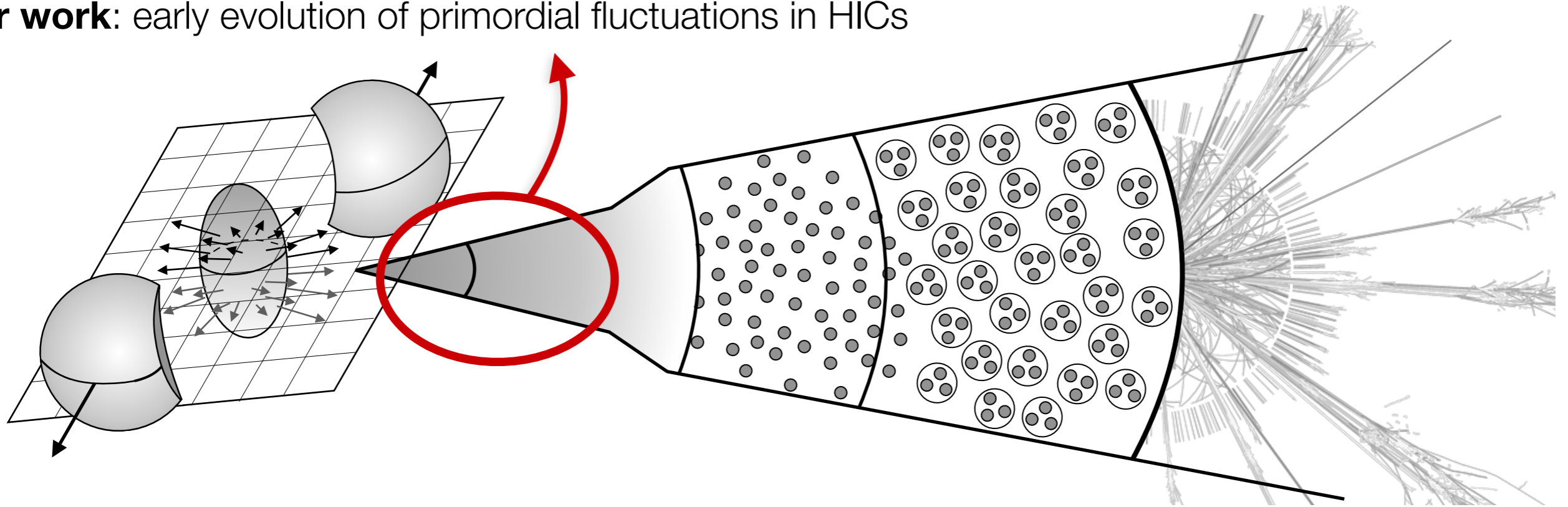


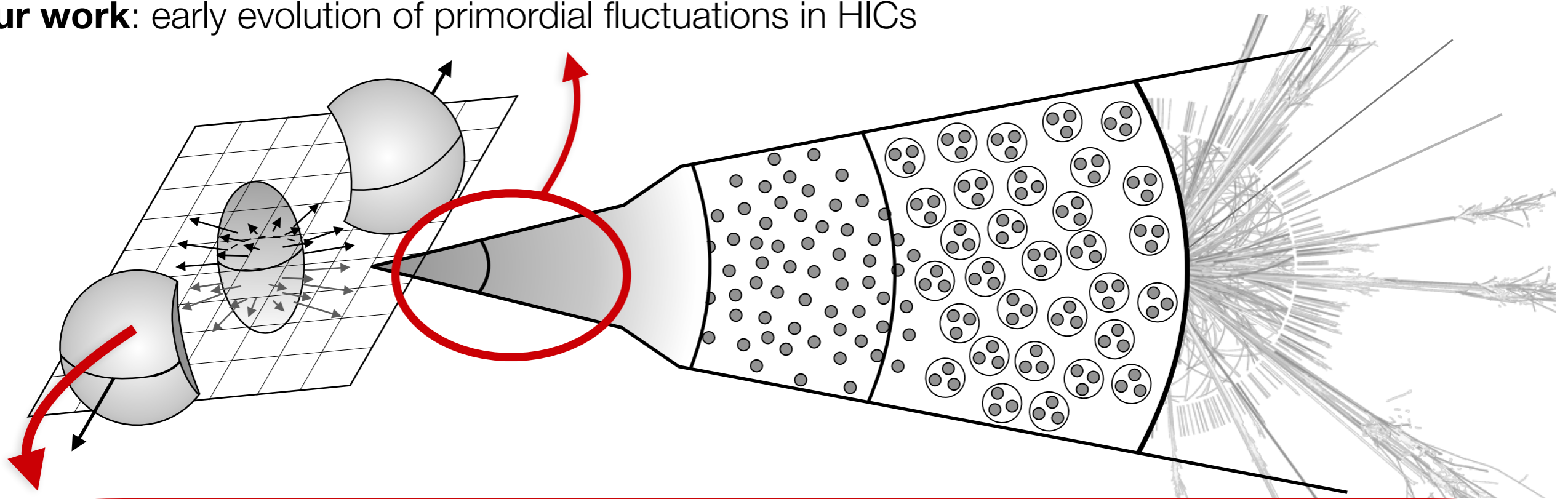


Our work: early evolution of primordial fluctuations in HICs

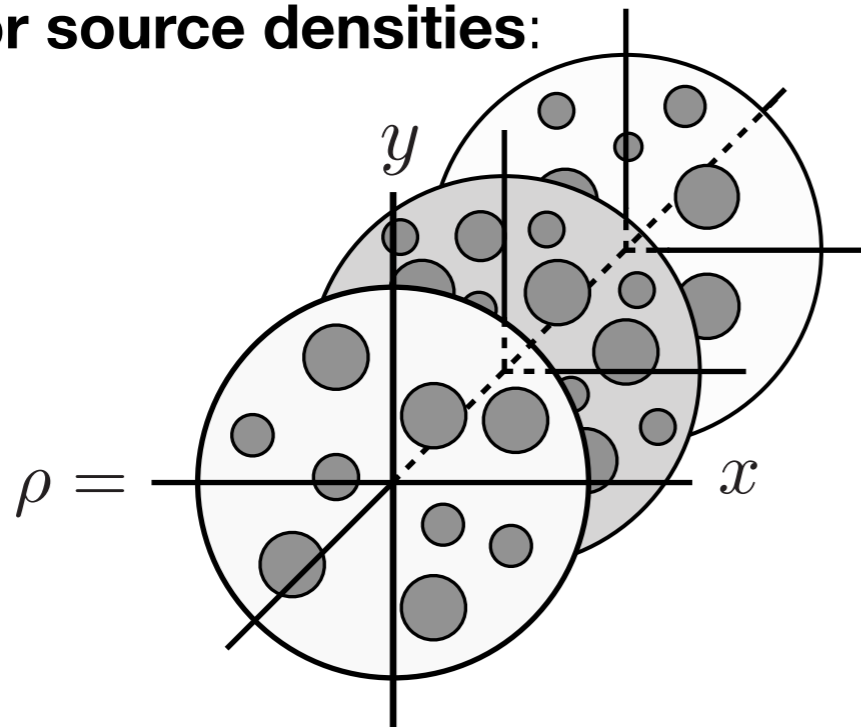




