Dynamical evolution of eletromagnetic field in out-of-equilibrium Quark-Gluon Plasma

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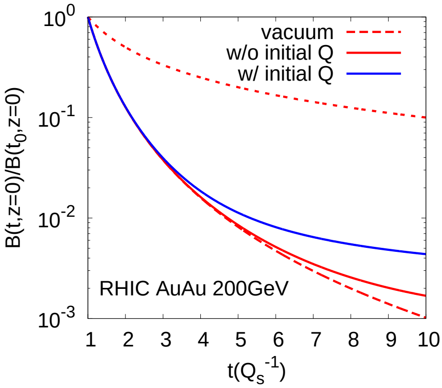
Work with Xu-Guang Huang

Highlights of the talk

- Goal: We solve dynamical evolution of $\vec{B}$ field in the early stages of realistic heavy-ion collisions, by coupling $\vec{B}$ field evolution and the thermalization process of QGP. We consider the weakly coupled QGP scenario, $\alpha_s \sim 0.2$.

- It is likely that the residual $\vec{B}$ field is less than 1% of initial field strength when hydro starts.



Coupled equations

$$\begin{cases} \partial_\mu A^{\mu\nu} = j^\nu \,, & \text{Maxwell} \\ \frac{1}{p^0}\left[p^\mu \partial_\mu + eQ_q p^\mu F^{\mu\nu}\partial_{p^\nu}\right] f_q = -\mathcal{C}[f_q]\,, & \text{Boltzmann} \end{cases}$$

$$\text{where } f_q = \overbrace{\bar{f}_q}^{\text{background}} + \overbrace{\delta f_q}^{\text{change due to EB}} \,,$$

$$j^\mu = e\sum_F Q_F s_F \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_p} p^\mu \underbrace{(f_q^F - f_{\bar{q}}^F)}_{g^F = \delta f_q^F - \delta f_{\bar{q}}^F}$$

* $g^F = \delta f_q^F - \delta f_{\bar{q}}^F$ difference in q and $\bar{q}$ due to coupling to EB.
* $\bar{f}_q = \bar{f}_{\bar{q}}$ because of QCD symmetry.

Simplification of the coupled equations

$$\frac{1}{p^0}p^\mu\partial_\mu g^F + \frac{1}{p^0}eQ_q p^\mu F^{\mu\nu}\partial_{p^\nu}[2\bar{f}_q + \delta f_q^F + \delta f_{\bar{q}}^F] = -\mathcal{C}[g^F]$$

$$\Rightarrow \quad \frac{1}{p^0}p^\mu\partial_\mu g^F + \underbrace{\frac{2}{p^0}eQ_q p^\mu F^{\mu\nu}\partial_{p^\nu}\bar{f}_q}_{\hat{\Gamma}[\bar{f}_q]} = \mathcal{O}(\delta f) \qquad (*)$$

- Assumptions and facts about Eq.(*):

1 EB-field has little effect on thermalization process of QGP:

$$|\delta f| \ll \bar{f}$$

2 If $\bar{f}_q$ highly anisotropic, $\hat{\Gamma}[\bar{f}]$ is sizable $\leftrightarrow$ very early stages.

3 Therefore,

$$\frac{1}{p^0}p^\mu\partial_\mu g^F = -\hat{\Gamma}[\bar{f}] + \underbrace{\mathcal{O}(\delta f)}_{\text{neglected}}$$

Configuration background QGP and EB-field

- Background QGP with respect to Bjorken symmetry,

$$(t, z) \quad \Leftrightarrow \quad (\tau, \xi), \qquad \text{no dep. on } \mathbf{x}_\perp$$

$$\text{hence, } \bar{f}_q(t, z, \mathbf{p})(= \bar{f}_{\bar{q}}(t, z, \mathbf{p})) \quad \Leftrightarrow \quad \bar{f}_q(\tau, \mathbf{p})$$

- EB-field does not obey Bjorken symmetry:

$$A^\mu(t, z) = (0, A^x(t, z), 0, 0) \quad \Rightarrow \quad \begin{cases} E_x = -\partial_t A^x \\ B_y = \partial_z A^x \end{cases}$$

We are able to study EB-field at $\vec{x}_\perp = 0$.

