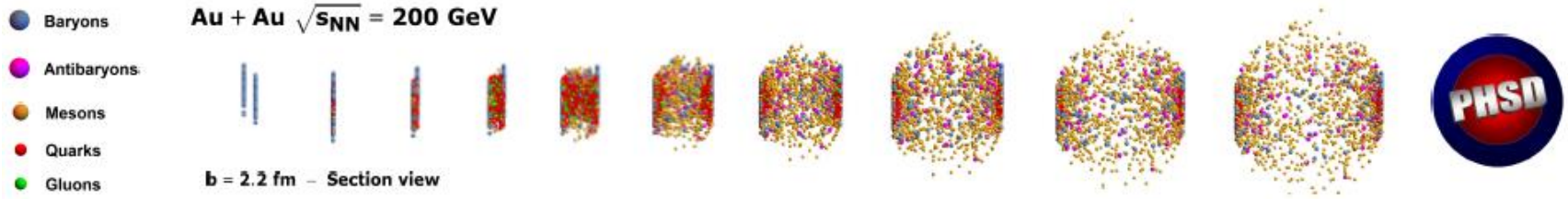
# Exploring the QCD phase diagram within a microscopic transport approach*

IS2021
The VI[th] International Conference on the INITIAL STAGES OF HIGH-ENERGY NUCLEAR COLLISIONS


## Pierre Moreau

**In collaboration with: Olga Soloveva, Taesoo Song, Elena Bratkovskaya, Steffen A. Bass**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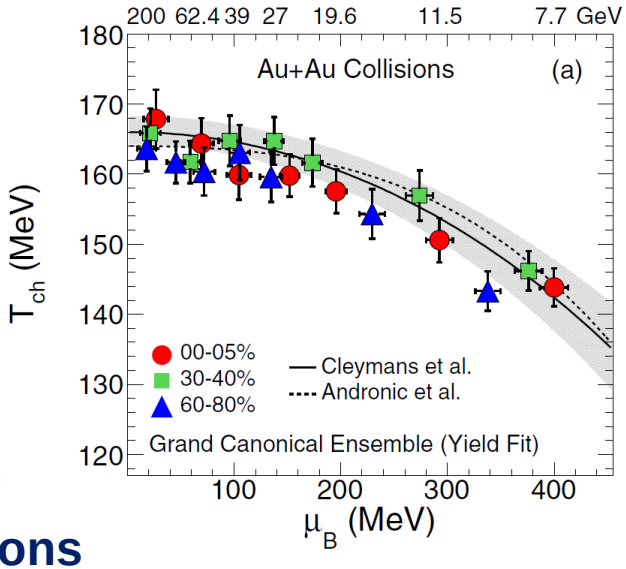
CRC-TR 211 Strong-interaction matter under extreme conditions


* work supported by the U.S. D.O.E. under Grant No. DE-FG02-05ER41367

- **The QCD phase diagram can be explored in heavy-ion collisions by varying the collisional energy and system size**
- **Information about freeze-out parameters is extracted using hadron resonance gas (HRG) model (simple model, no dynamics)**

BES STAR: PHYSICAL REVIEW C 96, 044904 (2017)




- **Goal: Use a fully microscopic transport approach to study the matter produced in heavy-ion collisions**

