Dynamical evolution of electromagnetic field in out-of-equilibrium Quark-Gluon Plasma

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2021. 01. 12, Initial stages 2021

Work with Xu-Guang Huang
Highlights of the talk

- **Goal:** We solve dynamical evolution of $\vec{B}$ field in the early stages of realistic heavy-ion collisions, by coupling $\vec{B}$ field evolution and the thermalization process of QGP. We consider the weakly coupled QGP scenario, $\alpha_s \sim 0.2$.

- It is likely that the residual $\vec{B}$ field is less than 1% of initial field strength when hydro starts.
Coupled equations

\[ \begin{cases} 
\partial_\mu A^{\mu\nu} = j^\nu, & \text{Maxwell} \\
\frac{1}{p^0} [p^\mu \partial_\mu + eQ_q p^\mu F^{\mu\nu} \partial_\nu] f_q = -C[f_q], & \text{Boltzmann}
\end{cases} \]

where \( f_q = \bar{f}_q + \delta f_q \),

\[ j^\mu = e \sum_F Q_F s_F \int \frac{d^3p}{(2\pi)^3 E_p} p^\mu (f_q^F - f_{\bar{q}}^F) \]

* \( g^F = \delta f_q^F - \delta f_{\bar{q}}^F \) difference in \( q \) and \( \bar{q} \) due to coupling to EB.
* \( \bar{f}_q = \bar{f}_{\bar{q}} \) because of QCD symmetry.
Simplification of the coupled equations

\[
\frac{1}{p^0} p^\mu \partial_\mu g^F + \frac{1}{p^0} e Q_q p^\mu F^{\mu \nu} \partial_\nu [2 \bar{f}_q + \delta f^F_q + \delta f^{F}_q] = -C[ g^F ]
\]

\[
\Rightarrow \quad \frac{1}{p^0} p^\mu \partial_\mu g^F + \frac{2}{p^0} e Q_q p^\mu F^{\mu \nu} \partial_\nu \bar{f}_q = \mathcal{O}(\delta f) \quad (\star)
\]

\[
\hat{\Gamma}[\bar{f}_q]
\]

- Assumptions and facts about Eq.(\star):

1. EB-field has little effect on thermalization process of QGP:
   \[ |\delta f| \ll \bar{f} \]

2. If \(\bar{f}_q\) highly anisotropic, \(\hat{\Gamma}[\bar{f}]\) is sizable \(\leftrightarrow\) very early stages.

3. Therefore,
   \[
   \frac{1}{p^0} p^\mu \partial_\mu g^F = -\hat{\Gamma}[\bar{f}] + \mathcal{O}(\delta f)
   \]
   neglected
Configuration background QGP and EB-field

• Background QGP with respect to Bjorken symmetry,

\[(t, z) \Leftrightarrow (\tau, \xi), \quad \text{no dep. on } x_\perp\]

hence, \(\bar{f}_q(t, z, p)(= \bar{f}_\bar{q}(t, z, p)) \Leftrightarrow \bar{f}_q(\tau, p)\)

• EB-field does not obey Bjorken symmetry:

\[A^\mu(t, z) = (0, A^x(t, z), 0, 0) \quad \Rightarrow \quad \begin{cases} 
E_x = -\partial_t A^x \\
B_y = \partial_z A^x
\end{cases}\]

We are able to study EB-field at \(\vec{x}_\perp = 0\).
Background QGP: Boltzmann & 2-2 scatterings

\[ D_t f^a_p \equiv \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{x} \right) f^a_p = C[f^a_p] \]

for very early stages, dominated by $2 \leftrightarrow 2$ scatterings in QCD

\[
C[f^a_p] = \frac{1}{2E_p \nu_a} \sum_{b,c,d} \frac{1}{s_{cd}} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 k}{(2\pi)^3 2E_k} \frac{d^3 k'}{(2\pi)^3 2E_{k'}} \times (2\pi)^4 \delta^{(4)}(P + P' - K - K') |M^{ab}_{cd}|^2 \\
\times \left[ f^c_k f^{d}_{k'}(1 + \epsilon_a f^a_p)(1 + \epsilon_b f^b_{p'}) - f^a_p f^b_{p'}(1 + \epsilon_c f^c_k)(1 + \epsilon_d f^{d}_{k'}) \right],
\]

where $|M|^2 \ni gg \leftrightarrow q\bar{q}, \ gq \leftrightarrow qg, \ g\bar{q} \leftrightarrow g\bar{q}, \ gg \leftrightarrow gg$
Initial conditions, parameters, etc.

- Background EB-field from two colliding nuclei,

\[ A^x \sim \left[ \frac{\tilde{z} + \tilde{v}\tilde{t}}{(\tilde{b}^2 / 4 + \gamma^2 (\tilde{z} + \tilde{v}\tilde{t})^2)^{1/2}} + \frac{\tilde{z} - \tilde{v}\tilde{t}}{(\tilde{b}^2 / 4 + \gamma^2 (\tilde{z} - \tilde{v}\tilde{t})^2)^{1/2}} \right] \]

- Background QGP: CGC inspired initial quark distribution, Romatschke, Strickland

\[ f_q(t = t_0, z = 0, p) f_0^q \Theta \left( 1 - \frac{\sqrt{p_{z}^2 \xi^2 + p_{\perp}^2}}{Q_s} \right) \]

* We have approximately \( \alpha_s \sim 0.2 \).

* We take \( f_0^q = 1 \) as an optimistic initialization. (\( f_0^q = 0 \)?)

  cf.1601.03576 (Gelfand, Hebenstreit, Berges)

* We take \( f_0^g \) according to the realistic mid-central AuAu (\( \sqrt{S_{NN}} = 0.2 \) GeV) and PbPb (\( \sqrt{S_{NN}} = 2.76 \) TeV).
B-field evolution ($z = 0$)

- Dotted line: Ideal Bjorken MHD: $\sigma \to \infty$ and $B(\tau) \sim 1/\tau$

PLB 750(2015)45-52(V.Roy et. al)
B-field evolution at fixed time

When hydro starts:

- RHIC AuAu (left): $eB \sim 10^{-5} Q_s^2 \sim 10$ MeV$^2$
- LHC PbPb (right): $eB \sim 10^{-7} Q_s^2 \sim 0.1$ MeV$^2$
Effective conductivity in the out-of-equilibrium QGP?

Effective conductivity: \( \sigma_{\text{eff}} = \frac{J_x}{E_x} \)
Summary and discussions

- We have solved the coupled evolution of QGP and EB-field.
- These calculations are done according to the realistic PbPb and AuAu in experiments at RHIC and the LHC.
- The remaining B field from the pre-equilibrium stage before hydro could be weak, even though QGP is a conducting medium.
Back-up slides
Backgound QGP evolution

- Gluon population taken: mid-central LHC PbPb 2.76TeV.
- System is away from equilibrium as expected.