Using Machine Learning to Understand the Properties of the QCD Critical Point

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Illinois Center for Advanced Studies of the Universe
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Outline

1. The search for the QCD critical point
   I. Current understanding + missing pieces
   II. 2D Equation of State with a critical point
   III. Defining a machine learning problem

2. Machine learning and the active learning framework
   I. Preparing the data
   II. Systematically comparing algorithm performance

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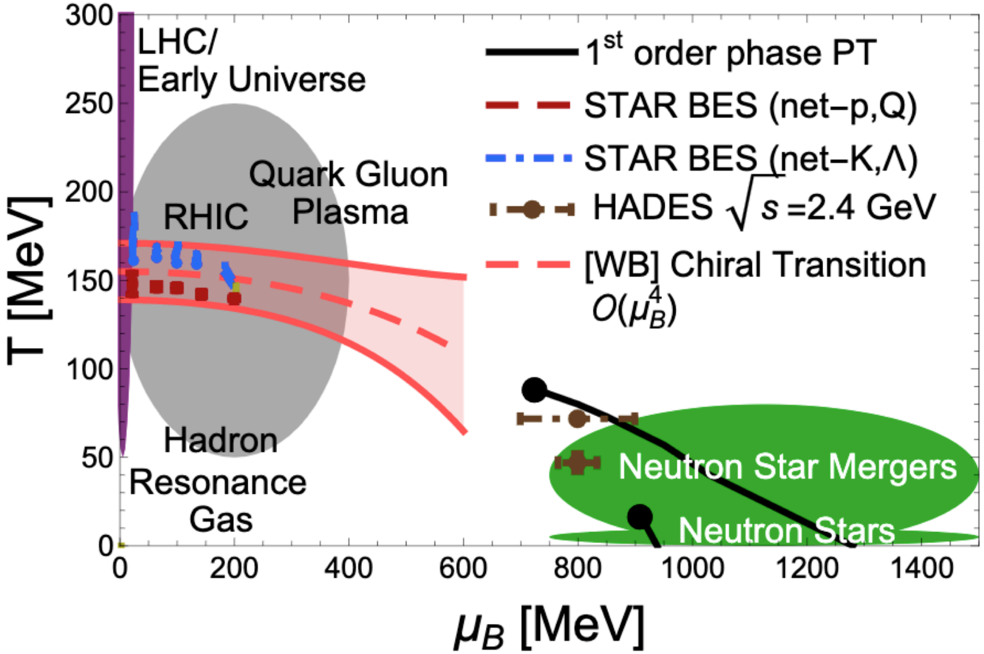
▪The QCD phase diagram: the one we don't know but love



WB Phys.Rev.Lett. (2020); P. Alba et al Phys.Lett. (2014);
Bellwied et al arXiv:1805.00088; V. Dexheimer ariXiv:1708.08342;
Critelli et al, Phys.Rev. D96 (2017);
HADES Nature Phys. (2019); Nucl.Phys.A (2014)

- Known with high precision at μ_B=0

S. Borsanyi et al, JHEP (2018)

- Sign problem at finite μ_B

M. Troyer and U.J. Wiese, Phys. Rev. Lett. (2005)

→ Challenges in interpreting recent/future experimental results.

To-do: Changes to IC + hydro + hadronization + transport are still needed in the vicinity of a CP.

Starting point: EOS with CP at finite baryon density matching Lattice at μ_B=0.

