Heavy Quark Radiation in Moliere Theory.

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The problem of the energy loss of heavy quark.

The problem was first raised

In Dokshitzer and Kharzeev Phys.Lett.B 519 (2001) 199-206, who argued that due to the dead cone effect

$$\omega \frac{dI^{\rm vac}}{d\omega dk_t^2} \sim \frac{\alpha_s C_F}{\pi^2} \frac{k_t^2}{(k_t^2 + \theta^2\omega^2)^2},$$

when the BDMPSZ maximum radiation angle falls into the dead cone region, the BDMPSZ radiation is strongly suppressed.

However experimental data shows that up to relatively low energies there is the same energy loss for charm and even for bottom quark and for bottom and charm jets.

The major progress was achieved in Armesto,Salgado, Wiedemann (2004) who showed that account of the phase space limitations on the BDMPSZ soft leads to the approximately the same energy loss of light and heavy quark up to the masses corresponding to dead cone angle $\theta = m/E = 0.05$.

On the other hand, an alternative approach to energy loss is based on opacity expansion Gyulassi,Levai,Vitev (2001), Wiedemann (2001). In particular , the N=1 opacity expansion leads to a compact analytical expression for heavy quark radiation. The radiation of heavy quark was considered for N=1 GLV case in Armesto, Salgado, Wiedemann (2004), and slightly after by Aurenche and Zakharov (2009). Such approach does not give a full treatment of interference efects as BDMPSZ, but explicitly takes into account large Coulomb corrections to BDMPSZ results. Quite remarkably this approach leads to similar result: for the masses corresponding to the dead cone angle θ =m/E up to $\theta = 0.05$ the energy loss remains independent of mass.

More recently another approach was developed aimed to include both the BDMPSZ interference and the large Coulombic Logarithmic effects Was developed in Mehtar-Tani (2019),Mehtar-Tani and Tywoniuk (2020). This approach is a generalization to the nonabelian case of the so called Moliere theory, known from the abelian (QED) LPM effect The basic idea is to consider the Coulombic interactions leading to large Coulombic Logarithms as a perturbation over LPM interference.

The aim of the present talk is to study the propagation and the energy loss of the heavy quark In the Moliere theory framework.
1)We shall see that large Coulomb logarithms play significant role numerically leading to the energy loss exceeding the BDMPSZ result but generally smaller than N=1 GLV.
2) If we do not take proper phase constraints we see that the energy loss is weakly dependent on the quark mass up to θ=0.05.
This result is quite similar to N=1 GLV estimate. The numerical differences due to Coulomb Logs are the largest for small frequencies,
3) For angular distributions we see that Coulombic gluons lead to rather large enhancement of the BDMSZ distributions. In particular for small angle radiation. Nevertheless generally The Coulomb corrections are rather smooth as a function of transverse momentum and the distribution approximately repeats the form of BDMPSZ approximation result due to Armesto, Salgado and Wiedemasnn, with the difference of a rather large increase for very small transverse momenta, inside the dead cone.

4)Our results show that the Coulombic logs give a large numerical contribution to the Quenching coefficients and energy loss as a function of frequency, also decreasing the quark mass dependence, nevertheless the weak dependence on the heavy quark mass comes from the proper taking into account of both the transverse and longitudinal phase constraints. Leading to rather weak dependence on the heavy quark mass.

Plan. of the talk :

1.Basic formalism of the Molier theory

.2. Energy loss and gluon radiation for heavy quark without phase constraints.

3.Angular/Transverse momentum distributions

4. Energy loss and radiation of heavy quark with proper longitudinal and transverse phase constraints

Moliere Theory

For processes that are dominated by large momentum transfer it is enough to take into account only the first terms in the Taylor expansion of Coulomb screened interaction potential $V(\rho)$. The first approximation corresponds to the quadratic term in the expansion. And is called HO or Harmonic Oscillator approximation. In this approximation the effective potential is given by

$$V(\rho) = \frac{1}{4}\hat{q}_{\rm eff}\rho^2. \qquad \hat{q}_{\rm eff} = \hat{q}\log(\frac{Q^2}{\mu^2}),$$

The HO approximation effectively describes the LPM bremsstralung.

More precise treatment of the energy loss includes also large Coulomb Logarithms and is called in the case of Abelian LPM the Moliere theory.

In the QCD framework the inclusion of Coulombic logarithms can be made using the perturbation theory (Mehtar-Tanni 2020,Mehtar-Tani,Tywonniuk 2020). Instead of a conventional GLV opacity expansion, we consider the Coulombic logarithms as a perturbation around the oscillator potential.

$$V(\rho) = V_{HO}(\rho) + V_{pert}(\rho), V_{HO}(\rho) = \frac{\hat{q}\log(Q^2/\mu^2)}{4}\rho^2, V_{\rm pert}(\rho) = \frac{\hat{q}}{4}\log(\frac{1}{Q^2\rho^2}),$$

$$Q \sim \sqrt{\hat{q}\omega} \text{ in the HO approximation}$$

$$K(\vec{y},t_1;\vec{x},t) = K_{HO}(\vec{y},t_1;\vec{x},t) - \int d^2z\int_t^{t_1} dsK_{HO}(\vec{y},t_1;\vec{z},s)V_{pert}(z)K_{HO}(\vec{z},s;\vec{x},t)$$

$$K_{HO}(\vec{y},t_1;\vec{x},t) = \frac{i\omega\Omega}{2\pi\sinh\Omega(t_1-t)}\exp(\frac{i\omega\Omega}{2}\{\coth\Omega(t_1-t)(\vec{x}^2+\vec{y}^2) - \frac{2\vec{x}\vec{y}}{\sinh\Omega(t_1-t)}\})\exp(-i\theta^2\omega(t_1-t)/2),$$

$$\Omega = \frac{(1+i)}{2}\sqrt{\frac{\hat{q}}{\omega}}$$

ò

Basic formalism:

$$\omega\frac{dI}{d\omega d^2k} = \frac{C_F\alpha_s}{(2\pi)^2\omega^2}2Re\int d^2y\int_0^\infty dt_1\int_0^{t_1} dte^{-i\vec{k}_t\vec{y}}$$

$$\times\ e^{-\int_{t_1}^\infty dsn(s)V(\vec{y}(s))}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y},t_1,\vec{x},t)-K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}.$$

Here we assume the brick like model for QGP feelig the brick of the finite length L. K is the heavy quark propagator, And the substraction of free part is to ensure zero in the vacuum, i.e. we consider only medium induced emissions. V is an effective scattering potential in 2 dimensions.