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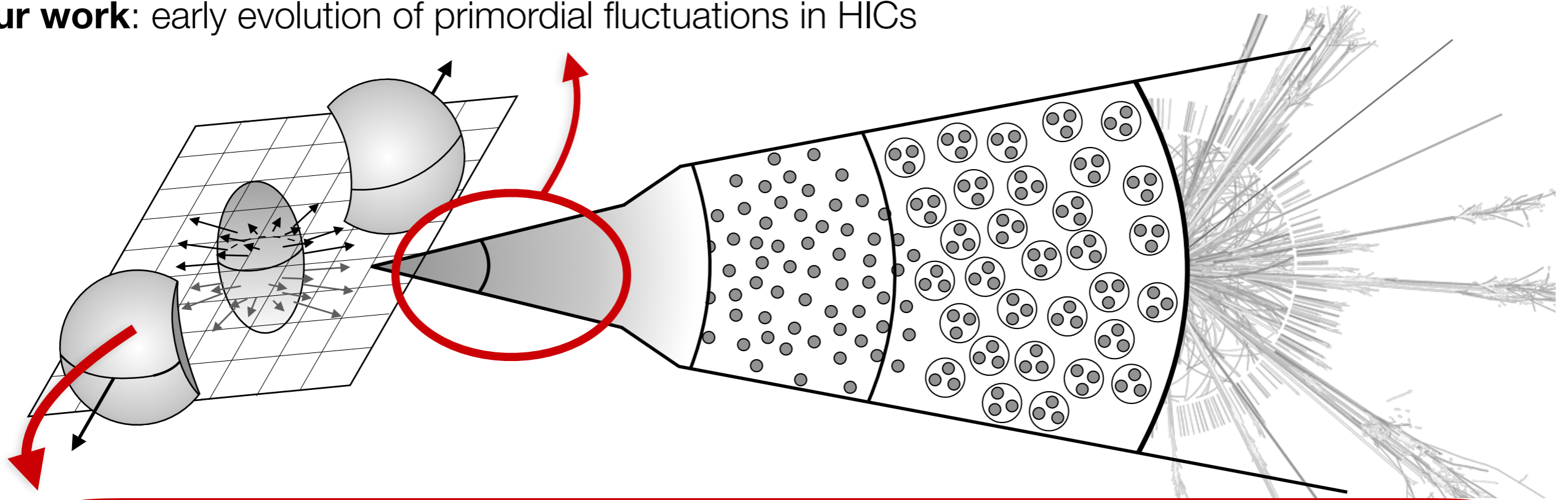
Event-by-event fluctuations of **color source densities:**