**Parton-Hadron-String-Dynamics (PHSD)**

**Initial state**
**String decay**

→ Dissolution →

**QGP**
**Dynamical Quasiparticles**

→ Hadronization →

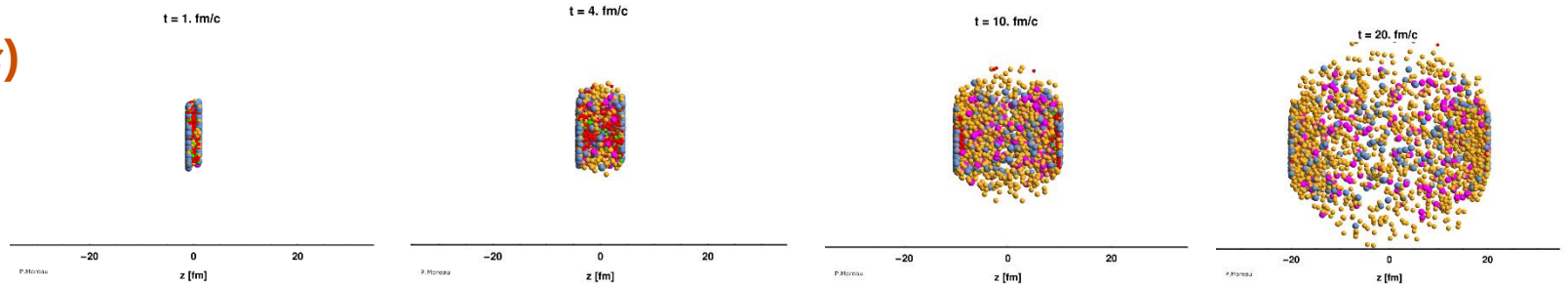
**hadronic phase**
**HSD**

**off-shell transport equations in phase-space representation for both partonic and hadronic phase**

W.Cassing, E.Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W.Cassing, EPJ ST 168 (2009) 3

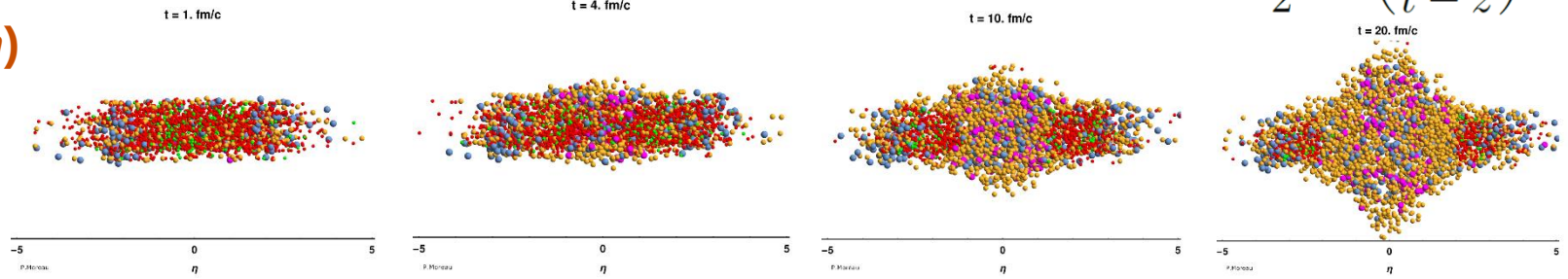
- **Calculation of the energy-momentum tensor ( $T^{\mu\nu}$ ) and charge currents ( $J^{\mu}_{B}$ , $J^{\mu}_{Q}$ , $J^{\mu}_{S}$ ) in ($\tau, x, y, \eta$) coordinates**

**($t, x, y, z$)**



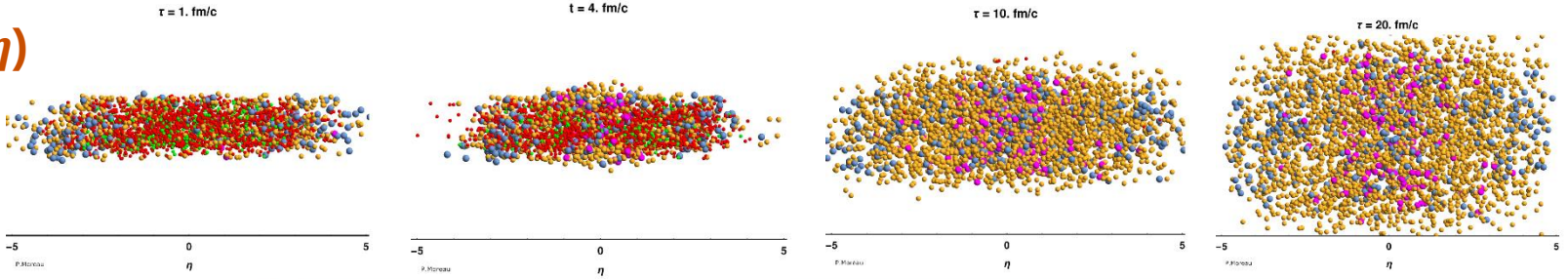

- **Transformation of $z$ to spatial rapidity (at fixed t):** $\eta_s = \frac{1}{2}\log\left(\frac{t+z}{t-z}\right)$