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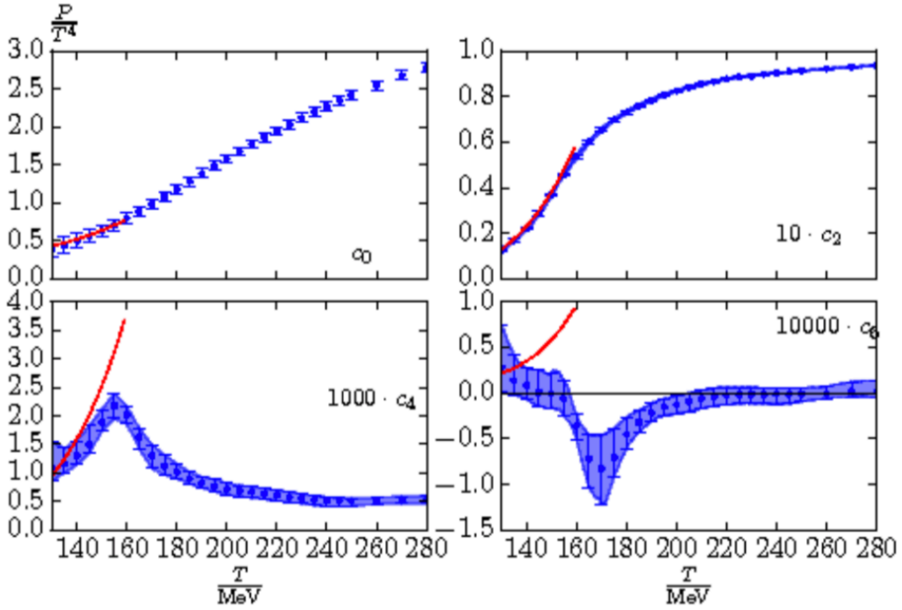
▪ Exploring the baryon dense regime

We can perform a Taylor expansion around $\mu_B = 0$

$$P(T,\mu_B) = T^4 \sum_n c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

$$c_n(T) = \frac{1}{n!} \frac{\partial^n P/T^4}{\partial(\mu_B/T)^n}\bigg|_{\mu_B=0} = \frac{1}{n!}\chi_n(T).$$



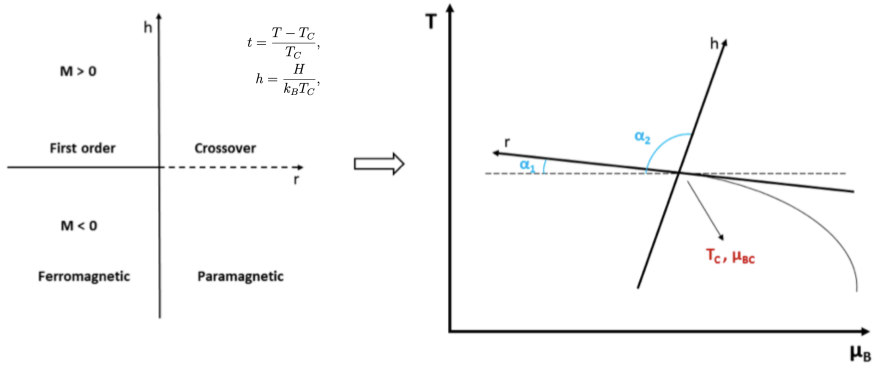
R. Bellwied et al, Phys. Lett. B (2015).

- Few coefficients from lattice → indication that $\mu_B/T < 2$ is disfavored
- No knowledge beyond CP ($\mu_B > \mu_{BC}$)

A. Bazavov et al Phys. Rev. D (2017).

▪ QCD phase diagram with a critical point from the 3D Ising model

3D Ising Model: We can borrow the critical region from a theory in the same universality class as QCD.



Up to $\mathcal{O}(\mu_B^4)$: P. Parotto, M. Bluhm, DM et al. Phys. Rev. C (2020)

Linear map Ising → QCD **requires 6 parameters:**

$$(\mathbf{r}, \mathbf{h}) \longleftrightarrow (\mathbf{T}, \mu_\mathbf{B}) : \quad \frac{T - \mathbf{T_C}}{\mathbf{T_C}} = \mathbf{w}\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$

$$\frac{\mu_B - \mu_{\mathbf{BC}}}{\mathbf{T_C}} = \mathbf{w}\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