$$V(\rho) = \frac{\hat{q}}{4N_c}(1-\mu\rho K_1(\mu\rho) = \frac{\hat{q}\rho^2}{4N_c}(\log(\frac{4}{\mu^2\rho^2})+1-2\gamma_E),$$

$$\hat{q} = 4\pi\alpha_s^2N_cn.$$

Is the bare quenching coefficient. The mass parameter $\boldsymbol{\mu}$ *is approximately equal to Debye mass The potential V is just the Fourier image of screened 2D Coulomb-like interaction.*

$$\frac{d\sigma(\vec{q}_t)}{d^2q_t} = \frac{4\pi\alpha_s m_D^2T}{(q_t^2+\mu^2)^2} \equiv \frac{g^4n}{(q_t^2+\mu^2)^2},$$

$$m_D \sim 4\pi\alpha_sT^2(1+N_f/6) = \frac{3}{2}g^2T^6$$

The calculations are done in Gyulassi-Wang model

$$n = \frac{3}{2}T^3,$$

$$n(s) = U(L-s)U(s)$$

The dynamics of heavy quark: *The expressions for energy loss and the angular distributions contain two characteristic oscillating scales: the one is the LPM length* $\sqrt{\omega}/q$

Another length is heavy quark one $2/\omega\theta^2$

The dynamics of the process is determined by the smallest of coherence length. For small frequencies the dynamics is determined by the LPM length which is the smallest of the two, while for large frequencies, for the radiation inside the dead cone, the relevant coherence length is the heavy quark one, which is actually independent from the media properties (the quenching coefficient). The corresponding integrals are cut by the heavy quark exponent, and the radiation into the dead cone becomes vacuum like. For the effective momentum acquired at the coherence length scale we
Now can have an interpolation formula:

$$Q^2 = \sqrt{\omega \hat{q}_{\text{eff}}}\, U(-\omega + \omega_{DC}) + \theta^2 \omega^2 U(\omega - \omega_{DC}),$$

$U(x)$ is a unit step function: $U(x) = 1$ if $x \geq 0$, and $U(x) = 0$ if $x \leq 0$.

$$\omega_{DC} = \sqrt[3]{qE^4/m^4}$$

Heavy quark energy loss without phase constraints.

In this case the energy loss can be easily calculated . The calculation, both for the leading (HO) term and for Coulombic corrections proceeds as a sum of bulk and boundary terms. For the leading term, i.e. HO approximation new have:

$$\omega\frac{dI^{\text{HO Bulk}}}{d\omega} = \frac{-2\alpha_s C_F}{\pi} 2Re\int_0^L dt_1 \int_0^{t_1} d\tau\left(\frac{\Omega^2}{\sinh(\Omega\tau)^2} - \frac{1}{\tau^2}\right)Exp(-i\tau\theta^2\omega/2)$$

$$= \frac{-2\alpha_s C_F}{\pi}\int_0^L ds(L-s)(\Omega^2/\sinh(\Omega s)^2 - 1/s^2)\exp(-is\theta^2\omega/2)$$

$$\omega\frac{dI^{\text{HO Boundary}}}{d\omega} = 2Re\frac{\alpha_s C_F}{\pi}\int_0^{l} - \left(\frac{1}{s} - \frac{i\theta^2\omega}{2}\exp(i\theta^2\omega s/2)\Gamma(0, i\theta^2\omega s/2)\right)$$

$$\times \frac{\Gamma(0, \frac{i\theta^2\omega\tanh(\Omega s)}{2\Omega})}{\cosh(\Omega s)^2}$$

$$\int_0^L dt_1 \int_0^{t_1} ds f(s) = \int_0^L (L-s)f(s)ds$$

For the Coulombic corrections we obtain

$$\omega\frac{dI^{\text{Bulk Coulomb}}}{d\omega} = Re\int_0^L dx(L-x)2i\frac{\alpha_s C_F\hat{q}}{\omega\pi}\frac{\exp(i\theta^2\omega(-i\theta*\omega x/2)}{\sinh\Omega(x)^3}$$
$$\times\ \cosh(\Omega x)(-((-2+A+Log(4)-2\log(\Omega*\omega/Q^2))$$
$$+\ 2*(\Omega x+\log(1-Exp(-2\Omega x)))-\log(2)))\tanh[\Omega x)$$
$$-\ (-\pi^2/6-(2+A)\Omega x-2Li_2(2,\exp(-2\Omega x))-Li_2(2,1)-(-i\pi+2\Omega x)^2/2$$
$$-\ \pi^2/3+2\Omega x(-i*\pi+\log((1-\exp(-2\Omega x))+2\Omega x+\log(-\Omega\omega/Q^2)), \quad (4$$

vhere

$$A=-i\pi+3-2\gamma_E. \quad (4$$

Here Li_2 is the dilogarithm (Spence) function [26]:

$$Li_2(z)=-\int_0^z\frac{\log(1-u)}{u}. \quad (4$$

$$
\begin{aligned}
\omega\frac{dI^{\text{Coulomb Boundary}}}{d\omega} &= \frac{i\alpha_s C_F\hat{q}}{2\pi\omega}\exp i\theta^2\omega(t-t_1) \\
&\times\ \cosh(\Omega(L-t))(2\log(((1-\exp(-2\Omega(L-t)))+\Omega(t_1-L)) \\
&\times\ (1+\exp(-2\Omega(L-t))))/(2\Omega(t_1-L))) \\
&+\ 2\Omega(L-t))\Omega(L-t_1)-((-2+A+\log(4)-2\log(\frac{\Omega\omega}{Q^2}))+2(\Omega(L-t) \\
&+\ \log((1-\exp(-2\Omega(L-t))))-\log(2))))\tanh(\Omega(L-t)) \\
&+\ (-\pi^2/6-(2+A)\Omega(L-t)-Li_2(\frac{\exp(-2\Omega(L-t))(1+\Omega(L-t_1))}{1-\Omega(L-t)} \\
&-\ Li_2(\exp(-2\Omega(L-t))-Li_2(-1+\frac{2}{1+\Omega(L-t_1))})-(-i\pi+2\Omega(L-t))^2/2
\end{aligned}
$$

$1+\Omega(L-t_1)$

$$
\begin{aligned}
&+\ 2\Omega(L-t)(\log((\exp(-2\Omega(L-t)+\frac{-1+\Omega(L-t_1)}{1+\Omega(L-t_1)})+2\Omega(L-t)+\log(-\frac{\Omega\omega}{Q^2}))) \\
&\times\ (-1+\Omega(L-t_1)\tanh(\Omega(L-t))))/(\Omega(t_1-L)\cosh(\Omega(L-t))+\sinh\Omega(L-t))^3.
\end{aligned}
$$

