Non-perturbative renormalization of the average color charge and multi-point correlators of color charges from a non-Gaussian small-x action

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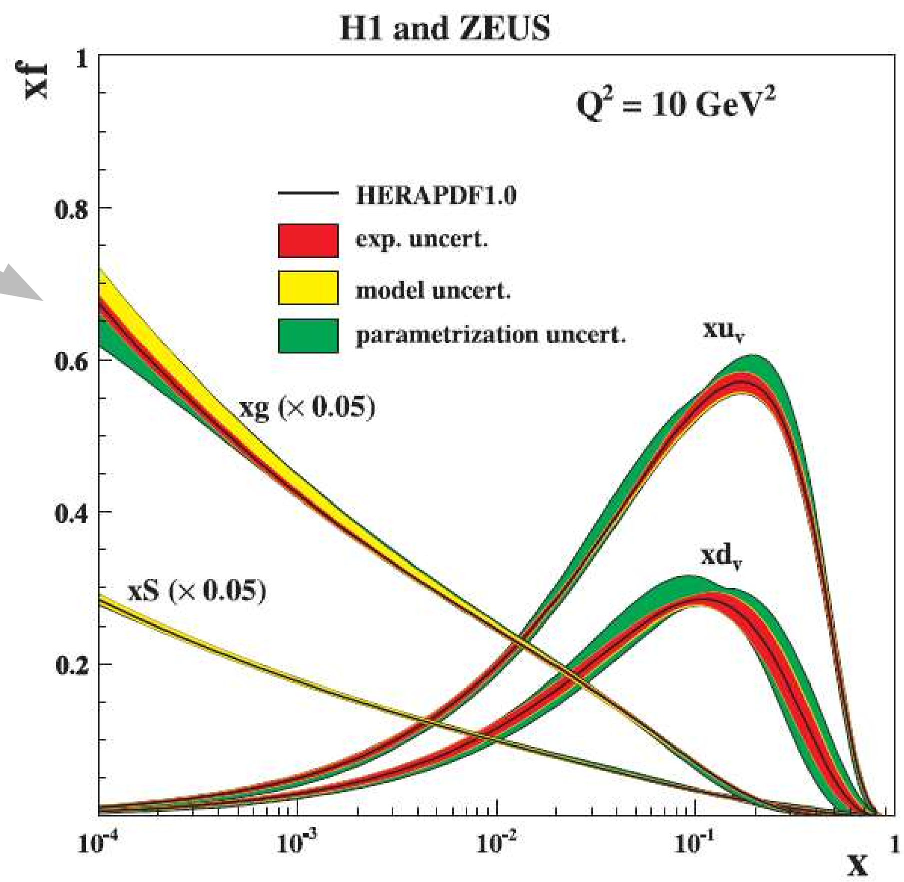
Bullet talk @ Initial Stages 2021

Parton Distribution Functions @ HERA

$x\,g(x, Q^2)$



Gluons dominate the hadronic wave function @ small-x

Small-x is accessible at high-energies: $x \sim p_T/\sqrt{s}$

QCD matter @ high gluon densities

Color Glass Condensate: E.F.T. of QCD @ small-x

McLerran, Venugopalan: PRD 49 (1994), 2233, ibid, 3352, PRD 50 (1994), 2225

Also: McLerran, Venugopalan, Jalilian-Marian, Kovner, Weigert, Iancu, Leonidov, Kovchegov, Balitski

Hadrons described as classical systems @ high gluon densities

Large-x partons → static randomly distributed color charges

Small-x gluon fields obtained from CYM equations for fixed color charge

$$\langle \mathcal{O}[\rho] \rangle_Y = \frac{\int [d\rho]\, W_Y[\rho]\, \mathcal{O}[\rho]}{\int [d\rho]\, W_Y[\rho]}$$

Here: $Y = 0$

Weight functions for color charge average

$A \to \infty$ **limit: weight function is Gaussian**

$$W_{MV}[\rho_x] = \exp\left\{ - \int d^2x \frac{\rho_x^a \rho_x^a}{2\mu^2} \right\}$$

$$a = 1, ..., N_c^2 - 1$$

$$\mu = constant$$

McLerran-Venugopalan (MV) model

PRD 49 (1994), 2233, ibid, 3352, PRD 50 (1994), 2225

average color charge per unit of area

Only non-trivial correlator:

$$\langle \rho_x^a \rho_y^b \rangle_{MV} = \delta^{ab} \delta^{(2)}(x-y) \mu^2$$

$$\langle \rho_x^a \rho_y^b \rho_u^c \rho_v^d \rangle_{\mathrm{MV}} = \langle \rho_x^a \rho_y^b \rangle \langle \rho_u^c \rho_v^d \rangle + permutations$$

Weight functions for color charge average

In reality: finite number of sources

$$W_{NG}[\rho_x] \simeq \exp\left\{ - \int d^2x \left[\frac{\rho_x^a \rho_x^a}{2\bar{\mu}^2} + \frac{3(\rho_x^a \rho_x^a)^2}{\kappa_4} \right] \right\}$$

Dumitru, Jalilian-Marian, Petreska, PRD 84 (2011), 014018

Deviations from MV model: $Z = \mu^2/\bar{\mu}^2$

$$\langle \rho_x^a \rho_y^b \rho_u^c \rho_v^d \rangle_{NG} \propto \delta^{(2)}(x-y)\delta^{(2)}(u-v)[1 - \mathcal{C}_{NG}\, \delta^{(2)}(x-u)] \; + perm.$$

non-factorizable & local

May explain initial condition for dipole evolution w/ $\gamma > 1$

Dumitru, Petreska, NP A879 (2012) 59-76

preferred by fits of HERA inclusive cross-section data

Multi-point Wilson line correlators w/ NG ensemble: ongoing calculation w/ Y. Nara

