\left\{-\frac{Q_s^2}{4N}\sum_{n=0}^{N-1}(\boldsymbol{z}_n-\bar{\boldsymbol{z}}_n)^2\right\}$$

The solution

$$\frac{1}{N_c^2-1}\langle Tr[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{G}^\dagger_{k^+}(x^+,\bar{\boldsymbol{x}};y^+,\bar{\boldsymbol{y}})]\rangle_T$$

$$=\left(\frac{k^+}{2\pi(x^+-y^+)}\right)^2\exp\left\{i\frac{k^+}{2(x^+-y^+)}[(\boldsymbol{x}-\boldsymbol{y})^2-(\bar{\boldsymbol{x}}-\bar{\boldsymbol{y}})^2]\right\}\exp\left\{-\frac{Q_s^2}{12}[(\boldsymbol{y}-\bar{\boldsymbol{y}})^2+(\boldsymbol{x}-\bar{\boldsymbol{x}})(\boldsymbol{y}-\bar{\boldsymbol{y}})+(\boldsymbol{x}-\bar{\boldsymbol{x}})^2]\right\}$$

Dipole operators beyond eikonal accuracy

$\mathcal{G}\,\mathcal{G}^\dagger$ operator

$$\frac{1}{N_c^2-1}\int_{x,\bar{x}} e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})}\left\langle Tr\left[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{G}^\dagger_{k^+}(x^+,\bar{\boldsymbol{x}};y^+,\bar{\boldsymbol{y}})\right]\right\rangle_T = exp\left\{-\frac{\alpha Q_s^2}{4}(\boldsymbol{y}-\bar{\boldsymbol{y}})^2 - i\,\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})\right\}$$

It is the same as the eikonal dipole operator! (only for single gluon production, not for double inclusive)

$\mathcal{G}\,\mathcal{U}^\dagger$ operator

$$\frac{1}{N_c^2-1}\int_{\boldsymbol{x}} e^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\left\langle Tr\left[\mathcal{G}_{k^+}(x^+,\boldsymbol{x};y^+,\boldsymbol{y})\mathcal{U}^\dagger_{\bar{\boldsymbol{y}}}(x^+,y^+)\right]\right\rangle_T$$

$$= A(\alpha^2\epsilon)\exp\left\{-iA(\alpha^2\epsilon)k^-L^+ - i\,\boldsymbol{k}\cdot\left[\frac{\boldsymbol{y}+\bar{\boldsymbol{y}}}{2}+\frac{(\boldsymbol{y}-\bar{\boldsymbol{y}})}{2}B(\alpha^2\epsilon)\right] - \frac{\alpha A(\alpha^2\epsilon)Q_s^2}{4}(\boldsymbol{y}-\bar{\boldsymbol{y}})^2\right\}$$

- $\alpha = \frac{x^+-y^+}{L^+}$ (fraction of the medium)
- Non-eikonal parameter: $\epsilon = \frac{Q_s^2 L^+}{k^+}$ (Eikonal approximation $\leftrightarrow \epsilon = 0$)

Harmonic oscillator:

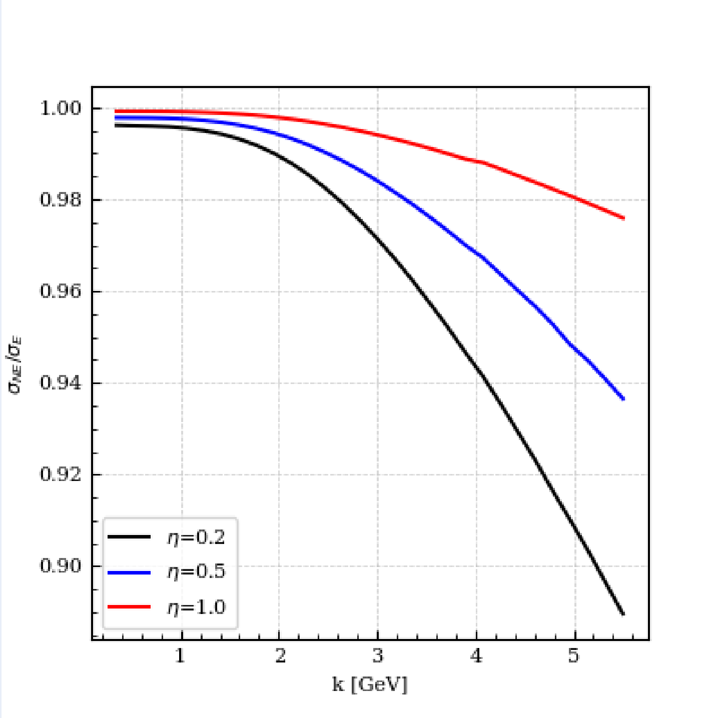
$$B(x) = \cosh^{-1}\left(\frac{1+i}{2}\sqrt{x}\right)$$

