A novel unintegrated gluon distribution from DIS

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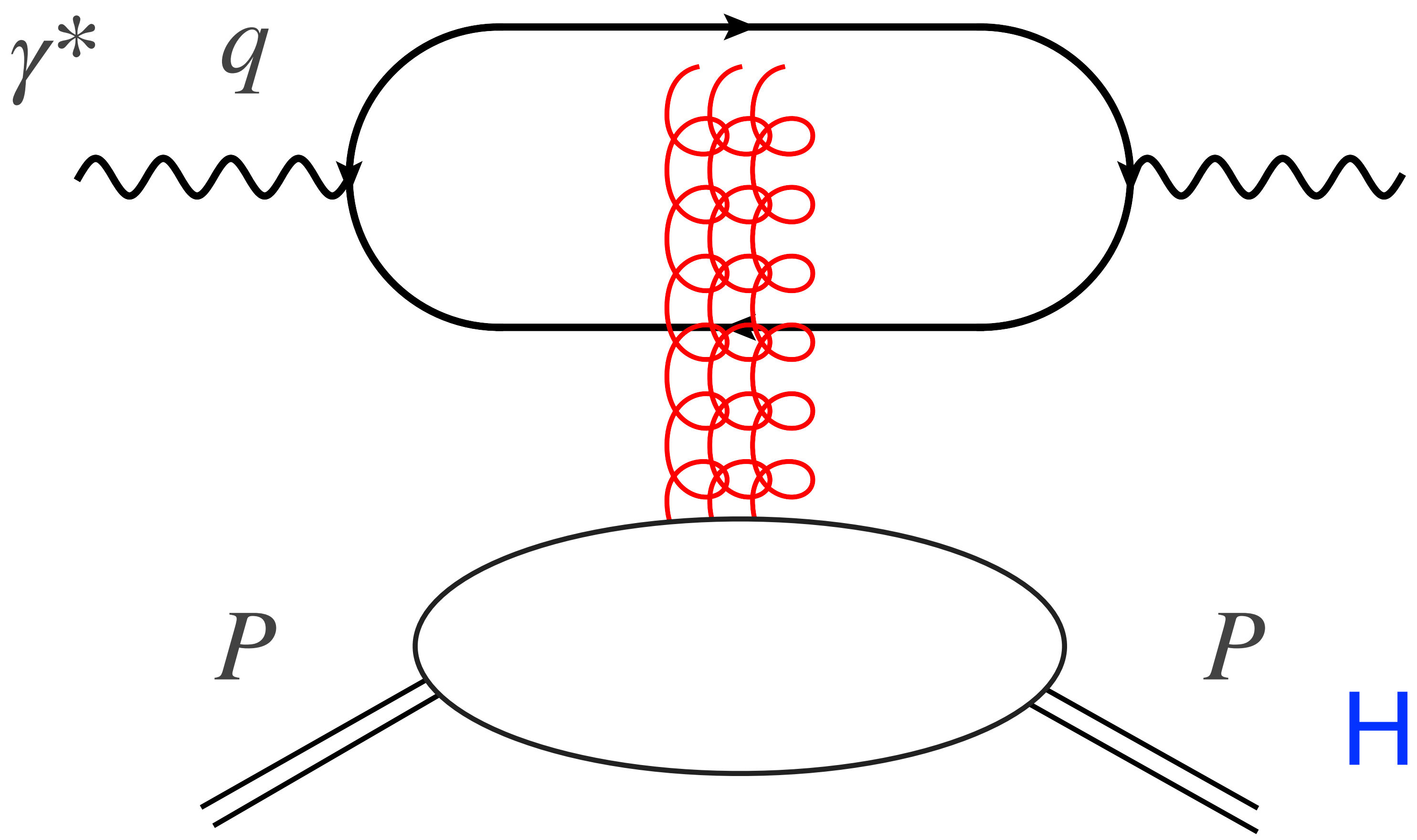
Based on: 2001.06449 and 2006.14569 [hep-ph]
In collaboration with Renaud Boussarie