Non-perturbative calculation on lattice

$$W_{\rm NG}[\rho] \to W_r = \frac{a^2\,Z\,r^2}{2\mu^2} + \frac{3\,a^2\,r^4}{\kappa_4}$$

$$r^2 \equiv \sum_a \rho^a \rho^a$$

$$\int d^2x \to a^2$$

No expansion of e^{-W_r}

$\langle \rho_x^a \rho_y^b \rangle_{\rm NG}$ **&** $\langle \rho_x^a \rho_y^b \rho_u^c \rho_v^d \rangle_{\rm NG}$ **to all orders in** $1/\kappa_4$

Perturbative results in the limit of small & large non-Gaussian fluctuations

$Z \sim 1$ ↙ ↘ $Z \sim 0$

Renormalization schemes (RS)

$$W_r = \frac{a^2 Z r^2}{2\mu^2} + \frac{3 a^2 r^4}{\kappa_4}$$

$$\langle \rho_x^a \rho_y^b \rangle_{\mathrm{NG}} = \langle \rho_x^a \rho_y^b \rangle_{\mathrm{MV}}$$

RS 1: $\frac{\kappa_4}{\bar{\mu}^6} = constant \quad \rightarrow \quad \kappa_4 \propto \mu^6 / Z^3 \quad \rightarrow \quad Z(a)$

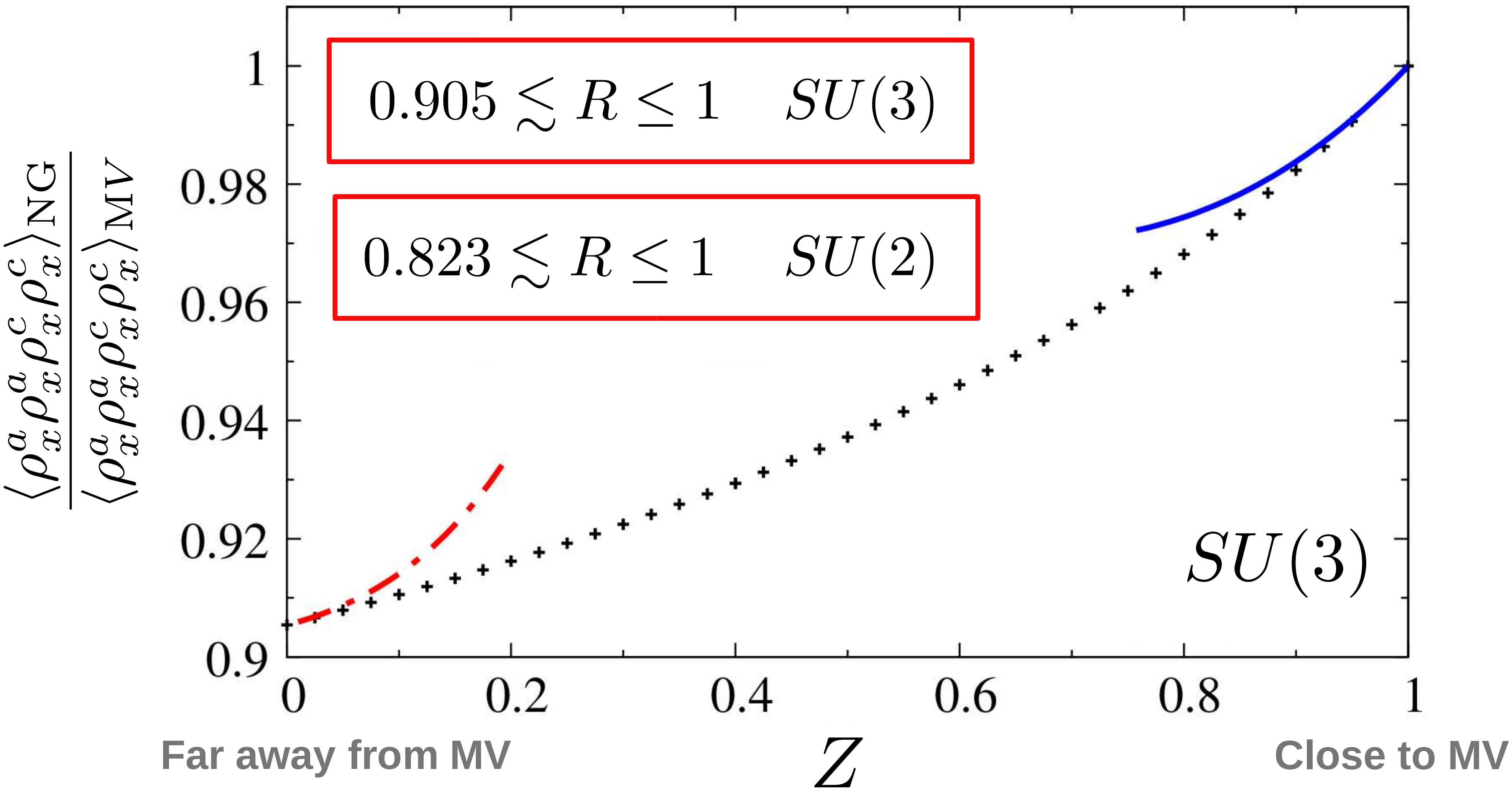
RS 2: $\kappa_4 = constant \quad \rightarrow \quad Z(a)$

No control over non-Gaussian fluctuations

RS 3: $\bar{\mu}^2 = constant \quad \rightarrow \quad Z = constant \quad \rightarrow \quad \kappa_4(a)$