**($t, x, y, \eta$)**




- **Transformation of time $t$ to proper time:** $\tau = \sqrt{t^2 - z^2}$

**($\tau, x, y, \eta$)**



- **Hadron resonance gas (HRG) model: equation of state of a noninteracting hadron gas, valid for $T < T_c$ and all $\mu_B$**

- **Lattice QCD EoS at arbitrary $T, \mu_B, \mu_Q, \mu_S$**

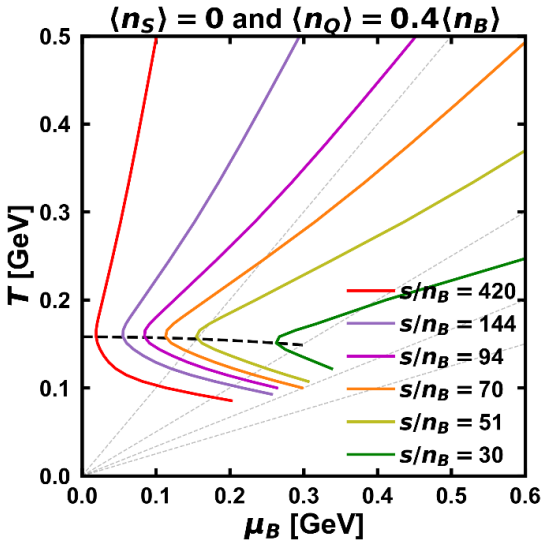
$$\frac{P(T,\mu)}{T^4} = \sum_{i,j,k} \frac{\chi^{B,Q,S}_{i,j,k}(T)}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

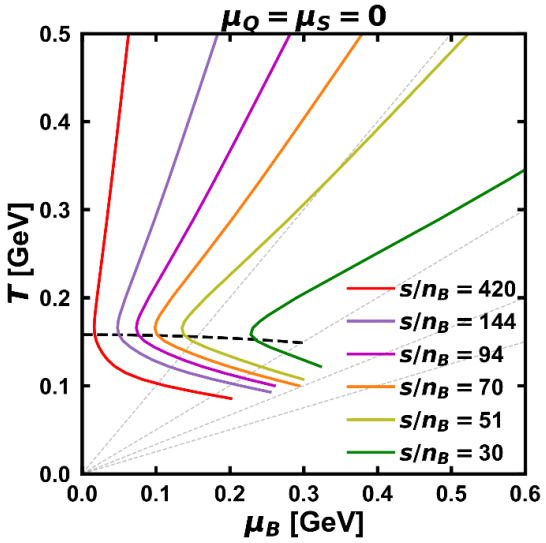
- **Match the HRG EoS and the lQCD EoS at the critical temperature $T_c(\mu)$**

