

# Fluctuations of anisotropic flow in transport

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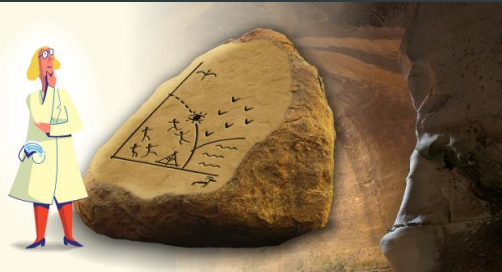
[arXiv:2012.02138](https://arxiv.org/abs/2012.02138)



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IS2021

The VI<sup>th</sup> International Conference on the  
INITIAL STAGES  
OF HIGH-ENERGY NUCLEAR  
COLLISIONS



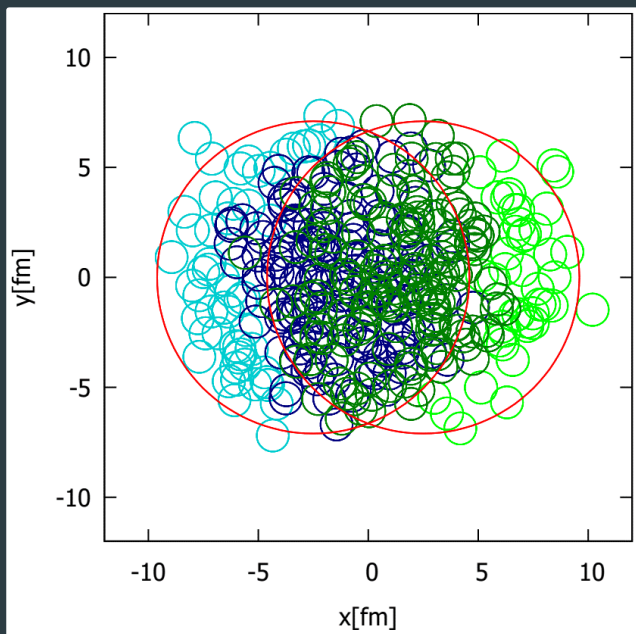
CRC-TR 211  
Strong-interaction matter  
under extreme conditions

# Motivation

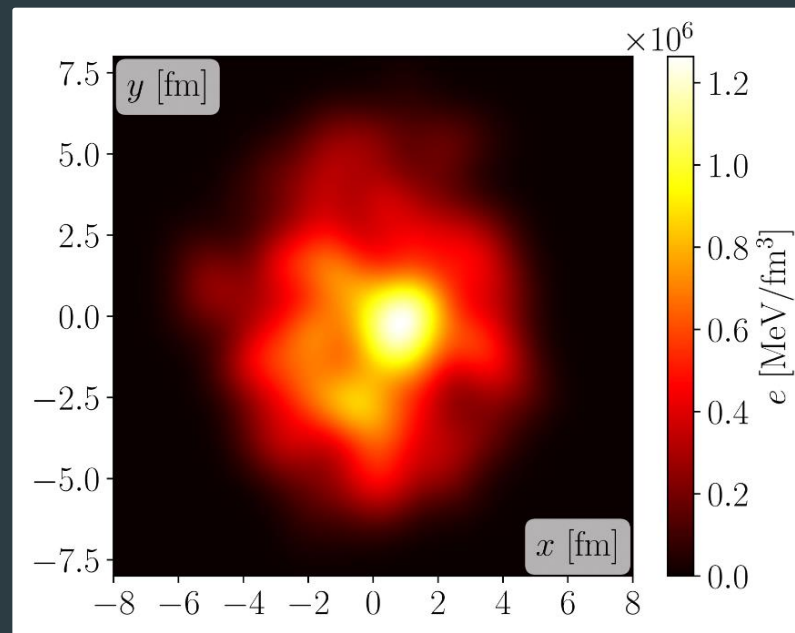
- ▶ Investigate how the number of scatterings influences the anisotropic flow for a given eccentricity