Background QGP: Boltzmann & 2-2 scatterings

$$D_t f^a_{\mathbf{p}} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f^a_{\mathbf{p}} = \mathcal{C}[f^a_{\mathbf{p}}]$$

for very early stages, dominated by $2 \leftrightarrow 2$ scatterings in QCD

$$\begin{aligned}\mathcal{C}[f^a_{\mathbf{p}}] =& \frac{1}{2E_p \nu_a} \sum_{b,c,d} \frac{1}{s_{cd}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}} \frac{d^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}} \frac{d^3\mathbf{k}'}{(2\pi)^3 2E_{\mathbf{k}'}} \\ & \times (2\pi)^4 \delta^{(4)}(P + P' - K - K') |\mathcal{M}^{ab}_{cd}|^2 \\ & \times \left[f^c_{\mathbf{k}} f^d_{\mathbf{k}'} (1 + \epsilon_a f^a_{\mathbf{p}})(1 + \epsilon_b f^b_{\mathbf{p}'}) - f^a_{\mathbf{p}} f^b_{\mathbf{p}'} (1 + \epsilon_c f^c_{\mathbf{k}})(1 + \epsilon_d f^d_{\mathbf{k}'})) \right] ,\end{aligned}$$

where $|\mathcal{M}|^2 \ni gg \leftrightarrow q\bar{q},\ gq \leftrightarrow gq,\ g\bar{q} \leftrightarrow g\bar{q},\ gg \leftrightarrow gg$

Initial conditions, parameters, etc.

- Background EB-field from two colliding nuclei,

$$A^x \sim \left[\frac{\tilde{z}+\tilde{v}\tilde{t}}{(\tilde{b}^2/4+\gamma^2(\tilde{z}+\tilde{v}\tilde{t})^2)^{1/2}} + \frac{\tilde{z}-\tilde{v}\tilde{t}}{(\tilde{b}^2/4+\gamma^2(\tilde{z}-\tilde{v}\tilde{t})^2)^{1/2}}\right]$$

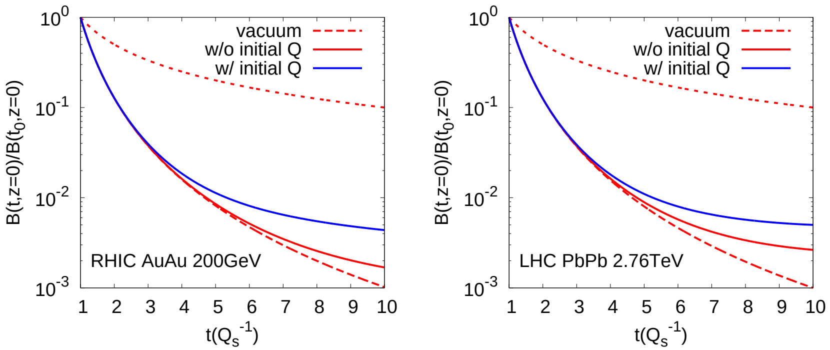
- Background QGP: CGC inspired initial quark distribution,

Romatschke, Strickland

$$f_q(t=t_0, z=0, \mathbf{p}) f_0^q \Theta\left(1 - \frac{\sqrt{p_z^2\xi^2+p_\perp^2}}{Q_s}\right)$$

  * We have approximately $\alpha_s \sim 0.2$.
  * We take $f_0^q = 1$ as an optimistic initialization. ($f_0^q = 0$?)
    cf.1601.03576 (Gelfand, Hebenstreit, Berges)
  * We take f_0^g according to the realistic mid-central AuAu ($\sqrt{S_{NN}} = 0.2$ GeV) and PbPb ($\sqrt{S_{NN}} = 2.76$ TeV).

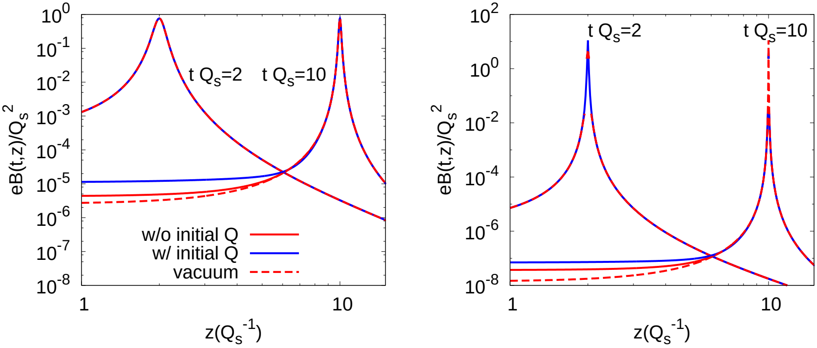
B-field evolution ($z = 0$)