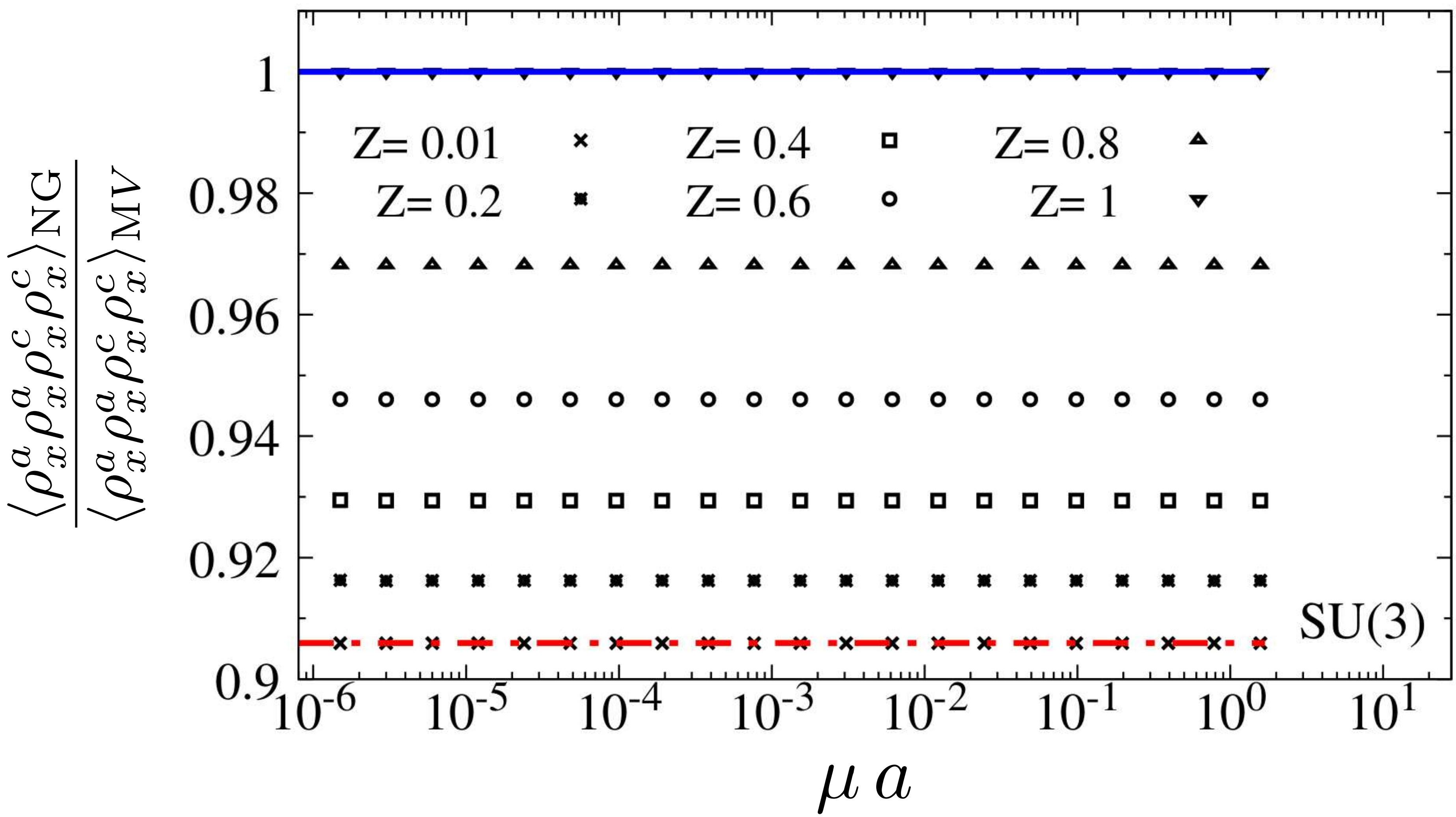
Can control deviations from MV model!

Ratio $R \equiv \langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\mathrm{NG}} / \langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\mathrm{MV}}$ in RS3



Ratio decreases by moving away from MV model

Ratio $R \equiv \langle\rho_x^a\rho_x^a\rho_x^c\rho_x^c\rangle_{\rm NG}/\langle\rho_x^a\rho_x^a\rho_x^c\rho_x^c\rangle_{\rm MV}$ in RS3



Fixed deviation from MV model as $a \to 0$

Conclusions & Remarks

- Studied non-perturbative effects of first (even C-parity) non-Gaussian correction to the Gaussian approximation of the CGC
- Considered different renormalization schemes for parameters appearing in the non-Gaussian small-x action. Found a way to control deviations from MV model

