Relativistic corrections to the vector meson light front wave function

Jani Penttala
In collaboration with Tuomas Lappi and Heikki Mäntysaari
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University of Jyväskylä

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Outline

1. Motivation
   - Exclusive vector meson production

2. Light front wave function
   - Relativistic corrections in the rest frame
   - Going from rest frame to light front

3. Phenomenology
Exclusive vector meson production in DIS

In the dipole picture:

\[
\frac{d}{dt} \sigma^{A+\gamma^*\rightarrow A+V} = \frac{1}{16\pi} R_g^2 (1 + \beta^2) |\text{Im} \mathcal{A}^{T,L}|^2
\]

\[
\text{Im} \mathcal{A}^\lambda = 2 \int d^2 b d^2 r d\frac{dz}{4\pi} e^{-i \left( b + \left( \frac{1}{2} - z \right) r \right) \cdot \Delta} \psi^\lambda_\gamma^*(r, z, Q^2) \psi^\lambda_V(r, z) \mathcal{N}(r, b, x_P)
\]

Dipole amplitude from the IPsat parametrization, fitted HERA data [1804.05311]

\[ z = \frac{k^+}{P^+} \], quark’s fraction of the meson’s momentum

\[ R_g^2 (1 + \beta^2) \] = phenomenological corrections (roughly constant)

\[ x_{IP} = \frac{M_V^2 + Q^2 - t}{W^2 + Q^2 - m_N^2} \]  \quad \[ Q^2 = \text{photon virtuality} \]  \quad \[ W^2 = \text{invariant mass of the } \gamma^*-\text{nucleon system} \]
Photon-meson overlap

- Production depends on $\sum_{h\bar{h}} \Psi_{\gamma,h\bar{h}}^{\lambda\ast} \Psi_{\gamma,h\bar{h}}^{\lambda}$
  - $h$ and $\bar{h}$ = helicities of the quark and antiquark
- From light cone perturbation theory:
  $$\Psi_{\gamma,h\bar{h}}^{\lambda=0}(r,z) = -ef e\sqrt{N_c} \delta_{h,\bar{h}} 2Qz(1 - z) \frac{K_0(\epsilon |r|)}{2\pi}$$
  $$\Psi_{\gamma,h\bar{h}}^{\lambda=\pm 1}(r,z) = -ef e\sqrt{2N_c} \left[ \pm ie^{\pm i\theta_r} \frac{\epsilon K_1(\epsilon |r|)}{2\pi} (z\delta_h \pm \delta_{\bar{h}} \mp 1 - z)\delta_h \pm \delta_{\bar{h}} + m \frac{K_0(\epsilon |r|)}{2\pi} \delta_h \pm \delta_{\bar{h}} \right]$$
  $$\epsilon = \sqrt{z(1 - z)Q^2 + m^2}$$

Vector meson wave function nonperturbative

- Consider $J/\psi$ production
  - A nonrelativistic system – one can try $\Psi_{J/\psi}(\vec{k}) \sim \delta^{(3)}(\vec{k}) \Rightarrow \Psi_{J/\psi}(r,z) \sim \delta(z - 1/2)$
  - However, relativistic corrections might be important as $\langle v^2 \rangle \approx 0.3!$
NRQCD

- NRQCD: An effective field theory for describing nonrelativistic quark-antiquark systems
  [hep-ph/9407339]

- NRQCD describes particle decay and production in terms of universal long-distance matrix elements (LDMEs) and perturbative hard factors

\[ \Gamma(\psi/J \rightarrow e^- e^+) = \frac{2 \text{Im} f_1(3S_1)}{m_c^2} \langle \psi/J | O_1(3S_1) | \psi/J \rangle + \frac{2 \text{Im} g_1(3S_1)}{m_c^4} \langle \psi/J | P_1(3S_1) | \psi/J \rangle + O(v^3) \]

- Relation to the rest frame wave function:

\[ \langle O_1(3S_1) \rangle \propto |\phi(0)|^2 \quad \langle P_1(3S_1) \rangle \propto - \text{Re}[\phi(0)^* \nabla^2 \phi(0)] \]

