Color charge correlations in the proton at moderately small x

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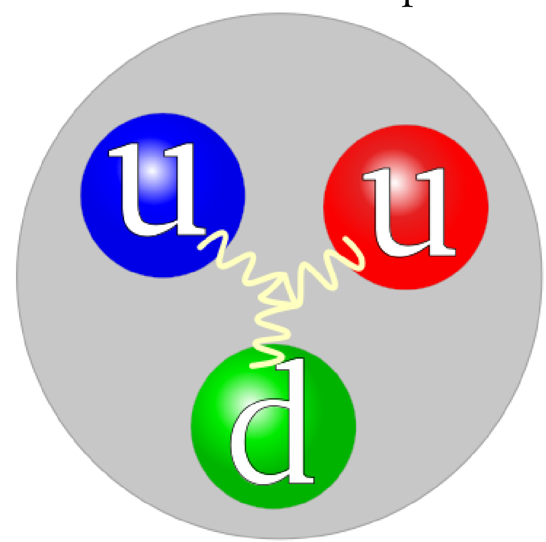
* Color charge correlators (and dipole scattering amplitudes) from the LFwf

wikipedia



talk based on collaborations with
H. Mäntysaari, R. Paatelainen: work in progress
R. Paatelainen: 2010.11245
T. Stebel, V. Skokov: 2001.04516
T. Stebel: 1903.07660
G. Miller, R. Venugopalan: 1808.02501

What's that ?

Motivation :

Accurate fits to HERA $F_2(x,Q^2)$ from BK / JIMWLK evolution:

– r.c. BK: Albacete et al, PRD 80 (2009), EPJ-C 71 (2011),
Mäntysaari & Schenke, 1806.06783

– coll. improved BK: Iancu et al, PLB 750 (2015), Ducloue et al, 1912.09196,
Beuf, Hänninen, Lappi, Mäntysaari, 2007, 2008

However,

– ad hoc initial conditions at $x_0 = 0.01$, parameters adjusted so that BK fit is optimal

– how do they depend on x_0 ? (important for NLO BK; Ducloue et al, 1902.06637)

– no b, r*b dependence (or model via MV)

BK evolution of C-odd amplitude:

– magnitude & sign of initial condition unknown
(e.g. Yao, Hagiwara, Hatta, PLB 790 (2019) compute
dipole gluon Sivers function at small x)

Our goal:

– relate (*x-dependent*) initial condition to light-front w.f. of proton, take advantage of "proton imaging" at EIC

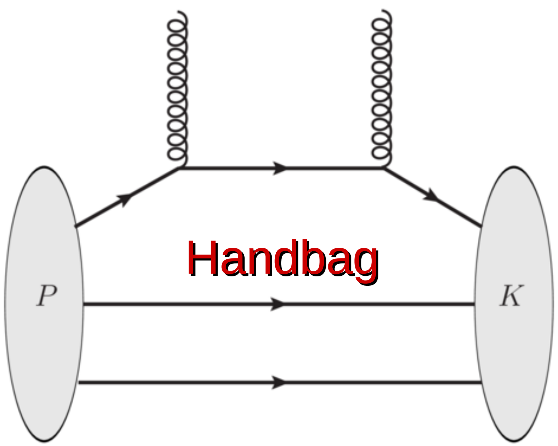
MV-like charge correlators :

$$\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\rangle = g^2\mu^2\,\delta^{ab}\,(2\pi)^2\,\delta(\vec{q}_1+\vec{q}_2)\,,\qquad \langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\rangle = 0$$

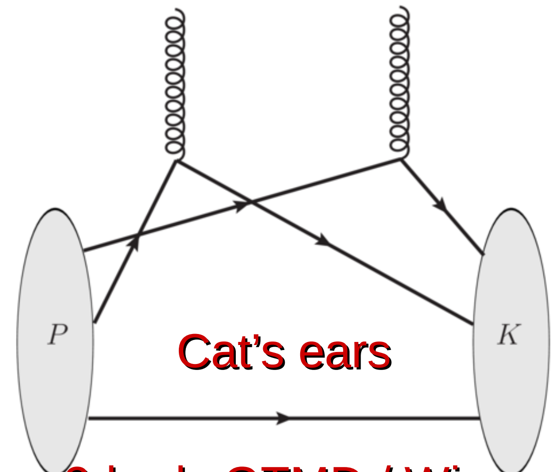
(note: μ^2 a positive constant)

proton :

$$\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\rangle \sim g^2\,\delta^{ab}\,G_2(\vec{q}_1,\vec{q}_2)$$



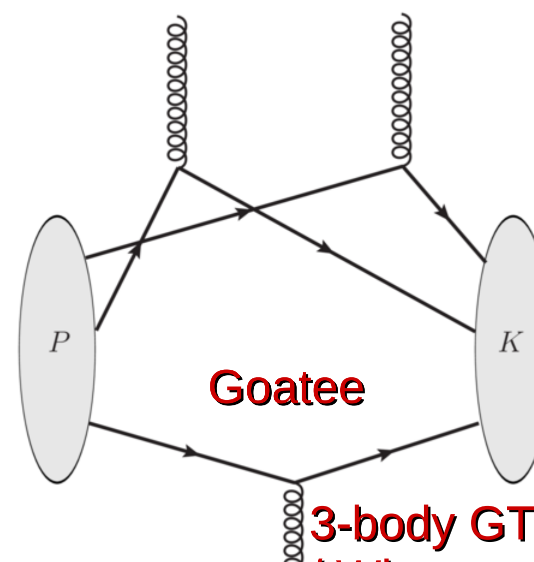
1-body GTMD / Wigner



2-body GTMD / Wigner

$$\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\rangle_{C=-} \sim g^3\,d^{abc}\,G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$

$$\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\rangle_{C=+} \sim ig^3\,f^{abc}\,G_3^+(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$



3-body GTMD / Wigner

The proton on the light front

The proton on the light front (valence quark Fock state; L.C. time $x^+ = 0$)

$$
\begin{aligned}
|P\rangle \;=\; & \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{\sqrt{x_1 x_2 x_3}} \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2k_1 d^2k_2 d^2k_3}{(16\pi^3)^3} 16\pi^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \\
& \times \psi(k_1, k_2, k_3) \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} |p_1, i_1; \, p_2, i_2; \, p_3, i_3\rangle
\end{aligned}
$$

+ higher Fock states

P. Lepage & Brodsky, 1979 - ....
Brodsky, Pauli, Pinsky, PR (1998)

* Fock space amplitude ψ is gauge invariant, universal, and process independent

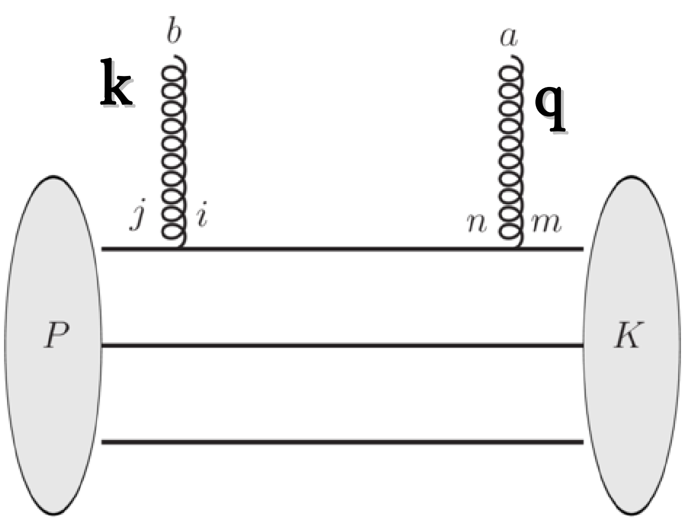
* encodes the non-perturbative structure of hadrons (QCD eigenstates)

$\longrightarrow$ Evaluate color charge $\overline{q}\gamma^+ t^a q$ correlators explicitly !