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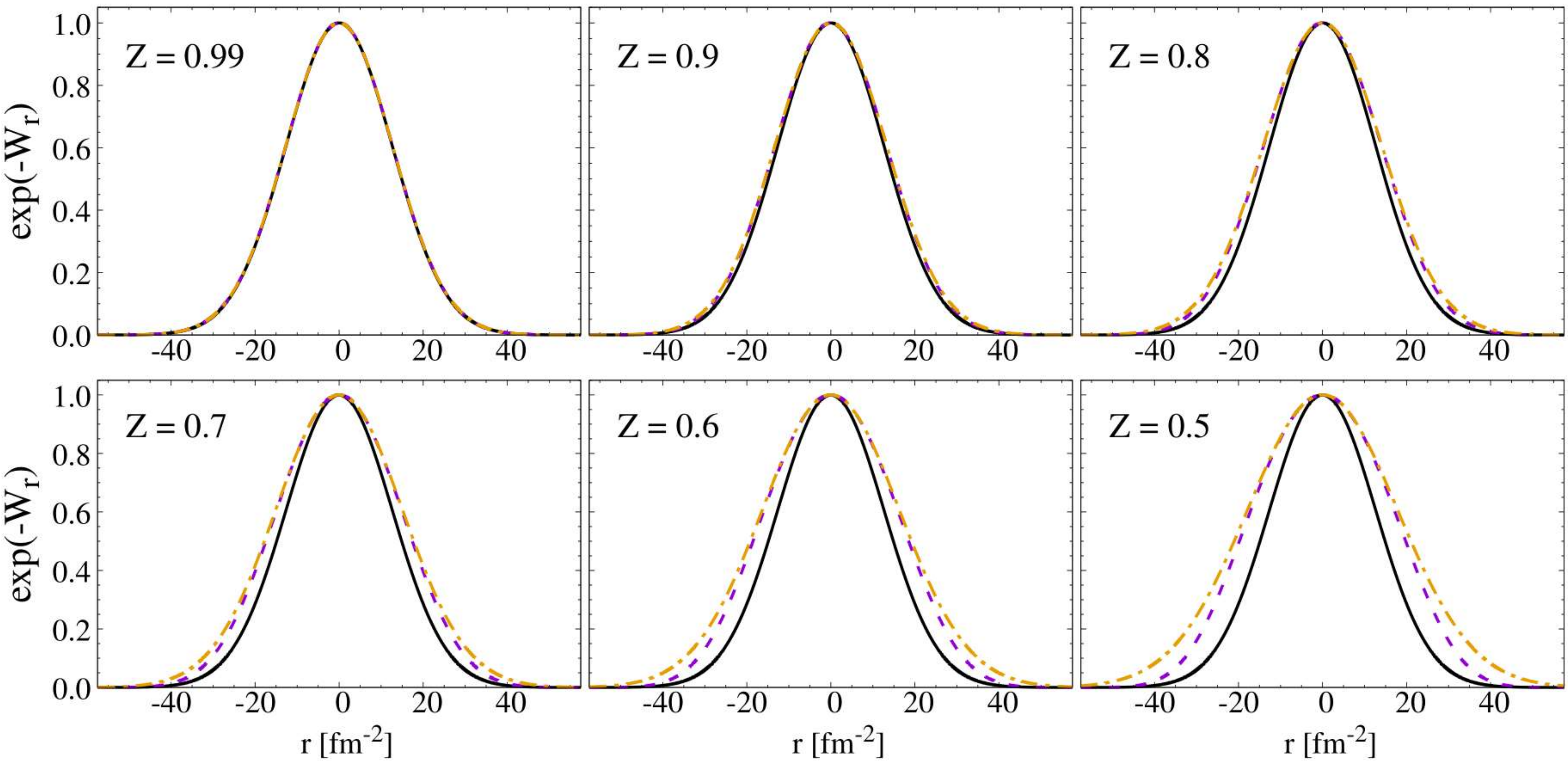
- 1st step to have lattice calculations beyond the Gaussian limit of CGC
- May lead to better characterization of small systems in CGC
- Gluon field fluctuations =/= Gaussian: affects multiplicity distributions
  Dumitru,Petreska 1209.4105
- Includes contributions missed by Gaussian averaging
  Kovner, Lublinsky, PRD83, 034017 (2011)   Dumitru, Jalilian-Marian, Petreska, PRD 84 (2011), 014018
- May impact particle production @ LHC as well as @ Electron – Ion

Thank you for your attention!

Questions / comments are welcome!

Backup slides

Weight functions $[\, \mu\, a \approx 0.024 \quad \& \quad N_s = 128 \,]$



MV model

$$W_r = \frac{a^2\, r^2}{2\mu^2}$$

Non-Gaussian

$$W_r = \frac{a^2\, r^2}{2\bar{\mu}^2} + 3\frac{r^4}{\kappa_4}$$

Gaussian envelope

$$W_r = \frac{a^2 r^2}{2\bar{\mu}^2}$$

Color charge correlators: main expressions

$$r^2 \equiv \sum_a \rho^a \rho^a$$

When $\mathcal{O}$ **is local:**

$$\langle \mathcal{O} \rangle = \frac{\int (\prod_x \prod_a d\rho_x^a)\, \mathcal{O}\, e^{-\sum_y W_y}}{\int (\prod_x \prod_a d\rho_x^a)\, e^{-\sum_y W_y}} = \frac{\int dr\, r^{N_c^2-2}\, \mathcal{O}_r\, e^{-W_r}}{\int dr\, r^{N_c^2-2}\, e^{-W_r}},$$

$$\langle \rho_x^a \rho_x^a \rangle_{\mathrm{NG}} = (N_c^2 - 1)\, \frac{\bar{\mu}^2 \sqrt{X}}{a^2}\, \frac{U\left(\frac{1}{4}\left(N_c^2+1\right), \frac{1}{2}, X\right)}{U\left(\frac{1}{4}\left(N_c^2-1\right), \frac{1}{2}, X\right)}$$

$$X = \frac{a^2\, \kappa_4}{48 \bar{\mu}^4}$$

$$\langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\mathrm{NG}} = \left(N_c^4 - 1\right)\, \frac{\bar{\mu}^4}{a^4}\, \frac{X U\left(\frac{1}{4}\left(N_c^2+3\right), \frac{1}{2}, X\right)}{U\left(\frac{1}{4}\left(N_c^2-1\right), \frac{1}{2}, X\right)}$$

$X \to \infty$ **limit: small non-Gaussian fluctuations,** $Z \to 1$

$X \to 0$ **limit: large non-Gaussian fluctuations,** $Z \to 0$

$$U(\alpha, \beta, \omega) = \frac{1}{\Gamma(\alpha)} \int_0^\infty e^{-\omega t}\, t^{\alpha-1}\, (1+t)^{\beta-\alpha-1}\, dt$$