- For \( \psi/J \), LDMEs have been determined by matching to decay widths [0710.0994]
NRQCD-motivated wave function

- Wave function in rest-frame
  - In terms of spin and 3-position $\vec{r} = (x^1, x^2, x^3)$
  - $J^{PC}$ conservation $\Rightarrow$ a combination of $S$ and $D$ waves

- From NRQCD: $D$ wave velocity suppressed $\Rightarrow$ consider only $S$ wave

$$\Psi_{s\bar{s}}^\lambda(\vec{r}) = \frac{1}{\sqrt{2}} \xi_s^\dagger \epsilon^\lambda \cdot \sigma \chi_{\bar{s}} \phi(r)$$

  - $\epsilon^\lambda$ = the polarization vector of $J/\psi$
  - $\xi_s$ and $\chi_{\bar{s}}$ = quark and antiquark spinors with spins $s$ and $\bar{s}$

- Our approach for the scalar part $\phi(r)$: expand around the origin

$$\phi(r) = A \underbrace{\mathcal{O}(v^0)}_{\mathcal{O}(v^0)} + B \underbrace{r^2 \mathcal{O}(v^2)}_{\mathcal{O}(v^2)}$$

- Use the LDMEs to determine the coefficients $A$ and $B$
From rest frame to light front

- This the wave function in the rest frame – we need the wave function in the light front
- Three major differences between the light front and the rest frame
  - Definition of the wave function
  - Coordinate system
  - Spinor basis
Light front: loop corrections defined as part of the wave function

\[
\Gamma(J/\psi \rightarrow e^- e^+) = \frac{16\pi e_f^2 \alpha}{M_V^2} \left( 1 - 2C_F \frac{\alpha_s}{\pi} \right)^2 |\Psi_{RF}(r=0)|^2
\]

\[
= \frac{16\pi e_f^2 \alpha}{M_V^2} \left| \int_0^1 dz \Psi_{LF}(r=0,z) \right|^2
\]

\[
\Rightarrow \Psi_{LF} = \left( 1 - 2C_F \frac{\alpha_s}{\pi} \right) \Psi_{RF}
\]

A loop correction to LFWF

[1911.01136]
From rest frame to light front: Coordinate system

- Different coordinates: \( \vec{r} \) in rest frame vs \((r, z)\) in light front

\[
\vec{r} \xrightarrow{3D \text{ Fourier}} \vec{k} \xrightarrow{k^3 \rightarrow z} (k, z) \xrightarrow{2D \text{ Fourier}} (r, z)
\]

- Relation between \( k^3 \) and \( z = k^+ / P^+ \), \( P^+ = M_V / \sqrt{2} \):
  - Assuming quarks on mass-shell, \( k^0 = \sqrt{m_c^2 + k^2} \)
  - \( k^3 = M(z - 1/2), \quad M^2 = \frac{k^2 + m_c^2}{z(1-z)} \) is the invariant mass of the quark-antiquark pair

- Coordinate transformation from probability conservation:

\[
\frac{d^3 \vec{k}}{(2\pi)^3} \psi(\vec{k}) \phi(\vec{k})^* = \frac{d^2 k d z}{(2\pi)^2 4\pi} \psi(k, z) \phi(k, z)^* \text{ for all wave functions } \psi \text{ and } \phi
\]
Different spinors in rest frame and light front:

- Rest frame: spin states \( u_s \) (Bjorken-Drell)
- Light front: light cone helicity states \( u_h \) (Lepage-Brodsky)

Define the Melosh rotation:

\[
R^{sh}(k, z) = \frac{1}{2m_c} \bar{u}_s(k, z) u_h(k, z)
\]

