Fluctuations of anisotropic flow in transport

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Motivation

- Investigate how the number of scatterings influences the anisotropic flow for a given eccentricity.
Energy density

Characterize density by eccentricities

Initial State

MC Glauber
\[ p^\mu \partial_\mu f = C[f] \]

**Massless test particles**

**2D covariant transport algorithm by C. Gombeaud & J.-Y. Ollitrault**
[Phys. Rev. C 77, 054904]

**Elastic 2 → 2 scatterings**

**Boltzmann Transport Equation**
Event-plane Angle Distributions

\[ p(n(\Psi_n - \Phi_n)) \]

Initial state: uncorrelated

Onset of correlation with inverse \( \langle K_n \rangle \) first for \( n=2 \), then for \( n=3 \)

Faster rise of \( \Psi_2 \) with decreasing \( \langle K_n \rangle \) than for \( \Psi_3 \)
Conditional Probabilities

\[ p_v | \varepsilon \left( v_n, c/s \right) | \varepsilon_{n,c/s} \]

**Fit:**

- **Calculate moments centered around fit function in bins**
- **Average moments over eccentricity bins**

\[ v_2 = \kappa_{2,2} \varepsilon_2 + \kappa_{2,222} \varepsilon_2^3 \]

\[ v_3 = \kappa_{3,3} \varepsilon_3 \]
Conditional Probabilities $p_v|\varepsilon(V_{n,c/s}|\varepsilon_{n,c/s})$

- Initial state $\rightarrow$ Gaussian fluctuations (numerical)
- Final state $\rightarrow$ almost Gaussian fluctuations for many scatterings
- Non-Gaussian fluctuations for low number of scatterings
- Same behavior for all tested impact parameters ($b=0,6,9$ fm) and for cos/sin parts
Further information


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- Computational resources have been provided by the Bielefeld GPU Cluster and the Center for Scientific Computing (CSC) at the Goethe-University of Frankfurt.
Backup
Initial state

- Pb - Pb collisions at $\sqrt{s_{\text{NN}}} = 5.02$ TeV
- Mixed scaling: $N(x, y) = (1 - \xi) N_{\text{part}} + \xi N_{\text{coll}}$ with $\xi = 0.15$
- Smear energy density with a Gaussian:

$$R_N = \frac{1}{2} \sqrt{\frac{\sigma_{\text{NN}}^{\text{inel}}}{\pi}}$$

- Standard eccentricities:

$$\varepsilon_n e^{in\Phi_n} = -\frac{\langle r^n e^{in\theta} \rangle}{\langle r^n \rangle}$$

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Transport algorithm

- For particlization we use 2D ideal gas equations to convert $e(x, y)$ to $n(x, y)$
- Initial state momentum anisotropy due to finite test particle number: $O((2N_D)^{-1/2})$
- Reduce stat. errors with multiple runs over one initial density
- Isotropic scattering angle in center-of-momentum frame is function of impact parameter
Knudsen number

- Defines the regime of the simulation:

\[ Kn \equiv \frac{\lambda_{mfp}}{R} = \frac{1}{n_0 \sigma R} \]

- Relation for anisotropic flow and Knudsen number:

\[ v_2 = \frac{v_{2,\text{hydro}}}{1 + \frac{Kn}{Kn_0}} \quad v_3 = \frac{v_{3,\text{hydro}} (1 + B_3 \cdot Kn)}{1 + (A_3 + B_3) \cdot Kn + C_3 \cdot Kn^2} \]

Centered moments

- **Mean:**
  \[
  \mu = \frac{1}{N} \sum_{i=1}^{N} (v_{n,i}(\varepsilon_{n,i}) - \bar{v}_{n,i}(\varepsilon_{n,i}))
  \]
  fit value

- **Variance (scaled):**
  \[
  \frac{\sigma^2}{K_{n,n}^2} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{v_{n,i}(\varepsilon_{n,i}) - \bar{v}_{n,i}(\varepsilon_{n,i})}{K_{n,n}} \right)^2
  \]

- **Skewness:**
  \[
  \gamma = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{v_{n,i}(\varepsilon_{n,i}) - \bar{v}_{n,i}(\varepsilon_{n,i})}{\sigma} \right)^3
  \]

- **Kurtosis:**
  \[
  \omega = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{v_{n,i}(\varepsilon_{n,i}) - \bar{v}_{n,i}(\varepsilon_{n,i})}{\sigma} \right)^4
  \]

i runs over events in one eccentricity bin
Outlook

- Analysis of $v_4$ fluctuations $\rightarrow$ more challenging due to $v_4 = K_{4,4} \varepsilon_4 + K_{4,22} \varepsilon_2^2$
- $p_T$- dependence of fluctuations
- Allowing for fluctuations in the impact parameters $\rightarrow$ Centrality bins