Tricomi's hypergeometric function

Renormalization eq.: $\langle \rho_x^a \rho_y^b \rangle_{\mathrm{NG}} = \langle \rho_x^a \rho_y^b \rangle_{\mathrm{MV}}$

Small non-Gaussian fluctuations (Z~1)

$$\mu^2 \equiv \bar{\mu}^2 \left(1 - 12 \frac{\bar{\mu}^4}{\kappa_4} \frac{(N_c^2+1)}{a^2} \right)$$

$$\bar{\mu}^2 > \mu^2 \quad \rightarrow \quad 0 < Z < 1$$

Large non-Gaussian fluctuations (Z~0)

$$\frac{\Gamma\left(\frac{1}{4}\left(N_c^2+1\right)\right)}{4\sqrt{3}\,\Gamma\left(\frac{1}{4}\left(N_c^2+3\right)\right)} \frac{\sqrt{\kappa_4}}{a} + \left(\frac{\Gamma\left(\frac{1}{4}\left(N_c^2+1\right)\right)^2}{\Gamma\left(\frac{1}{4}\left(N_c^2-1\right)\right)\Gamma\left(\frac{1}{4}\left(N_c^2+3\right)\right)} - 1 \right) \frac{Z\,\kappa_4}{24\,\mu^2} = \frac{\mu^2}{a^2}$$

Renormalization eq. in RS1 ($\kappa_4/\bar{\mu}^6 = \gamma/g^2$ const.)

$$\bar{\mu}^2 \sim \frac{g^2 A}{\pi R^2} \qquad \kappa_4 \sim \frac{g^4 A^3}{(\pi R^2)^3} \quad \rightarrow \quad \kappa_4 = \frac{\gamma}{g^2}\bar{\mu}^6$$

SU(2)

$\gamma = 48$

SU(3)

$\gamma = 144$

Small non-Gaussian fluctuations (Z~1)

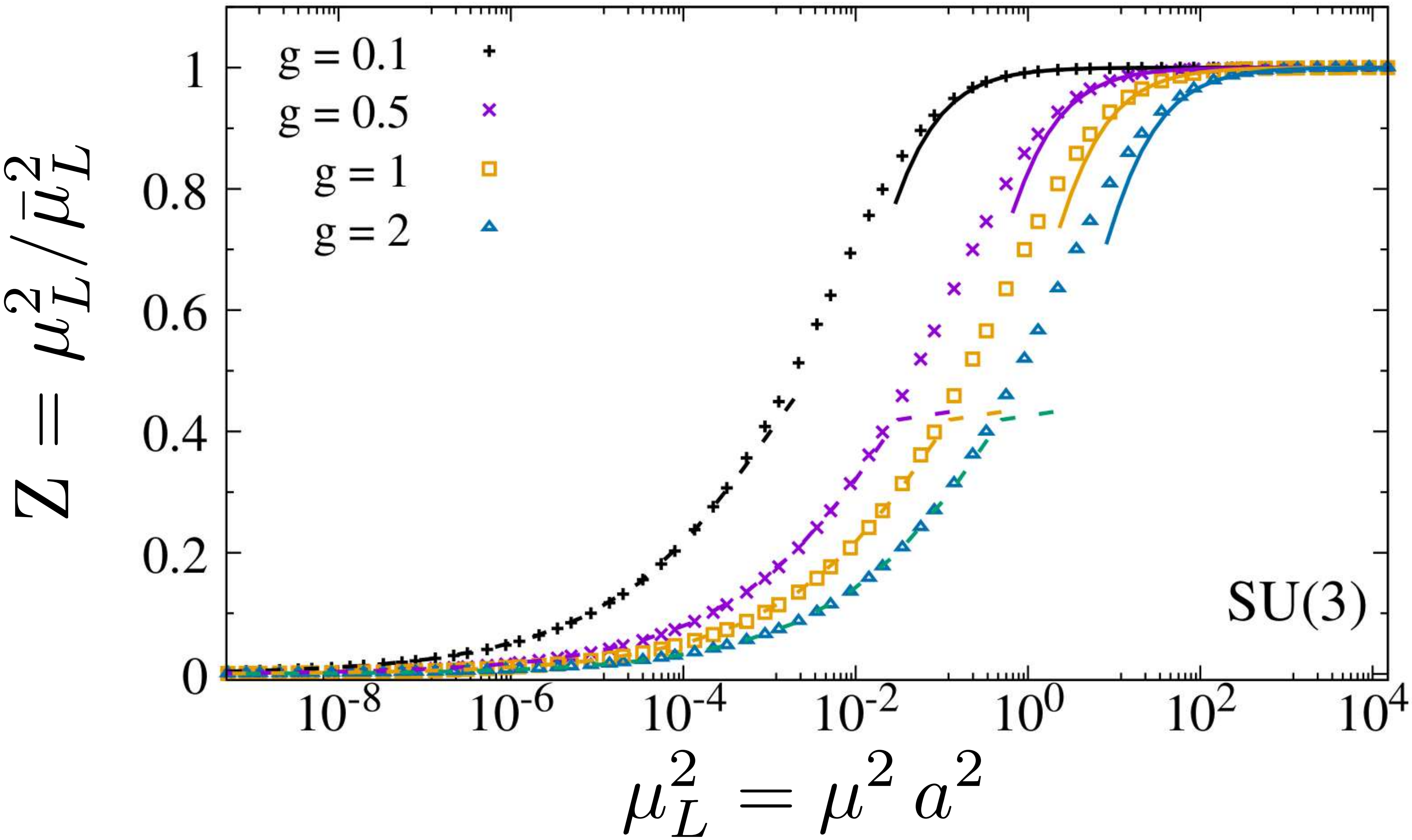
$$Z(a) = \frac{\mu^2 a^2}{12(N_c^2+1)\, g^2/\gamma + \mu^2 a^2}$$

