The dipole picture and the non-relativistic expansion

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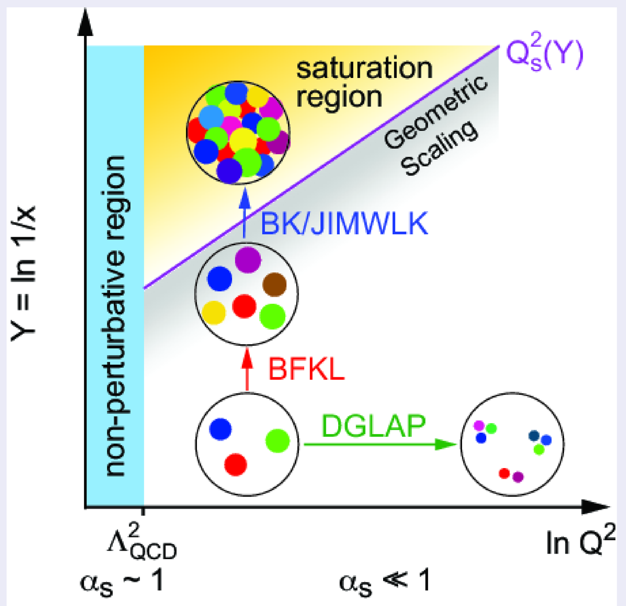
Why is exclusive quarkonium production interesting?



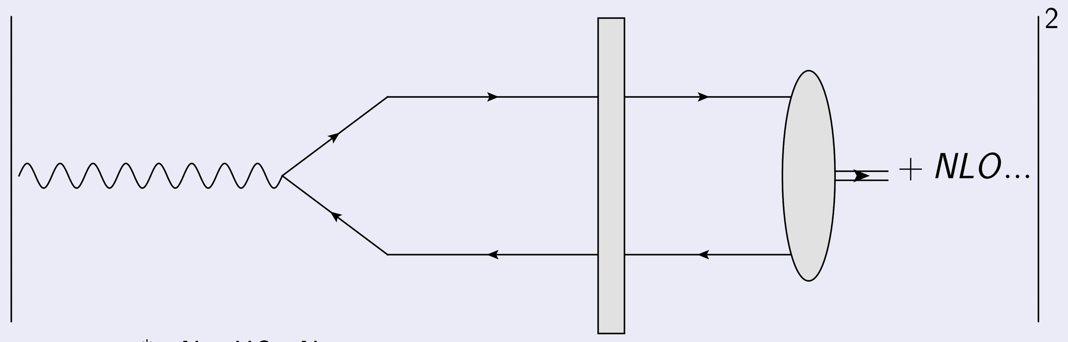
Picture taken from Marquet (2013)

Exclusive quarkonium production is an ideal way to study the gluon distribution at low x in DIS and UPCs

- Exclusive processes depend on the gluon density quadratically.
- Non-perturbative contributions are suppressed with respect to other exclusive processes.

The dipole picture

Application of light cone perturbation theory to exclusive processes



(1)

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt} = \frac{1}{16\pi}\left|\int d^2r_\perp \int_0^1 \frac{dz}{4\pi}\left(\Psi^*_{HQ}\Psi_{\gamma^*}\right)_{T,L}\sigma_{q\bar{q}}\right|^2 \quad (2)$$

- Ψ_{γ^*} is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality Q. Dominated by perturbative physics.
- $\sigma_{q\bar{q}}$ is the dipole cross-section. Contains the information about the gluon content of the nucleus and the saturation scale Q_s.
- Ψ_{HQ} is the quarkonium wave function.
- At higher orders the wave functions have to take into account the presence of gluons inside the photon and quarkonium. We also need to take into account $\sigma_{q\bar{q}g}$, $\sigma_{q\bar{q}gg}$ and so on.
- Our aim is to determine the properties of the nucleus. But in order to do this we need an accurate description of quarkonium wave function.

Non-relativistic assumption

- Widely used in the literature in other contexts, generally combined with EFT approaches (NRQCD, pNRQCD). Inclusive production, spectroscopy, decays.
- Well-defined limit of QCD. Theoretically interesting.
- It has already been used in the dipole model in its simplest form [1, 2].