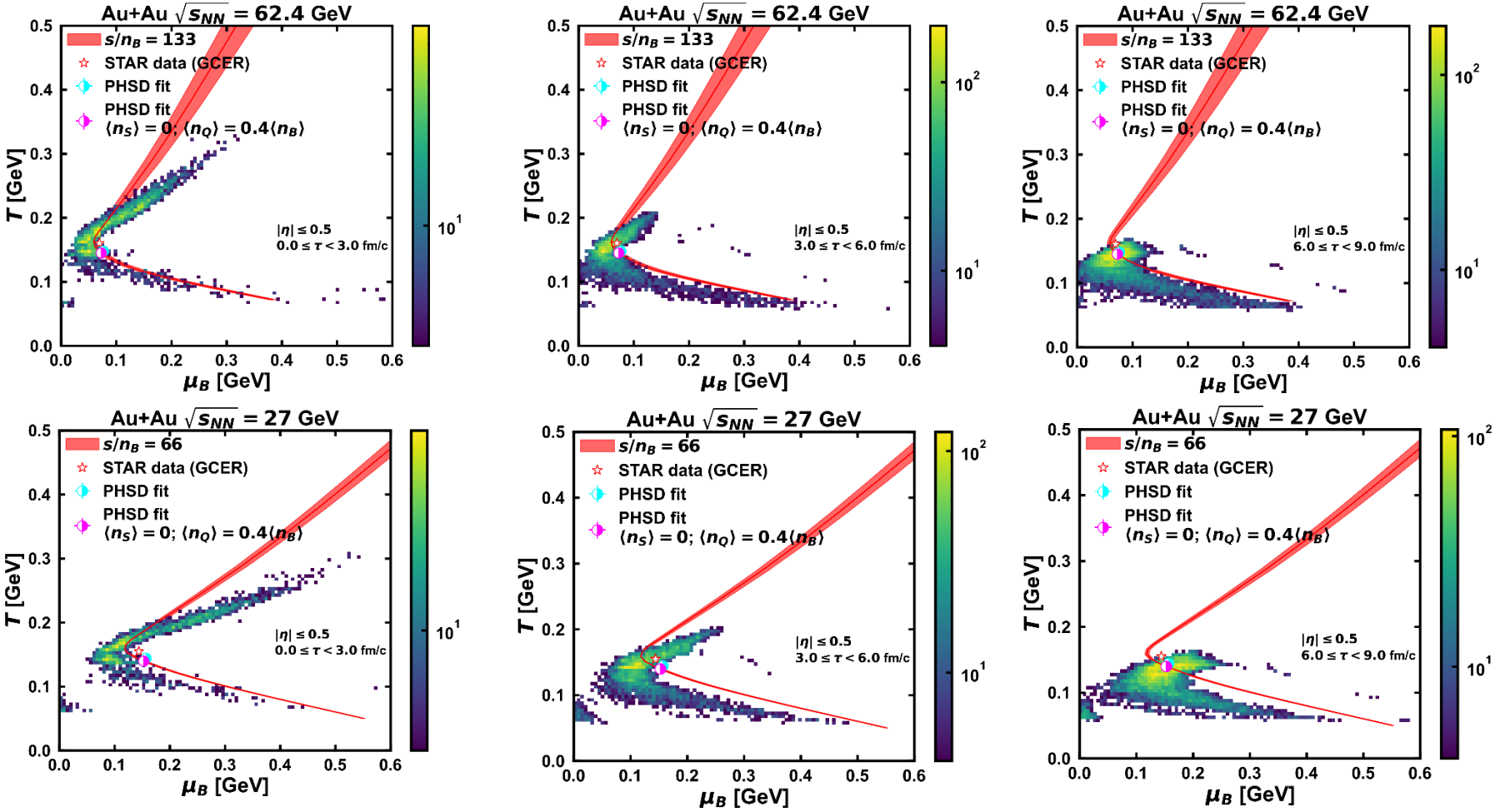
$$\frac{P}{T^4} = \frac{1}{2}\left[1 - f(T,\mu_J)\right]\frac{P_{HRG}}{T^4} + \frac{1}{2}\left[1 + f(T,\mu_J)\right]\frac{P_{lQCD}}{T^4}$$

- **With the QCD EoS, our next steps are to:**
  - **calculate $(T, \mu_B, \mu_Q, \mu_S)$ from $(\epsilon, n_B, n_Q, n_S)$ in PHSD**
  - **compare with freeze out parameters and isentropic trajectories**