Large non-Gaussian fluctuations (Z~0)

$$Z^3(a) = \frac{\gamma\, \Gamma\left(\frac{1}{4}\left(N_c^2+1\right)\right)^2}{48\, g^2\, \Gamma\left(\frac{1}{4}\left(N_c^2+3\right)\right)^2}\, a^2 \mu^2$$

$Z(a) \rightarrow 0$ **as** $a \rightarrow 0$

Z(a) vs a in RS1



Increasing deviations from MV model as $a \to 0$

4-point function in RS1

Z → 1:

$$\frac{\langle \rho^a_x \rho^a_x \rho^c_x \rho^c_x \rangle_{\rm NG}}{\langle \rho^a_x \rho^a_x \rho^c_x \rho^c_x \rangle_{\rm MV}} = 1 - \frac{24\,(g^2/\gamma)\,\mu^4\,a^4}{\left[\,\mu^2\,a^2 + 12\,(N_c^2+1)\,g^2/\gamma\,\right]^3}$$

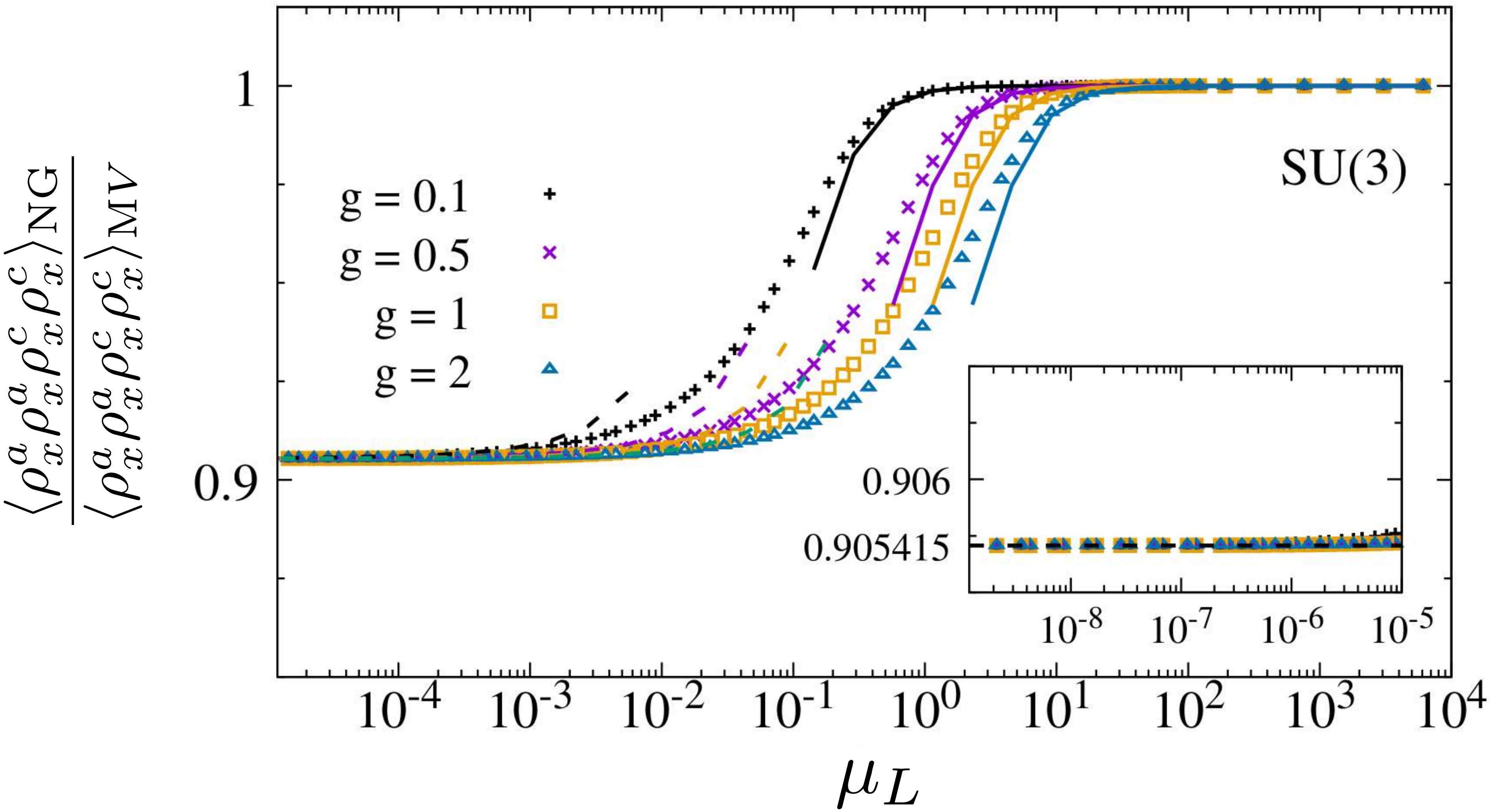
Ratio equal to 1 when $a \rightarrow 0$ **(inconsistent w/ previous result)**

Z → 0:

$$\frac{\langle \rho^a_x \rho^a_x \rho^c_x \rho^c_x \rangle_{\rm NG}}{\langle \rho^a_x \rho^a_x \rho^c_x \rho^c_x \rangle_{MV}} = \begin{cases} 0.822504, & \text{for SU(2)} \\ 0.905415, & \text{for SU(3)} \end{cases}$$

Maximum deviation from Gaussian theory (for this ratio!)