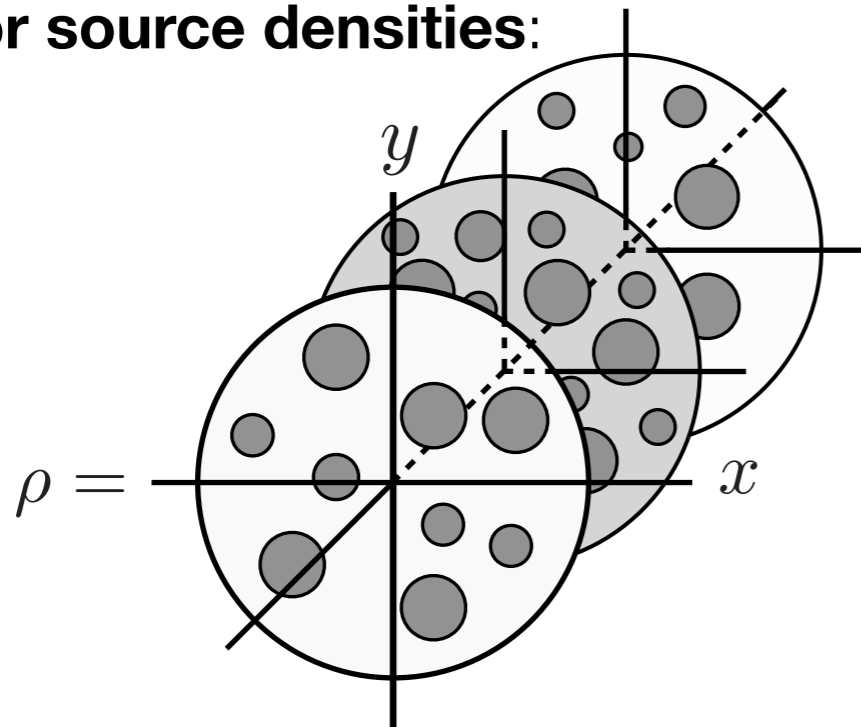


Primordial Fluctuations

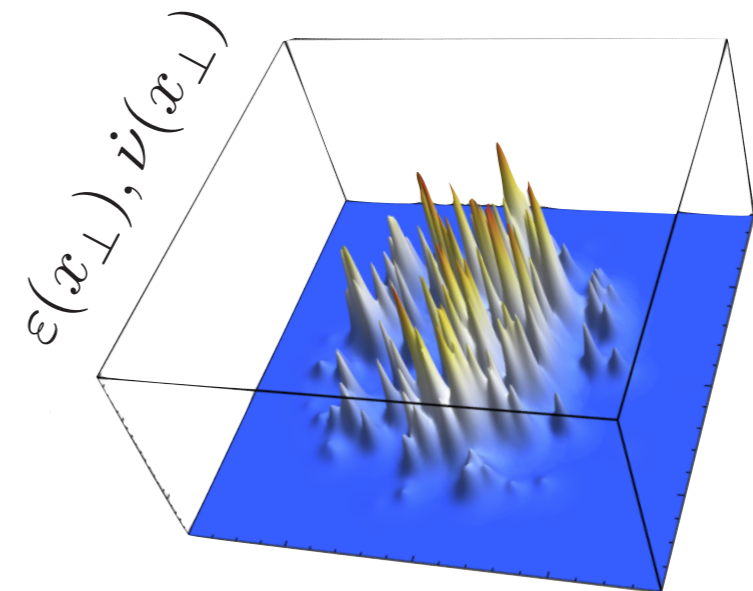
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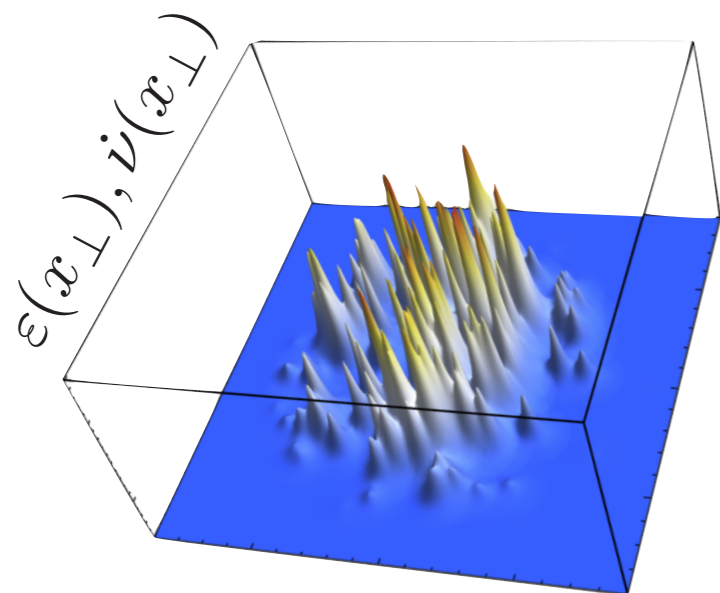
Fluctuating distributions of **energy density** and **axial charge**





CGC correlators

Fluctuating distributions of **energy density** and **axial charge** characterized by the following **correlators**:

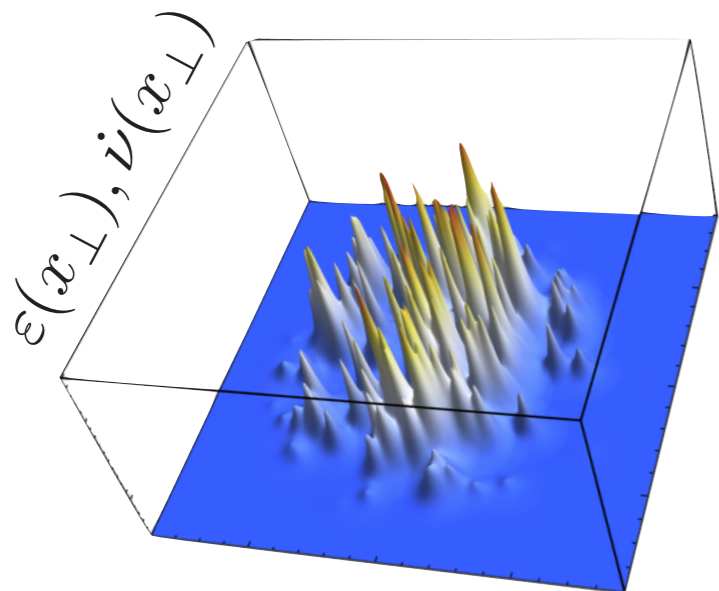


$\langle \varepsilon(x_\perp) \rangle$	}	Average/ Expectation value
$\langle \dot{\nu}(x_\perp) \rangle$		
$\langle \varepsilon(x_\perp) \varepsilon(y_\perp) \rangle$	}	Variance
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Color Glass Condensate: appropriate theoretical framework to compute these correlators in HICs

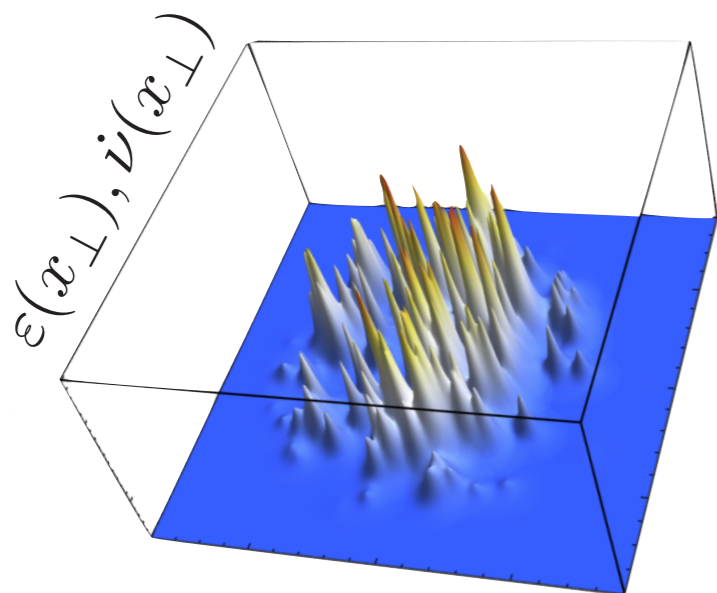
$$\langle \rho(x_\perp)\rho(y_\perp) \rangle \propto \delta(x_\perp - y_\perp)$$

Gaussian density correlations (MV model)



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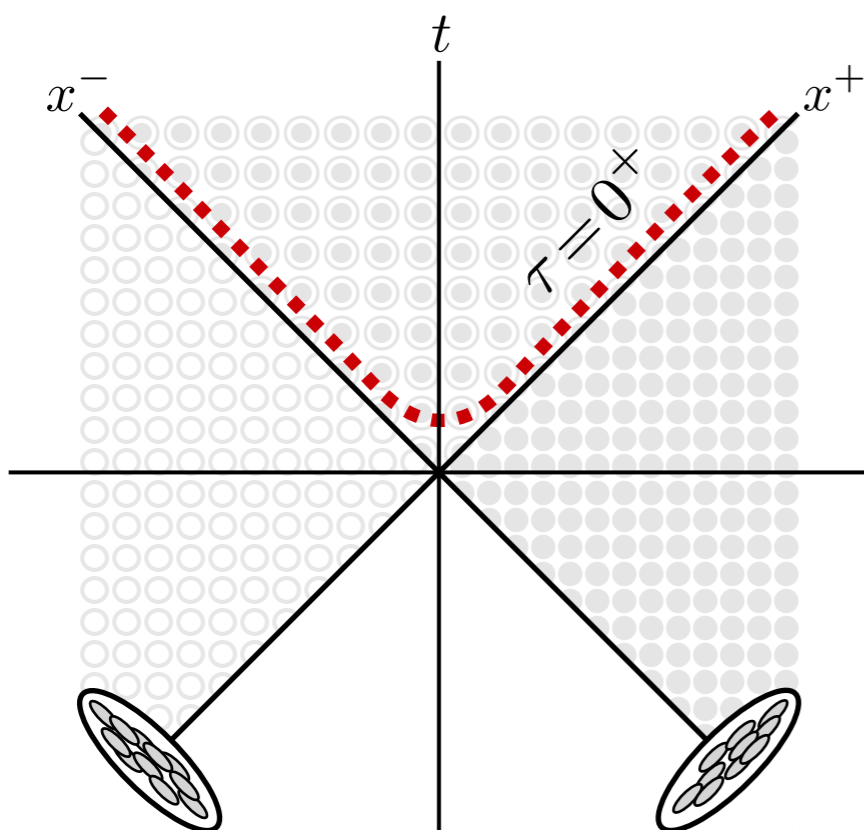
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Analytical calculations performed for the instant **right after the collision**