In the case of the zero heavy quark mass our results coincide with the ones obtained earliar in Mehtar-Tanni (2020), Mehtar-Tani and Tywoniuk (2020)

$$\omega \frac{dI^{\text{Coulomb}}}{d\omega} = \int_0^L ds \frac{1}{k(s)} \log(k(s) + \gamma),$$

$$k(s) = \frac{i\omega\Omega}{2} (\coth(\Omega(s) + \tanh(\Omega(L - s))$$

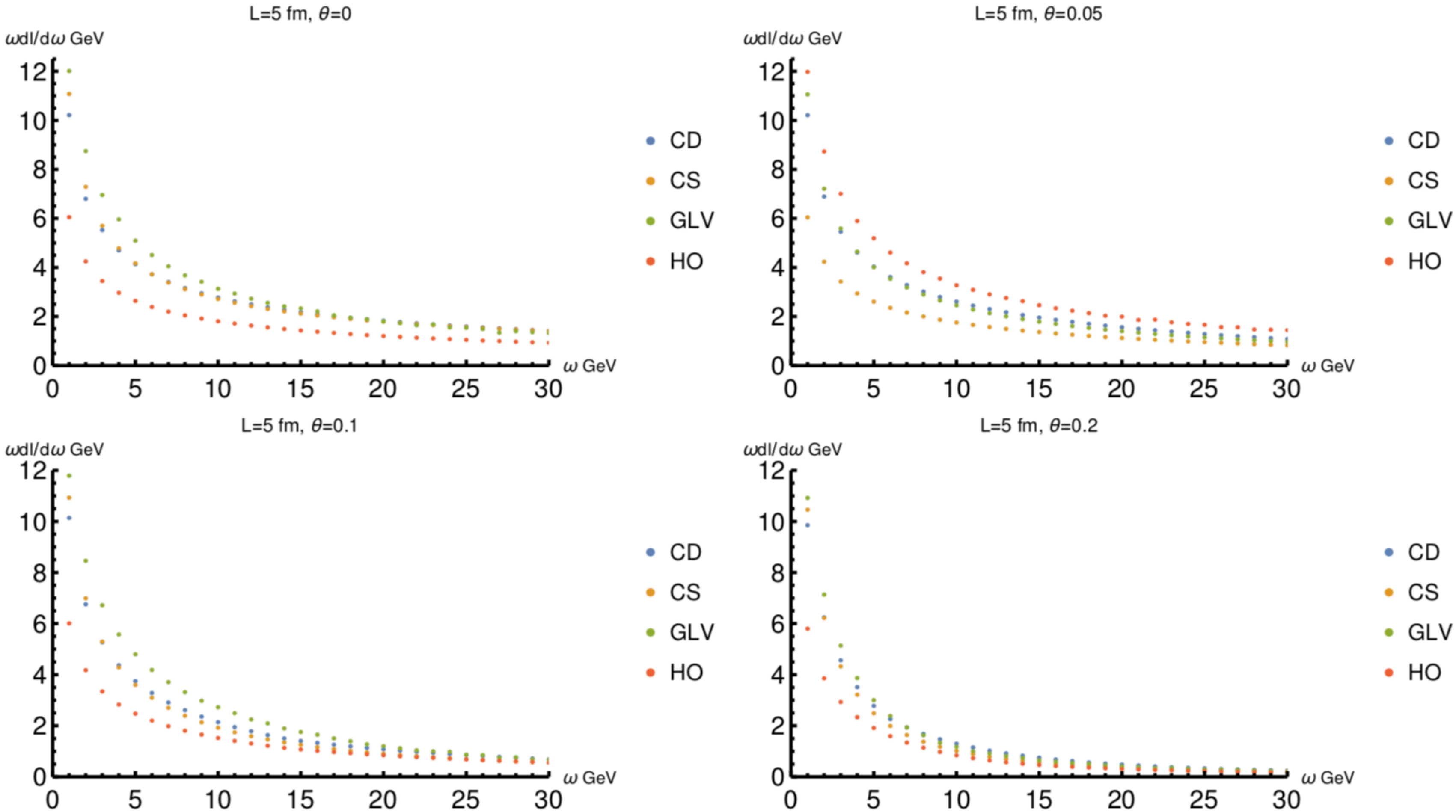


FIG. 2: . *The energy loss in the leading order in* α_s *for L=5 fm for different values of* θ *as a function of the radiated gluon energy* ω*, divided by* $\alpha_s C_F$ *for different parameterisations of* $Q(\hat{q}, \omega, \theta)$ *for HO and total (HO+Coulomb) contributions: CD, CS are the total (HO+Coulomb) energy losses for typical momenta Q given by Eqs. 19,20 respectively.HO refers to HO approximation with Q given by Eq. 56, and GLV refers to N=1 GLV expression with Q independent* $\hat{q}$.

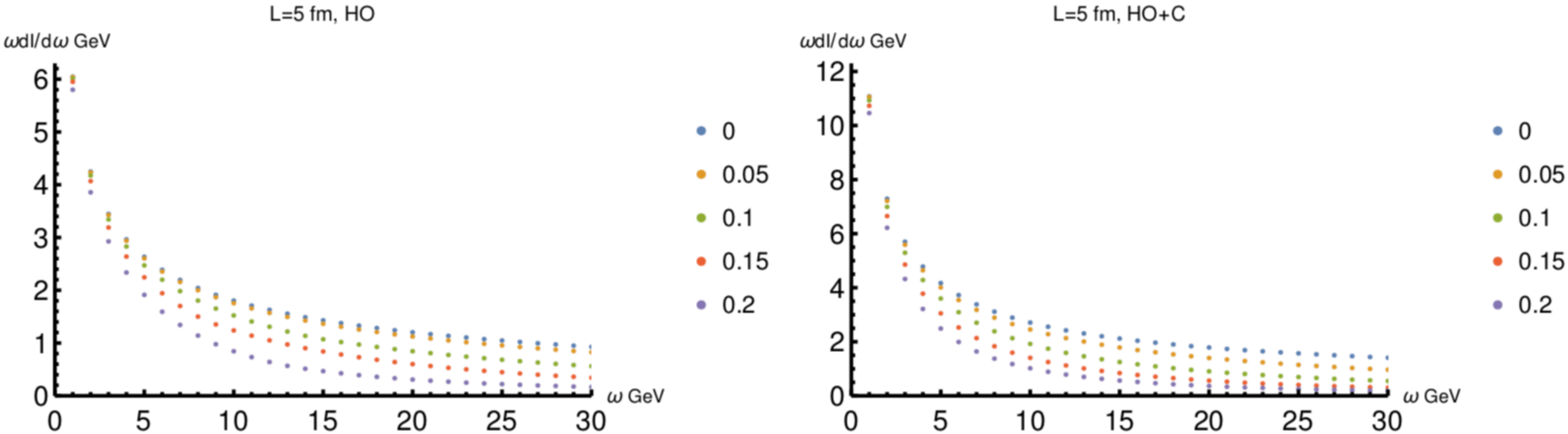


FIG. 4: . *The energy loss in the leading order in* α_s *for L=5 fm for different values of* θ *as a function of the radiated gluon energy* ω, *divided by* $\alpha_s C_F$ *right-total energy loss in Moller theory, left-HO approximation.We use Eq.* 20 *for* Q.