Ratio $\langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\rm NG} / \langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\rm MV}$ in RS1



Increasing deviations from MV model as $\mu_L \to 0$

Similar results for SU(2)

Renormalization eq. in RS2 (κ_4 const.)

When Z~1:

$$Z^2(1-Z) = \frac{12\,(N_c^2+1)\,\mu^4}{\kappa_4\,a^2}$$

$$Z(a) = \frac{1}{3}\left[1-(\nu(a)-1)^{-1/3}-(\nu(a)-1)^{1/3}\right]$$

$$\nu(a) \equiv \frac{18\,\mu^2\left(\sqrt{(N_c^2+1)\,(81\,(N_c^2+1)\,\mu^4-a^2\kappa_4)}+9\,(N_c^2+1)\,\mu^2\right)}{a^2\,\kappa_4}$$

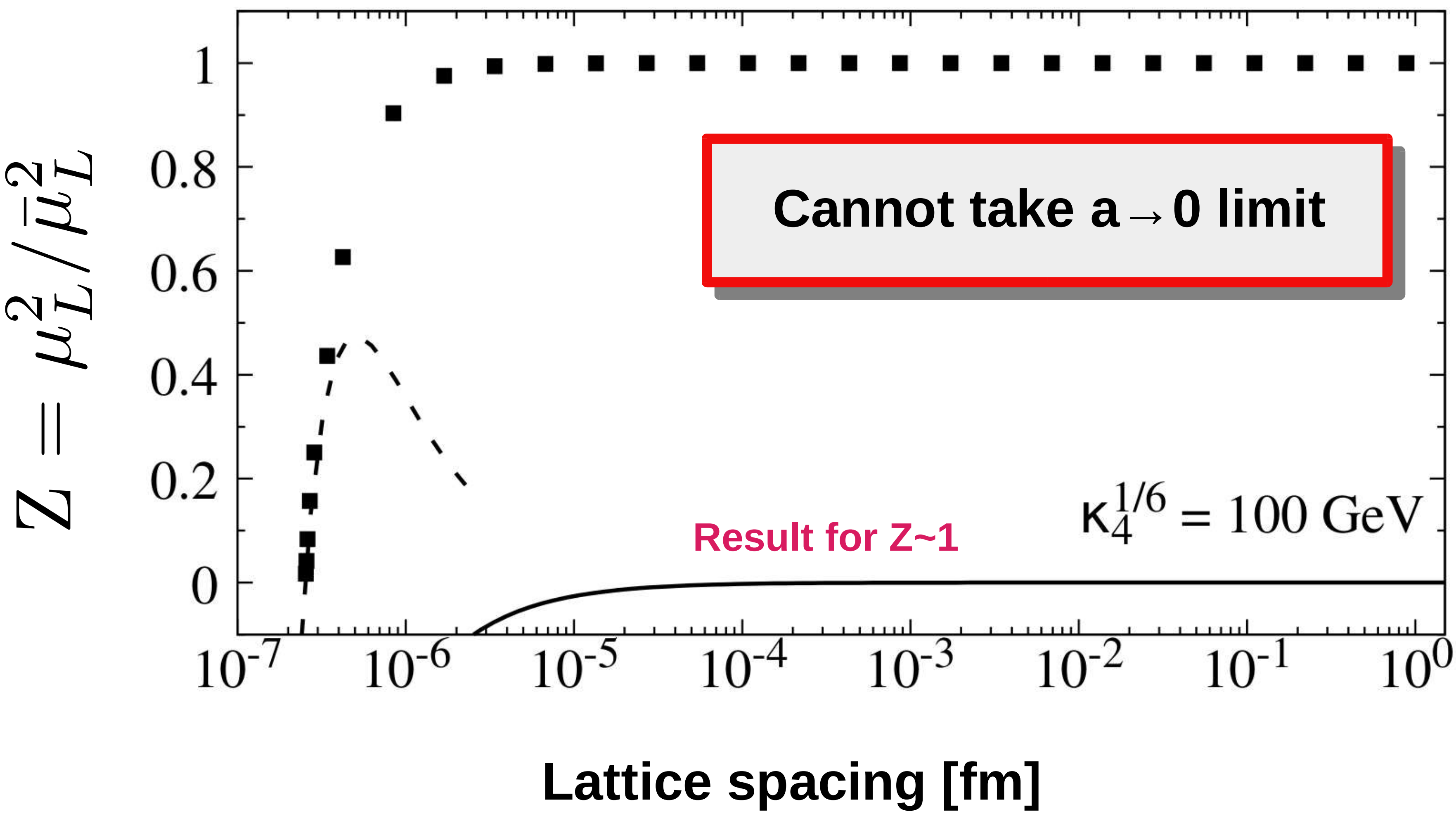
$Z(a) < 0$ **as** $a \to 0$

RS2: pert. calculation for Z~1 not applicable!

