Relativistic corrections to the vector meson light front wave function

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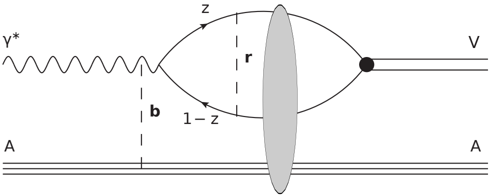
Outline

1. Motivation
   - Exclusive vector meson production
2. Light front wave function
   - Relativistic corrections in the rest frame
   - Going from rest frame to light front
3. Phenomenology

Exclusive vector meson production in DIS

- In the dipole picture:

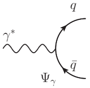
$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma^{A+\gamma^*\to A+V} = \frac{1}{16\pi}R_g^2(1+\beta^2)|\,\mathrm{Im}\,\mathcal{A}^{T,L}|^2$$

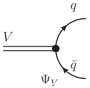


- $\mathrm{Im}\,\mathcal{A}^\lambda = 2\int \mathrm{d}^2\mathbf{b}\mathrm{d}^2\mathbf{r}\frac{\mathrm{d}z}{4\pi}e^{-i\left(\mathbf{b}+\left(\frac{1}{2}-z\right)\mathbf{r}\right)\cdot\mathbf{\Delta}}\underbrace{\Psi_\gamma^{\lambda *}(\mathbf{r},z,Q^2)\Psi_V^\lambda(\mathbf{r},z)}_{\text{photon-meson overlap}}\underbrace{N(\mathbf{r},\mathbf{b},x_{\mathbb{P}})}_{\text{dipole amplitude}}$
- Dipole amplitude from the IPsat parametrization, fitted HERA data [1804.05311]
- $z = k^+/P^+$, quark's fraction of the meson's momentum
- $R_g^2(1+\beta^2)$ = phenomenological corrections (roughly constant)
- $x_{\mathbb{P}} = \frac{M_V^2+Q^2-t}{W^2+Q^2-m_N^2}$
- Q^2 = photon virtuality
- W^2 = invariant mass of the γ^*-nucleon system

Photon-meson overlap

- Production depends on $\sum_{h\bar{h}} \Psi^{\lambda *}_{\gamma, h\bar{h}} \Psi^{\lambda}_{V, h\bar{h}}$
  - h and $\bar{h}$ = helicities of the quark and antiquark





- From light cone perturbation theory:

$$\Psi^{\lambda=0}_{\gamma, h\bar{h}}(\mathbf{r}, z) = -e_f e \sqrt{N_c} \delta_{h,-\bar{h}} 2Qz(1-z) \frac{K_0(\epsilon|\mathbf{r}|)}{2\pi}$$

$$\Psi^{\lambda=\pm 1}_{\gamma, h\bar{h}}(\mathbf{r}, z) = -e_f e \sqrt{2N_c} \left[\pm i e^{\pm i\theta_r} \frac{\epsilon K_1(\epsilon|\mathbf{r}|)}{2\pi} (z\delta_{h\pm}\delta_{\bar{h}\mp} - (1-z)\delta_{h\mp}\delta_{\bar{h}\pm}) + m \frac{K_0(\epsilon|\mathbf{r}|)}{2\pi} \delta_{h\pm}\delta_{\bar{h}\pm} \right]$$

$$\epsilon = \sqrt{z(1-z)Q^2 + m^2}$$

- Vector meson wave function nonperturbative
- Consider J/ψ production
  - A nonrelativistic system – one can try $\Psi_{\mathrm{J}/\psi}(\vec{k}) \sim \delta^{(3)}(\vec{k}) \quad \Rightarrow \quad \Psi_{\mathrm{J}/\psi}(\mathbf{r}, z) \sim \delta(z - 1/2)$
  - However, relativistic corrections might be important as $\langle v^2 \rangle \approx 0.3$!