$<\rho^a\rho^b>$ correlator

(color charge operators ~ $b^\dagger b$, not classical charge densities)

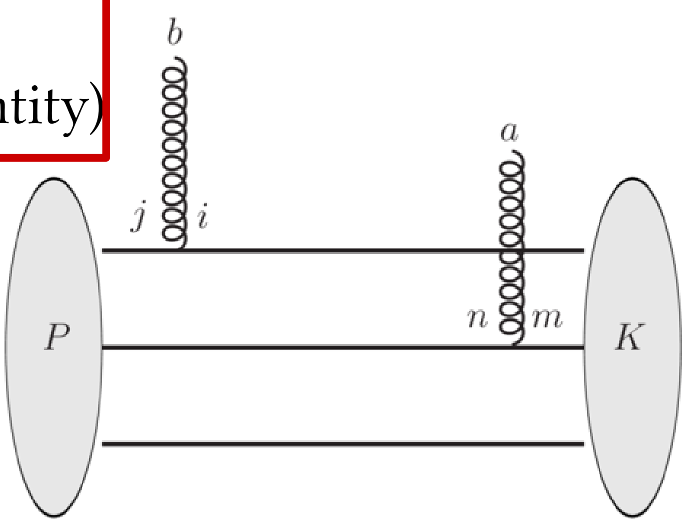
$$\begin{aligned}
\langle\, \rho^a(\vec{q})\, \rho^b(\vec{k})\,\rangle_{K_\perp} \quad &= \quad g^2\, \mathrm{tr}\; t^a t^b \int [dx_i][d^2p_i] \\
&\quad \Big\{\psi^* \left(\vec{p}_1 + (1-x_1)\vec{K}_T, \vec{p}_2 - x_2\vec{K}_T, \vec{p}_3 - x_3\vec{K}_T\right) \\
&\quad -\psi^* \left(\vec{p}_1 - \vec{q} - x_1\vec{K}_T, \vec{p}_2 - \vec{k} - x_2\vec{K}_T, \vec{p}_3 - x_3\vec{K}_T\right)\Big\} \\
&\quad \psi\left(\vec{p}_1, \vec{p}_2, \vec{p}_3\right) \\
&\sim \quad g^2\, \delta^{ab}\, G_2(\vec{q}, \vec{k}) \qquad\qquad (\vec{q}+\vec{k}+\vec{K}_T = 0)
\end{aligned}$$



involve 1- and 2-particle GTMDs,
sum vanishes in IR
(color neutrality / Ward identity)