Z(a) vs a in RS2

Dashed line: $$Z(a,\kappa_4) = \frac{2\,\mu^2\,\Gamma\left(\frac{1}{4}\left(N_c^2-1\right)\right)\left[\sqrt{3\,\kappa_4}\,a\,\Gamma\left(\frac{1}{4}\left(N_c^2+1\right)\right) - 12\,\mu^2\,\Gamma\left(\frac{1}{4}\left(N_c^2+3\right)\right)\right]}{\left[\Gamma\left(\frac{1}{4}\left(N_c^2-1\right)\right)\Gamma\left(\frac{1}{4}\left(N_c^2+3\right)\right) - \Gamma\left(\frac{1}{4}\left(N_c^2+1\right)\right)^2\right]\,a^2\,\kappa_4}$$

Result for Z~0



Renormalization eq. RS3 (fixed Z)

Small non-Gaussian fluctuations (Z~1)

$$a^2 \kappa_4 = 12 \frac{\mu^4}{Z^2 - Z^3} (N_c^2 + 1)$$

Large non-Gaussian fluctuations (Z~0)

$$\kappa_4(a, Z) \equiv \frac{\kappa_4(Z)}{a^2} = \frac{6\,\mu^4\,[\alpha(Z) - \beta(Z)]}{a^2\, Z^2\, \Gamma^2_{\frac{1}{4},1}\, (\Gamma^2_{\frac{1}{4},1} - \Gamma_{\frac{1}{4},-1}\, \Gamma_{\frac{1}{4},3})^2}$$

$$\Gamma_{k,m} \equiv \Gamma\left(k\,(N_c^2 + m)\right)$$

$$a^2 \kappa_4 = constant \quad \textbf{for fixed Z}$$

4-point function in RS3

Z~1:

$$\frac{\langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\mathrm{NG}}}{\langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\mathrm{MV}}} = 1 - \frac{2\,Z^2(1-Z)}{(N_c^2+1)}$$

Z~0:

$$\frac{\langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\mathrm{NG}}}{\langle \rho_x^a \rho_x^a \rho_x^c \rho_x^c \rangle_{\mathrm{MV}}} = Z^{-2} \quad [4.72132 - 0.505056\,\Psi(Z) - 3.55162\,\epsilon(Z) + 0.37993\,\Psi(Z)\,\epsilon(Z) + Z\ (2.45171\,\Psi(Z) + 0.532404\,\Psi(Z) + 1.71781\,Z - 8.23612)]$$

$$\Psi(Z) \equiv \sqrt{-14.0948\,\epsilon(Z) - 19.7514\,Z + 18.7368} \qquad \epsilon(Z) \equiv \sqrt{1.76715 - 3.72567\,Z}$$

Independent of lattice spacing → Fixed deviation from MV model

4-point function $\langle \rho^a_x \rho^a_y \rho^b_u \rho^b_v \rangle$

$$\langle \rho^a_x \rho^b_y \rho^c_u \rho^d_v \rangle_{\rm NG} = \delta^{ab}\delta^{cd}\frac{\delta_{xy}}{a^2}\frac{\delta_{uv}}{a^2}\mu^4\left(1 - \mathcal{C}_{\rm NG}\frac{\mu^4}{\kappa_4}\frac{\delta_{xu}}{a^2}\right) \quad + \quad permutations$$

Z~1: $\mathcal{C} = 24$

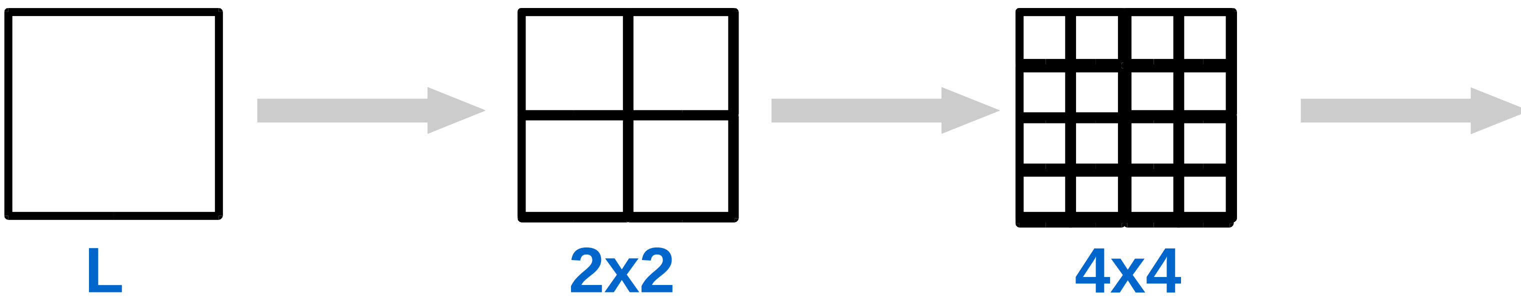
Z~0: $\mathcal{C} = 48\frac{\Gamma(\frac{1}{4}(N_c^2+3))^2}{\Gamma(\frac{1}{4}(N_c^2+1))^2}\left[1 - \frac{\Gamma\left(\frac{1}{4}\left(N_c^2+3\right)\right)^2}{\Gamma\left(\frac{1}{4}\left(N_c^2+1\right)\right)\Gamma\left(\frac{1}{4}\left(N_c^2+5\right)\right)}\right]$

$\langle \rho^4 \rangle_{\rm NG} = \langle \rho^4 \rangle_{\rm MV}$ **when** $x = y,\; u = v,\; x \neq u$

$\langle \rho^4 \rangle_{\rm NG} < \langle \rho^4 \rangle_{\rm MV}$ **when** $x = y = u = v$

Lattice regularization

2D transverse space ~ lattice with N_s^2 sites of length a



... $N_s \times N_s$

Continuum recovered by: $a \to 0$ **with fixed** $L = N_s\, a$

$$\int d^2x \to a^2$$

$$\mu \to \mu_L = \mu\, a$$

$$\rho \to \rho_L = \rho\, a^2$$

$$\delta^{(2)}(x-y) \to \frac{\delta_{xy}}{a^2}$$

$$\langle \rho_x^a \rho_y^b \rangle = \delta^{ab} \delta^2(x-y) \mu^2 \quad \longrightarrow \quad \langle \rho_x^a \rho_y^b \rangle = \frac{\delta^{ab} \delta_{xy}}{a^2} \mu^2$$

Krasnitz, Venugopalan: Nucl. Phys. B 557 (1999), 237