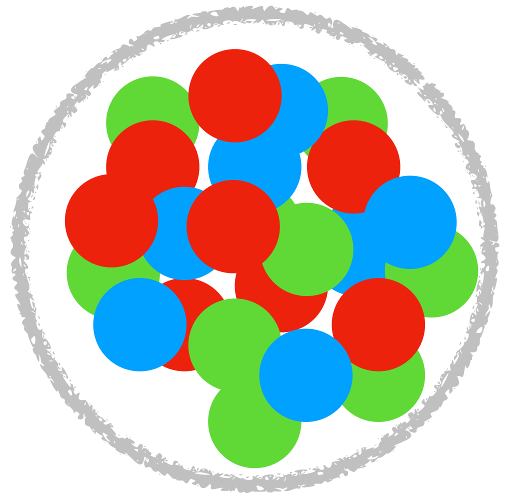
Gluon content of the proton

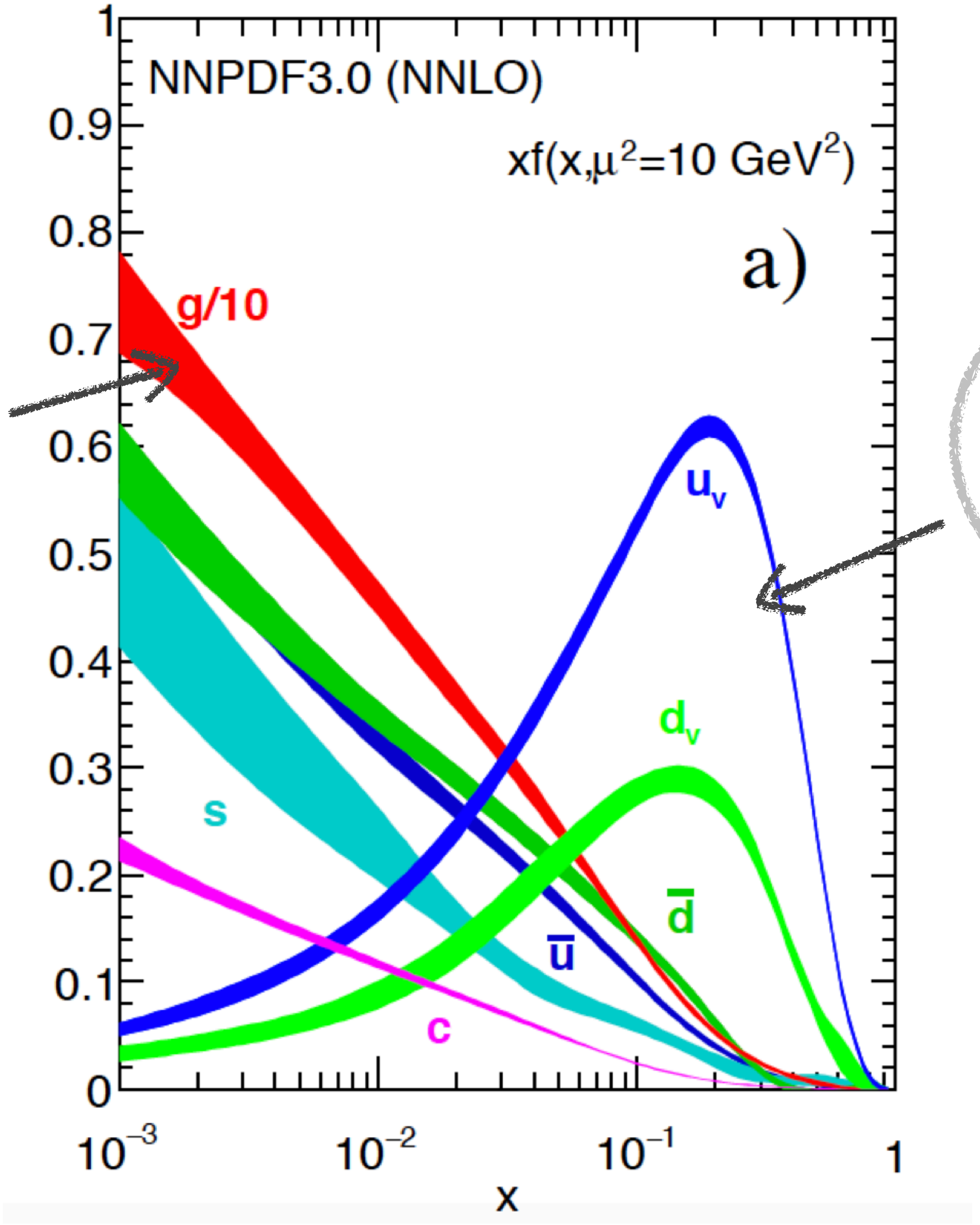
Wave ↔ Particle

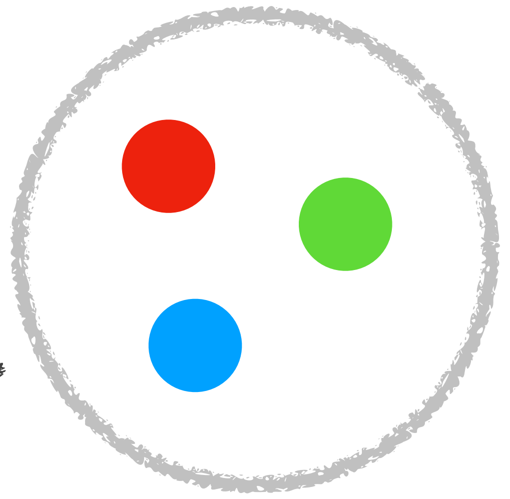
Regge limit (small x):