← dominates when $q^2, k^2 \gg K_T^2$

→ dominates when $\vec{q} \sim \vec{k} \sim -\vec{K}_T/2$

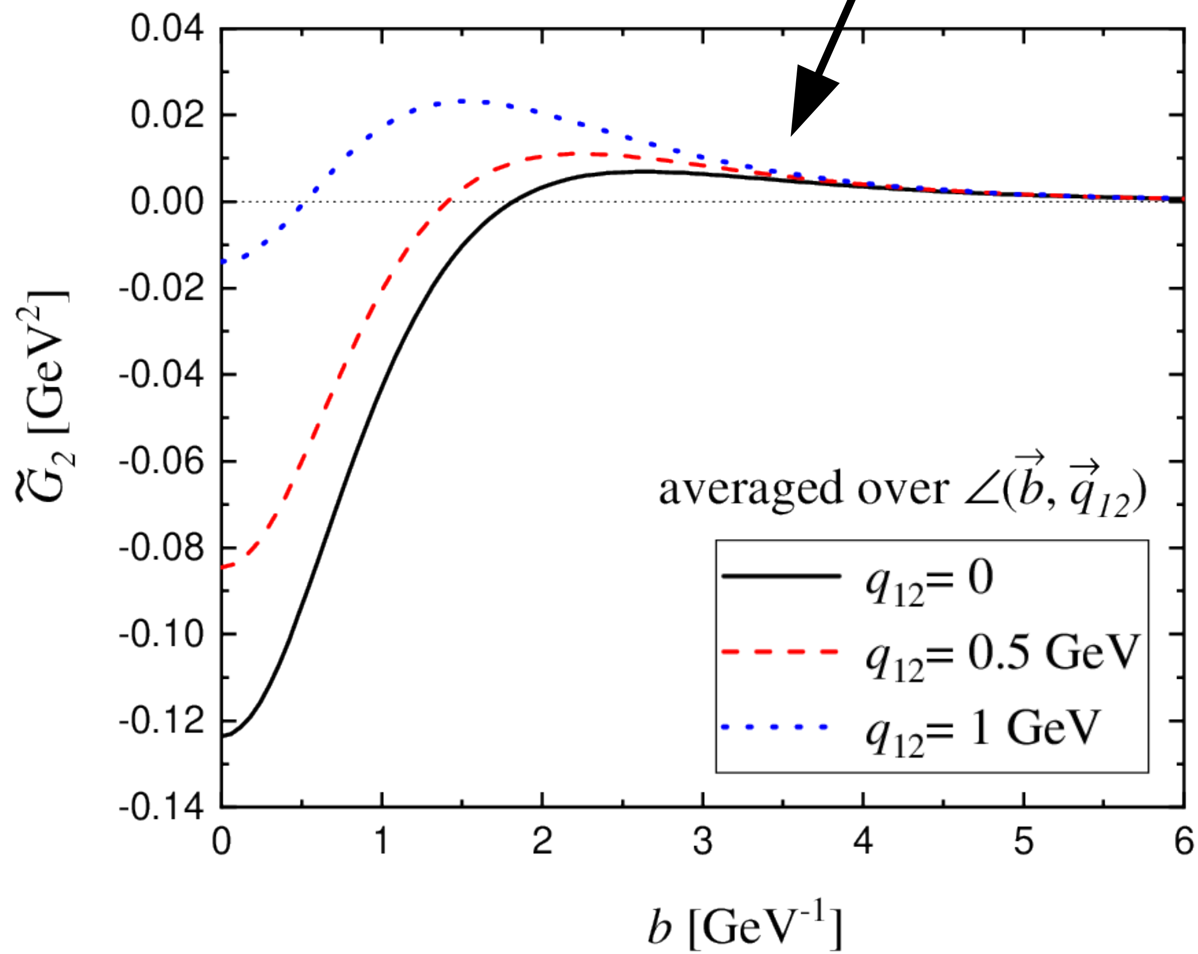


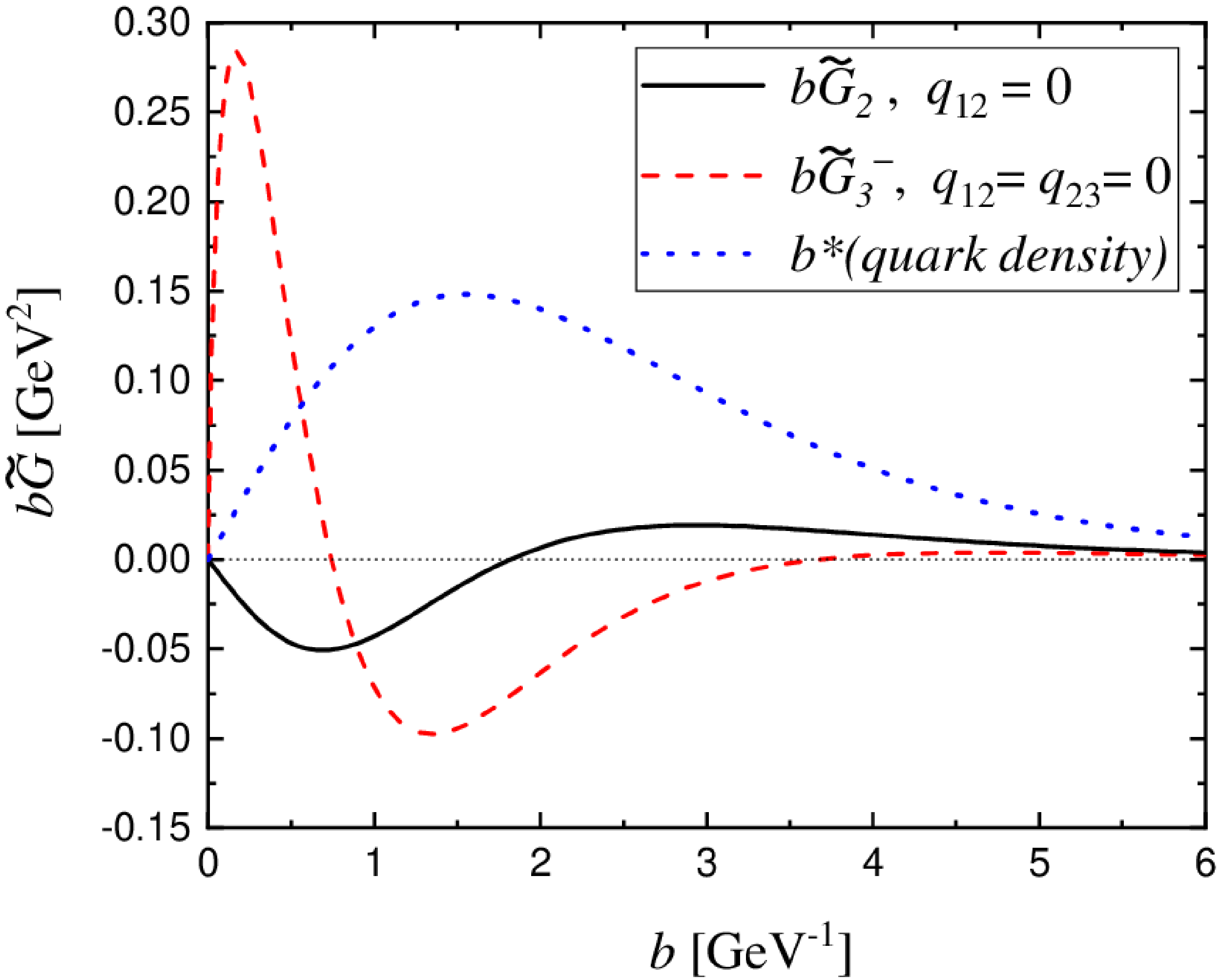
Numerical results for G_2 correlator at LO

(using Brodsky & Schlumpf LFwf)

- depends on b as well as on q_{12} = q1-q2 (resp. r)
- and on their relative angle
- note: small b is large K_T, "cat's ears" dominates !

resembles "shape function"





* G_2, G_3 are n-body GTMDs, compare to 1-body quark density (thickness func.)

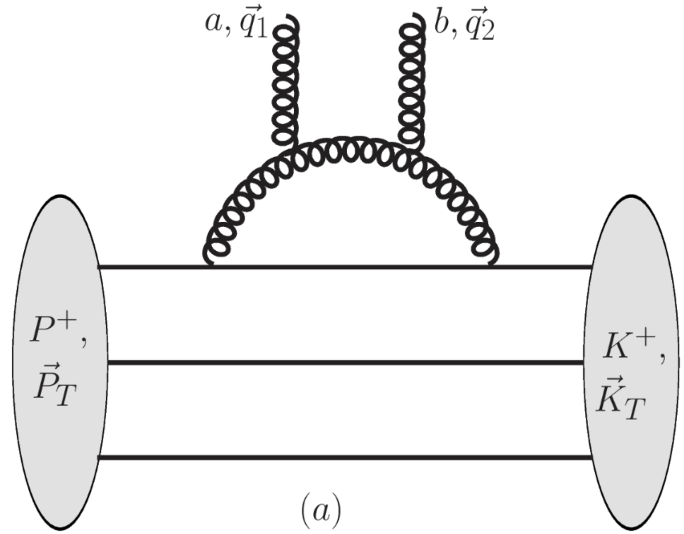
b-dependent parton densities $\sim T_p(\vec{b})$??

[not even for a nucleus, see EPS09s, JHEP 2012]

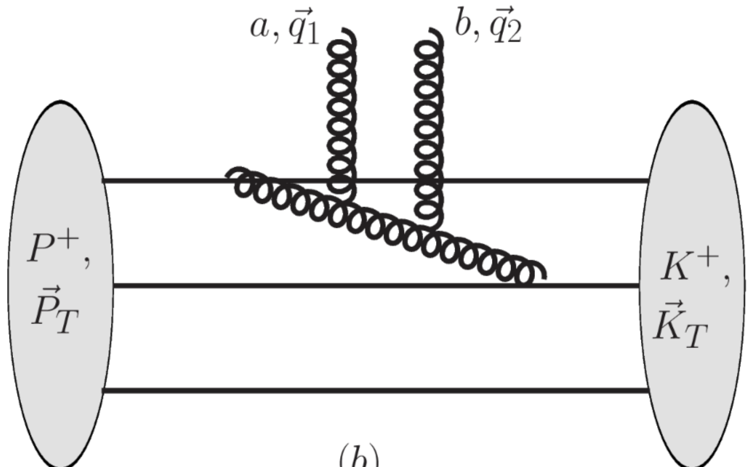
Now to $|P\rangle \sim \psi_{qqq}|qqq\rangle + \psi_{qqqg}|qqqg\rangle$

computed in perturbation theory, 1-gluon emission / exchange,
w/o employing small-x approximation

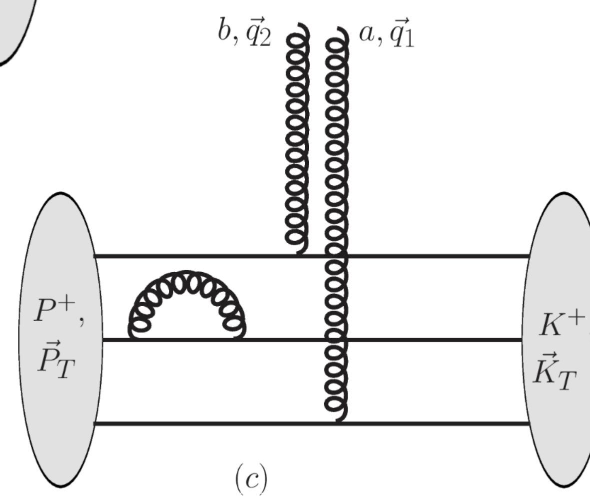
notes:
* soft & collinear div.
* UV divergences cancel
* Ward satisfied



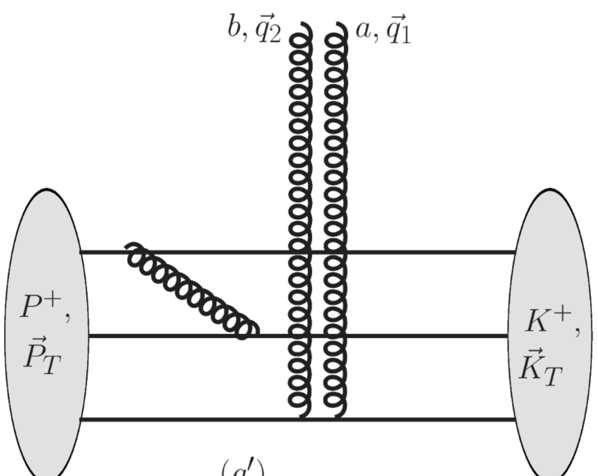
(a)



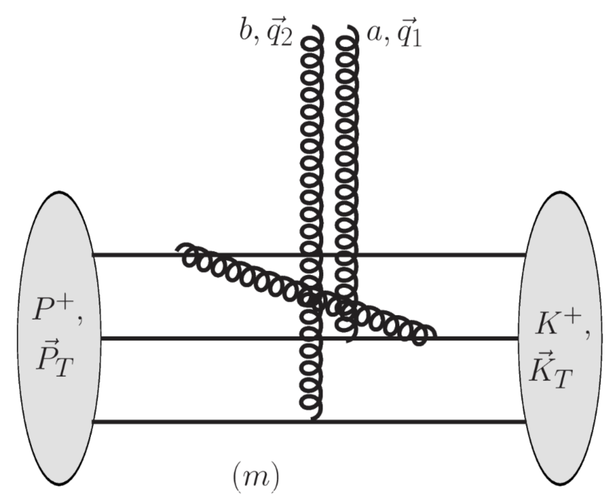
(b)