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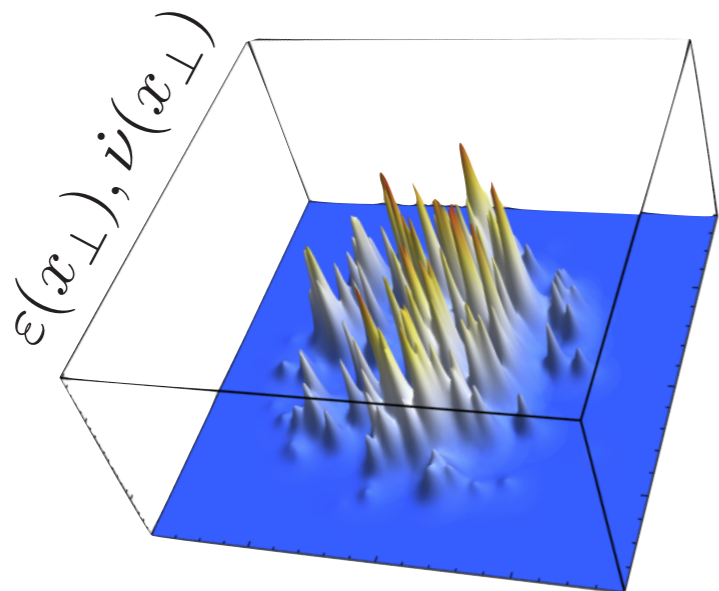
(J.L.Albacete, Cyrille Marquet and PGR, JHEP 1901 (2019) 073)
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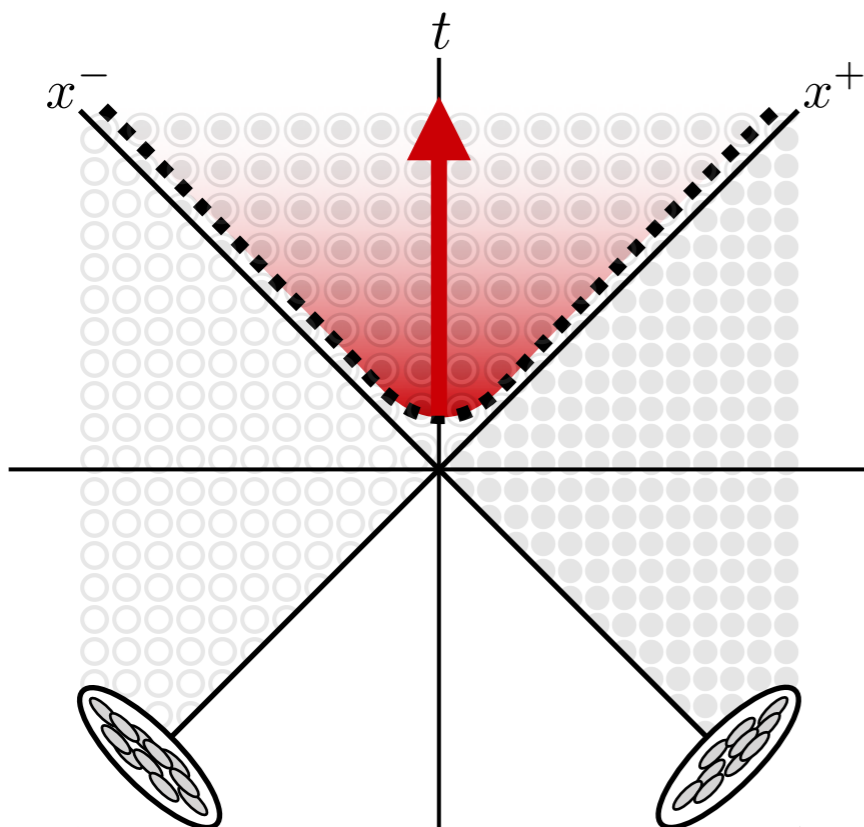
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Evolution: **classical Yang-Mills equations**
(no analytical solution)



Our approach

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Analytical approximation: **linearizing Yang-Mills equations**

$$\partial_\tau \frac{1}{\tau} \partial_\tau (\tau^2 \alpha) = \tau \partial^i \partial_i \alpha$$

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In previous approaches: **weak field limit / dilute-dense regime**

$$U(x_\perp) = 1 + igA(x_\perp)$$

Wilson lines corresponding to weak source are expanded

(A. Kovner, L. McLerran, H. Weigert, *Phys.Rev.D*52 (1995) 6231-6237)

(L. McLerran, V. Skokov, [10.1016/j.nuclphysa.2016.12.011](https://arxiv.org/abs/1612.011))

(A. Dumitru, L. McLerran, *Nucl.Phys.A*700:492-508,2002)

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- Good approximation for **UV-dominated quantities** such as:

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- We compute evolution linearly, but we conserve the **non-linear features of the initial conditions** (saturation)
- Our approach is equivalent to a resummation of the contributions of high momentum modes that would make a power series expansion in proper time divergent.