NRQCD

- NRQCD: An effective field theory for describing nonrelativistic quark-antiquark systems [hep-ph/9407339]
- NRQCD describes particle decay and production in terms of universal long-distance matrix elements (LDMEs) and perturbative hard factors

$$\Gamma(\mathrm{J}/\psi \to e^- e^+) = \frac{2\,\mathrm{Im}\, f_1(^3S_1)}{m_c^2} \langle \mathrm{J}/\psi | \mathcal{O}_1(^3S_1) | \mathrm{J}/\psi \rangle + \frac{2\,\mathrm{Im}\, g_1(^3S_1)}{m_c^4} \langle \mathrm{J}/\psi | \mathcal{P}_1(^3S_1) | \mathrm{J}/\psi \rangle + \mathcal{O}(v^3)$$

- Relation to the rest frame wave function:

$$\langle \mathcal{O}_1(^3S_1) \rangle \propto |\phi(0)|^2 \qquad\qquad \langle \mathcal{P}_1(^3S_1) \rangle \propto -\mathrm{Re}[\phi(0)^* \nabla^2 \phi(0)]$$

- For J/ψ, LDMEs have been determined by matching to decay widths [0710.0994]

NRQCD-motivated wave function

- Wave function in rest-frame
  - In terms of spin and 3-position $\vec{r} = (x^1, x^2, x^3)$
  - J^{PC} conservation $\Rightarrow$ a combination of S and D waves
- From NRQCD: D wave velocity suppressed $\Rightarrow$ consider only S wave
  $\Rightarrow \Psi^{\lambda}_{s\bar{s}}(\vec{r}) = \frac{1}{\sqrt{2}} \xi^{\dagger}_{s} \epsilon^{\lambda} \cdot \sigma \chi_{\bar{s}} \, \phi(r)$
  - ϵ^{λ} = the polarization vector of J/ψ
  - ξ_s and $\chi_{\bar{s}}$ = quark and antiquark spinors with spins s and $\bar{s}$
- Our approach for the scalar part $\phi(r)$: expand around the origin

$$\phi(r) = \underbrace{A}_{\mathcal{O}(v^0)} + \underbrace{Br^2}_{\mathcal{O}(v^2)}$$

- Use the LDMEs to determine the coefficients A and B

From rest frame to light front

- This the wave function in the rest frame – we need the wave function in the light front
- Three major differences between the light front and the rest frame
  1. Definition of the wave function
  2. Coordinate system
  3. Spinor basis

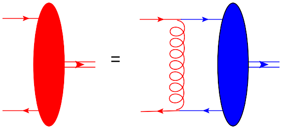
From rest frame to light front: Definition of the wave function

- Light front: loop corrections defined as part of the wave function

$$\Gamma(\mathrm{J}/\psi \to e^- e^+) = \frac{16\pi e_f^2 \alpha}{M_V^2}\left(1 - 2C_F\frac{\alpha_s}{\pi}\right)^2 |\Psi_{\mathrm{RF}}(\vec{r}=0)|^2$$

$$= \frac{16\pi e_f^2 \alpha}{M_V^2}\left|\int_0^1 \mathrm{d}z \Psi_{\mathrm{LF}}(\mathbf{r}=0, z)\right|^2$$

$$\Rightarrow \Psi_{\mathrm{LF}} = \left(1 - 2C_F\frac{\alpha_{\mathrm{s}}}{\pi}\right)\Psi_{\mathrm{RF}}$$



A loop correction to LFWF [1911.01136]

From rest frame to light front: Coordinate system

- Different coordinates: $\vec{r}$ in rest frame vs $(\mathbf{r}, z)$ in light front

$$\vec{r} \xrightarrow{\text{3D Fourier}} \vec{k} \xrightarrow{k^3 \to z} (\mathbf{k}, z) \xrightarrow{\text{2D Fourier}} (\mathbf{r}, z)$$

- Relation between k^3 and $z = k^+/P^+$, $P^+ = M_V/\sqrt{2}$:
  - Assuming quarks on mass-shell, $k^0 = \sqrt{m_c^2 + k^2}$
    $\Rightarrow k^3 = M(z - 1/2)$, $M^2 = \frac{\mathbf{k}^2 + m_c^2}{z(1-z)}$ is the invariant mass of the quark-antiquark pair
- Coordinate transformation from probability conservation:
  $\frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\psi(\vec{k})\phi(\vec{k})^* = \frac{\mathrm{d}^2\mathbf{k}\mathrm{d}z}{(2\pi)^2 4\pi}\psi(\mathbf{k}, z)\phi(\mathbf{k}, z)^*$ for all wave functions ψ and ϕ