- $x_{Bj} \sim Q^2/s \to 0$
- strong classical gauge fields A^μ
- dipole model



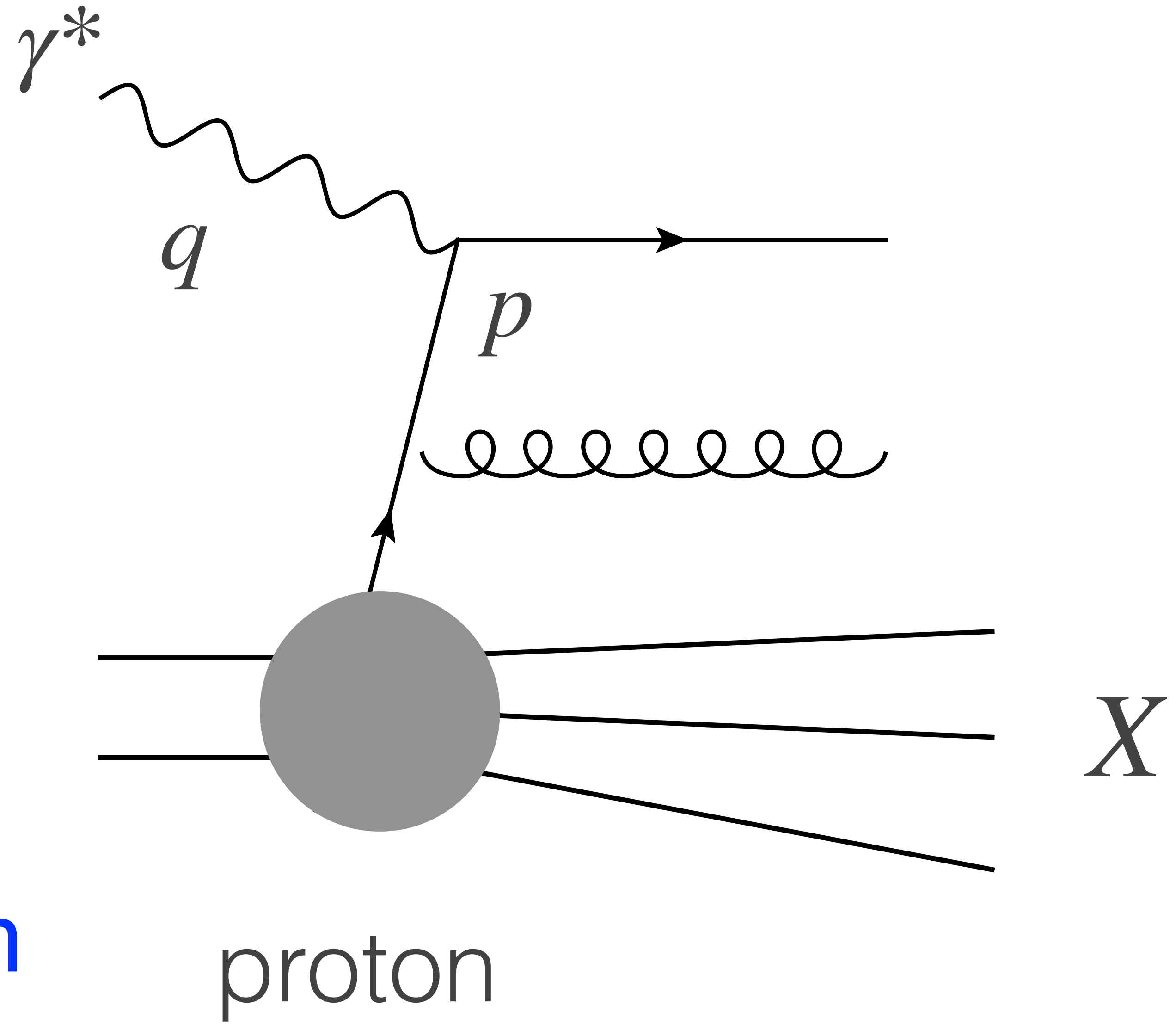






Bjorken limit (moderate x):

- $x_{Bj} \sim Q^2/s \sim 1$
- partonic picture
- QCD factorization



How can one reconcile the two pictures from first principles?

Diagnosis: searching for (small) x

- In the Regge limit distributions evaluated in the strict x=0 limit

$$f(k_\perp, x = 0)$$

- No x dependence at LO: quantum evolution generates rapidity dependence. Ambiguous connection to x.

- The dipole model (with locality in transverse space) is inconsistent with x dependence

Bialas, Navlet and Peschanski (2000)

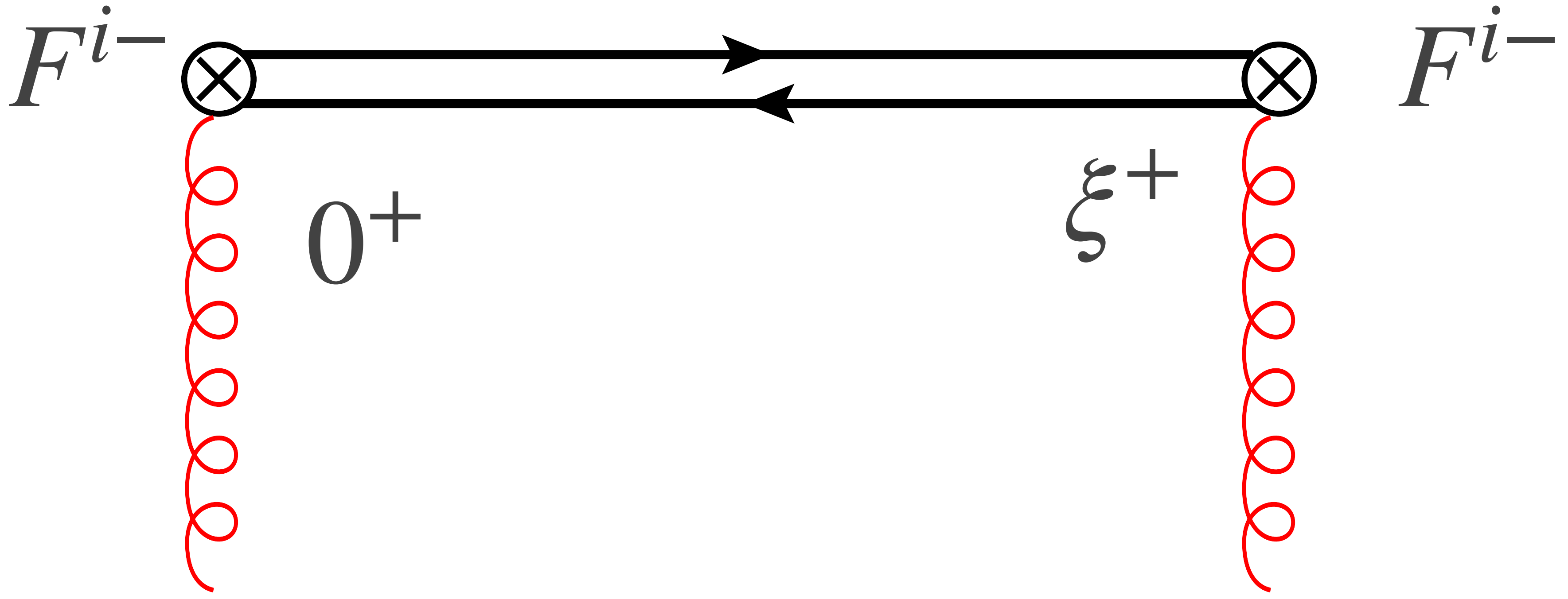
- Large collinear logs in NLO BK. Numerically unstable. Several fixes proposed: modification of the evolution kernel, better choice of the evolution variable, etc $\rightarrow$ need to address the factorization scheme itself