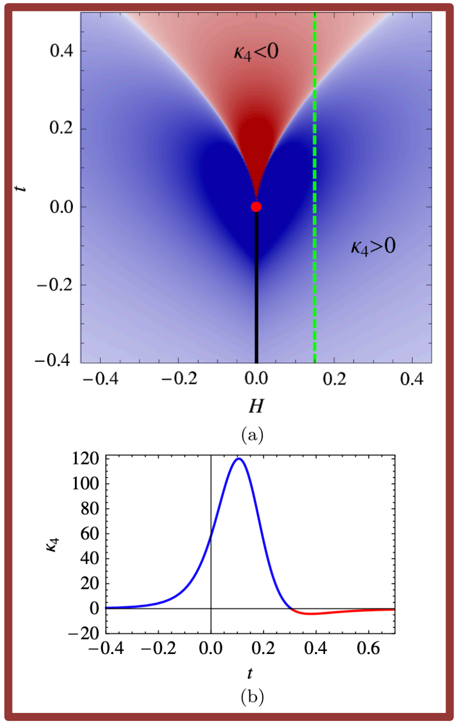
Safe assumption: transition line is a parabola with curvature **κ**

$$T = T_0 + \kappa T_0 \left(\frac{\mu_B}{T_0}\right)^2 + O(\mu_B^4). \qquad \alpha_1 = \tan^{-1}\left(2\frac{\kappa}{T_0}\mu_{BC}\right)$$

κ and T_0 (T at which the transition line crosses the T axis) – Estimates from Lattice
α_1 – Obtained from parabola
μ_{BC} – Critical baryon chemical potential
α_{diff} – Angle difference between the Ising axes
w, **ρ** – Scaling parameters (size and shape of the critical region)

▪ Thermodynamic Stability

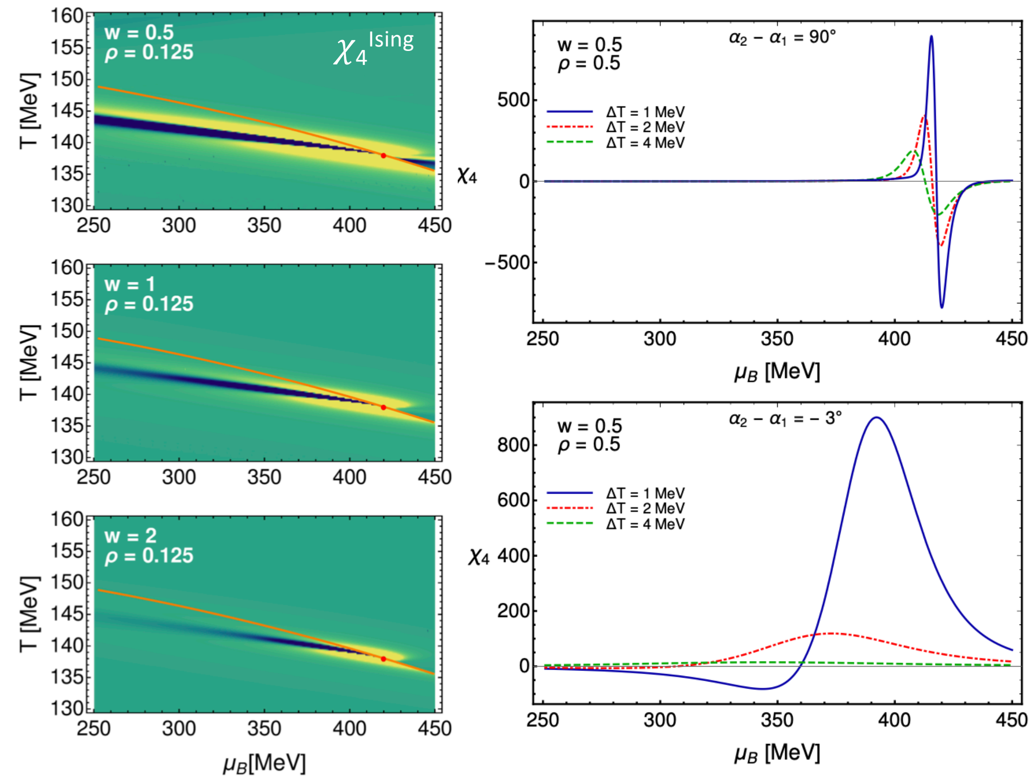
Not an unambiguous signature of CP!



Stephanov, Phys. Rev. Lett. (2011)

Any choices that display the dip thermodynamically consistent?