MC Glauber



Energy density



Initial  
State

Characterize density by  
eccentricities

$$p^\mu \partial_\mu f = \mathcal{C}[f]$$

Massless test  
particles

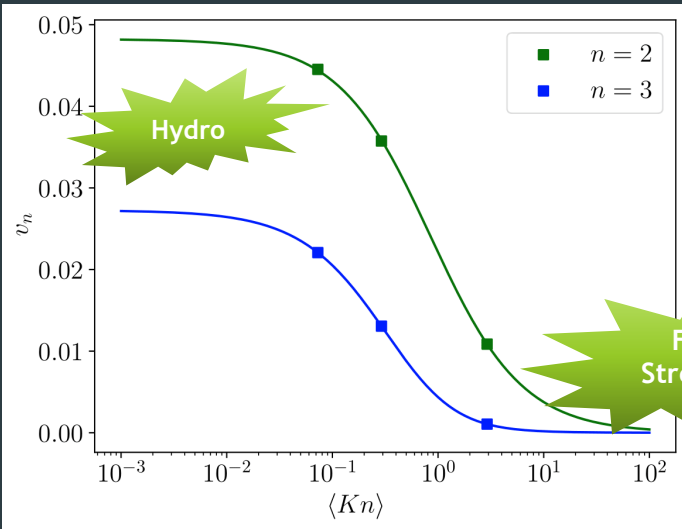
2D covariant transport  
algorithm by C. Gombeaud  
& J.-Y. Ollitrault  
[[Phys. Rev. C 77, 054904](#)]