Isentropic trajectories at fixed $s/n_B$



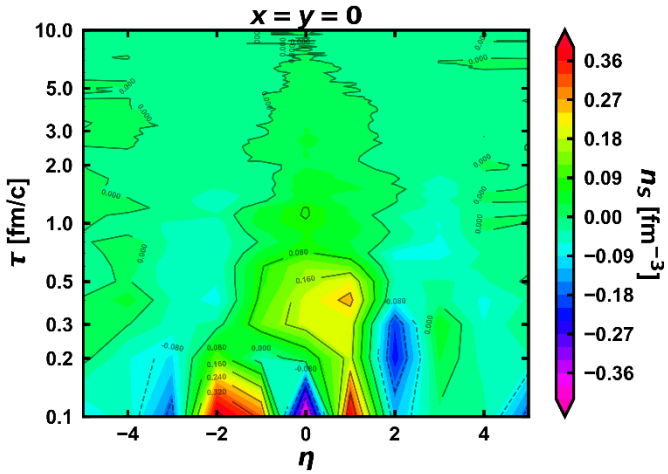





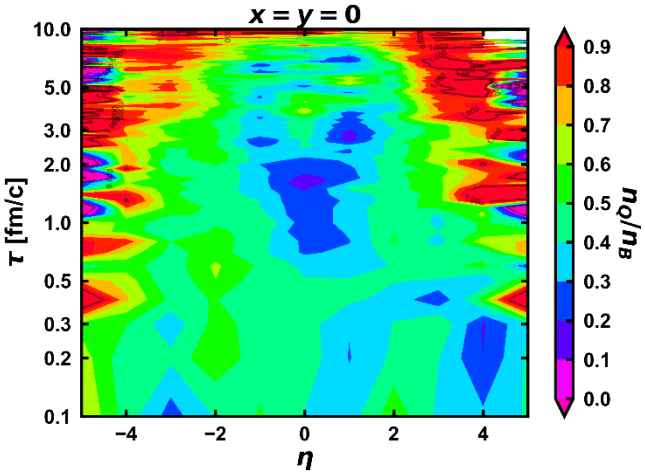

- **A significant portion of the matter passes through the freeze-out parameters extracted from the final particle spectra**
- **Small deviations from the isentropic trajectories are seen at all times**

# Illustrations at $\sqrt{s_{NN}} = 62.4$ GeV

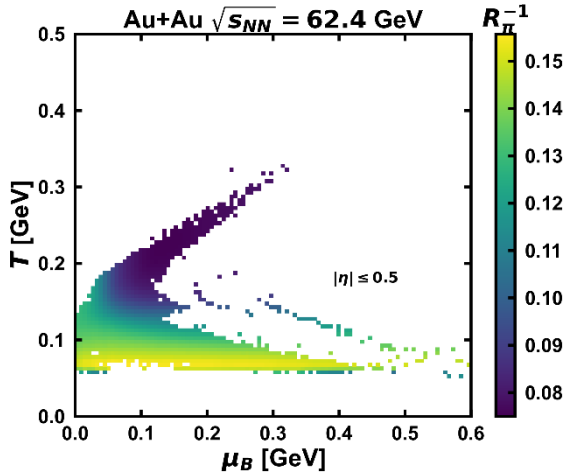
- **Isentropic trajectories at fixed $s/n_B$ assume $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$, which is not strictly realized during the whole dynamics of the PHSD**




Strangeness density




Ratio of charge density over baryon density




Inverse Reynolds number associated with the shear-stress tensor

- **Large dissipative currents (bulk viscous pressure, shear-stress tensor) can modify the trajectories in the phase diagram**

T. Dore, J. Noronha-Hostler, E. McLaughlin, Phys.Rev.D 102 (2020) 7, 074017

- **Coarse graining in $\eta$ enhances deviations from equilibrium at late times**

# Backup slides

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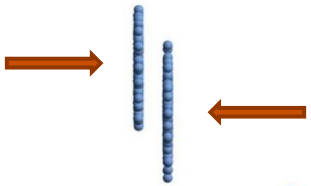
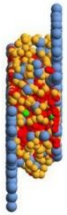
# Stages of a collision in PHSD

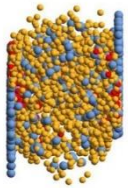
**Initial A+A collision**



Partonic phase



Hadronization



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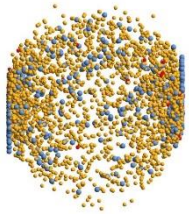
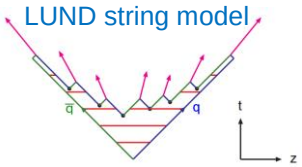