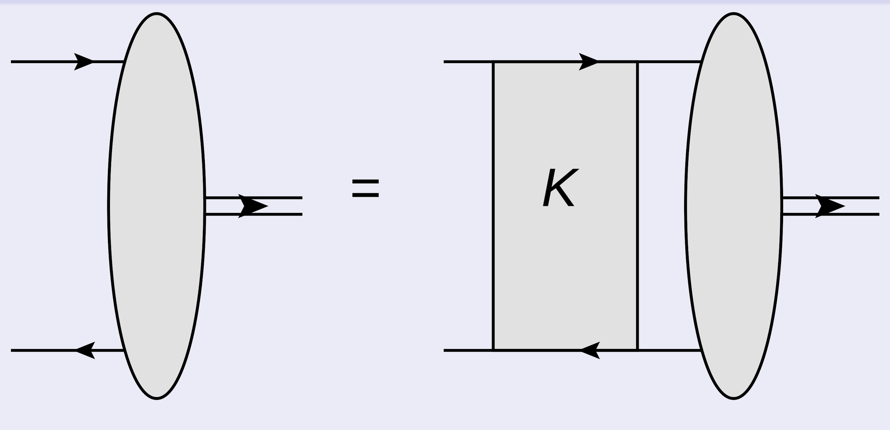
The non-relativistic limit simplifies the treatment because...

- It allows to separate the computation of scale m effects, which are pertubative.
- From the point of view of the scales smaller than m the production of heavy quarks is a local process.

Basic assumptions

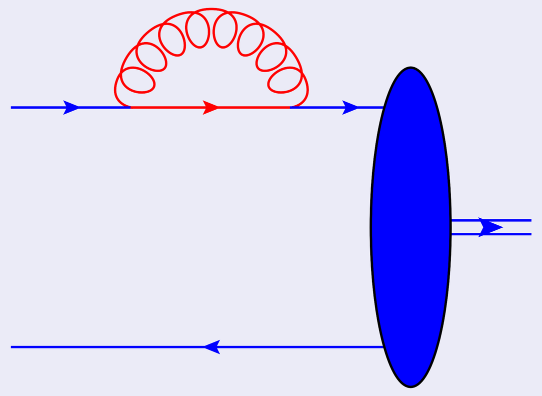
- The leading order light cone wave function can be computed taking into account only non-relativistic quarks and their interaction.
- Relativistic degrees of freedom can appear, but they are a perturbation $\to$ can appear during small times.
- Non-relativistic quarks (in light-cone perturbation theory) are defined by having a $p_\perp$ much smaller than m and a momentum fraction very close to $\frac{1}{2}$.

The leading order wave function



- Contains only non-relativistic components.
- Fulfils a Bethe-Salpeter equation which can be expressed as a Schrödinger equation.
- Relation between this and the wave function in potential models/NRQCD studied in [3].

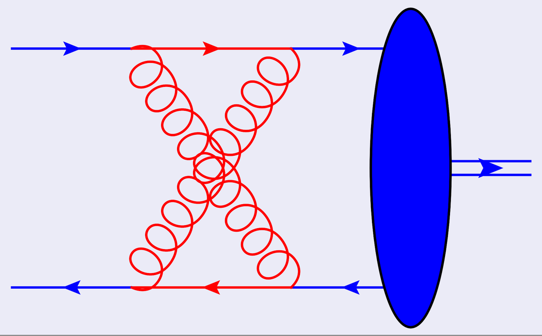
Relativistic corrections to the non-relativistic wave function



Type 1. Contribution of the relativistic degrees of freedom to the wave function renormalization of the non-relativistic quark. Computed in [4].

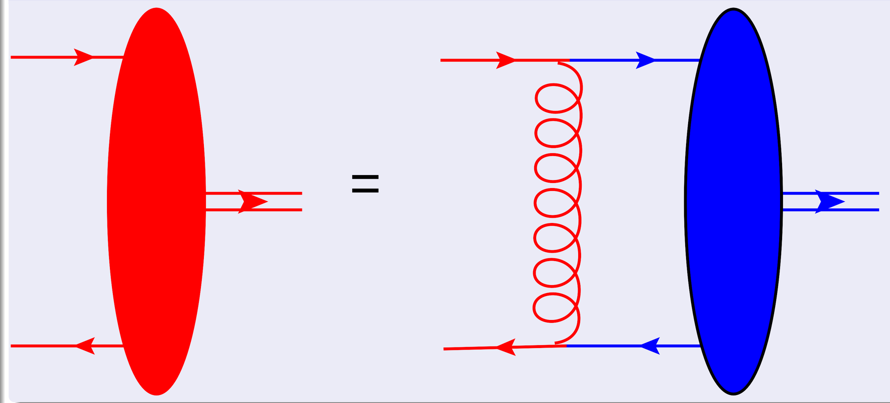
Color code

- Relativistic particles and gluons with virtuality of order m^2.
- Non-relativistic particles and softer gluons.