[Lappi and Mäntysaari (2015)]

[Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)]

Gluon PDF and the gauge choice

$$xg(x,\mu^2) = 2\int \frac{d\xi^+}{(2\pi)P^-} e^{ixP^-\xi^+} \langle P|\, \mathrm{Tr}\,[0,\xi^+]\, F^{i-}(\xi^+)\, [\xi^+,0]\, F^{i-}(0)\, |P\rangle$$



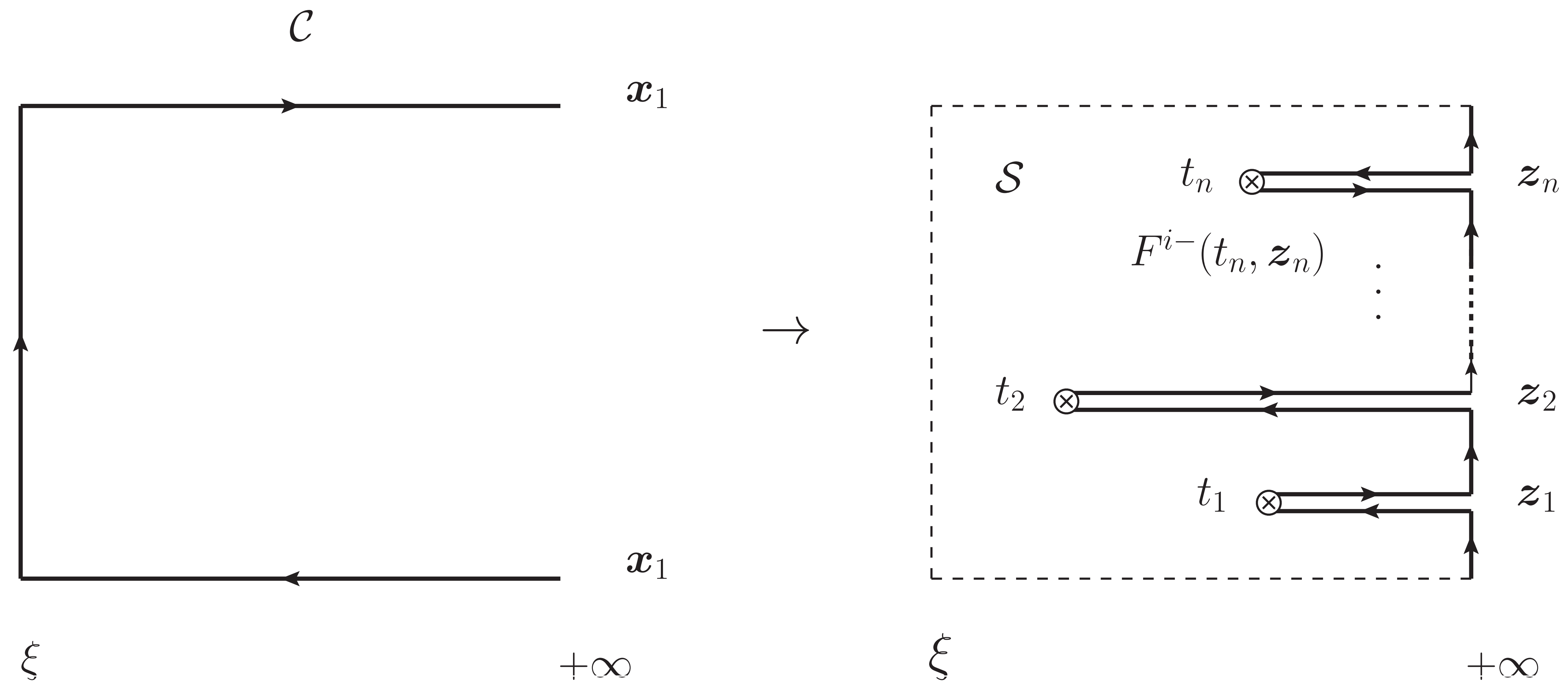
- N.B.: the partonic picture is manifest in the LC-gauge $A^- = 0$ (with $A_\perp \neq 0$)
- small x observable: naturally expressed in the wrong LC-gauge $A^- \neq 0$ (with $A_\perp = 0$),

$$U_x \equiv [+\infty, -\infty]_x = P\exp\left[ig\int_{-\infty}^{+\infty} dx^+ A^-(x^+, \boldsymbol{x})\right]$$