Elastic 2 → 2  
scatterings

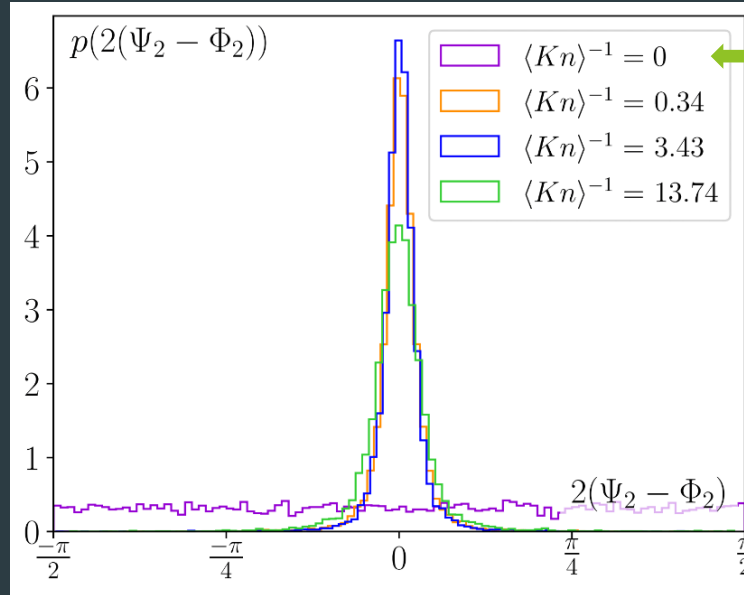
Boltzmann  
Transport  
Equation

# Event-plane Angle Distributions

$$p(n(\Psi_n - \Phi_n))$$

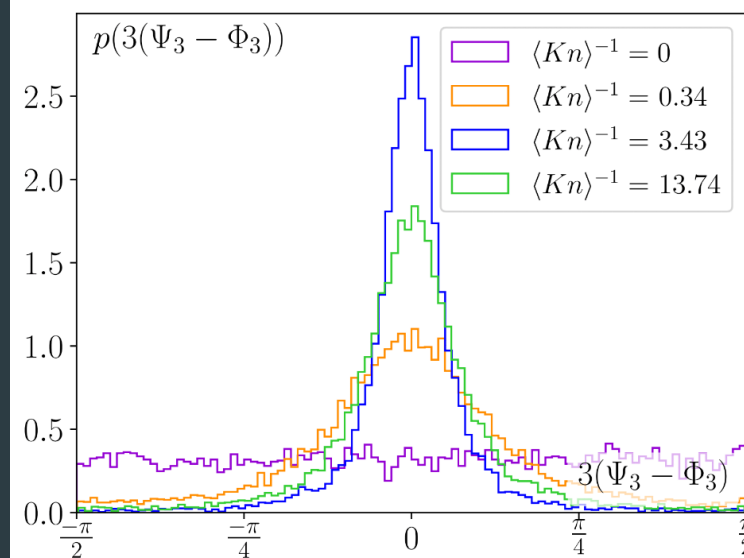


Faster rise of  $v_2$  with decreasing  $\langle Kn \rangle$  than for  $v_3$



Initial state: uncorrelated

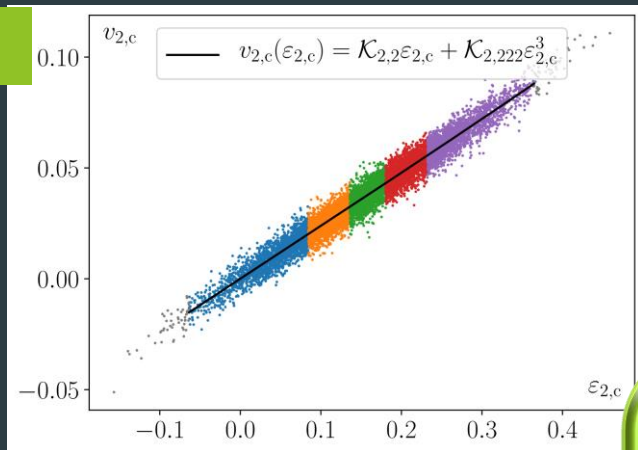
Onset of correlation with inverse  $\langle Kn \rangle$  first for  $n=2$ , then for  $n=3$



# Conditional Probabilities

$$p_{v|\varepsilon}(v_{n,c}/s | \varepsilon_{n,c}/s)$$

Fit:



$$v_2 = \mathcal{K}_{2,2}\varepsilon_2 + \mathcal{K}_{2,222}\varepsilon_2^3$$

$$v_3 = \mathcal{K}_{3,3}\varepsilon_3$$

Conditional  
Probability  
Distribution

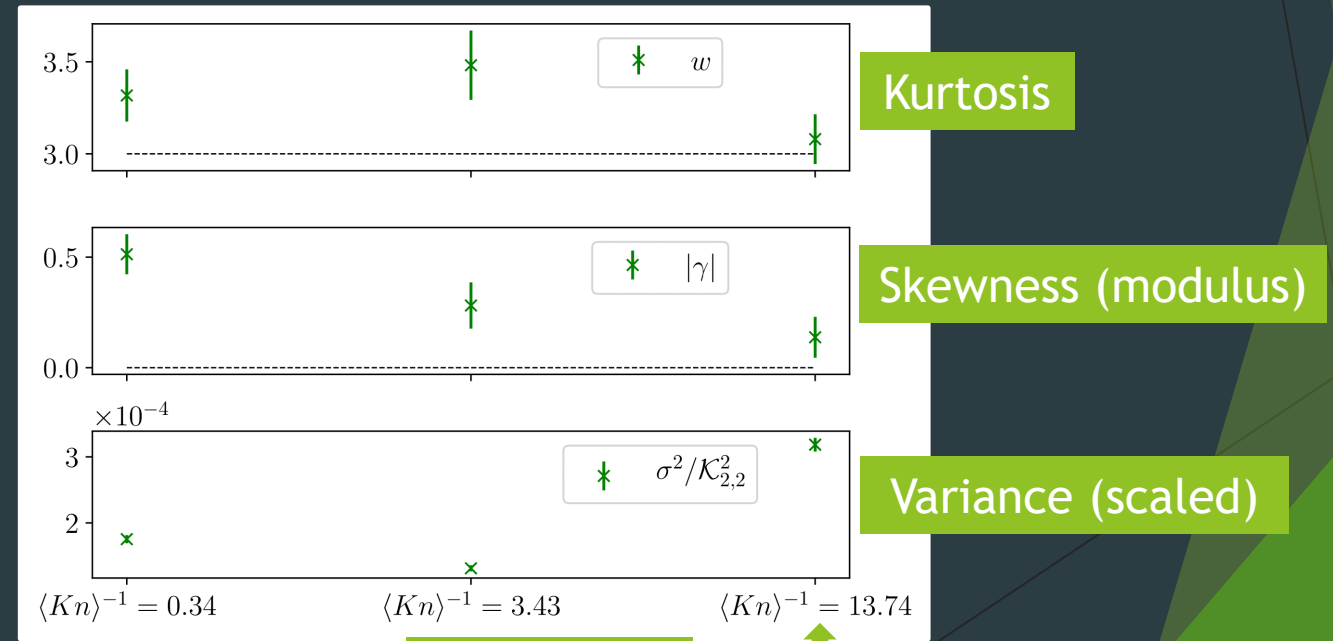
Calculate moments  
centered around fit  
function in bins