From rest frame to light front: Spinor basis

- Different spinors in rest frame and light front:
  - Rest frame: spin states u_s (Bjorken-Drell)
  - Light front: light cone helicity states u_h (Lepage-Brodsky)
- Define the Melosh rotation:
  $R^{sh}(\mathbf{k}, z) = \frac{1}{2m_c} \bar{u}_s(\mathbf{k}, z) u_h(\mathbf{k}, z)$
- Change of basis: $u_h = \sum_s R^{sh} u_s$

$$\Rightarrow \Psi^{\lambda}_{h\bar{h}}(\mathbf{k}, z) = \sum_{s\bar{s}} R^{*sh}(\mathbf{k}, z) R^{*\bar{s}\bar{h}}(-\mathbf{k}, 1-z) \Psi^{\lambda}_{s\bar{s}}(\mathbf{k}, z)$$

$$u_{s=\uparrow} = \frac{1}{\sqrt{2m_c(E+m_c)}} \begin{pmatrix} E+m_c \\ 0 \\ k^3 \\ k^1 + ik^2 \end{pmatrix}$$

$$u_{h=+} = \frac{1}{\sqrt{2m_c(E+k^3)}} \begin{pmatrix} E+m_c+k^3 \\ k^1 + ik^2 \\ E-m_c+k^3 \\ k^1 + ik^2 \end{pmatrix}$$

NRQCD expansion ansatz

- Taking all these differences into account, we get the *NRQCD expansion* light front wave function

$$
\begin{aligned}
\Psi^{\lambda=0}_{+-}(\mathbf{r},z) &= \Psi^{\lambda=0}_{-+}(\mathbf{r},z) = \frac{\pi\sqrt{2}}{\sqrt{m_c}}\left[A\delta(z-1/2) + \frac{B}{m_c^2}\left(\left(\frac{5}{2}+\mathbf{r}^2 m_c^2\right)\delta(z-1/2) - \frac{1}{4}\partial_z^2\delta(z-1/2)\right)\right] \\
\Psi^{\lambda=1}_{++}(\mathbf{r},z) &= \Psi^{\lambda=-1}_{--}(\mathbf{r},z) = \frac{2\pi}{\sqrt{m_c}}\left[A\delta(z-1/2) + \frac{B}{m_c^2}\left(\left(\frac{7}{2}+\mathbf{r}^2 m_c^2\right)\delta(z-1/2) - \frac{1}{4}\partial_z^2\delta(z-1/2)\right)\right] \\
\Psi^{\lambda=1}_{+-}(\mathbf{r},z) &= -\Psi^{\lambda=1}_{-+}(\mathbf{r},z) = \left(\Psi^{\lambda=-1}_{-+}(\mathbf{r},z)\right)^* = \left(-\Psi^{\lambda=-1}_{+-}(\mathbf{r},z)\right)^* = -\frac{2\pi i}{m_c^{3/2}}B\delta(z-1/2)(r_1+ir_2) \\
\Psi^{\lambda=1}_{--}(\mathbf{r},z) &= \Psi^{\lambda=-1}_{++}(\mathbf{r},z) = \Psi^{\lambda=0}_{++}(\mathbf{r},z) = \Psi^{\lambda=0}_{--}(\mathbf{r},z) = 0
\end{aligned}
$$

- Here $A = 0.213\ \mathrm{GeV}^{3/2}$ and $B = -0.0157\ \mathrm{GeV}^{7/2}$
- For comparison: try also the fully nonrelativistic wave function $\phi(r) = A'$
- A' from the decay width: $A' = 0.211\ \mathrm{GeV}^{3/2}$ (and $B' = 0$)
- Gives us the *Delta* LFWF: corresponds to $\Psi \sim \delta(z-1/2)\delta^2(\mathbf{k})$ in the momentum space

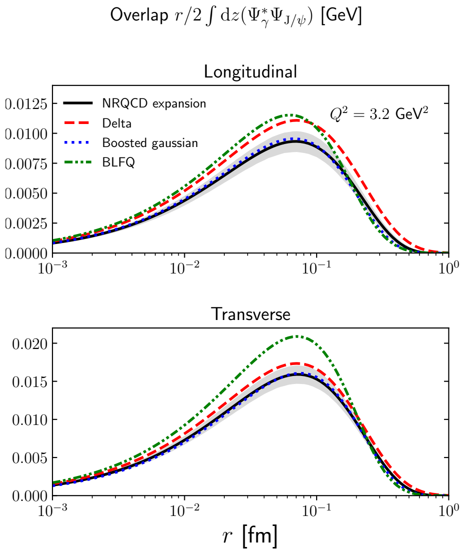
Summary of the wave functions

- *Delta* = fully nonrelativistic
- *NRQCD expansion* = v^2-corrections included

Two phenomenological wave functions:

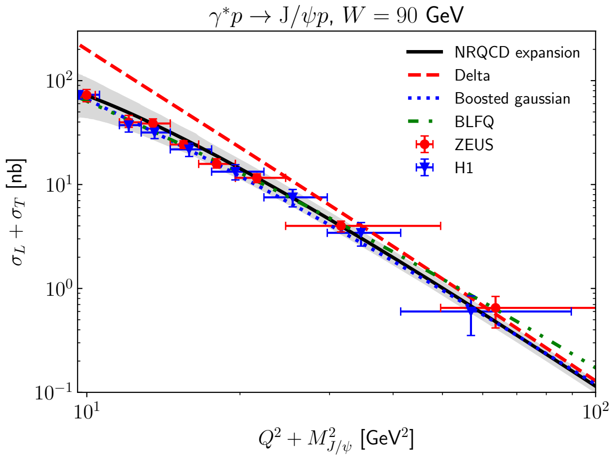
- *Boosted gaussian* = The meson has the same helicity structure as the photon [hep-ph/0606272]
  $\Rightarrow$ the wave function has both S and D waves
- *BLFQ* = Wave functions determined from the mass spectrum of charmonium [1704.06968]

Photon-meson overlaps



- Relativistic corrections $\sim -\mathbf{r}^2$
  - Brings the wf closer to Boosted gaussian
- Large **r**: NRQCD, BLFQ and Boosted gaussian similar
- Note: cross section $\sim$ overlap $\otimes$ dipole amplitude
  $\Rightarrow$ larger **r** behavior more important for cross sections

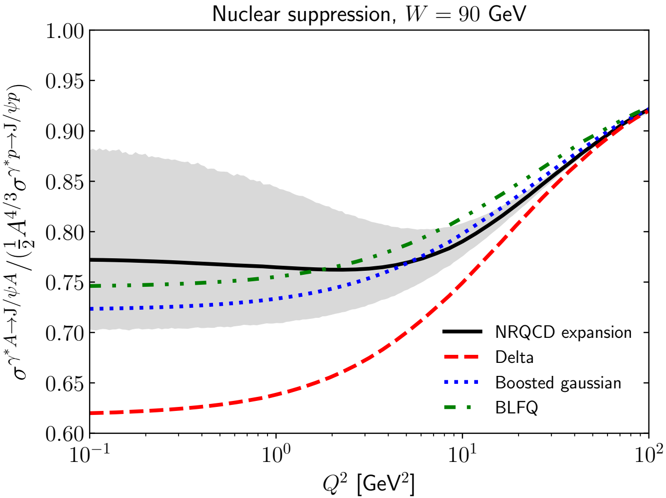
Cross sections: Total



HERA data from [hep-ex/0510016] and [hep-ex/0404008]

- Focus on the Q^2 dependence
  - Overlap peak at $\mathbf{r} \sim 1/(Q^2 + M_{J/\psi}^2)$
  - Small Q^2: larger effect from relativistic correction $-\mathbf{r}^2$
- Q^2 dependence of Delta in disagreement with the data
- Other wave functions describe the data well
- Similar effects also in the ratio σ_L/σ_T (see backup)

Nuclear suppression in EIC kinematics

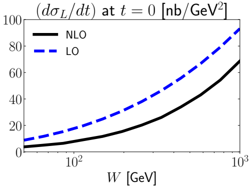


- Heavy ion vs proton in the initial state
  - Identically 1 without non-linear effects

    $\Rightarrow$ Measures nuclear suppression
- The wave function effects might be expected to cancel
  - At low Q^2 this does not happen!
  - Important to use a realistic wave function when comparing to EIC data

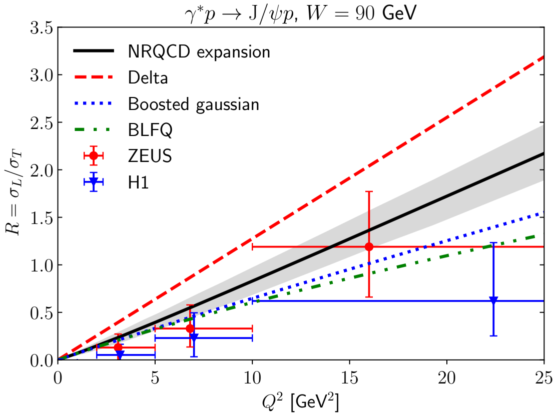
Summary

- We included the relativistic corrections at order v^2 to the wave function
- Wave function coefficients determined from decay width using NRQCD
  - Two independent parameters (compare to Boosted gaussian: only one independent parameter)
- Relativistic corrections found to be important for J/ψ production
  - Similar results as with Boosted gaussian and BLFQ
  - Needed for good agreement with HERA data
- The wave function doesn't completely cancel in the nuclear suppression – need to take relativistic effects into account
- Future work: NLO calculations on the way