We see that first HO+Coulomb does not depend on the heavy qaurk mass up to m/E less than 0.05, as it is inn N=1 GLV. Second numerically the results for H)+Coulomb are quite close to N=1 GLV especially for large frequencies beyond dead cone angle, the agreement improves with the decrease of L

Angular Distributions

In this case we use the same strategy as before: we repeat the calculation of the HO bulk and boundary contributions as it was done in ASW (2004) and then calculate analytically the bulk and boundary coulombic corrections. Essentially we are able to calculate analytically the inntegrals over transverse coordinates, while time integrals must be taken numerically. We obtain

$$\omega\frac{dI^{HO\ \ Bulk}}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y\int_0^L dt_1\int_0^{t_1} dt e^{-i\vec{k}_t\vec{y}}$$

$$\times\ e^{-1/4\hat{q}(L-t_1)y^2}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y},t_1,\vec{x},t)-K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}.,$$

$$\omega\frac{dI^{\rm HO\ \ Bulk}}{d\omega d^2k_t} = -2{\rm Re}\frac{\alpha_s C_F\Omega^2}{\pi^2R^2\sinh\Omega(t_1-t)^2}(q(L-t_!)-\frac{2i\omega\Omega\cosh\Omega(t_1-t)}{R})$$

$$\times\ \exp(i\theta^2\omega(t-t_1)/2)\exp(-k_t^2/R),$$

$$R = q(L-t_1)-2i\omega\Omega\coth\Omega(t_1-t),$$

$$\omega\frac{dI^{\rm HO\ \ boundary}}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y\int_L^\infty dt_1\int_0^L dt e^{-i\vec{k}_t\vec{y}}$$

$$\times\ \partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y},t_1,\vec{x},t)-K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0},$$

$$K(\vec{y},t_1;\vec{x},t) = \int d^2z K_0(\vec{y},t_1;\vec{z},L)K_{HO}(\vec{z},L;\vec{x},t).$$

$$\omega\frac{dI^{\text{HO boundary}}}{d\omega d^2k_t} = \frac{-i\alpha_s C_F k_t^2}{(k_t^2+\theta^2\omega^2)(2\pi)^2\omega}\frac{\exp(\frac{-ik_t^2\tanh\Omega(L-t)}{2\omega\Omega})\exp(i\theta^2\omega(t-L)/2}{\cosh\Omega(L-t)^2}$$

Coulombic corrections: there are two sources of corrections: one correction comes from the expansion of the exponent the first correction exists only for bulk term, while the corrections due to propagator are present in both bulk and boundary contributions.

$$\omega\frac{dI}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y\int_0^\infty dt_1\int_0^{t_1} dte^{-i\vec{k}_t\vec{y}}$$

$$K = K_{HO} + K_{HO}V_{pert}K_{HO}$$

$$\times\; e^{-\int_{t_1}^\infty dsn(s)(V_{HO}+V_{pert})(\vec{y}(s))}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y},t_1,\vec{x},t)-K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}.$$

Exponent expansion

$$\omega\frac{dI^{\text{Coulomb one}}}{d\omega d^2k} = -\frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y\int_0^L dt_1\int_0^{t_1} dte^{-i\vec{k}_t\vec{y}}$$

$$\times\; (L-t_1)V_{pert}(\vec{y})\partial_{\vec{x}}\partial_{\vec{y}}(K_{HO}(\vec{y},t_1,\vec{x},t)-K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}.$$

$$\omega\frac{dI^{\text{Coulomb one}}}{d\omega d^2k_t} = \frac{\hat{q}}{4}2Re\frac{\alpha_s C_F\Omega^2}{(2\pi)^2}\int_0^L dt_1\int_0^{t_1} dt(2F_2(R/4,k_t,Q)+i\omega\Omega\coth(\Omega(t_1-t))F_3(R/4,k_t,Q))$$

$$R = = \hat{q}(L-t_1) - 2i\omega\Omega\coth\Omega(t_1-t)$$

$$F_2(p,c,Q) \;=\; \int_0^\infty x^3 \log(x^2Q^2) J_0(cx) \exp(-px^2)$$

$$F_3(p,c,Q) \;=\; \int_0^\infty x^5 \log(x^2Q^2) J_0(cx) \exp(-px^2).$$

$$\omega \frac{dI^{\text{Coulomb bulk}}}{d\omega d^2k} \;=\; \frac{\alpha_s C_F}{(2\pi)^2\omega^2} \int d^2z \int d^2u \int_0^L dt_1 \int_0^{t_1} dt \int_t^{t_1} ds \frac{e^{-\hat{q}(L-t_1)u^2/4 - i\vec{k}_t\vec{u}}}{\sinh\Omega(t_1-s)\sinh\Omega(s-t)}$$

$$\times \;\; \partial_{\vec{y}}\partial_{\vec{u}} K_{HO}(u,t_1;\vec{z},s)(\frac{\hat{q}}{4} z^2 \log(1/z^2Q^2)) K_{HO}(\vec{z},s;t,\vec{y}=0)$$

$$\omega \frac{dI^{\text{Coulomb bulk}}}{d\omega d^2k_t} \;=\; \alpha_s C_F \int_0^L dt_1 \int_0^{t_1} dt \int_t^{t_1} ds \frac{q}{4} \frac{i\omega^2\Omega^4}{2\pi^2} \frac{\exp(-k_t^2/R(t_1,s))}{\sinh\Omega(t_1-s)^2 \sinh\Omega(s-t)^2}$$

$$\times \;\; (\hat{q}(L-t_1)F_3(p,c,Q) - 2k_t \cosh\Omega(t_1-s) F_4(p,c,Q)) \exp(i\theta^2\omega(t-t_1)/2)$$

$$p \;=\; -\frac{\omega\Omega\sinh\Omega(t_1-t)}{2\sinh\Omega(t_1-s)\sinh\Omega(s-t)} \frac{R(t_1,t)}{R(t_1,s)}$$

$$c \;=\; k_t \frac{2i\omega\Omega}{R(t_1,s)\sinh\Omega(t_1-s)}$$

$$F_4(p,c,Q) = \int_0^\infty dz z^4 J_1(cz) \exp(-pz^2) \log(z^2Q^2)$$

$$\omega\frac{dI^{\text{Coulomb boundary}}}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}\int d^2z\int d^2r\int d^2u\int_L^\infty dt_1\int_0^L dt\int_t^L ds\frac{\exp(-i\vec{k}_t\vec{u})}{\sinh\Omega(t_1-s)\sinh\Omega(s-t)}$$

$$\times\ \partial_{\vec{y}}\partial_{\vec{u}}K_0(t_1,\vec{u};,L,\vec{r})K_{HO}(\vec{r},L;\vec{z},s)(\frac{\hat{q}}{4}z^2\log(1/z^2Q^2)K_{HO}(\vec{z},s;t,\vec{y}=0)$$

$$\omega\frac{dI^{\text{Coulomb boundary}}}{d\omega d^2k_t} = \frac{\hat{q}}{4}2\alpha_s C_F Re\frac{\omega\Omega^2}{(2\pi)^2}\frac{ik_t}{k_t^2+\theta^2\omega^2}\exp(-i\frac{k_t^2\tanh(\Omega(L-s))}{2\omega\Omega})$$

$$\times\ F_4(\frac{-i\omega\Omega\cosh(\Omega(L-t)}{\cosh(\Omega(L-s)\sinh\Omega(s-t))},\frac{k_t}{\cosh\Omega(L-s)},Q)\exp(i\theta^2\omega(t-L)/2).$$