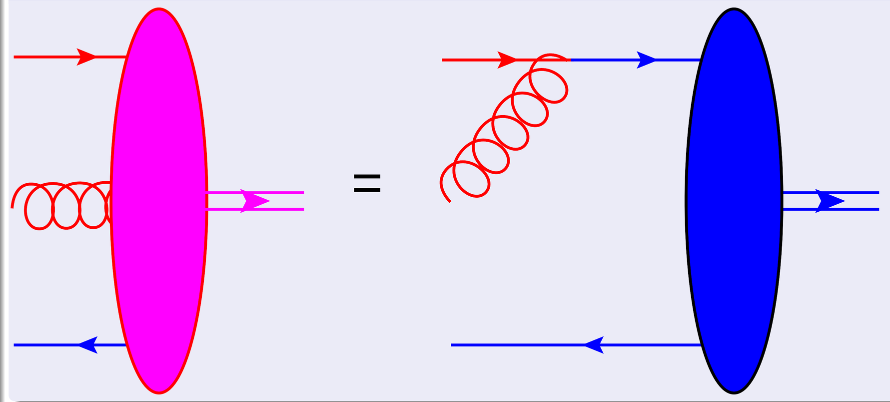
Type 2. Contributions that can be encoded as a redefinition of the potential between non-relativistic quarks.
In our case, it will not appear explicitly in our equations. However, it modifies the value of the non-relativistic wave-function at the origin.

Relativistic quark-antiquark pair component



- Can be computed in perturbation theory. $\to$ Proportional to $\alpha_s(m)$.
- Point like interaction from the point of view of non-relativistic quarks. $\to$ Proportional to the leading order non-relativistic wave function at the origin.

Relativistic quark, hard gluon and non-relativistic antiquark



- Proportional to $g(m)$.
- Cross-check. One can recover the wave-function renormalization by computing the square of this contribution.

Mathematical structure

$$\int dz f(z)\Psi^n_{HQ}(z,x_\perp) = \sum_{m,k}\int dz f(z) C^k_{n\leftarrow m}(z,x_\perp)\left(\frac{\nabla}{m}\right)^k \int \frac{d\lambda}{4\pi}\phi^m(\lambda,0) \quad (3)$$

- In this formula it is assumed that $x_\perp \sim \frac{1}{m}$ or smaller. The momentum fraction of a non-relativistic quark is $\lambda + \frac{1}{2}$ where $\lambda \ll 1$.
- ϕ represents the non-relativistic part. n and m label de components of the Fock space. For example, $C^k_{q\bar{q}\leftarrow q\bar{q}}(z,x_\perp)$ means how the $q\bar{q}$ component of the full wave function depends on the same component of the non-relativistic wave function.
- The terms in the rhs scale as v^k. Note that if $mv^2 \gg \Lambda_{QCD}$ then $v \sim \alpha_s(mv)$.
- $C^k_{n\leftarrow m}(z,x_\perp)$ can be computed as and expansion in $\alpha_s(m)$.

Power counting

- The first correction from terms with $k \neq 0$ will enter at NNLO in α_s.
- At NLO we only need to take into account $C^0_{q\bar{q}\leftarrow q\bar{q}}$ and $C^0_{q\bar{q}g\leftarrow q\bar{q}}$.
- In this power counting we did not consider the difference between $\alpha_s(m)$ and $\alpha_s(mv)$.

Cross check:Quarkonium decay into leptons

This is a quantity that can be computed knowing the light-cone wave function. It has also been computed at one loop using NRQCD and related approaches [5] (in dimensional regularization). Therefore, we know

$$\int_0^1 dz \sum_n \Psi^n_{HQ}(z,0_\perp) = \left(1 - \frac{2\alpha_s C_F}{\pi}\right)\int d\lambda\phi(\lambda,0) \quad (4)$$

However, we are not using dimensional regularization. Instead we get

$$\int_0^1 dz \Psi^{q\bar{q}}_{HQ}(z,0_\perp) = \left(1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{x_0} - 2\right)\right)\int d\lambda\phi(\lambda,0) \quad (5)$$

- We are using a cut-off x_0 to regulate the integration in z and DR to regulate the transverse component.
- Power like divergences do not appear in dimensional regularization (DM) but they can appear in our case.
- The divergence comes from the region in which $p_\perp \sim mx_0 \ll m$ and $\left|z - \frac{1}{2}\right| \sim \frac{x_0}{2} \ll 1$. Coulomb singularity.

We need that

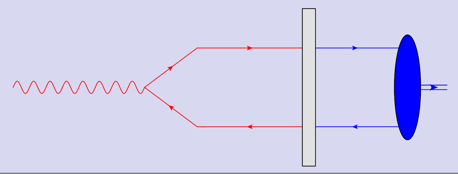
$$\frac{d}{dx_0}\int_0^1 dz \Psi^{q\bar{q}}_{HQ}(z,0_\perp) = -\frac{\alpha_s C_F}{\pi x_0^2}\int d\lambda\phi(\lambda,0) + \frac{d}{dx_0}\int d\lambda\phi(\lambda,0) = 0 \quad (6)$$

Looking at the leading order Bethe-Salpeter equation, we can check that it is indeed the case.