- in order to connect to the partonic interpretation one needs to deal with transverse fields

Connecting small x and the parton picture

- Non-Abelian Stokes' theorem: the dipole operator rewritten as a path ordered tower of "twisted" field strength tensor (i.e. dressed with future pointing Wilson lines)

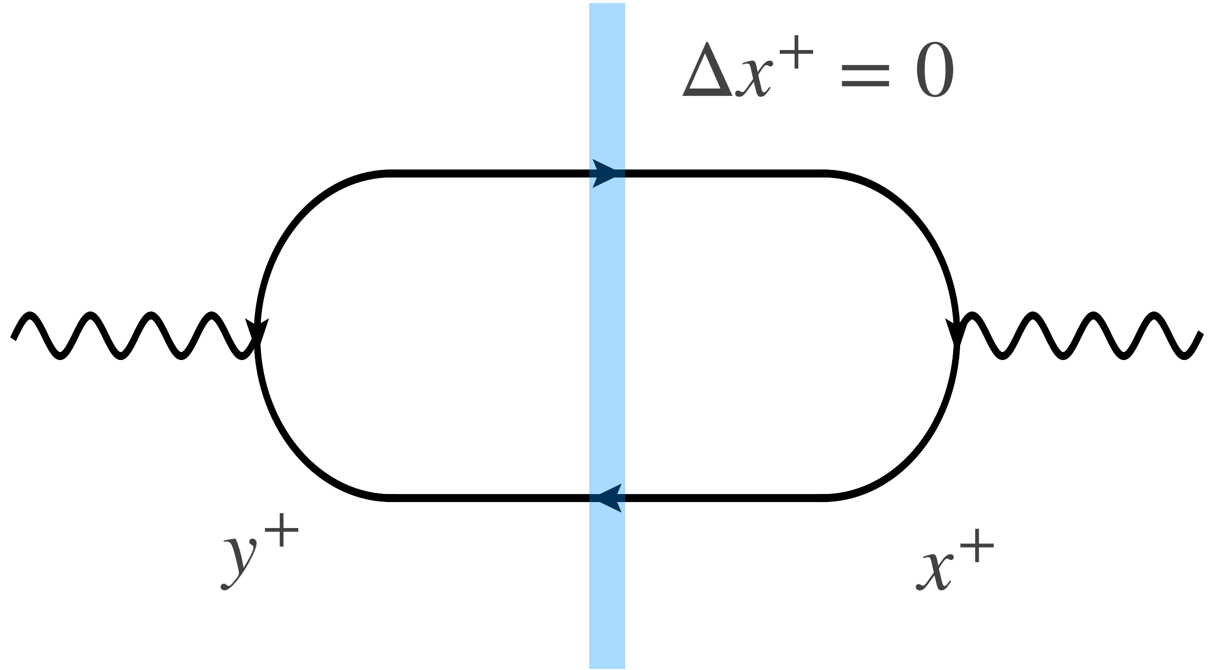


[Fishbane, Gasiorowicz, Kaus (1981) Wiedemann (2000)
YMT, Boussarie (2020)]

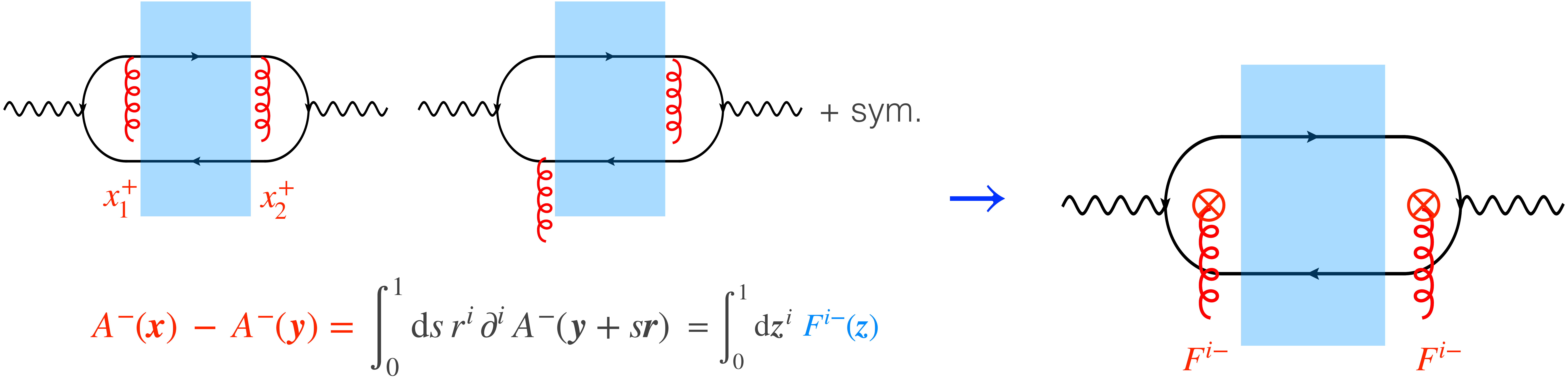
$$U_{x_2} U^\dagger_{x_1} \equiv P\exp\left[-ig\int_S \mathrm{d}t\mathrm{d}z\,[+\infty, x^+]_x\, F^{i-}(x^+, x)\,[x^+, +\infty]_x\right]$$

Inclusive DIS beyond shock wave

Relax the shock wave approximation $\Delta x^+ = 0$:
what is the longitudinal extent of the shock wave ?