$$\omega\frac{dI(\omega,\hat{q},\theta,\mu,L)}{d\omega d^2k_t} = \omega\frac{dI^{HO}(w,q_{eff},\theta,\mu,Q_{eff},L)}{d\omega d^2k_t} + \frac{\hat{q}}{4}\omega\frac{dI^{\text{Coulomb}}(\omega q_{eff},\theta,Q_{eff},\mu,L)}{d\omega d^2k_t}$$

$$\omega\frac{dI^{HO}}{d\omega d^2k_t} = \omega\frac{dI^{\text{HO Bulk}}}{d\omega d^2k_t} + \omega\frac{dI^{\text{HO Boundary}}}{d\omega d^2k_t}$$

$$\omega\frac{dI^{\text{Coulomb}}}{d\omega d^2k_t} = \omega\frac{dI^{\text{Coulomb one}}}{d\omega d^2k_t} + \omega\frac{dI^{\text{Coulomb Bulk}}}{d\omega d^2k_t} + \omega\frac{dI^{\text{Coulomb Boundary}}}{d\omega d^2k_t}$$

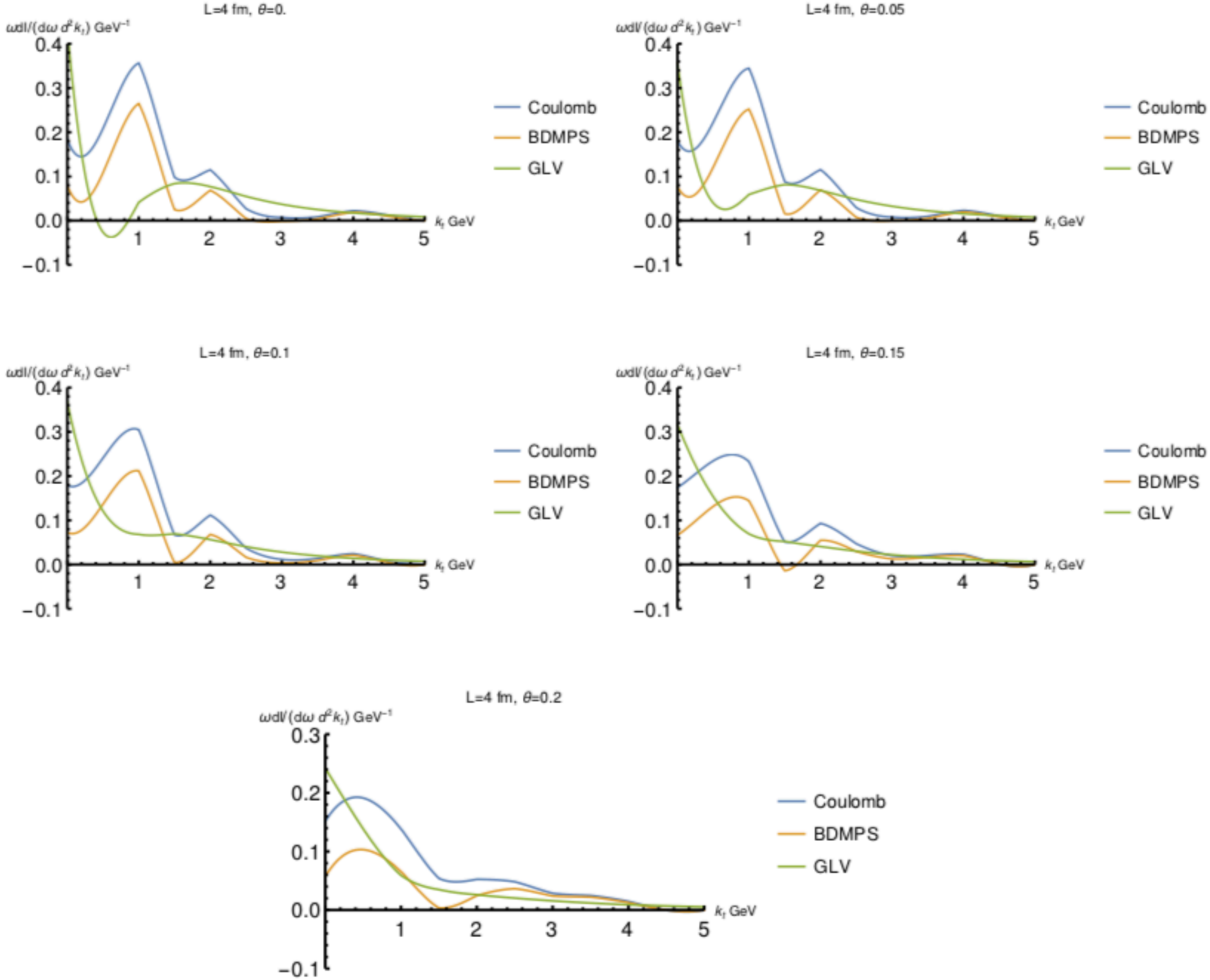
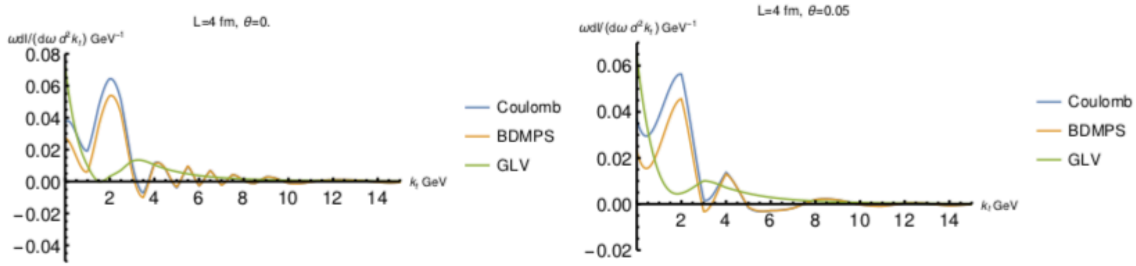


FIG. 1: *The angular distribution of radiated gluons for* $\omega = 5$ *GeV* for different $\theta = 0, 0.05, 0.1, 0.15, 0.2,$ Here and in the Figs. 1,2 BDMPS means the BDMPS angular distribution in the Harmonic Oscillator approximation given by Eq. 42, Coulomb means the angular distribution in the Moller theory given by Eq. 41, All graphs here and below are presented divided by $\alpha_s C_F$