(c)



(g′)

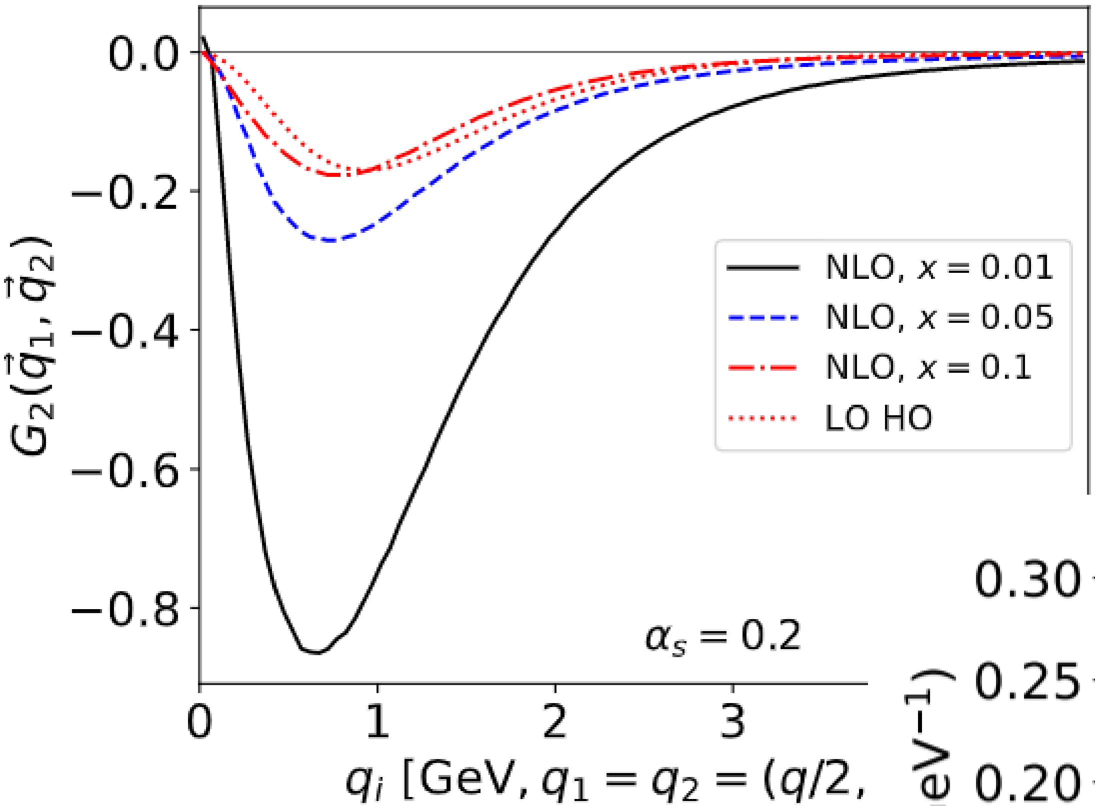


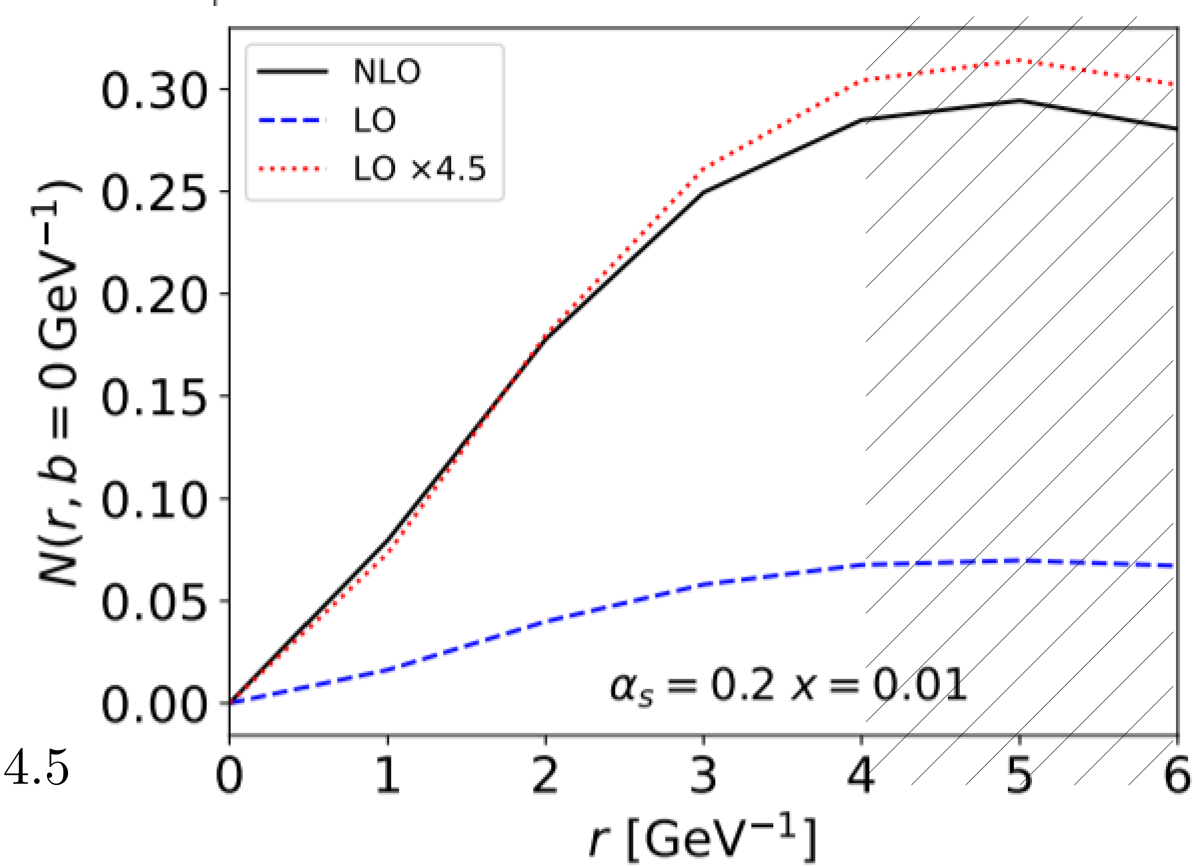
(m)

+ a bunch more

- I will spare you the explicit expressions for these ~40 diagrams (see A.D. + R. Paatelainen 2020, if you need them)
- But they have all been coded by H. Mäntysaari !
- So now we can look at a few initial preliminary plots !

The effect of adding the gluon:

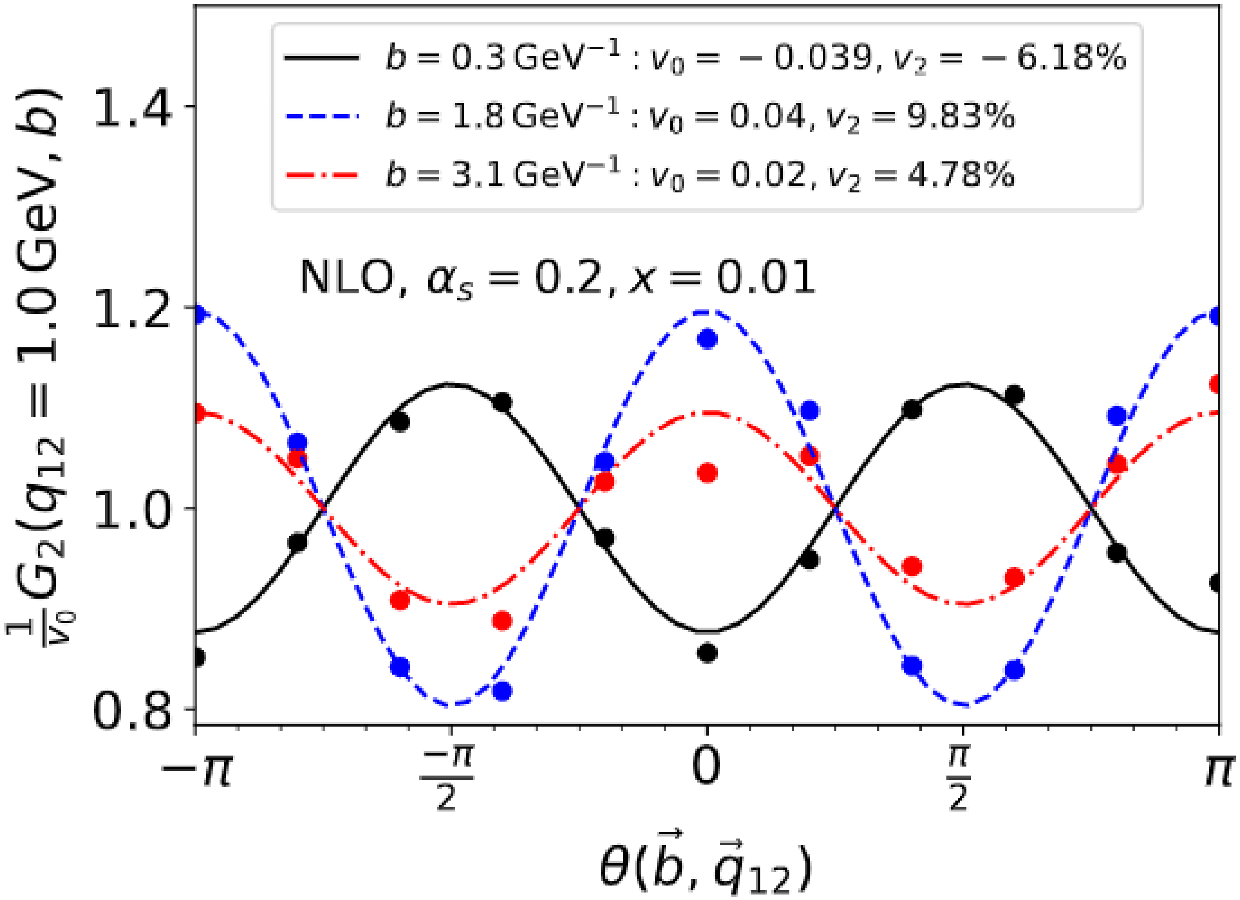




* modest effect at x=0.1
* huge effect at x = 0.01 !
  we see factor

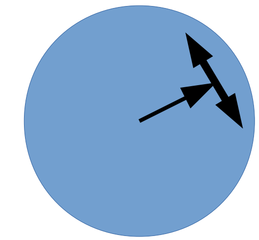
$$1 + 3\alpha_s \log \frac{x_0}{x} = 1 + 3 \cdot 0.2 \log 10 \simeq 4.5$$

Color charge correlator exhibits angular dependence:



* sign and magnitude of <cos 2θ> change drastically with b, q_{12}

Azimuthal anisotropy of $\mathcal{T}_{gg}(\vec{r},\vec{b})$

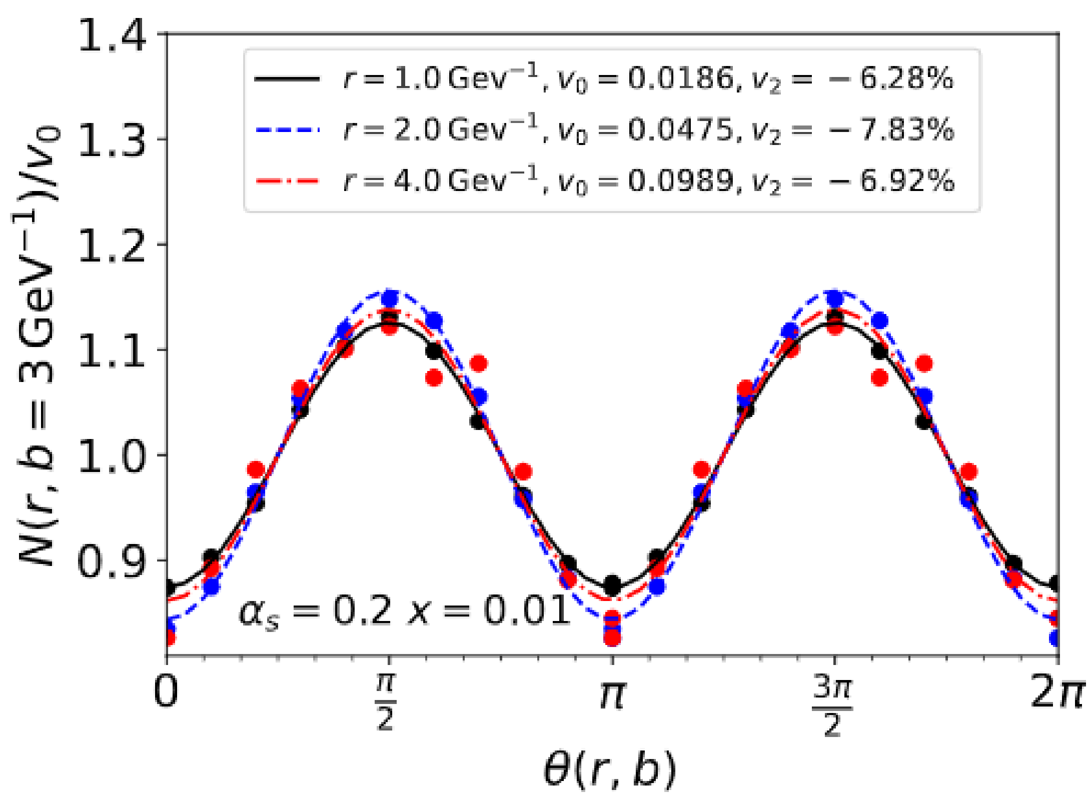


If the color charge correlator is simply proportional to the "proton shape function" $\langle \rho^a(\vec{x})\,\rho^b(\vec{y})\rangle \sim \delta^{ab}\,\mu^2(\vec{b})\,\delta(\vec{x}-\vec{y})$

then $\mathcal{T}_{gg}(\vec{r},\vec{b}) \sim 1 - \#(\vec{r}\cdot\vec{b})^2$
at small r, b.

e.g. A. Rezaeian & E. Iancu, 1702.03943
Kovner & Lublinsky, 1211.1928
E. Levin & A. Rezaeian, 1105.3275

- Recall that "cat's ears" diagram involves 2-body correlations and breaks <ρρ> ~ T_p(b)
- v2 from Tgg (=N) is negative
- in this plot, the *magnitude of v2 is not proportional to r²*



Summary

* *Quantum* color charge fluctuations in the proton at $x \geq 0.01$ are not Gaussian: $<\rho^3> \neq 0$, $<\rho^4> \neq <\rho^2> <\rho^2>$ etc
  $<\rho^2>$ very different from MV at small b (cat's ears !)

* color charge correlators $<\rho^2>$, $<\rho^3>$ etc related to n-body GTMDs / Wigners. Many-body diagrams dominant at high |t| or small b

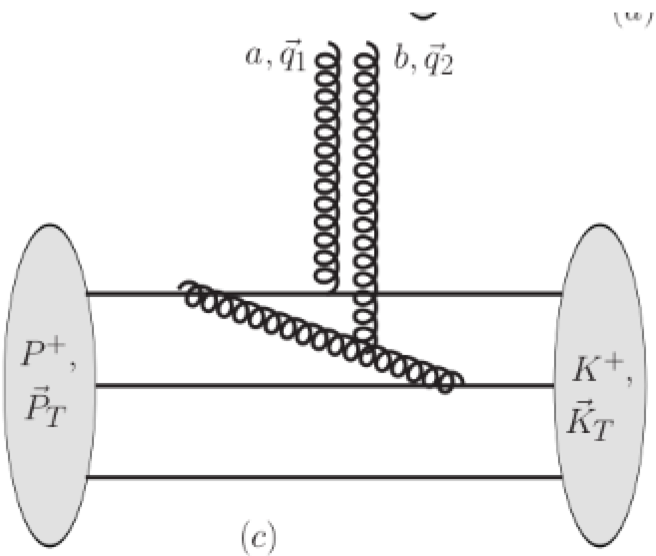
* Explicit relations to LFwf of the proton

* initial conditions for (NLO) BK incl. b, $\vec{r}\cdot\vec{b}$, x_0 dependence (and C-odd contribution from 3g exchange)

* Angular dependence of $\mathcal{T}_{gg}(\vec{r},\vec{b})$ appears to be quite different from models assuming $<\rho^2> \sim T_p(b)$

* To do list: |qqqg> corrections to 3g exchange amplitude, numerical results, fluctuations of proton configuration, ...

