Multiparticle correlations from direct calculation of cumulants using particle azimuthal angles

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Motivations

\[
\langle 4 \rangle_{n,n|n,n} \equiv \frac{1}{(4)^{4!}} \sum_{i,j,k,l=1 \atop (i\neq j \neq k \neq l)}^{M} e^{in(\phi_i + \phi_j - \phi_k - \phi_l)}
\]

\[
c_n\{4\} = \langle \langle 4 \rangle \rangle - 2 \cdot \langle \langle 2 \rangle \rangle^2
\]

\[
v_n\{4\} = 4\sqrt{-c_n\{4\}}
\]

How much nonflow left in the multiparticle correlations?


CMS pPb √sNN = 5.02 TeV

0.3 < p_T < 3.0 GeV/c; |η| < 2.4
We start by introducing the quantities we will use throughout this report.

**1.1 Q-vector**

\[ Q_n \overset{\text{vector evaluated in harmonic}}{=} e^{i \eta_n} \]

\[ Q_n = \sum_{i=1}^{M} e^{i \phi_i} \]

The summation in above definition goes over all particles in an event with multiplicity \( M \) and \( \phi_i \) is the azimuthal angle of the \( i \)-th particle measured in the laboratory frame.

**1.2 Multi-particle azimuthal correlations**

2-, 4- and 6-particle azimuthal correlations by definition are obtained through the averaging procedure which consists of two distinct steps. First we define the average multi-particle correlations for each event in the following way:

\[ h_2^{n,n} = \frac{1}{(4^4)} \sum_{i,j=1}^{M} e^{i \eta_n (\phi_i + \phi_j - \phi_k - \phi_l)} \]

\[ h_4^{n,n,n} = \frac{1}{(4^4)} \sum_{i,j,k,l=1}^{M} e^{i \eta_n (\phi_i + \phi_j + \phi_k + \phi_l)} \]

\[ h_6^{n,n,n,n} = \frac{1}{(4^4)} \sum_{i,j,k,l,m,n=1}^{M} e^{i \eta_n (\phi_i + \phi_j + \phi_k + \phi_l + \phi_m + \phi_n)} \]

\[ Q_n \equiv \sum_{i=1}^{M} e^{i \phi_i} \]

\[ \langle 4 \rangle = f (Q_n) \]

\[ \langle 4 \rangle_{2\text{sub}} = f (Q_{nA}, Q_{nB}) \]

\[ \langle 4 \rangle_{3\text{sub}} = f (Q_{nA}, Q_{nB}, Q_{nC}) \]

Nonflow suppressed with $\eta$ gaps between subevents?

Introducing $\eta$ gaps between subevents:

2-sub

$\eta = -2.5$

3-sub

$\eta = -2.5$

$\eta = 2.5$

The problem with $\eta$ gaps between subevents

- Need to study correlations vs. $\eta$ gap
- Run out of statistics quickly
- Biases are introduced when using gaps between subevents because not all possible combinations are included
1 Definitions, terminology, notation...

We start by introducing the quantities we will use throughout this report.

1.1 $Q$-vector

$Q$-vector evaluated in harmonic $n$ is a complex quantity denoted by $Q_n$ and defined as

$$Q_n \equiv \frac{1}{(M)_4 4!} \sum_{i,j,k,l=1}^{M} e^{in(\phi_i + \phi_j - \phi_k - \phi_l)}$$

(1)

The summation in above definition goes over all particles in an event with multiplicity $M$ and $i$ is the azimuthal angle of the $i$-th particle measured in the laboratory frame.

1.2 Multi-particle azimuthal correlations

2-, 4- and 6-particle azimuthal correlations by definition are obtained through the averaging procedure which consists of two distinct steps. First we define the average multi-particle correlations for each event in the following way:

$$h_{2i}^{n,n} \equiv \frac{1}{(M)^2 2!} \sum_{i,j=1}^{M} e^{in(\phi_i - \phi_j)}$$

(2)

$$h_{4i}^{n,n} \equiv \frac{1}{(M)^4 4!} \sum_{i,j,k,l=1}^{M} e^{in(\phi_i + \phi_j - \phi_k - \phi_l)}$$

(3)

- Calculation of cumulants directly from particle $\phi$ angles, without $Q$ vectors
- All possible combinations are included, even when using $\eta$ gaps

Time complexity:

- $O(M^n)$, for $n$ particle cumulant with $M$ total number of particles per event
- The 8 particle cumulant for an event with 1000 particles takes $\sim$1 billion years
- However, our interest is in small collisions systems with $M$ less than 100
- It takes a few seconds to calculate 4 particle cumulant with $M=100$
- Much faster when applying $\eta$ gaps between particles
Results and Conclusions

Simulated 200 million Pythia 8 events

- The looping method includes all possible combinations from multiparticle correlations, with no biases introduced when flow has $\eta$ dependence
- The method suppresses more nonflow and could also be used for calculating symmetric and asymmetric cumulants
Thank you for your attention!

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