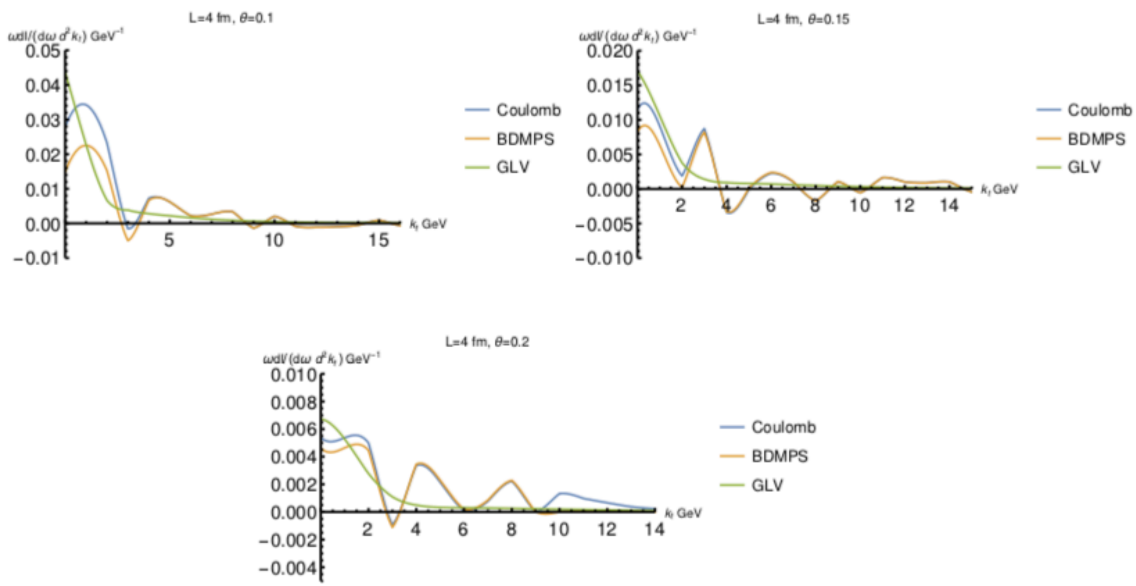


FIG. 3: *The angular distribution of radiated gluons for $\omega = 20$ GeV* for different $\theta = 0, 0.05, 0.1, 0.15, 0.2$

We see that the Coulomb distribution is rather smooth, has maximum for small transverse momenta, Increasing the filling of the dead cone, on the other hand beyond very small angles Coulomb contribution has the form very similar to BDMPSZ distribution, slightly shifting it to smaller transverse momenta, but larger

Energy loss.

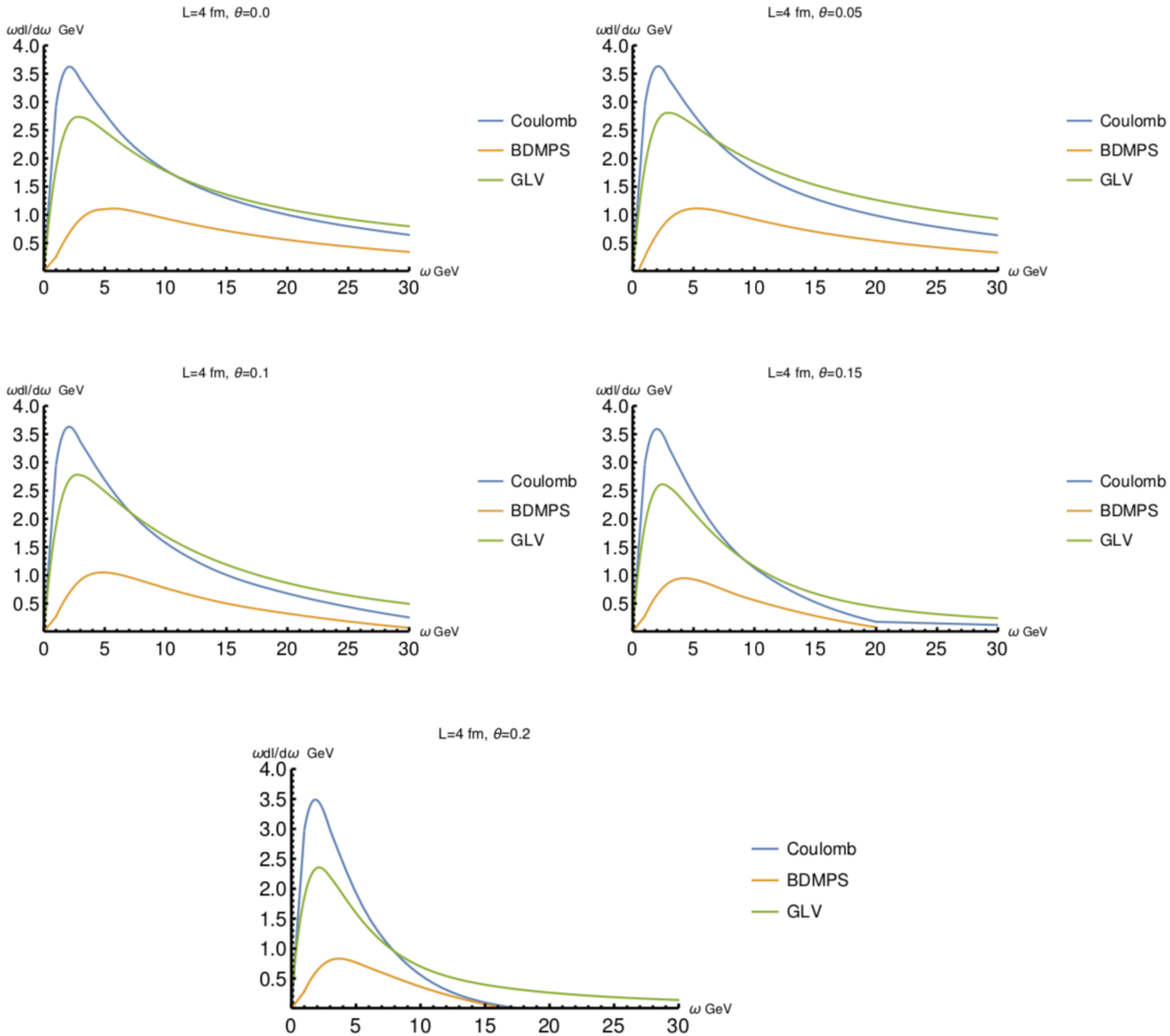


FIG. 4: *The energy loss $\omega dI/d\omega$ with Coulomb gluons for different $\theta = 0, 0.05, 0.1, 0.15, 0.2$, The energy loss $\omega dI/d\omega$ was calculated by integrating the corresponding angular distribution over k_t in the finite interval of k_t from $k_t = 0$ to the kinematical bound $k_t \leq \omega$. BDMPS means the expression for soft gluon emission in Harmonic Oscillator approximation, and Coulomb means the full result including Coulomb logarithms*