Choice of parameters affects behavior of baryon kurtosis



DM et al arXiv:2008.04022

Identifying thermodynamic stability

Not every parameter choice will result in a thermodynamically stable EOS.

stable

unstable
- $C_s^2 < 0$ or $C_s^2 > 1$
- acausal + $s, \varepsilon, n_B, P < 0$

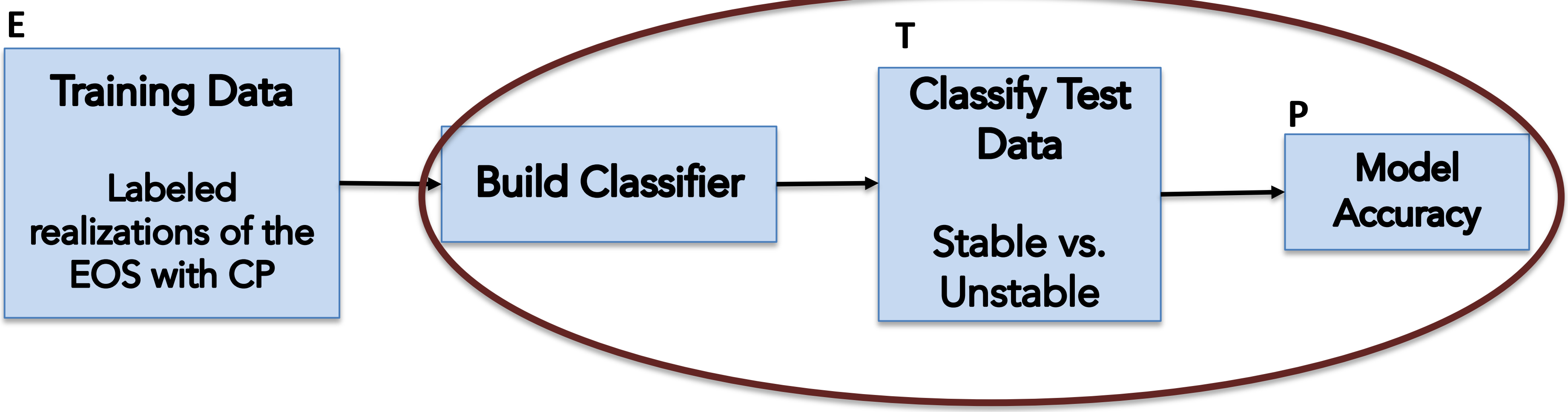
$s, \varepsilon, n_B, c_s^2$: combination of derivatives of the pressure.

Stability is encoded in P: <u>how do we use that in our favor and reduce the computational costs of constraining our model</u>?

Can we determine thermodynamic stability <u>without taking derivatives</u>?

- **Formulating a well-posed machine learning problem**

*A computer learns from **experience** E with respect to some class of **tasks** T and **performance measure** P, if its performance at tasks in T improves, as measured by P, with experience E.*



→ Final version of the classifier can be used in large scans of the parameter space to understand underlying physics (coming soon).

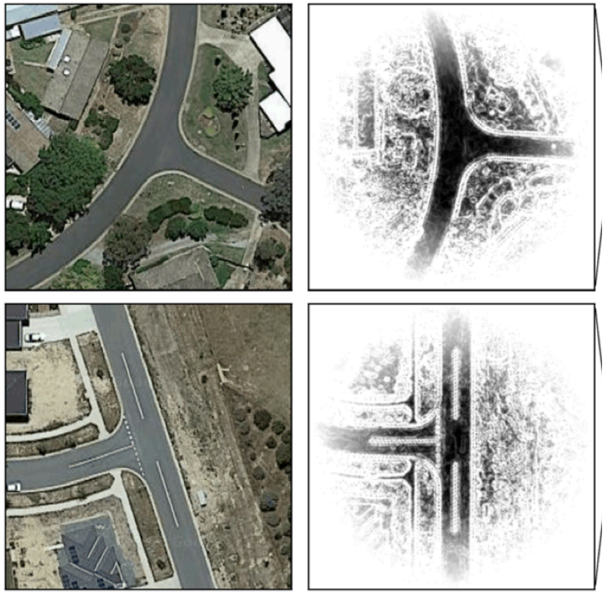
▪ Preprocessing

Goals of preprocessing stage:

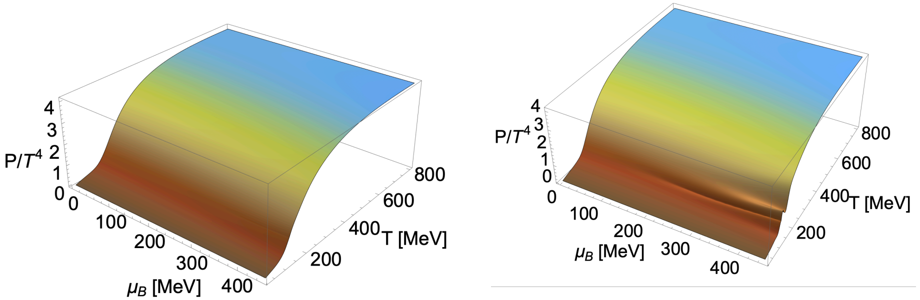
i) **Reduce** input dimension.

ii) Obtain **separation** between the classes.

Preprocessing filters out relevant features

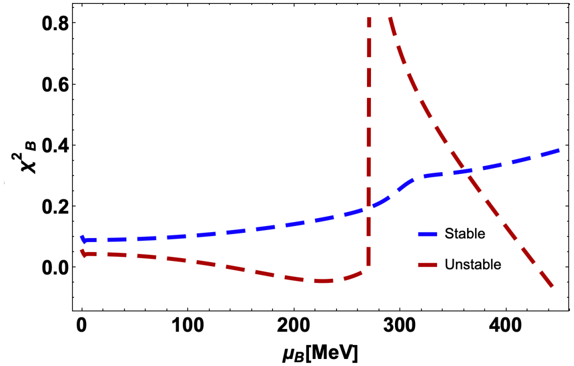


J. Wijnands et al Comput. Aided Civ. Inf. (2020)



Pressure contains extra information

We want to detect features such as:



Second baryon cumulant at T = 148 MeV

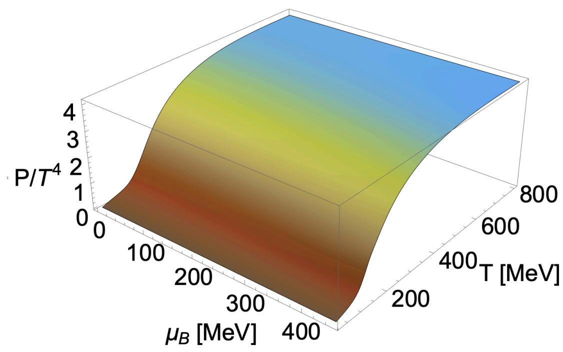
Preprocessing

Combination of SVD and averaging yields class separation described by **3 features**.

$$\boldsymbol{\Gamma}: P(T_0, \kappa, \mu_{BC}, w, \rho) \longrightarrow \mathbf{v}(T_0, \kappa, \mu_{BC}, w, \rho)$$

$$\text{Dim}(P) = 771\text{x}451 \text{ – grid size}$$
$$\text{Dim}(\mathbf{v}) = 3$$

$\boldsymbol{\Gamma}$:



→ $\{v_1 , v_2, v_3\}$

Choosing the right model & training strategy

Collection of classifiers

1) Random Forest

L. Breiman *ML* (2001)

2) Support Vector Machine

C. Cortes, V. Vapnik ML (1995)

3) Gradient Boosting

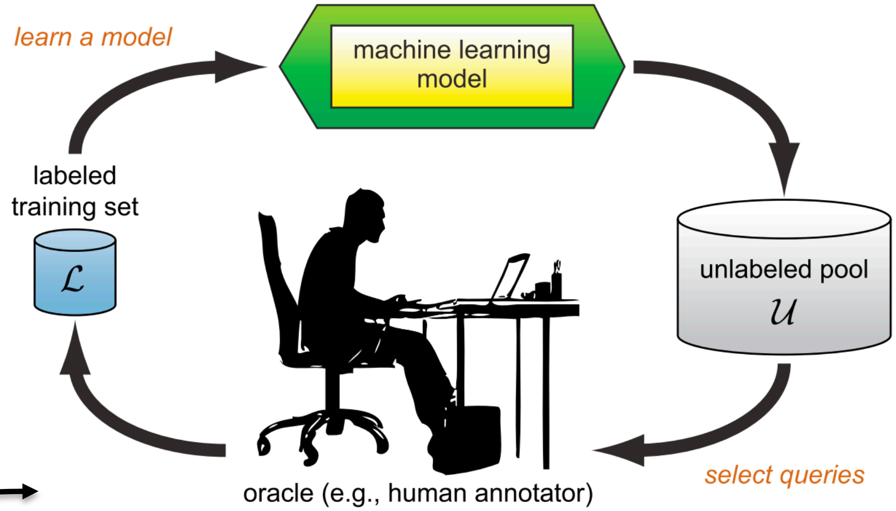
J. Friedman *Annals of Statistics* (2001)