Backup

Backup - Cross sections: L/T ratio



HERA data from [hep-ex/0510016] and [hep-ex/0404008]

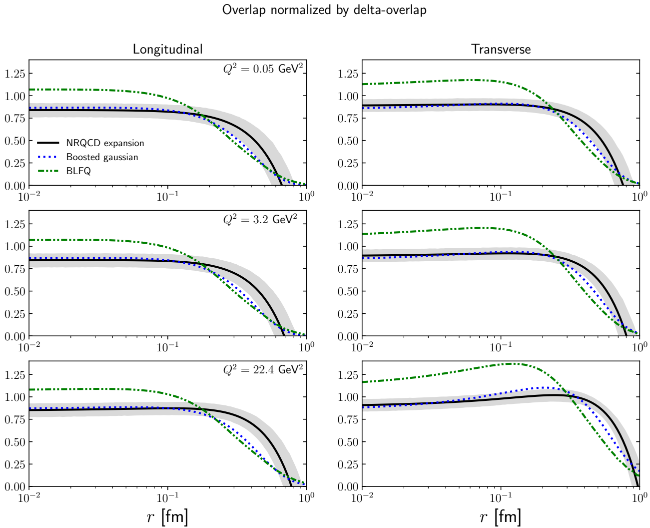
- Overall normalization cancels
- Relativistic corrections bring the ratio closer to the data

Backup - Expressions for overlaps

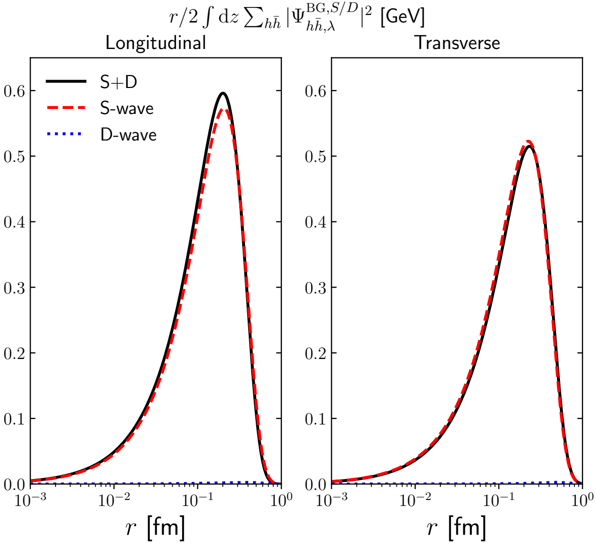
$$r\sum_{h\bar{h}}\int_0^{2\pi}\mathrm{d}\theta_r\int_0^1\frac{\mathrm{d}z}{4\pi}(\Psi^L_{\mathrm{J}/\psi})^*\Psi^L_\gamma e^{i(z-1/2)\mathbf{r}\cdot\mathbf{\Delta}}=\frac{ree_fQ}{2}\sqrt{\frac{N_c}{2m_{c,NR}}}\left[AK_0(r\bar{\epsilon})+\frac{B}{m^2_{c,NR}}\left(\frac{9}{2}K_0(r\bar{\epsilon})+m^2_{c,NR}r^2K_0(r\bar{\epsilon})-\frac{Q^2r}{4\bar{\epsilon}}K_1(r\bar{\epsilon})+\frac{1}{4}\Delta^2r^2K_0(r\bar{\epsilon})\right)\right]$$

$$r\sum_{h\bar{h}}\int_0^{2\pi}\mathrm{d}\theta_r\int_0^1\frac{\mathrm{d}z}{4\pi}(\Psi^T_{\mathrm{J}/\psi})^*\Psi^T_\gamma e^{i(z-1/2)\mathbf{r}\cdot\mathbf{\Delta}}=ree_f\sqrt{\frac{N_cm_{c,NR}}{2}}\left[AK_0(r\bar{\epsilon})+\frac{B}{m^2_{c,NR}}\left(\frac{7}{2}K_0(r\bar{\epsilon})+m^2_{c,NR}r^2K_0(r\bar{\epsilon})-\frac{r}{2\bar{\epsilon}}(Q^2+2m^2_{c,NR})K_1(r\bar{\epsilon})+\frac{1}{4}\Delta^2r^2K_0(r\bar{\epsilon})\right)\right]$$

Backup - Overlap comparison



Backup - Decomposition into S- and D-waves



Backup - Decomposition into S- and D-waves