1. extract the first and last interactions. 4 contributions that reduce to a single one:



$$A^-(\mathbf{x}) - A^-(\mathbf{y}) = \int_0^1 \mathrm{d}s\, r^i \partial^i A^-(\mathbf{y} + s\mathbf{r}) = \int_0^1 \mathrm{d}z^i\, F^{i-}(\mathbf{z})$$

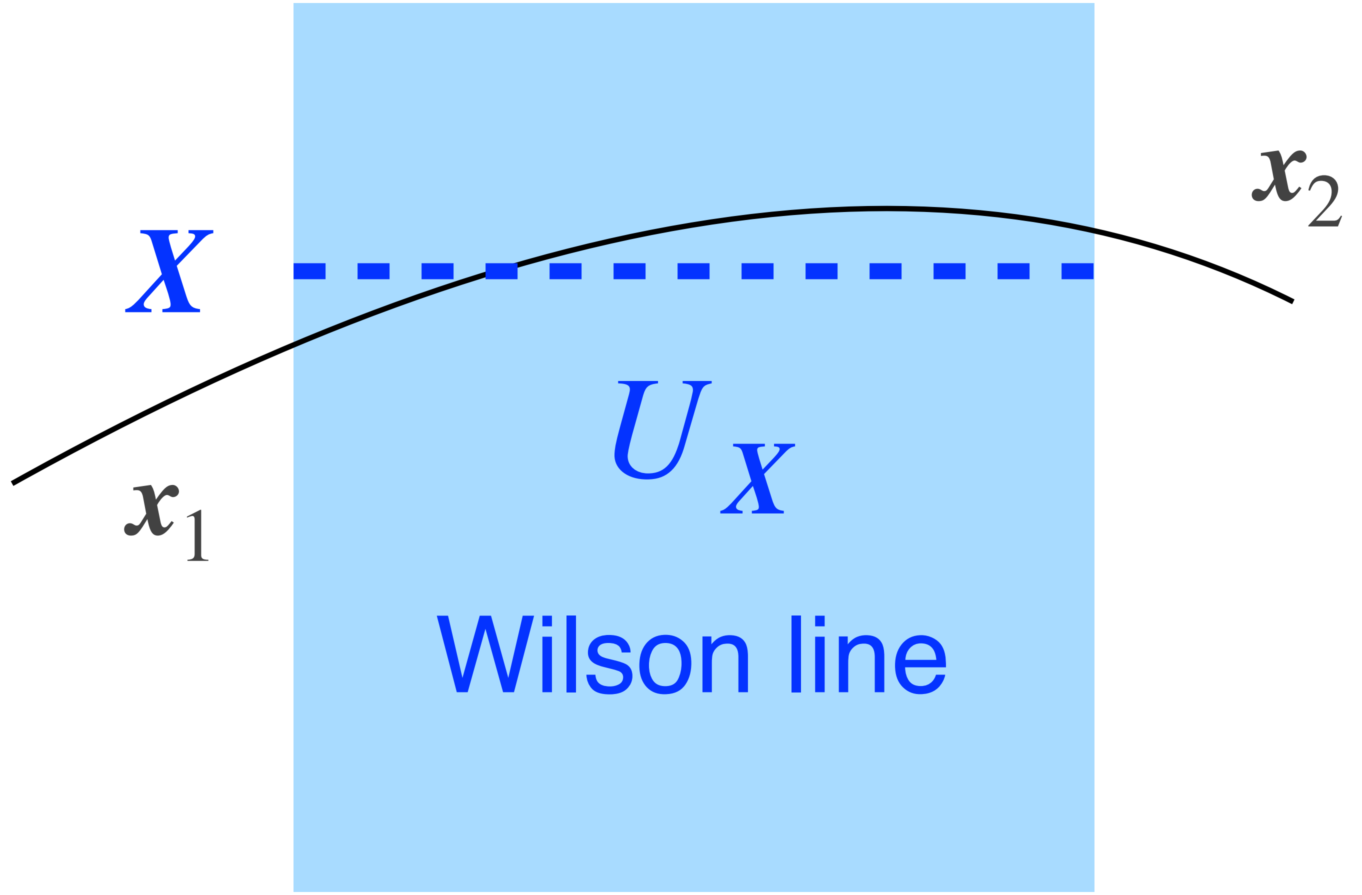
Inclusive DIS beyond shock wave

2. Expansion around the eikonal trajectory for the propagator

$$\mathcal{G}_{p^+}(x^+,\boldsymbol{x}_2;\, y^+,\boldsymbol{x}_1) \;= \mathcal{G}_0(\boldsymbol{x}_2-\boldsymbol{x}_1, x_2^+-y_1^+)\, U_X(x_2^+,x_1^+)+\dots$$

- On may Fourier transform w.r.t. $\boldsymbol{u}=\boldsymbol{x}_2-\boldsymbol{x}_1$

$$\mathcal{G}_{p^+}(x_2,x_1^+,\boldsymbol{X};\boldsymbol{\ell}) \;= e^{i\frac{\ell^2}{2zq^+}\Delta x^+}\, U_X(x_2^+,x_1^+)+\dots$$