4) K-Nearest Neighbors

E. Fix, *J.L. Hodges, (1951).*

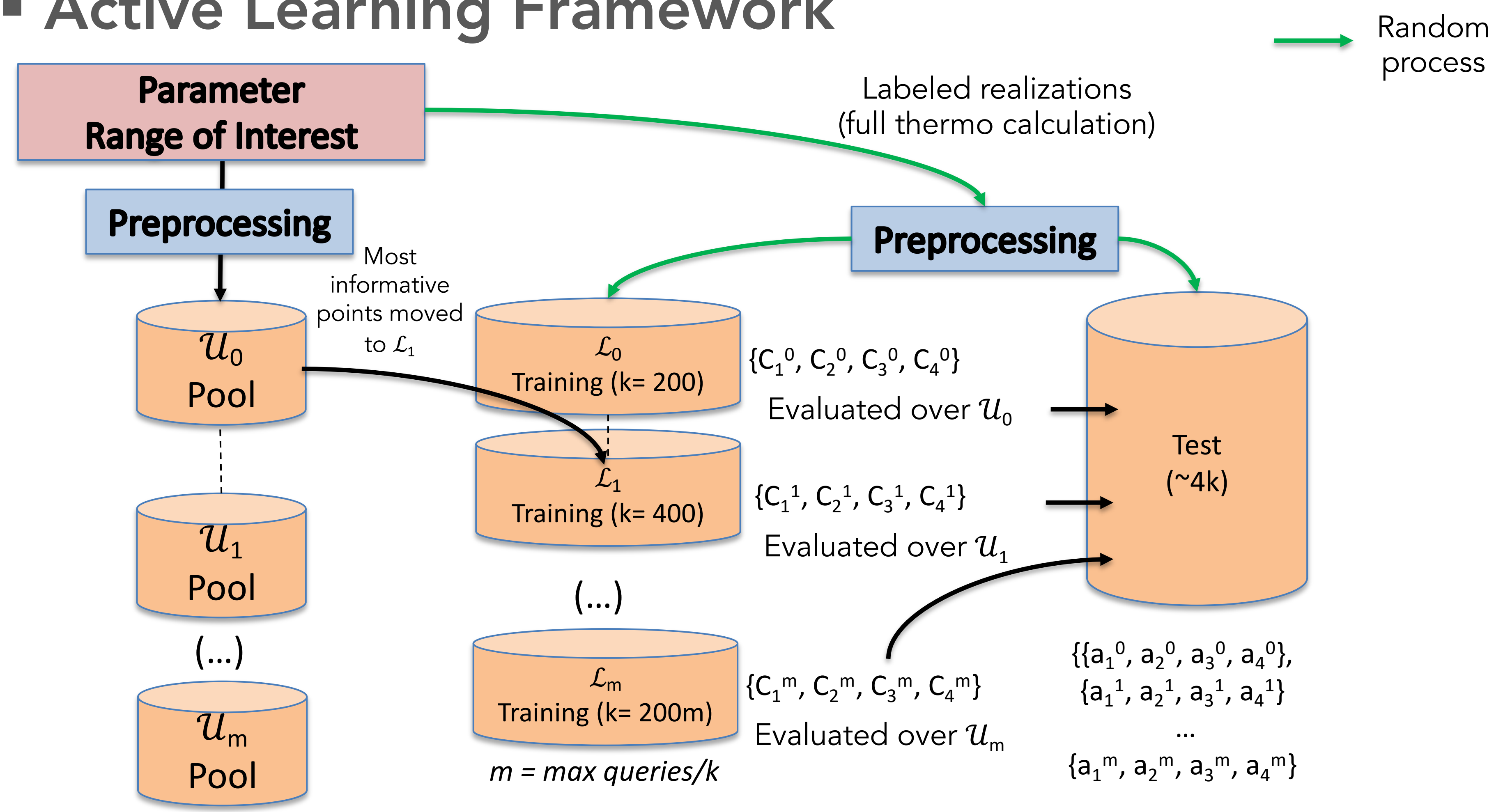
Sampling

1) Passive (random)
2) Active
   i. Margin
   ii. Entropy



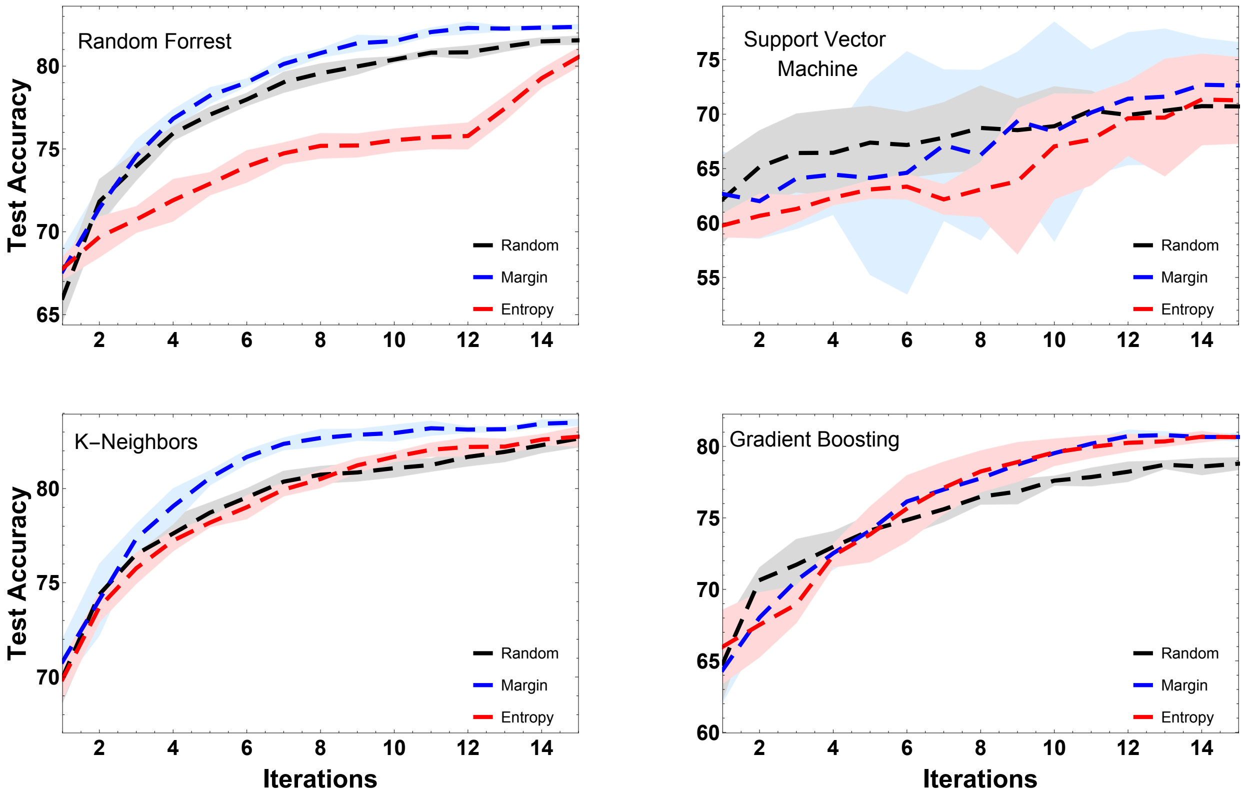
B. Settles, Active Learning Lit. Survey (2009)