Longitudinal phase constraints

$$V(\rho) = \frac{1}{8}\hat{q}(\rho^2 \log(1/(\rho^2\mu^2) + ((1-z)^2\rho^2 \log 1/((1-z)^2\rho^2\mu^2) - z^2/9x^2 \log(1/(z^2\rho^2\mu^2)$$

$$V_0(\vec{\rho}) = \frac{1}{8}\hat{q}\rho^2(\log(Q^2/\mu^2) + ((1-z)^2 \log Q^2/((1-z)^2\mu^2) - z^2/9 \log(Q^2/(z^2\mu^2)$$

$$V_{pert}(\vec{\rho}) = \frac{\hat{q}}{8}(1 + (1-z)^2 - z^2/9)\rho^2 \log 1/(Q^2\rho^2)$$

$$\hat{q}_{eff} = \hat{q}\frac{1}{2}(\log(Q^2/\mu^2) + ((1-z)^2 \log Q^2/((1-z)^2\mu^2) - z^2/9 \log(Q^2/(z^2\mu^2)$$

$$z\frac{dI}{dzd^2k_t} = \frac{(1+(1-z)^2)}{2}\frac{C_F\alpha_s}{(2\pi)^2(z(1-z))^2}2Re\int d^2y\int_0^\infty dt_1\int_0^{t_1} dt e^{-i\vec{k}_t\vec{y}}$$

$$\times\ e^{-\int_{t_1}^\infty dsn(s)V(\vec{y}(s))}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y},t_1,\vec{x},t) - K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}.$$

$$(i\frac{\partial}{\partial t} + \frac{\vec{\partial}^2}{2z(1-z)E} + iV(x) + m^2)K(\vec{y},t_1;\vec{x},t)) = i\delta(\vec{x}-\vec{y})\delta(t-t_1)$$

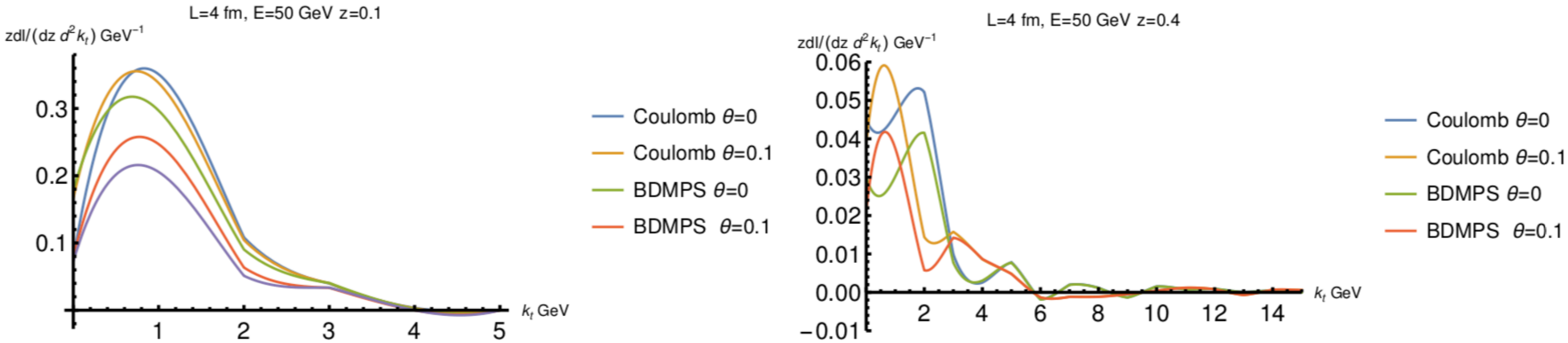


FIG. 5: The angular distributions $z\frac{dI}{dzd^2k_t}$ for different θ=0,0.05,0.1 for E=50 GeV. As above BDMPS means the angular gluon distribution calculated in the Harmonic Oscillator approximation but including finite gluon energy, Coulomb means the angular distribution in the Moller theory (i.e. BDMPS+Coulomb Logarithms) calculated taking into account finite gluon energy.

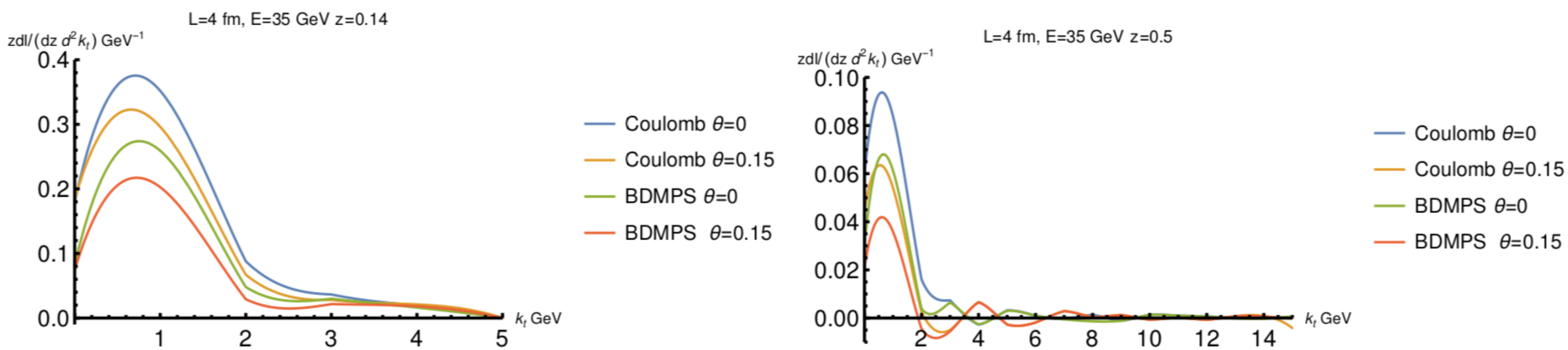
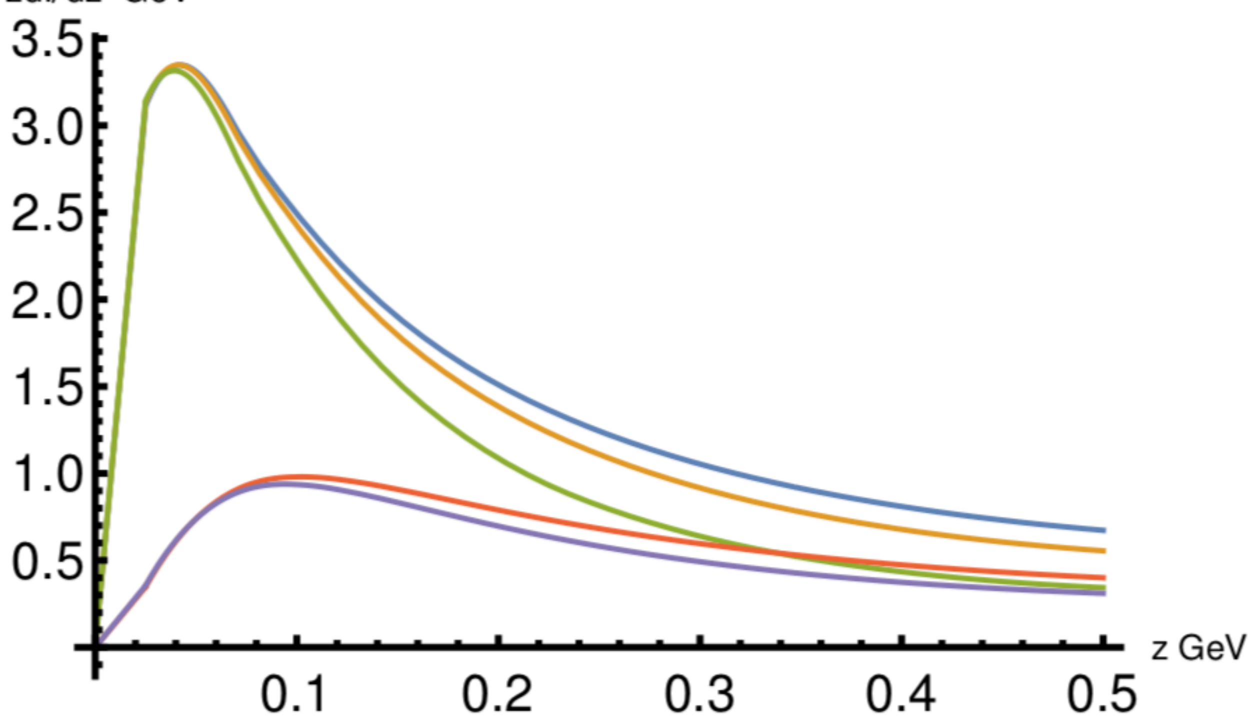