- **String formation in primary NN collisions**

→ **decays to pre-hadrons (baryons and mesons)**



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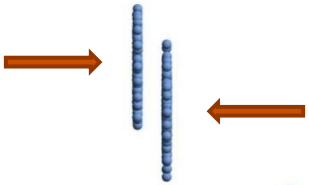

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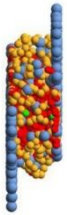

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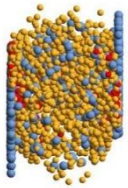

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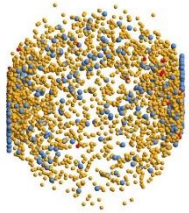

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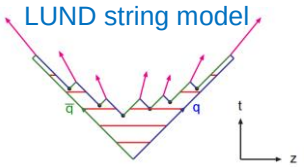


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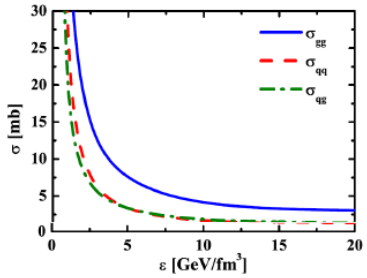
  **Dissolution of pre-hadrons → DQPM**

  → **massive quarks/gluons and mean-field energy**




**(quasi-)elastic collisions :**

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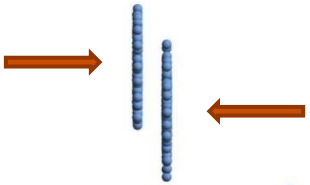

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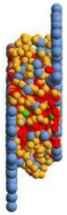

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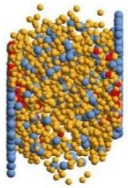

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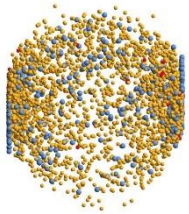

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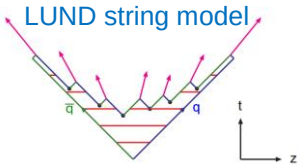


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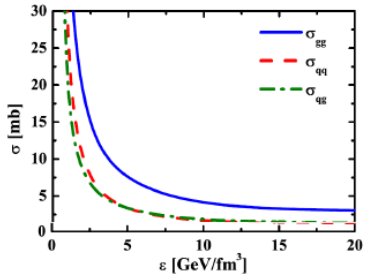


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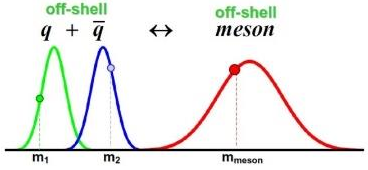
**Strict 4-momentum and quantum number conservation**



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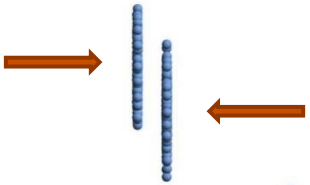

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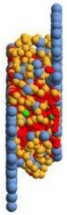

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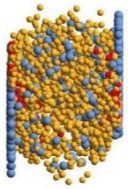

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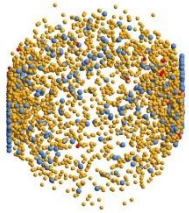

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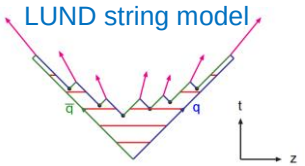


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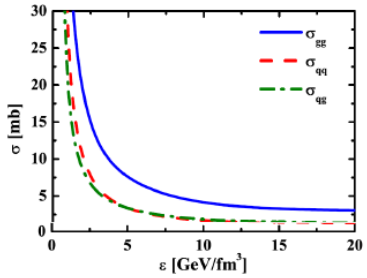


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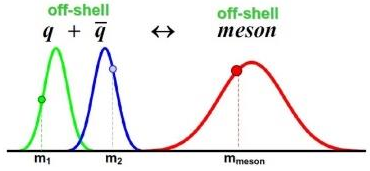