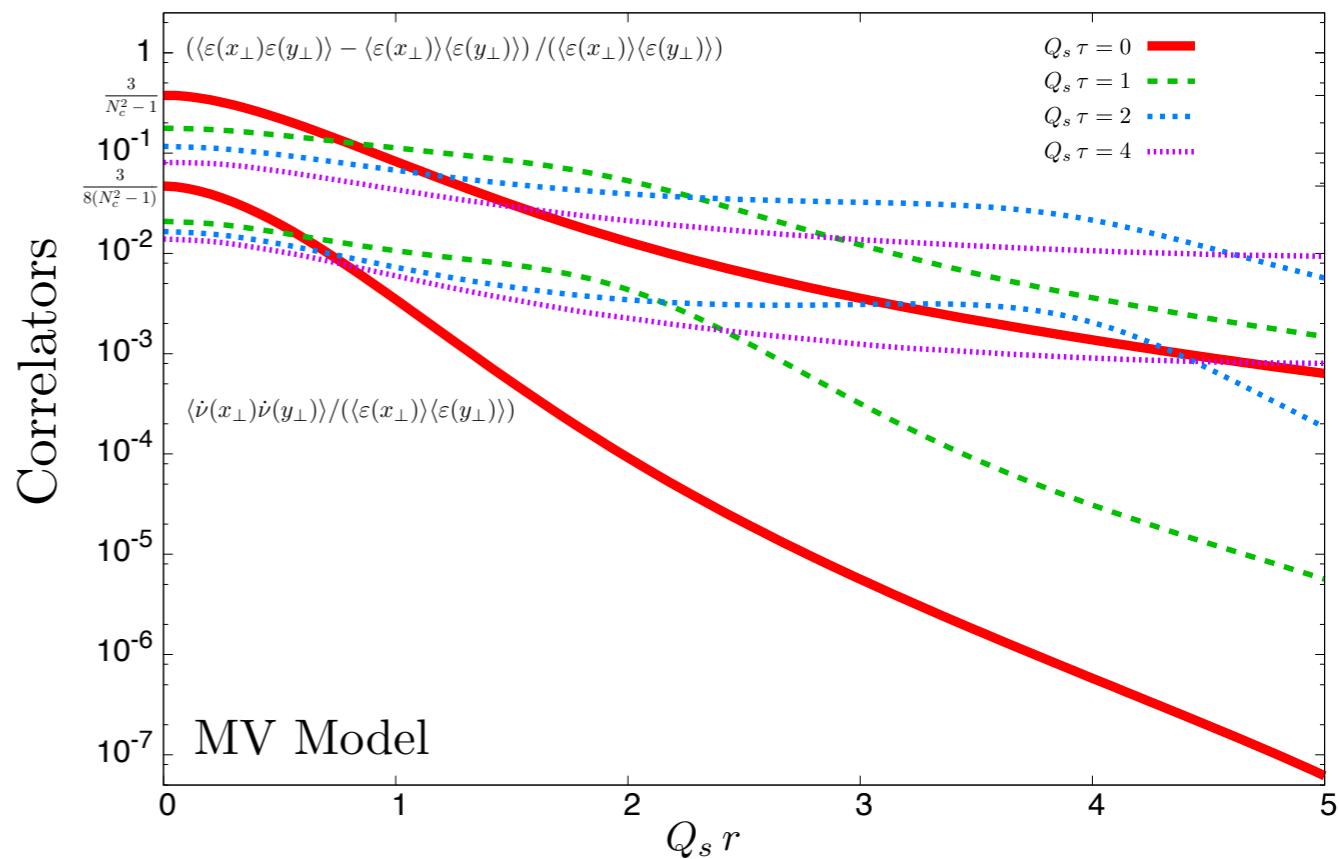
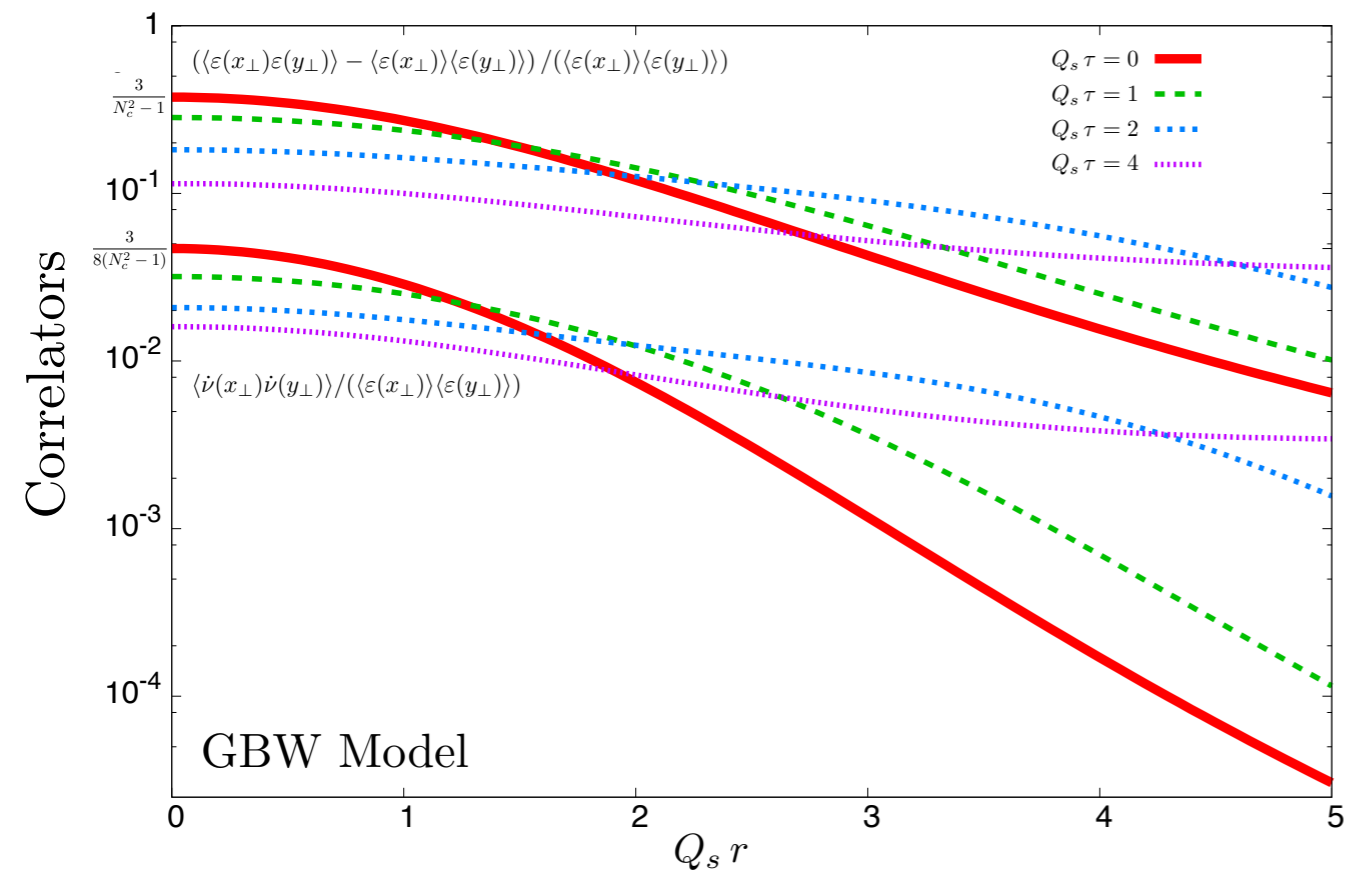
$$A(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(\vec{x}_\perp), \quad A_\perp^i(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A_{\perp(n)}^i(\vec{x}_\perp),$$