FIG. 6: The angular distributions $z\frac{dI}{dzd^2k_t}$ for different θ=0.05,0.1 for E=35 GeV

that the inclusions of the longitudinal phase space constraints, i.e. the finite glu



Energy loss calculated as a function of z with integration over transverse momenta carried up to Ez(1-z). We see that small frequency spectra are very close . Similar picture for other energies

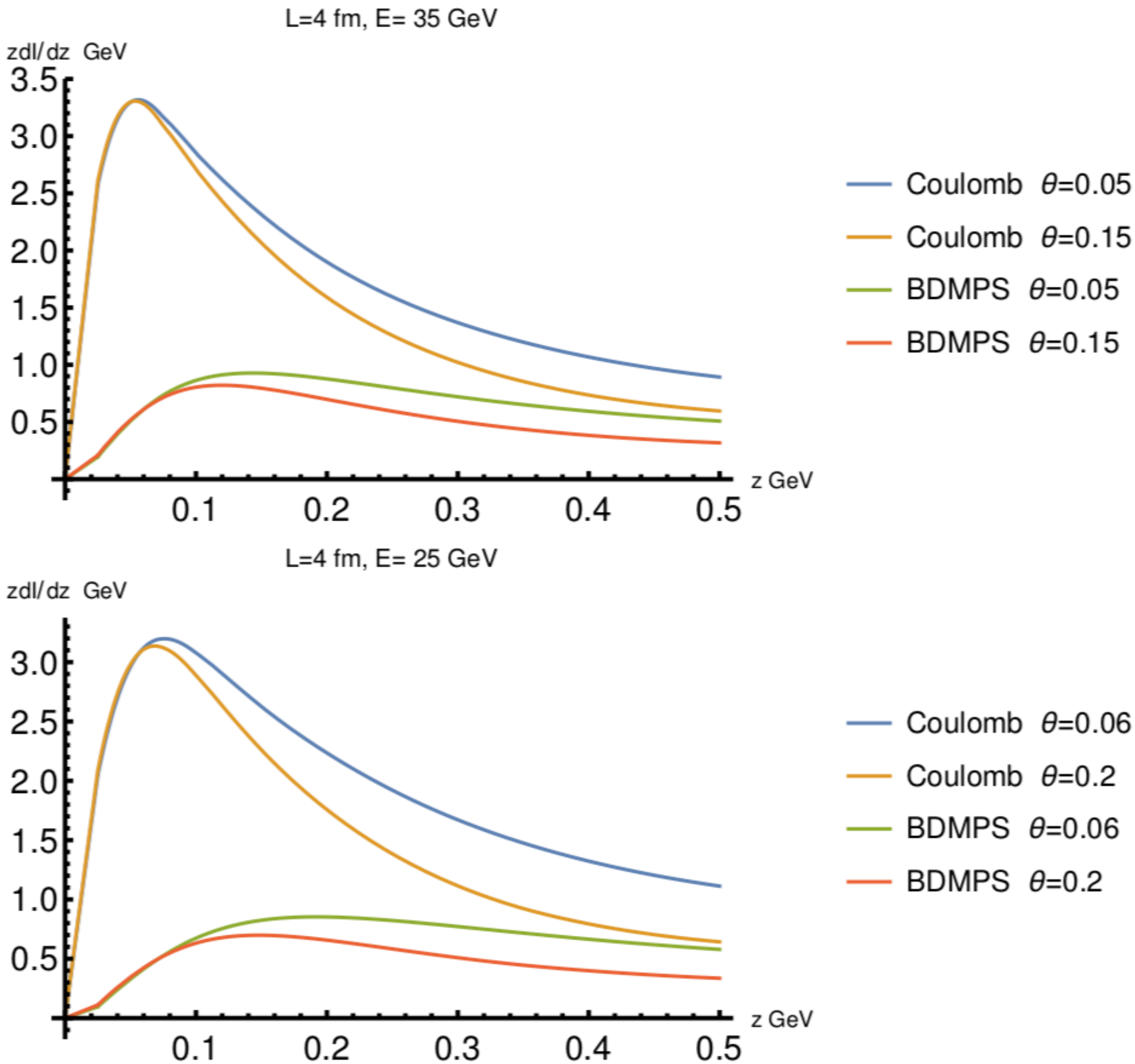


FIG. 7: The energy loss zdI/dz with ic gluons and BDMPS with phase constraints for different Energies and heavy quark masses $m = 1.5$ and 5 GeV.

	$E = 25$ GeV	$E = 35$ GeV	$E = 50$ GeV
BDMPS	- S(E)	- S(E)	-S(E)
Light quark m=0	0.57	0.6	0.6
Heavy quark $m_b = 5$ GeV	0.43	0.49	0.54
Coulomb	-S(E)	- S(E)	-S(E)
Light quark m=0	1.89	1.76	1.55
Heavy quark $m_b = 5$ GeV	1.59	1.56	1.47

TABLE II: *The estimate for quenching coefficients S(E) for light and heavy quarks, for $L = 4$ fm widths. The jet quenching factor $Q(E) = \exp(-S(E))$ Here all S(E) are divided on $\alpha_s C_F$*

Here

$$S(E) = \int_0^{1/2} z dI/dz (e^{-nz} - 1)/z \tag{55}$$

and we assume $\alpha_s = 0.3$. Here the quenching weight $Q(E) = \exp(S(E))$.

Conclusions:

1. *The Coulombic contributions for total energy loss are rather large, and without phase space constraints lead to energy loss and quenching coefficients quite close to N=1 GLV.*

2. For angular distributions the Coulomb contributions are large for very small transverse momenta, and for higher transverse momenta

thei total spectrum is similar in form to BDMPSZ spectrum for heavy quark.

3. The close values for Quenching Weights for heavy and light quarks beyond m/E=0.05 (and this always includes charmed quarks`) is actually the question of correct imposition of phase constraints, both longitudinal and transverse.