- Active Learning Framework



Results

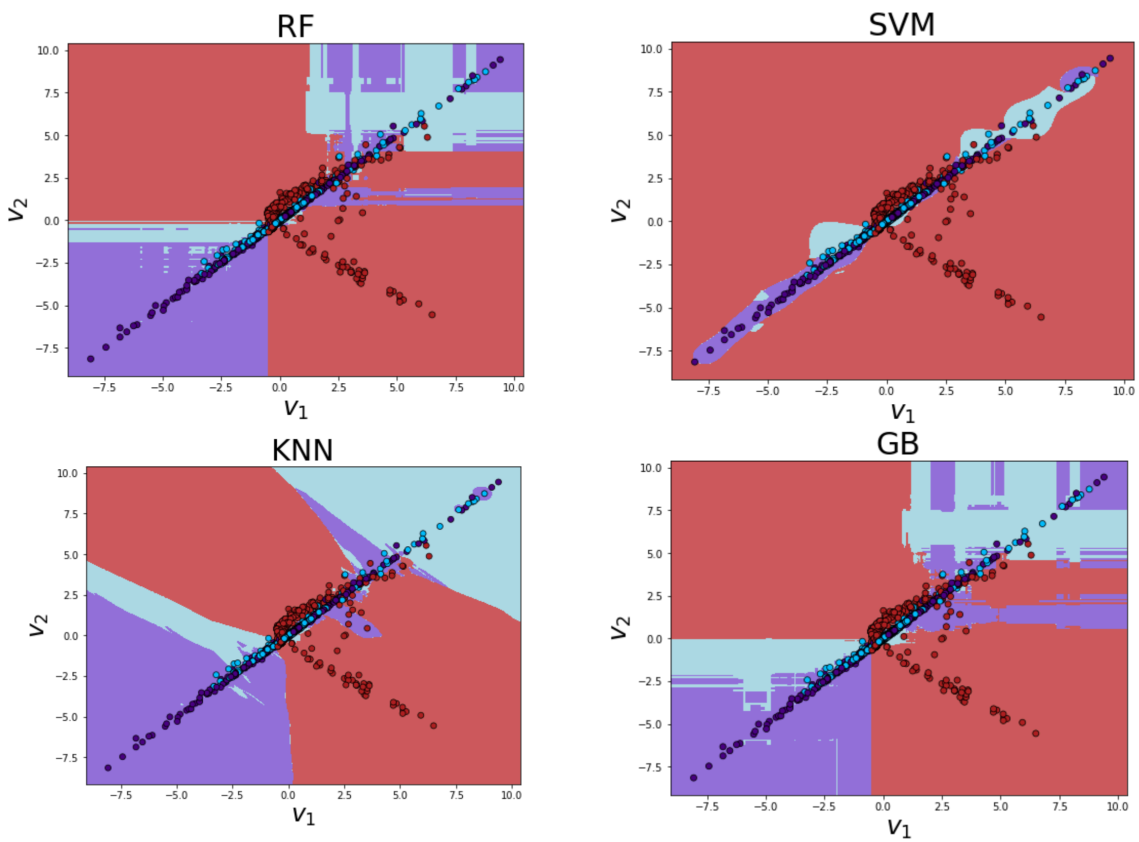
Same process is repeated 5x per model per sampling method.



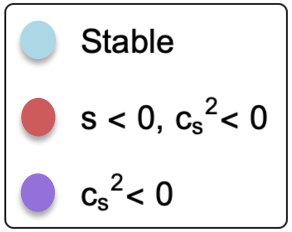
- Active learning outperforms random sampling.
- Non-parametric classifiers perform better → irregular boundaries

Test accuracy evolution with training set expansion with 1-σ uncertainty bands.

▪ 2D projected class boundaries



- Consistent features across different classifiers → **class separation is present**
- Success in creating a mechanism to classify Equations of State



Conclusions & future work

- Need better understanding of CP influence on EOS.
- We have built a tool to constrain parametrically complex models → not just BEST EOS, can be extended to higher-dimensional models
- We have demonstrated through systematic analysis that

  1) ML can be used to constrain EOS

  2) Active learning significantly cuts back on sampling requirements