(G. Chen, R. J. Fries, J. I. Kapusta and Y. Li, Phys. Rev. C92 (2015) No.6 064912)

(H. Fujii, K. Fukushima and Y. Hidaka, Physical Review C 79 (Feb, 2009))



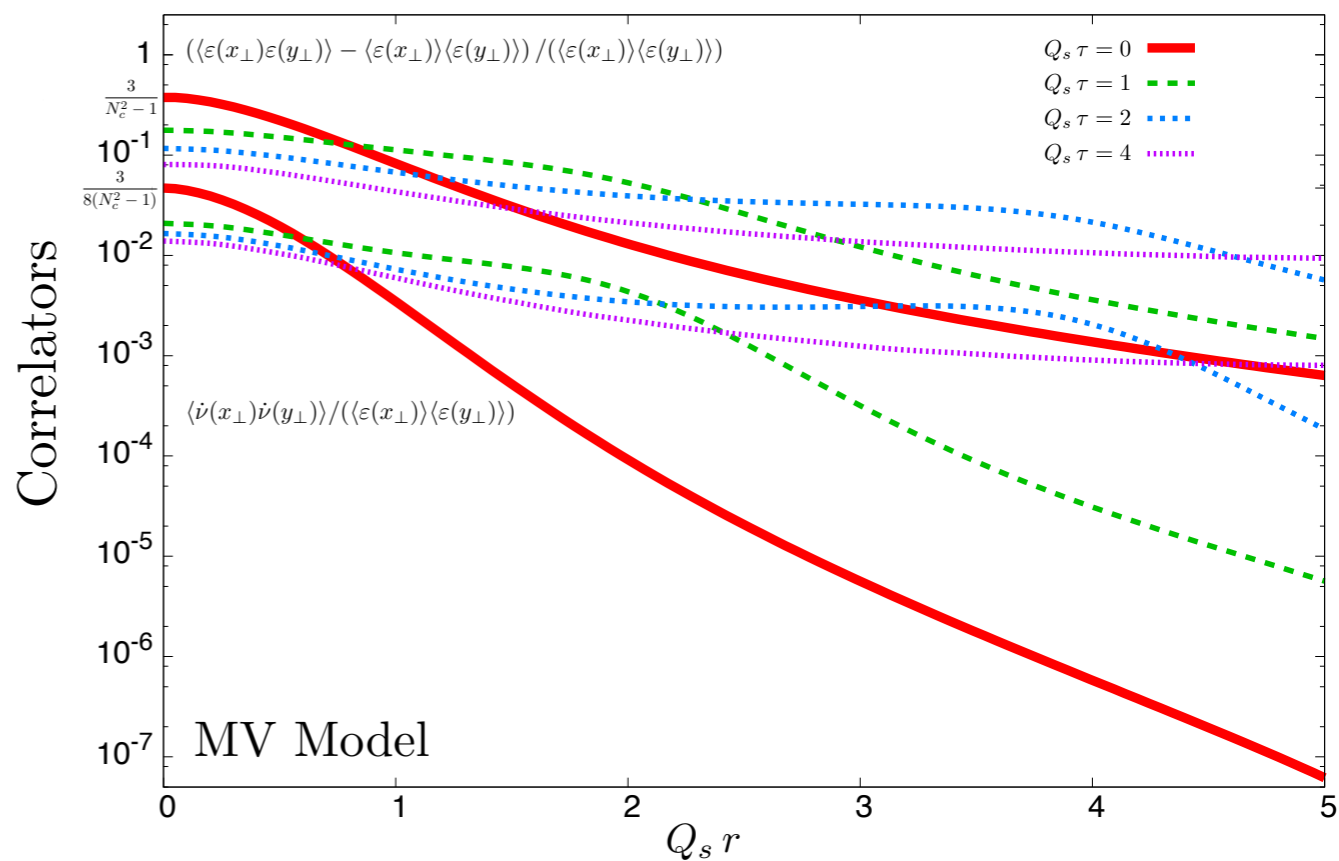
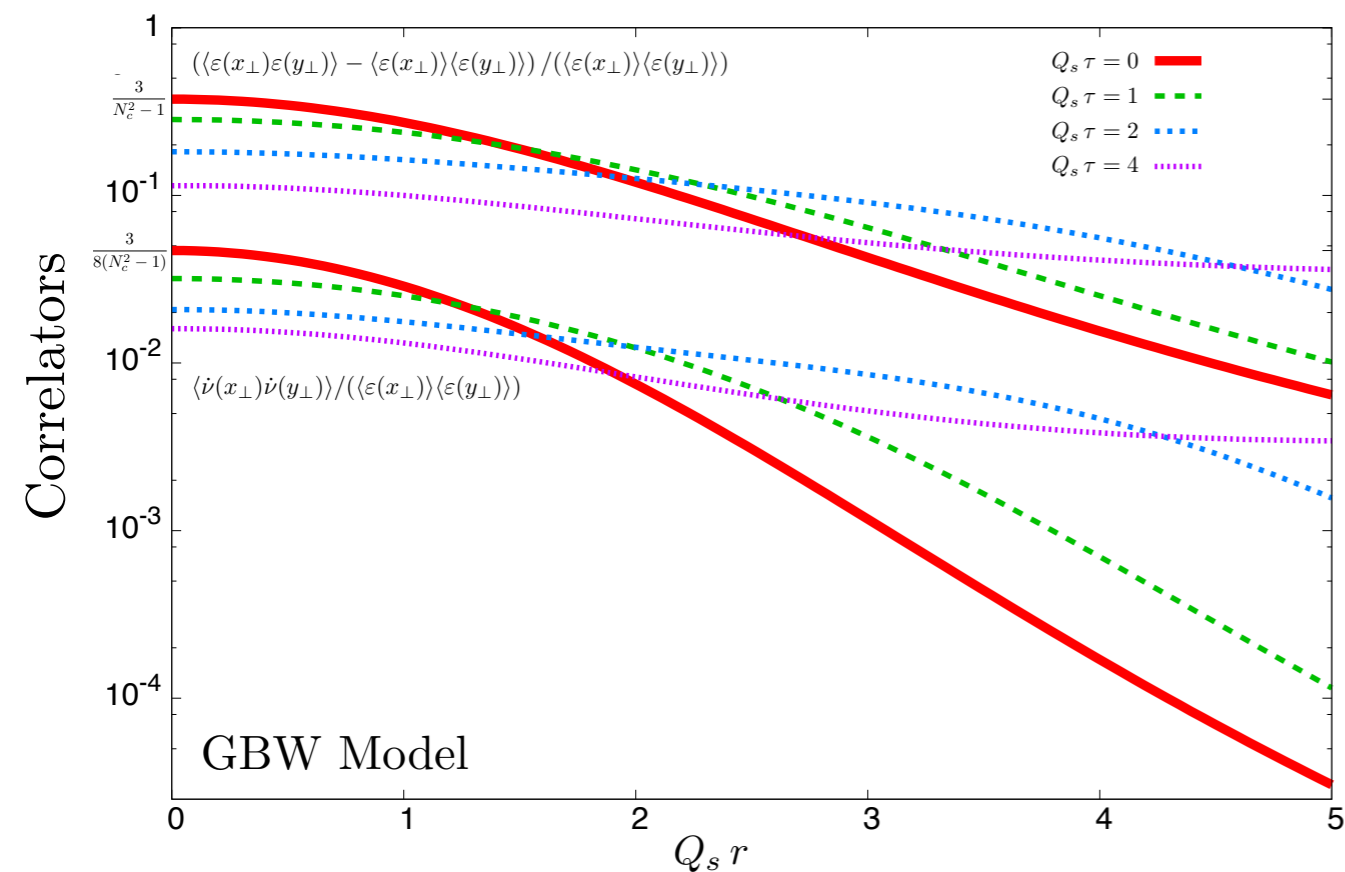
(Preliminary) results



- “Correlation length” growth: sign of a system that is approaching the hydrodynamical regime
- Theoretical insight into thermalization phase of HICs (during classical regime)
- More refined calculations (i.e. beyond Glasma Graph approximation) needed for the description of IR-sensitive quantities like eccentricity fluctuations
- Potential application: saving computing time in models that rely on numerical solutions of Yang-Mills equations



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Thank you for your attention