Average moments  
over eccentricity bins

# Conditional Probabilities $p_{v|\varepsilon}(v_{n,c/s}|\varepsilon_{n,c/s})$

- ▶ Initial state → Gaussian fluctuations (numerical)
- ▶ Final state → almost Gaussian fluctuations for many scatterings
- ▶ Non-Gaussian fluctuations for low number of scatterings
- ▶ Same behavior for all tested impact parameters (b=0,6,9 fm) and for cos/sin parts

Conditional Probability Distribution

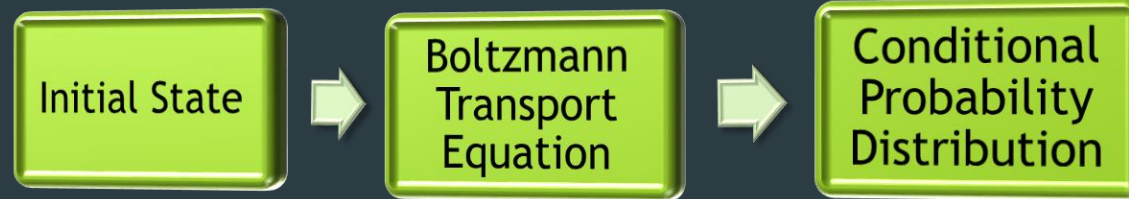


Hydro limit



# Further information

- ▶ arXiv:2012.02138 [nucl-th]



- ▶ We acknowledge support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the CRC-TR 211 'Strong-interaction matter under extreme conditions' - project number 315477589 - TRR 211.
- ▶ Computational resources have been provided by the Bielefeld GPU Cluster and the Center for Scientific Computing (CSC) at the Goethe-University of Frankfurt.



**Backup**

# Initial state

- ▶ Pb - Pb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV
- ▶ Mixed scaling:  $N(x, y) = (1 - \xi)N_{\text{part}} + \xi N_{\text{coll}}$  with  $\xi = 0.15$
- ▶ Smear energy density with a Gaussian:

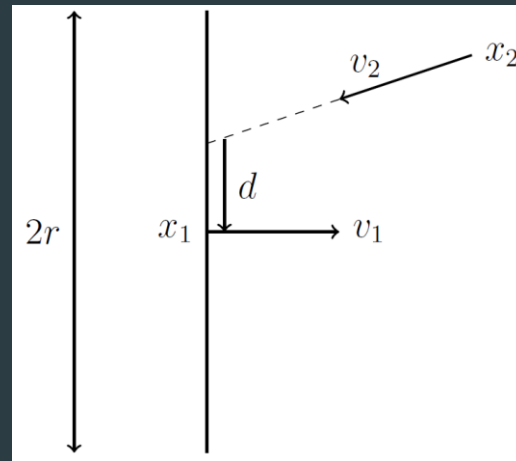
$$R_N = \frac{1}{2} \sqrt{\frac{\sigma_{\text{inel}}^{\text{NN}}}{\pi}}$$

- ▶ Standard eccentricities:

$$\varepsilon_n e^{in\Phi_n} = -\frac{\langle r^n e^{in\theta} \rangle}{\langle r^n \rangle}$$

# Transport algorithm

- ▶ For particlization we use 2D ideal gas equations to convert  $e(x, y)$  to  $n(x, y)$
- ▶ Initial state momentum anisotropy due to finite test particle number:  $\mathcal{O}((2N_p)^{-1/2})$
- ▶ Reduce stat. errors with multiple runs over one initial density
- ▶ Isotropic scattering angle in center-of-momentum frame is function of impact parameter



# Knudsen number

- Defines the regime of the simulation:

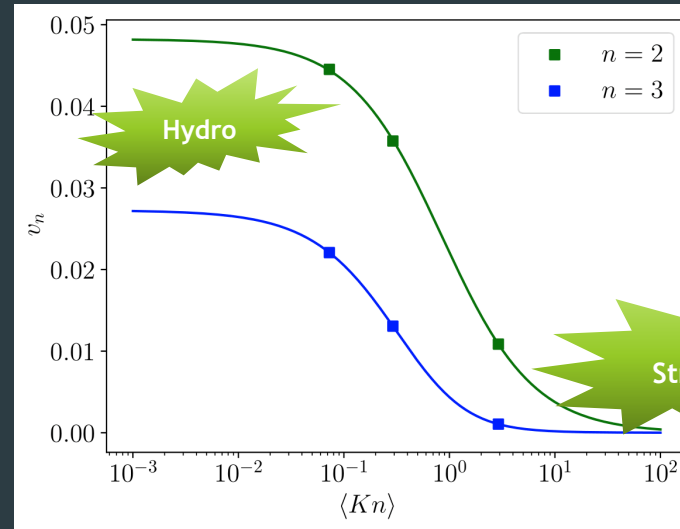
$$Kn \equiv \frac{\lambda_{\text{mfp}}}{R} = \frac{1}{n\sigma R}$$

