A novel unintegrated gluon distribution from DIS

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Gluon content of the proton

Regge limit (small $x$):

- $x_{Bj} \sim \frac{Q^2}{s} \to 0$
- strong classical gauge fields $A^\mu$
- dipole model

Bjorken limit (moderate $x$):

- $x_{Bj} \sim \frac{Q^2}{s} \sim 1$
- partonic picture
- QCD factorization

How can one reconcile the two pictures from first principles?
Diagnosis: searching for (small) $x$

- In the **Regge limit** distributions evaluated in the strict $x=0$ limit
  \[ f(k_\perp, x = 0) \]

- No $x$ dependence at LO: quantum evolution generates rapidity
  dependence. **Ambiguous connection to $x$**.

- The dipole model (with locality in transverse space) is inconsistent with $x$
  dependence

- Large collinear logs in NLO BK. Numerically unstable. Several fixes
  proposed: modification of the evolution kernel, better choice of the
  evolution variable, etc → need to **address the factorization scheme itself**

Bialas, Navlet and Peschanski (2000)

[Lappi and Mäntysaari (2015)]

Gluon PDF and the gauge choice

\[ xg(x, \mu^2) = 2 \int \frac{d\xi^+}{(2\pi)P^-} e^{ixP^-\xi^+} \langle P \mid \text{Tr} \left[ 0 , \xi^+ \right] F^{i-} (\xi^+) \left[ \xi^+ , 0 \right] F^{i-} (0) \mid P \rangle \]

- N.B.: the partonic picture is manifest in the LC-gauge \( A^- = 0 \) (with \( A_\perp \neq 0 \))
- small \( x \) observable: naturally expressed in the wrong LC-gauge \( A^- \neq 0 \) (with \( A_\perp = 0 \)),

\[ U_x \equiv [+\infty, -\infty]_x = P \exp \left[ ig \int_{-\infty}^{+\infty} dx^+ A^-(x^+, x) \right] \]
- in order to connect to the partonic interpretation one needs to deal with transverse fields
Connecting small $x$ and the parton picture

- **Non-Abelian Stokes’ theorem**: the dipole operator rewritten as a path ordered tower of “twisted” field strength tensor (i.e. dressed with future pointing Wilson lines)

\[
U_{x_2} U_{x_1}^\dagger \equiv P \exp \left[ -i g \int_S dt dz \left[ + \infty, x^+ \right]_x F^{i-}(x^+, x) \left[ x^+, + \infty \right]_x \right]
\]

Inclusive DIS beyond shock wave

Relax the shock wave approximation $\Delta x^+ = 0$: what is the longitudinal extent of the shock wave?

1. extract the first and last interactions. 4 contributions that reduce to a single one:

$$A^-(x) - A^-(y) = \int_0^1 ds \ r^i \ \partial^i A^-(y + sr) = \int_0^1 dz^i \ F^{i-}(z)$$
2. Expansion around the eikonal trajectory for the propagator

\[ G_{p^+}(x^+, x_2, y^+, x_1) = G_0(x_2 - x_1, x_2^+ - y_1^+) \, U_X(x_2^+, x_1^+) + \ldots \]

- On may Fourier transform w.r.t. \( u = x_2 - x_1 \)

\[ G_{p^+}(x_2, x_1^+, X; \ell) = e^{i \frac{\ell^2}{2q^+} \Delta x^+} \, U_X(x_2^+, x_1^+) + \ldots \]

- \( \ell \) is the average transverse momentum of the quark

Altiloniuk, Armesto, Beuf, Martinez, Salgado (2015)
Factorization formula for DIS at arbitrary $x$ (2006.14569 [hep-ph])

$x$ dependent unintegrated gluon distribution

$$xG^{ij}(x, k_\perp) \equiv 2 \int \frac{d\xi^+ d\xi}{(2\pi)^3 P^-} e^{ix P^- \xi^+ - ik \cdot \xi} \langle P | \text{Tr} [0, \xi^+]_\xi F^i-(\xi^+, s'\xi) [\xi^+, 0]_0 F^j-(0, s\xi) | P \rangle$$

→ Recovers gluon PDF and the dipole amplitude

- integrating over $k_\perp$ yields $\xi_\perp = 0$ and we recover the gluon PDF
- at small $x$ we recover shock wave

$$\xi^i \xi^j G^{ij}(x = 0, \xi) \rightarrow \langle P | \text{Tr} U_\xi U_0^\dagger | P \rangle$$

$(\xi, \xi^+)$
Factorization formula for DIS at arbitrary $x$ (2006.14569 [hep-ph])

- **factorization formula in momentum space**: minimally improved shock wave approximation

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 dz P(z) \int_0^1 dx \int d\ell d\ell' \partial^i \phi \left( \ell - \frac{k}{2} \right) \partial^i \phi^* \left( \ell + \frac{k}{2} \right) \delta \left( x - x_{Bj} - \frac{\ell^2}{2z\bar{z}q^+} \right) \times xG^{ij}(x,k) + O \left( k_\perp^2/s \right)$$

- $\phi(\ell)$ is the Fourier transform of the photon wave function (shock wave)
- The **delta function** relates $x$ in the gluon distribution to $x_{Bj}$
• We revisited the shock wave approximation in high energy scattering by performing a gradient expansion around the classical trajectory of partons

• The leading power interpolates between small and moderate x limits

• We have calculated in this framework gluon induced DIS and obtained in particular a new factorization formula involving a novel unintegrated gluon distribution

• Outlook: quantum evolution, application to other observables such as DVCS

• Potential probe of gluon saturation on the lattice