$$A(x) = \frac{\tan\sqrt{-\frac{ix}{2}}}{\sqrt{-\frac{ix}{2}}}$$

Results

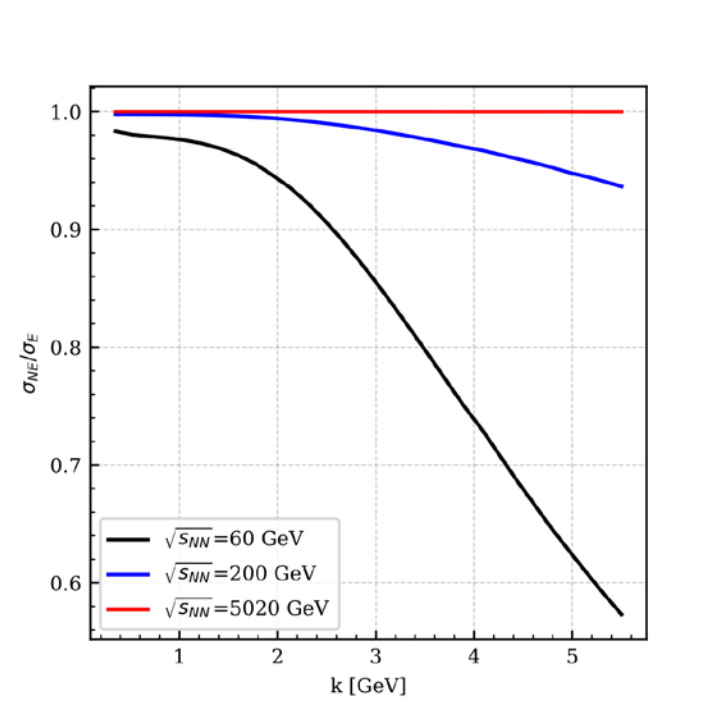
Sizeable at mid-rapidty and large p_T

$\sqrt{s_{NN}} = 200$ GeV, $Q_s = 1\ GeV\ and\ L_0 = 6\ fm$



Sizeable at RHIC energies

$\eta = 0.5$, $Q_s = 1\ GeV\ and\ L_0 = 6\ fm$



Results

Comparing with data

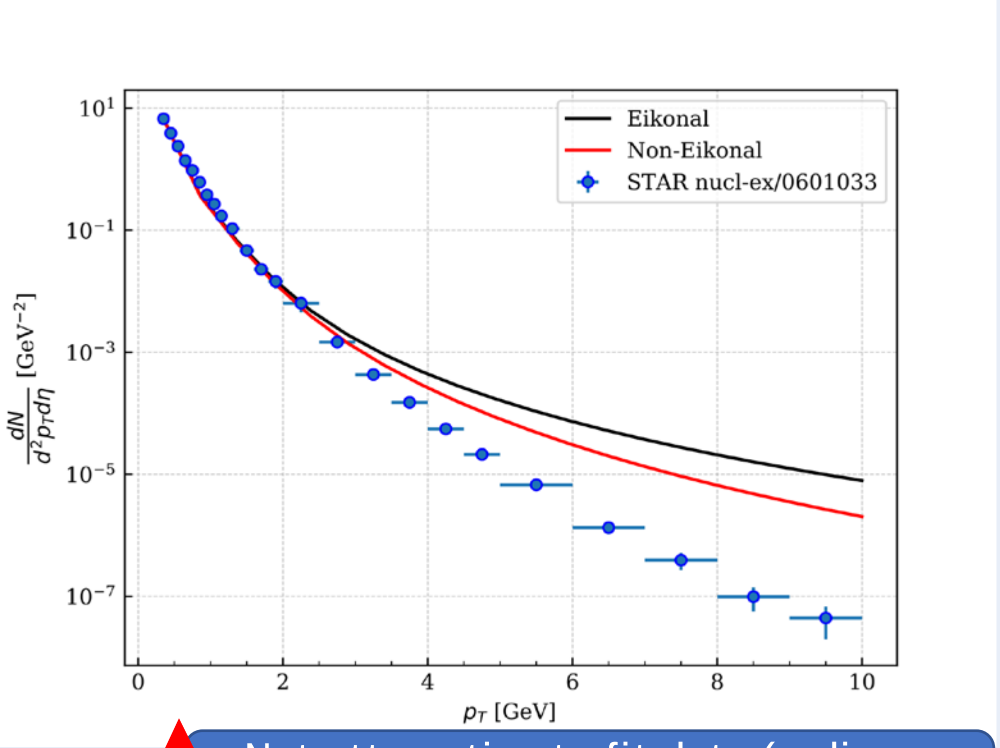
We convolute our result with a FF:

$$\frac{dN^{p+A\to\pi^{+}+X}}{d\eta d^2\boldsymbol{k}} = \int_{z_{min}}^{1} \frac{dz}{z^2} D_g^{\pi^+}(z, Q=|\boldsymbol{k}|) \frac{dN^{g}}{d\eta d^2\boldsymbol{k}}$$

[NLO NNFF1.0 arXiv:1706.07049]

- STAR $d + Au \to \pi^+ + X$ at $\sqrt{s_{NN}} = 200$ GeV
  $|y| < 0.5$ and centrality 0-20%
- Our parameters: $\eta = 0.2, Q_s = 1\ GeV$
  $L_0^+ = 7$ fm

The improvement with respect to the eikonal approximation is a factor 2 at large $p_\perp$!



! Not attempting to fit data (ordinary NLO corrections are needed)

Conclusions

We have computed the dipole operators and single particle production beyond eikonal accuracy in dilute-dense scattering (pA)

Non-eikonal corrections are sizeable when $\sqrt{s_{NN}} < 200\ GeV$, $\eta < 1$ and $p_\perp > 2\ GeV$ ➡ relevant for **RHIC** and **EIC**

We are working on computing double inclusive gluon production beyond eikonal accuracy in dilute-dense scattering and studying its azimuthal harmonics

Thank you!