- Relation for anisotropic flow and Knudsen number:

$$v_2 = \frac{v_2^{\text{hydro}}}{1 + \frac{Kn}{Kn_0}} \quad v_3 = \frac{v_2^{\text{hydro}} (1 + B_3 \cdot Kn)}{1 + (A_3 + B_3) \cdot Kn + C_3 \cdot Kn^2}$$

[Phys. Lett. B 627 (2005) 49]

[Phys. Rev. C 82 (2010) 034913]



# Centered moments

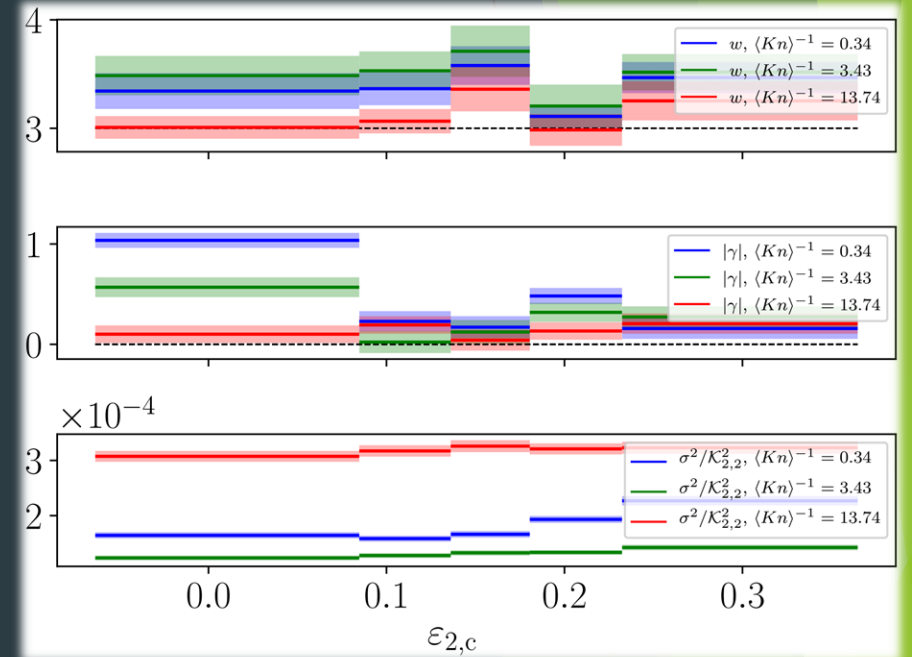
► Mean: 
$$\mu = \frac{1}{N} \sum_{i=1}^N (v_{n,i}(\varepsilon_{n,i}) - \underbrace{\bar{v}_{n,i}(\varepsilon_{n,i})}_{\hat{=} \text{fit value}})$$

► Variance (scaled): 
$$\frac{\sigma^2}{\mathcal{K}_{n,n}^2} = \frac{1}{N} \sum_{i=1}^N \left( \frac{v_{n,i}(\varepsilon_{n,i}) - \bar{v}_{n,i}(\varepsilon_{n,i})}{\mathcal{K}_{n,n}} \right)^2$$

► Skewness: 
$$\gamma = \frac{1}{N} \sum_{i=1}^N \left( \frac{v_{n,i}(\varepsilon_{n,i}) - \bar{v}_{n,i}(\varepsilon_{n,i})}{\sigma} \right)^3$$

► Kurtosis: 
$$w = \frac{1}{N} \sum_{i=1}^N \left( \frac{v_{n,i}(\varepsilon_{n,i}) - \bar{v}_{n,i}(\varepsilon_{n,i})}{\sigma} \right)^4$$

i runs over events in one eccentricity bin



# Outlook

- ▶ Analysis of  $v_4$  fluctuations → more challenging due to  $v_4 = \mathcal{K}_{4,4}\varepsilon_4 + \mathcal{K}_{4,22}\varepsilon_2^2$
- ▶  $p_T$  - dependence of fluctuations
- ▶ Allowing for fluctuations in the impact parameters → Centrality bins