Exclusive quarkonium production in the $Q \gg m$ limit at NLO

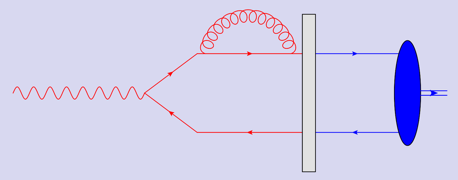
- Our final goal is to compute this process in the general case $Q \sim m$.
- For this we need the NLO photon wave function with massive quarks. This is being investigated at the moment, see talks by Beuf in HP2018 and Penttala in IS2020.
- At the moment, we check that all divergences cancel in the $Q \gg m$ limit. We focus on the simpler, longitudinal polarization case.
- We get consistent results compatible with B-JIMWLK evolution.

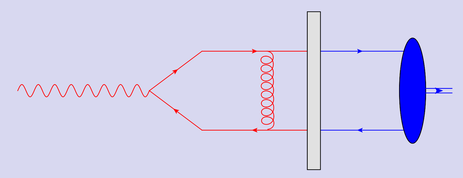
Tree level



Dependence on x_0 hidden in two terms. $\sigma_{q\bar{q}}$, which fulfils B-JIMWLK evolution, and the non-relativistic wave function, which depends on x_0 (see eq. (6)).

One loop corrections to photon wave function

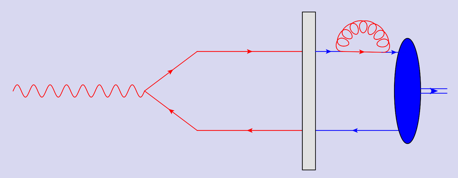


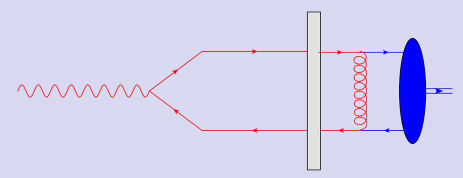


$$\Psi_\gamma(z,r_\perp)\big|_{NLO} = \Psi_\gamma(z,r_\perp)\big|_{LO}\left(1 + \delta Z_\gamma(z,r_\perp)\right)$$

Recently computed in [6]. In our case we need the value at $z = \frac{1}{2}$.

One loop corrections to quarkonium wave function





Dependence on μ

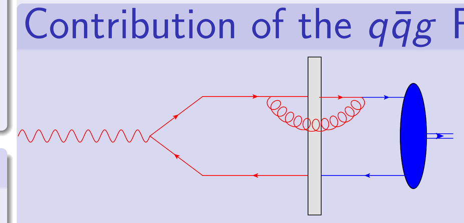
Comes only from the wave function renormalization and fulfils that $\frac{d\delta Z}{d\mu} = \frac{d\delta Z_\gamma\left(\frac{1}{2},r_\perp\right)}{d\mu}$

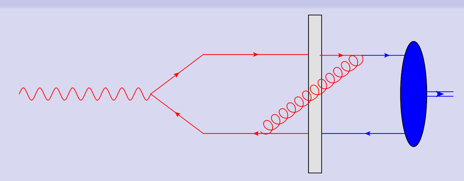
Dependence on x_0

Can be divided into two pieces:

- One which cancels the x_0 dependence of the non-relativistic wave function (eq.(6)).
- One whose derivative is proportional to $\frac{d\delta Z_\gamma\left(\frac{1}{2},r_\perp\right)}{dx_0}$.

Contribution of the $q\bar{q}g$ Fock state





Dependence on μ

Note that in the ultraviolet $\sigma_{q\bar{q}g} \to \sigma_{q\bar{q}}$. It has a divergence that cancels that of the wave functions of the photon and quarkonium.

Dependence on x_0

Can be divided in two terms:

- One proportional to $(\sigma_{q\bar{q}g} - \sigma_{q\bar{q}})$ which cancels the B-JIMWLK evolution of the target.
- One proportional to $\sigma_{q\bar{q}}$ which cancels the divergences of the wave functions of the photon and quarkonium, except for the piece related with the Coulomb singularity

Conclusions

- We have computed the NLO corrections to the quarkonium wave function in the non-relativistic limit.
- We have checked that the light-cone distribution amplitude obtained in this framework fulfils ERBL equation.
- We recovered known results for the decay of quarkonium into leptons. To our knowledge, first computation in light-cone gauge.
- We have checked that when the wave function is applied to compute exclusive quarkonium production all divergences will cancel.
- For preliminary phenomenological applications, see the talk by Penttala on Wednesday.

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