- $\boldsymbol{\ell}$ is the average transverse momentum of the quark

Altiloniuk, Armesto, Beuf, Martinez, Salgado (2015)

Factorization formula for DIS at arbitrary x (2006.14569 [hep-ph])

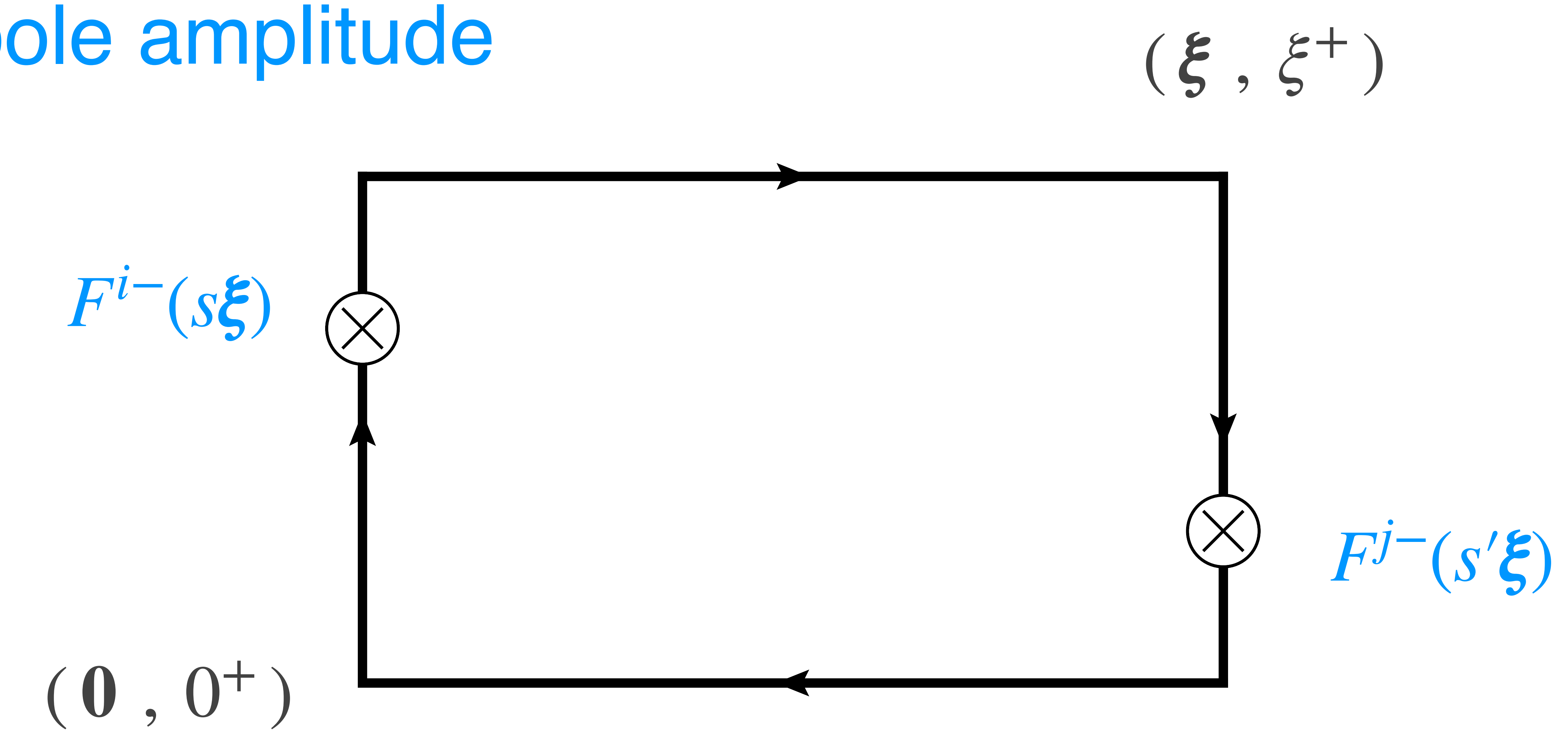
x dependent unintegrated gluon distribution

$$xG^{ij}(x,k_\perp) \equiv 2\int \frac{d\xi^+ d\boldsymbol{\xi}}{(2\pi)^3 P^-}\, e^{\,ixP^-\xi^+ - ik\cdot\boldsymbol{\xi}} \left\langle P \,|\, \mathrm{Tr}\,[\,0\,,\xi^+]_{\boldsymbol{\xi}}\, F^{j-}(\xi^+, s'\boldsymbol{\xi})\,[\,\xi^+\,,0]_{\boldsymbol{0}}\, F^{i-}(0\,, s\boldsymbol{\xi}) \,|\, P \right\rangle$$

$\rightarrow$ Recovers gluon PDF and the dipole amplitude

- integrating over $k_\perp$ yields $\xi_\perp = 0$ and we recover the gluon PDF
- at small x we recover shock wave

$$\xi^i \xi^j G^{ij}(x=0,\boldsymbol{\xi}) \rightarrow \langle P \,|\, \mathrm{Tr}\ U_{\boldsymbol{\xi}}\ U^\dagger_{\boldsymbol{0}} \,|\, P \rangle$$