- Dotted line: Ideal Bjorken MHD: $\sigma \to \infty$ and $B(\tau) \sim 1/\tau$

PLB 750(2015)45-52(V.Roy et. al)

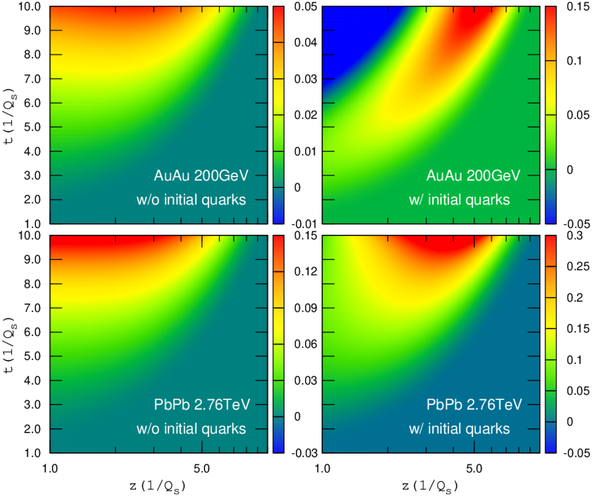
B-field evolution at fixed time



When hydro starts:

- RHIC AuAu (left): $eB \sim 10^{-5} Q_s^2 \sim 10\ \mathrm{MeV}^2$
- LHC PbPb (right): $eB \sim 10^{-7} Q_s^2 \sim 0.1\ \mathrm{MeV}^2$

Effective conductivity in the out-of-equilibrium QGP?



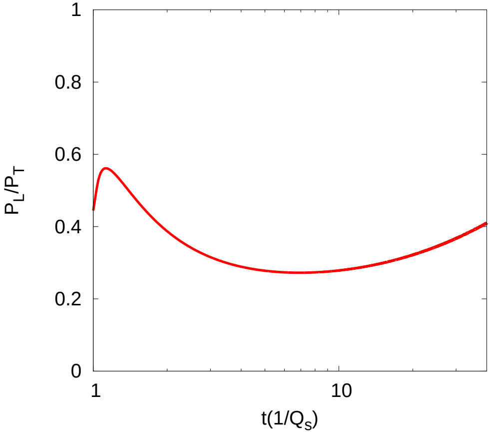
Effective conductivity: $\sigma_{\text{eff}} = \dfrac{J_x}{E_x}$

Summary and discussions

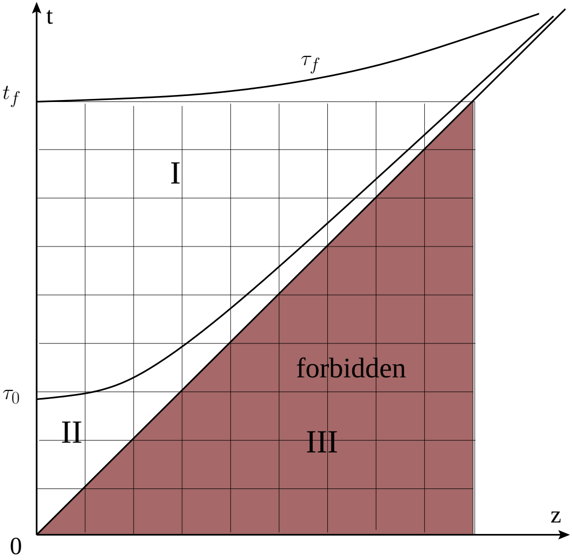
- We have solved the coupled evolution of QGP and EB-field.
- These calculations are done according to the realistic PbPb and AuAu in experiments at RHIC and the LHC.
- The remaining B field from the pre-equrilibrium stage before hydro could be weak, even though QGP is a conducting medium.

Back-up slides

Backgound QGP evolution



- Gluon population taken: mid-central LHC PbPb 2.76TeV.
- System is away from equilibrium as expected.


t
t_f
τ_f
I
forbidden
τ_0
II
III
z
0