Change of basis: \( u_h = \sum_s R^{sh} u_s \)

\[
\Rightarrow \psi^\lambda_{hh}(k, z) = \sum_{s\bar{s}} R^{*sh}(k, z) R^{*\bar{s}h}(-k, 1 - z) \psi^\lambda_{s\bar{s}}(k, z)
\]

\[
u_{s=\uparrow} = \frac{1}{\sqrt{2m_c(E + m_c)}} \begin{pmatrix} E + m_c \\ 0 \\ k^3 \\ k^1 + ik^2 \end{pmatrix}
\]

\[
u_{h=+} = \frac{1}{\sqrt{2m_c(E + k^3)}} \begin{pmatrix} E + m_c + k^3 \\ k^1 + ik^2 \\ E - m_c + k^3 \\ k^1 + ik^2 \end{pmatrix}
\]
NRQCD expansion ansatz

• Taking all these differences into account, we get the NRQCD expansion light front wave function

\[
\Psi_{\lambda=0}(r, z) = \Psi_{\lambda=0}(r, z) = \frac{\pi \sqrt{2}}{\sqrt{m_c}} \left[ A \delta(z - 1/2) + \frac{B}{m_c^2} \left( \frac{5}{2} + r^2 m_c^2 \right) \delta(z - 1/2) - \frac{1}{4} \partial_z^2 \delta(z - 1/2) \right]
\]

\[
\Psi_{\lambda=1}(r, z) = \Psi_{\lambda=-1}(r, z) = \frac{2\pi}{\sqrt{m_c}} \left[ A \delta(z - 1/2) + \frac{B}{m_c^2} \left( \frac{7}{2} + r^2 m_c^2 \right) \delta(z - 1/2) - \frac{1}{4} \partial_z^2 \delta(z - 1/2) \right]
\]

\[
\Psi_{\lambda=1}(r, z) = -\Psi_{\lambda=-1}(r, z) = (\Psi_{\lambda=-1}(r, z))^* = (-\Psi_{\lambda=-1}(r, z))^* = -\frac{2\pi i}{m_c^{3/2}} B \delta(z - 1/2)(r_1 + ir_2)
\]

\[
\Psi_{\lambda=-1}(r, z) = \Psi_{\lambda=-1}(r, z) = \Psi_{\lambda=0}(r, z) = \Psi_{\lambda=0}(r, z) = 0
\]

• Here \( A = 0.213 \text{ GeV}^{3/2} \) and \( B = -0.0157 \text{ GeV}^{7/2} \)

• For comparison: try also the fully nonrelativistic wave function \( \phi(r) = A' \)

• \( A' \) from the decay width: \( A' = 0.211 \text{ GeV}^{3/2} \) (and \( B' = 0 \))

• Gives us the Delta LFWF: corresponds to \( \Psi \sim \delta(z - 1/2)\delta^2(k) \) in the momentum space
Summary of the wave functions

- \textit{Delta} = fully nonrelativistic

- \textit{NRQCD expansion} = $v^2$-corrections included

Two phenomenological wave functions:

- \textit{Boosted gaussian} = The meson has the same helicity structure as the photon [hep-ph/0606272]
  \Rightarrow the wave function has both \textit{S} and \textit{D} waves

- \textit{BLFQ} = Wave functions determined from the mass spectrum of charmonium [1704.06968]
Photon-meson overlaps

Overlap \( r/2 \int dz (\Psi^* \gamma J/\psi) \) [GeV]

Longitudinal

- Relativistic corrections \( \sim -r^2 \)
  - Brings the wf closer to Boosted gaussian

- Large \( r \): NRQCD, BLFQ and Boosted gaussian similar

Transverse

- Note: cross section \( \sim \) overlap \( \otimes \) dipole amplitude
  \( \Rightarrow \) larger \( r \) behavior more important for cross sections
Focus on the $Q^2$ dependence

- Overlap peak at $r \sim 1/(Q^2 + M^2_{J/\psi})$
- Small $Q^2$: larger effect from relativistic correction $-r^2$

$Q^2$ dependence of Delta in disagreement with the data

Other wave functions describe the data well

Similar effects also in the ratio $\sigma_L/\sigma_T$
(see backup)
Nuclear suppression in EIC kinematics

Nuclear suppression, $W = 90$ GeV

- Heavy ion vs proton in the initial state
  - Identically 1 without non-linear effects
  - $\Rightarrow$ Measures nuclear suppression

- The wave function effects might be expected to cancel
  - At low $Q^2$ this does not happen!
  - Important to use a realistic wave function when comparing to EIC data
Summary