Backup Slides



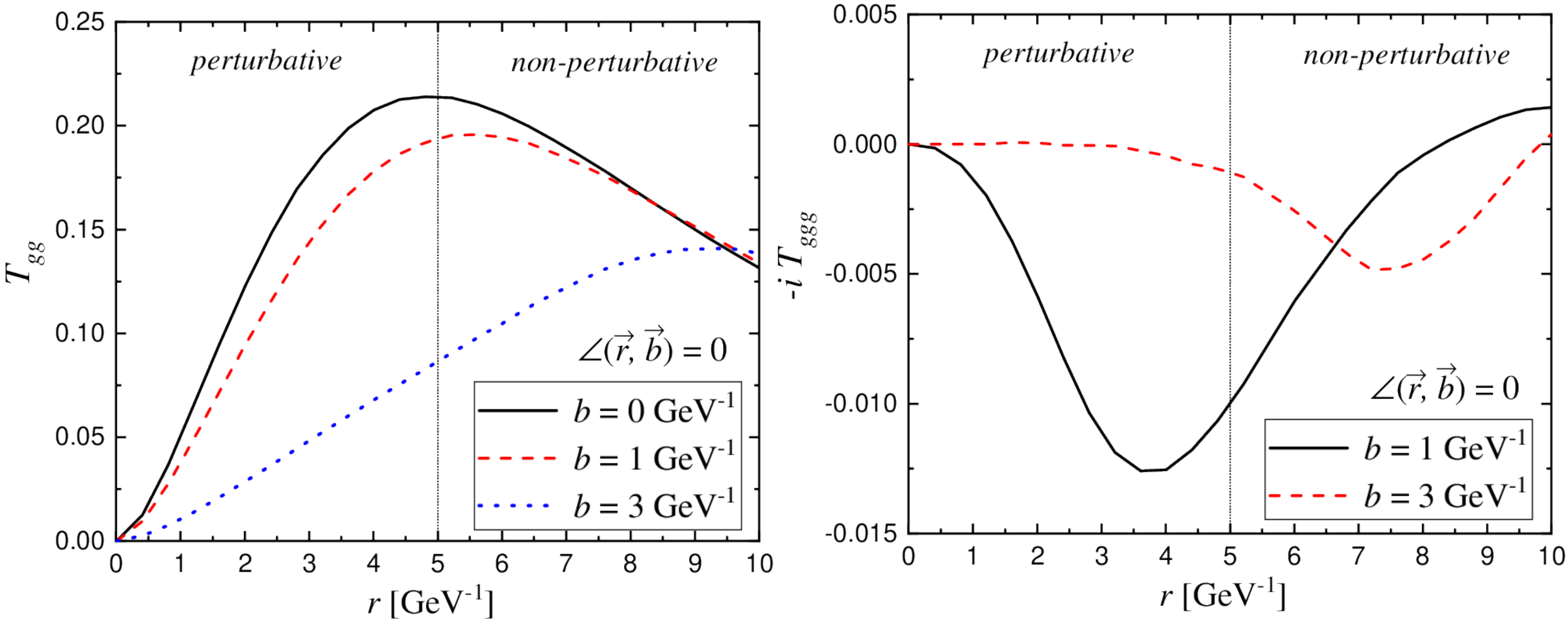
$$
\begin{aligned}
\text{fig. 3c} \; = \; & \frac{g^4}{12 \cdot 16\pi^3} \; \mathrm{tr}\, T^a T^b \int [\mathrm{d}x_i] \int [\mathrm{d}^2 k_i] \;\; \Psi_{qqq}(x_1, \vec{k}_1; x_2, \vec{k}_2; x_3, \vec{k}_3) \\
& \int\limits_{x}^{\min(x_1, 1-x_2)} \frac{\mathrm{d}x_g}{x_g} \left(1 - \frac{z_1 + z_2}{2} + \frac{z_1 z_2}{6}\right) \sqrt{\frac{x_1}{x_1 - x_g}} \sqrt{\frac{x_2}{x_2 + x_g}} \\
& \int \mathrm{d}^2 k_g \frac{z_1 \vec{p}_1 - \vec{k}_g}{\left(z_1 \vec{p}_1 - \vec{k}_g\right)^2} \cdot \frac{z_2 \vec{p}_2 - (1 - z_2)(\vec{k}_g - \vec{q}_2)}{\left(z_2 \vec{p}_2 - (1 - z_2)(\vec{k}_g - \vec{q}_2)\right)^2} \\
& \Psi^*_{qqq}(x_1 - x_g, \vec{k}_1 + x_1 \vec{q} - \vec{q}_1 - \vec{k}_g + x_g \vec{K}; x_2 + x_g, \vec{k}_2 + x_2 \vec{q} - \vec{q}_2 + \vec{k}_g - x_g \vec{K}; x_3, \vec{k}_3 + x_3 \vec{q})
\end{aligned}
$$

(see arXiv:2010.11245 for details and other diagrams)

LO Dipole scattering amplitude (2g vs. 3g singlet exchanges)

$$\mathcal{T}(\vec{r},\vec{b}) = \left\langle 1 - \frac{1}{N_c} \mathrm{tr}\, U\left(\vec{b} + \frac{1}{2}\vec{r}\right) U^\dagger\left(\vec{b} - \frac{1}{2}\vec{r}\right)\right\rangle$$

$$\mathcal{T}_{gg}(\vec{r},\vec{b}) = \mathrm{Re}\,\mathcal{T}(\vec{r},\vec{b}) \quad , \quad \mathcal{T}_{ggg}(\vec{r},\vec{b}) = \mathrm{Im}\,\mathcal{T}(\vec{r},\vec{b})$$



* Tggg(r,b) << Tgg(r,b)

* not because color charge fluct. are Gaussian, or because of extra α_s, or because of extra power of r;   but because Tggg is P-odd

Aside: <ρ^a ρ^b ρ^c> correlator (C odd part)

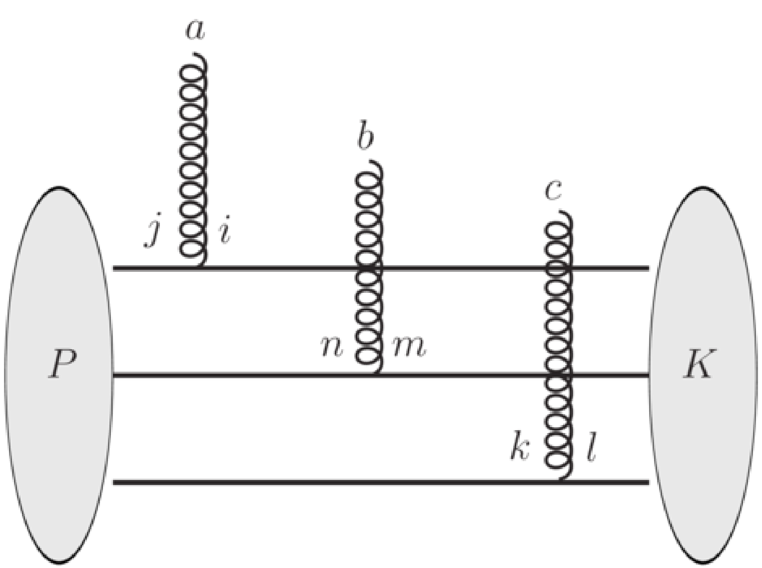
does not vanish (color charge fluct. <u>not Gaussian</u>) :