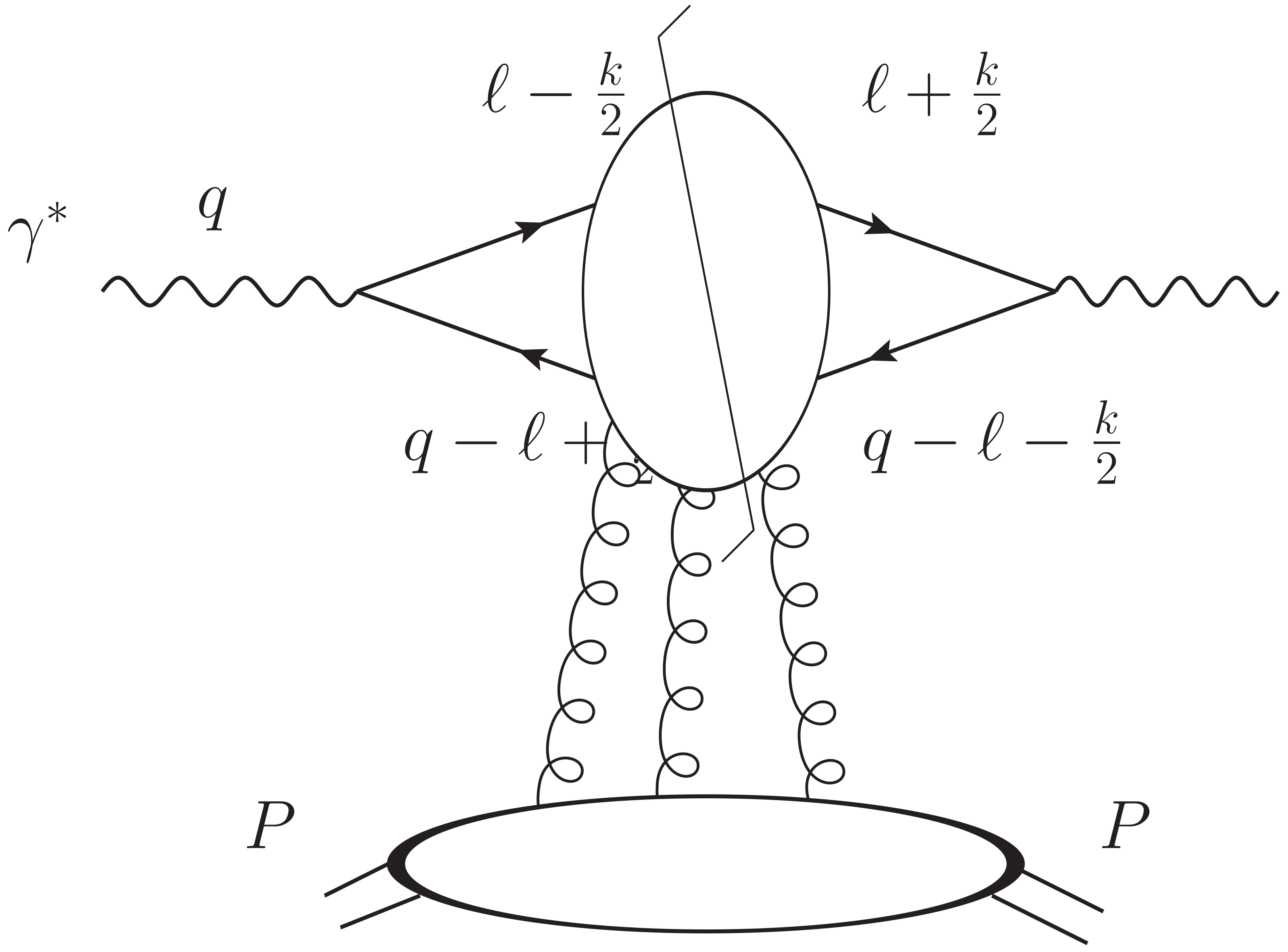


Factorization formula for DIS at arbitrary x (2006.14569 [hep-ph])

- factorization formula in momentum space: minimally improved shock wave approximation

$$\sigma(x_{Bj}, Q^2) \sim e^2 \int_0^1 \mathrm{d}z\, P(z) \int_0^1 \mathrm{d}x \int_{\boldsymbol{\ell},\boldsymbol{k}} \partial^i \varphi\left(\boldsymbol{\ell} - \frac{\boldsymbol{k}}{2}\right) \partial^j \varphi^*\left(\boldsymbol{\ell} + \frac{\boldsymbol{k}}{2}\right) \delta\left(x - x_{Bj} - \frac{\boldsymbol{\ell}^2}{2z\bar{z}q^+}\right)$$

$$\times\, xG^{ij}(x, \boldsymbol{k}) \;+\; O\left(k_\perp^2/s\right)$$



- $\varphi(\boldsymbol{\ell})$ is the Fourier transform of the photon wave function (shock wave)
- The delta function relates x in the gluon distribution to x_{Bj}

Summary and outlook

- We revisited the shock wave approximation in high energy scattering by performing a gradient expansion around the classical trajectory of partons
- The leading power interpolates between small and moderate x limits
- We have calculated in this framework gluon induced DIS and obtained in particular a new factorization formula involving a novel unintegrated gluon distribution
- Outlook: quantum evolution, application to other observables such as DVCS
- Potential probe of gluon saturation on the lattice