- We included the relativistic corrections at order $v^2$ to the wave function.
- Wave function coefficients determined from decay width using NRQCD:
  - Two independent parameters (compare to Boosted gaussian: only one independent parameter).
- Relativistic corrections found to be important for $J/\psi$ production:
  - Similar results as with Boosted gaussian and BLFQ.
  - Needed for good agreement with HERA data.
- The wave function doesn’t completely cancel in the nuclear suppression – need to take relativistic effects into account.
- Future work: NLO calculations on the way.
\[ \gamma^* p \to J/\psi p, \ W = 90 \text{ GeV} \]

\[ R = \frac{\sigma_L}{\sigma_T} \]

- Overall normalization cancels
- Relativistic corrections bring the ratio closer to the data

HERA data from [hep-ex/0510016] and [hep-ex/0404008]
\[ r \sum_{h\bar{h}} \int_0^{2\pi} \int_0^1 \frac{dz}{4\pi} (\psi_{\lambda\psi}^L)^* \psi_{\lambda} e^{i(z-1/2)r\cdot\Delta} = \text{ree} \sqrt{\frac{N_c}{2m_{c,\text{NR}}} \left[ A K_0(r\bar{\epsilon}) + \frac{B}{m_{c,\text{NR}}^2} \left( \frac{9}{2} K_0(r\bar{\epsilon}) + m_{c,\text{NR}}^2 r^2 K_0(r\bar{\epsilon}) - \frac{Q^2 r}{4\bar{\epsilon}} K_1(r\bar{\epsilon}) + \frac{1}{4} \Delta^2 r^2 K_0(r\bar{\epsilon}) \right) \right]} \]

\[ r \sum_{h\bar{h}} \int_0^{2\pi} \int_0^1 \frac{dz}{4\pi} (\psi_{\lambda\psi}^T)^* \psi_{\gamma} e^{i(z-1/2)r\cdot\Delta} = \text{ree} \sqrt{\frac{N_c m_{c,\text{NR}}}{2} \left[ A K_0(r\bar{\epsilon}) + \frac{B}{m_{c,\text{NR}}^2} \left( \frac{7}{2} K_0(r\bar{\epsilon}) + m_{c,\text{NR}}^2 r^2 K_0(r\bar{\epsilon}) - \frac{r}{2\bar{\epsilon}} (Q^2 + 2m_{c,\text{NR}}^2) K_1(r\bar{\epsilon}) + \frac{1}{4} \Delta^2 r^2 K_0(r\bar{\epsilon}) \right) \right]} \]
Overlap normalized by delta-overlap

**Longitudinal**

- $Q^2 = 0.05 \text{ GeV}^2$
- $Q^2 = 3.2 \text{ GeV}^2$
- $Q^2 = 22.4 \text{ GeV}^2$

**Transverse**

- $Q^2 = 0.05 \text{ GeV}^2$
- $Q^2 = 3.2 \text{ GeV}^2$
- $Q^2 = 22.4 \text{ GeV}^2$

$r$ [fm]
Backup - Decomposition into S- and D-waves

\[ r/2 \int dz \sum_{hh} |\Psi_{hh,\lambda}^{BG,S/D}|^2 \text{ [GeV]} \]

**Longitudinal**

- S+D
- S-wave
- D-wave

**Transverse**
Backup - Decomposition into S- and D-waves

\[ r/2 \int dz \sum_{hh} |\Psi_{hh,\lambda}^{S/D}|^2 [GeV] \]

\begin{align*}
\text{Longitudinal} & \quad Q^2 = 0.05 \text{ GeV}^2 \\
\text{Transverse} & \quad Q^2 = 22.4 \text{ GeV}^2
\end{align*}

\begin{align*}
\text{Longitudinal} & \quad r/2 \int dz \sum_{hh} |\Psi_{hh,\lambda}^{S/D}|^2 [GeV] \\
\text{Transverse} & \quad r/2 \int dz \sum_{hh} |\Psi_{hh,\lambda}^{S/D}|^2 [GeV]
\end{align*}