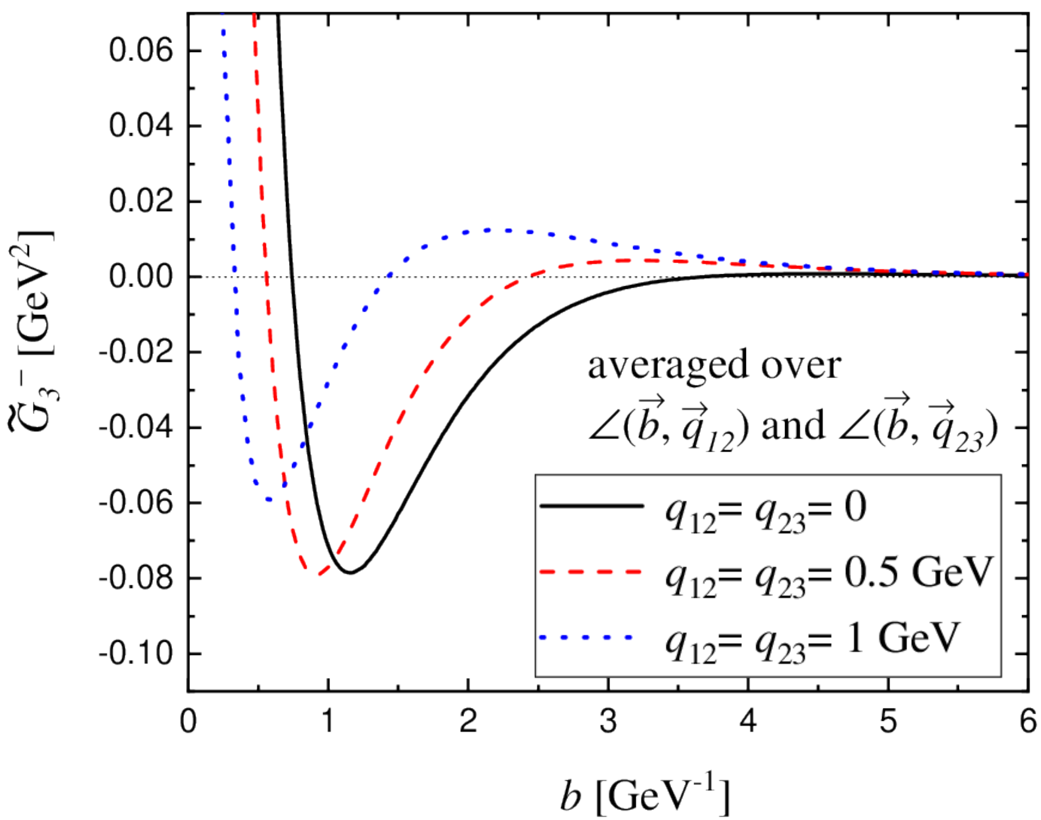
$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\right\rangle_{K_\perp}\Big|_{\mathcal{C}=-} \equiv \frac{g^3}{4}\,d^{abc}\,G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$

$$\begin{aligned} G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3) = & \int [dx_i][dp_i] \\ & \Big[\psi^*(\vec{p}_1+(1-x_1)\vec{K}_\perp,\vec{p}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\ & -\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_\perp,\vec{p}_2+\vec{q}_1+(1-x_2)\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\ & -\psi^*(\vec{p}_1+\vec{q}_2+(1-x_1)\vec{K}_\perp,\vec{p}_2-\vec{q}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\ & -\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-x_1\vec{K}_\perp,\vec{p}_2+\vec{q}_1+\vec{q}_2+(1-x_2)\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\ & +2\,\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_\perp,\vec{p}_2+\vec{q}_1+\vec{q}_2+(1-x_2)\vec{K}_\perp,\vec{p}_3-\vec{q}_2-x_3\vec{K}_\perp) \\ & \Big]\,\psi(\vec{p}_1,\vec{p}_2,\vec{p}_3) \end{aligned}$$

* 1-, 2- and 3-particle GTMDs,
  sum vanishes when either $q_i \longrightarrow 0$
  (Ward identities)

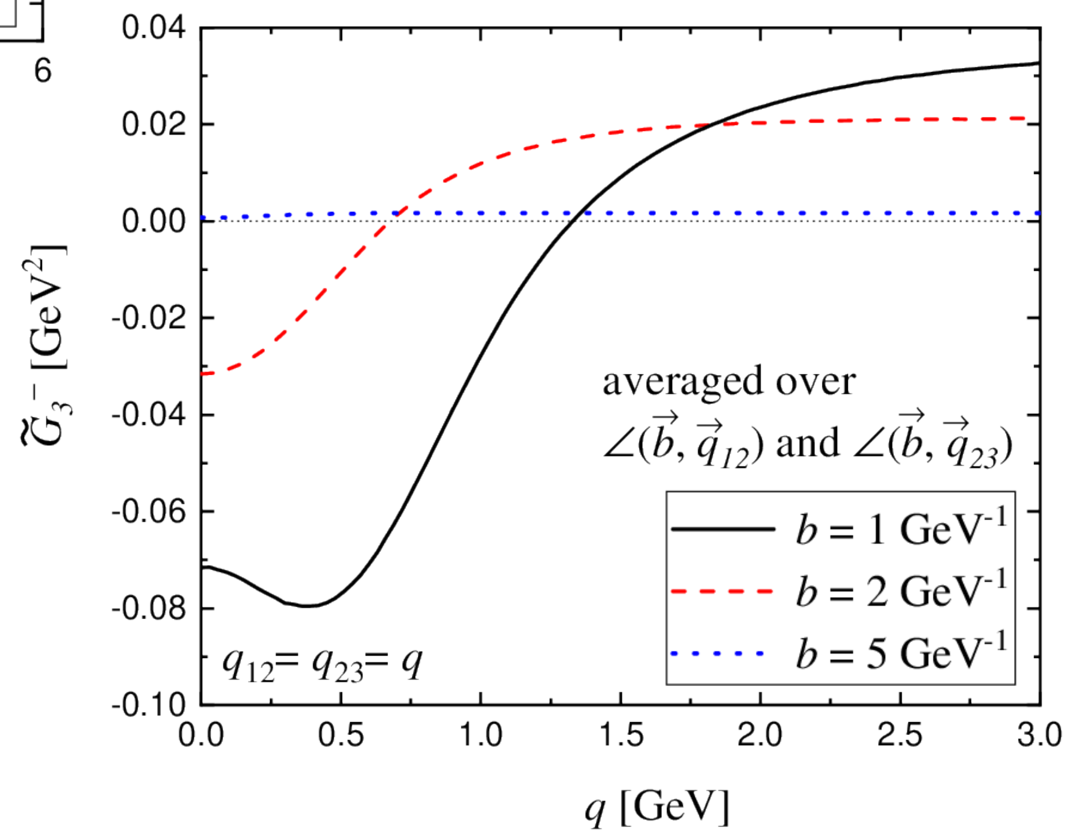
* "3-body" diagrams not (power-) suppressed
  when $\vec{q}_1 \sim \vec{q}_2 \sim \vec{q}_3 \sim -\vec{K}_T/3 \gg \Lambda_{\rm QCD}$
  but actually dominant !





- magnitude for generic b, q comparable to G_2
- diverges for $b \longrightarrow 0$ due to contribution from high K_T

$$\left(\int \frac{\mathrm{d}^2 K_T}{K_T^2} \; e^{-i\vec{b}\cdot\vec{K}_T} \right)$$



$<\rho^a\rho^b\rho^c>$ correlator (C even part)

$$\left\langle \rho^a(\vec{q}_1)\, \rho^b(\vec{q}_2)\, \rho^c(\vec{q}_3) \right\rangle_{K_\perp} \Big|_{\mathcal{C}=+} \quad \equiv \quad \frac{g^3}{4}\, i f^{abc}\, G_3^+(\vec{q}_1, \vec{q}_2, \vec{q}_3)$$

$$\begin{aligned}
G_3^+(\vec{q}_1, \vec{q}_2, \vec{q}_3) \quad &= \quad \int [dx_i] \int [d^2p_i] \\
& \Big[\psi^*(\vec{p}_1 - \vec{q}_1 - \vec{q}_2 - \vec{q}_3 - x_1\vec{K}_\perp, \vec{p}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\
& -\psi^*(\vec{p}_1 - \vec{q}_2 - \vec{q}_3 - x_1\vec{K}_\perp, \vec{p}_2 - \vec{q}_1 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\
& +\psi^*(\vec{p}_1 - \vec{q}_1 - \vec{q}_3 - x_1\vec{K}_\perp, \vec{p}_2 - \vec{q}_2 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\
& -\psi^*(\vec{p}_1 - \vec{q}_1 - \vec{q}_2 - x_1\vec{K}_\perp, \vec{p}_2 - \vec{q}_3 - x_2\vec{K}_\perp, \vec{p}_3 - x_3\vec{K}_\perp) \\
& \Big]\, \psi(\vec{p}_1, \vec{p}_2, \vec{p}_3) \\
&= \quad G_2(\vec{q}_1 + \vec{q}_2, \vec{q}_3) - G_2(\vec{q}_1 + \vec{q}_3, \vec{q}_2) + G_2(\vec{q}_1, \vec{q}_2 + \vec{q}_3)
\end{aligned}$$

(like Reggeized 2-gluon exchange)

e.g. C. Ewerz, hep-ph/0103260, hep-ph/0306137

G_3^+ vanishes when $q_1 \longrightarrow 0$ or $q_3 \longrightarrow 0$ but not for $q_2 \longrightarrow 0$

$<\rho^a\rho^b\rho^c\rho^d>$ correlator

$$\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\,\rho^d(\vec{q}_4)\,\rangle = g^4 \int [dx_i] \int [d^2p_i]\,\psi(\vec{p}_1,\vec{p}_2,\vec{p}_3)$$

$$\Big\{\mathrm{tr}\, t^at^bt^ct^d\;\; \psi^*(\vec{p}_1+(1-x_1)\vec{K}_T,\vec{p}_2-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$+\left(\mathrm{tr}\, t^at^b\,\mathrm{tr}\, t^ct^d-\mathrm{tr}\, t^at^bt^ct^d\right)\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-x_1\vec{K}_T,\vec{p}_2-\vec{q}_3-\vec{q}_4-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$+\left(\mathrm{tr}\, t^at^c\,\mathrm{tr}\, t^bt^d-\mathrm{tr}\, t^at^ct^bt^d\right)\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_3-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-\vec{q}_4-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$+\left(\mathrm{tr}\, t^at^d\,\mathrm{tr}\, t^bt^c-\mathrm{tr}\, t^at^dt^bt^c\right)\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_4-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-\vec{q}_3-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$-\mathrm{tr}\, t^at^bt^ct^d\;\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-\vec{q}_3-x_1\vec{K}_T,\vec{p}_2-\vec{q}_4-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$-\mathrm{tr}\, t^at^bt^ct^d\;\;\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-\vec{q}_3-\vec{q}_4-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$-\mathrm{tr}\, t^at^bt^dt^c\;\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-\vec{q}_4-x_1\vec{K}_T,\vec{p}_2-\vec{q}_3-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$-\mathrm{tr}\, t^at^ct^dt^b\;\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_3-\vec{q}_4-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-x_2\vec{K}_T,\vec{p}_3-x_3\vec{K}_T)$$

$$+\left(\mathrm{tr}\, t^at^bt^ct^d+\mathrm{tr}\, t^at^bt^dt^c-\mathrm{tr}\, t^at^b\,\mathrm{tr}\, t^ct^d\right)\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-x_1\vec{K}_T,\vec{p}_2-\vec{q}_3-x_2\vec{K}_T,\vec{p}_3-\vec{q}_4-x_3\vec{K}_T)$$

$$+\left(\mathrm{tr}\, t^at^ct^bt^d+\mathrm{tr}\, t^at^ct^dt^b-\mathrm{tr}\, t^at^c\,\mathrm{tr}\, t^bt^d\right)\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_3-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-x_2\vec{K}_T,\vec{p}_3-\vec{q}_4-x_3\vec{K}_T)$$

$$+\left(\mathrm{tr}\, t^at^dt^bt^c+\mathrm{tr}\, t^at^dt^ct^b-\mathrm{tr}\, t^at^d\,\mathrm{tr}\, t^bt^c\right)\;\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_4-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-x_2\vec{K}_T,\vec{p}_3-\vec{q}_3-x_3\vec{K}_T)$$

$$+\left(\mathrm{tr}\, t^at^bt^ct^d+\mathrm{tr}\, t^at^dt^bt^c-\mathrm{tr}\, t^at^d\,\mathrm{tr}\, t^bt^c\right)\;\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-\vec{q}_3-x_2\vec{K}_T,\vec{p}_3-\vec{q}_4-x_3\vec{K}_T)$$

$$+\left(t^at^bt^ct^d+t^at^ct^dt^b-t^at^b\,t^ct^d\right)\;\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-x_2\vec{K}_T,\vec{p}_3-\vec{q}_3-\vec{q}_4-x_3\vec{K}_T)$$

$$+\left(t^at^bt^dt^c+t^at^ct^bt^d-t^at^c\,t^bt^d\right)\;\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_T,\vec{p}_2-\vec{q}_2-\vec{q}_4-x_2\vec{K}_T,\vec{p}_3-\vec{q}_3-x_3\vec{K}_T)\Big\}\;,$$

where $\vec{K}_T=-(\vec{q}_1+\vec{q}_2+\vec{q}_3+\vec{q}_4)$

Note: $\neq \langle\rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\rangle\,\langle\rho^c(\vec{q}_3)\,\rho^d(\vec{q}_4)\rangle + \text{perm.}$

Model LFwf for the proton (Brodsky & Schlumpf, PLB 329, 1994)

$$\psi_{\rm H.O.}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) = N_{\rm H.O.}\exp(-\mathcal{M}^2/2\beta^2)\ ,$$
$$\psi_{\rm Power}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) = N_{\rm Power}(1+\mathcal{M}^2/\beta^2)^{-p}\ .$$

$$\mathcal{M}^2 = \sum_{i=1}^{3}\frac{\vec{k}_{\perp i}^2 + m^2}{x_i}$$

m = 0.26 GeV, β = 0.55 for H.O. wf

m = 0.263, β = 0.607, p = 3.5 for PWR wf

With these parameters they fit:

- proton radius $R^2 = -6\,\frac{dF_1(Q^2)}{dQ^2}\Big|_{Q^2=0} = (0.76\ \mathrm{fm})^2$
- proton / neutron magnetic moments $1 + F_2(Q^2\to 0) = 2.81\ /\ -1.66$
- axial vector coupling g_A = 1.25

Dipole scattering amplitude

two gluon exchange:

$$\mathcal{T}_{gg}(\vec{r},\vec{K}_\perp) = -\frac{g^4 C_F}{2}\int_q \frac{1}{q^2\,(\vec{q}+\vec{K}_\perp)^2} \left(e^{i\vec{r}\cdot(\vec{q}+\vec{K}_T/2)} - \cos\left(\frac{\vec{r}\cdot\vec{K}_T}{2}\right)\right) G_2(\vec{q},-\vec{q}-\vec{K}_T)$$

even under $\vec{r}\to-\vec{r}$ or $\vec{K}_T\to-\vec{K}_T$

three gluon exchange:

$$\mathcal{T}_{ggg}(\vec{r},\vec{K}_\perp) = g^6\frac{5}{18}\int_{q_1}\int_{q_2}\frac{1}{q_1^2}\frac{1}{q_2^2}\frac{1}{(\vec{q}_1+\vec{q}_2+\vec{K}_\perp)^2} \times \left[\sin\left(\vec{r}\cdot(\vec{q}_1+\frac{\vec{K}_T}{2})\right) - \frac{1}{3}\sin\left(\frac{\vec{r}}{2}\cdot\vec{K}_T\right)\right] \times G_3^-(q_1,q_2,-K_\perp-q_1-q_2)$$

odd under $\vec{r}\to-\vec{r}$ or $\vec{K}_T\to-\vec{K}_T$