- **Hadron-string interactions – off-shell HSD**

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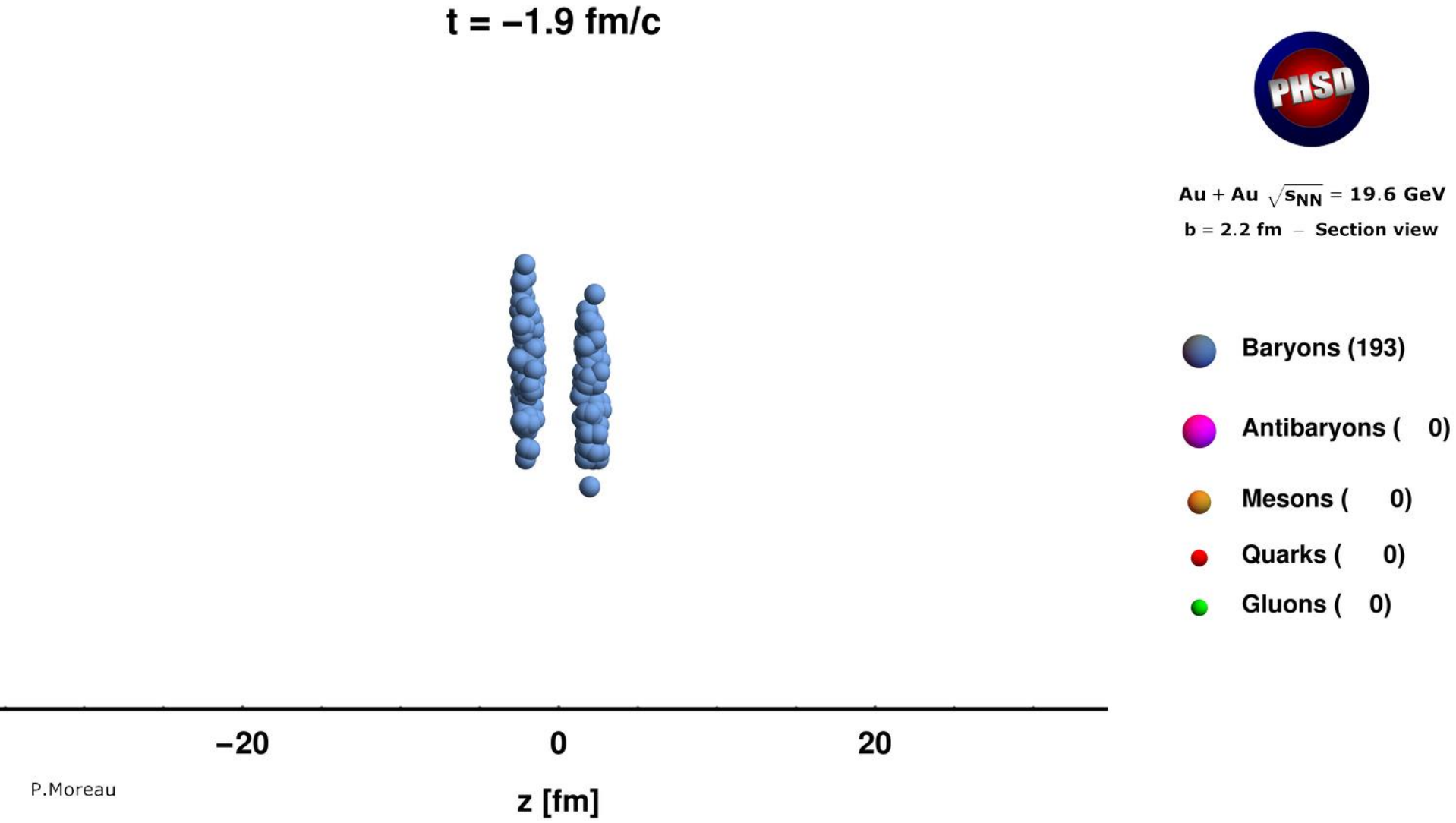

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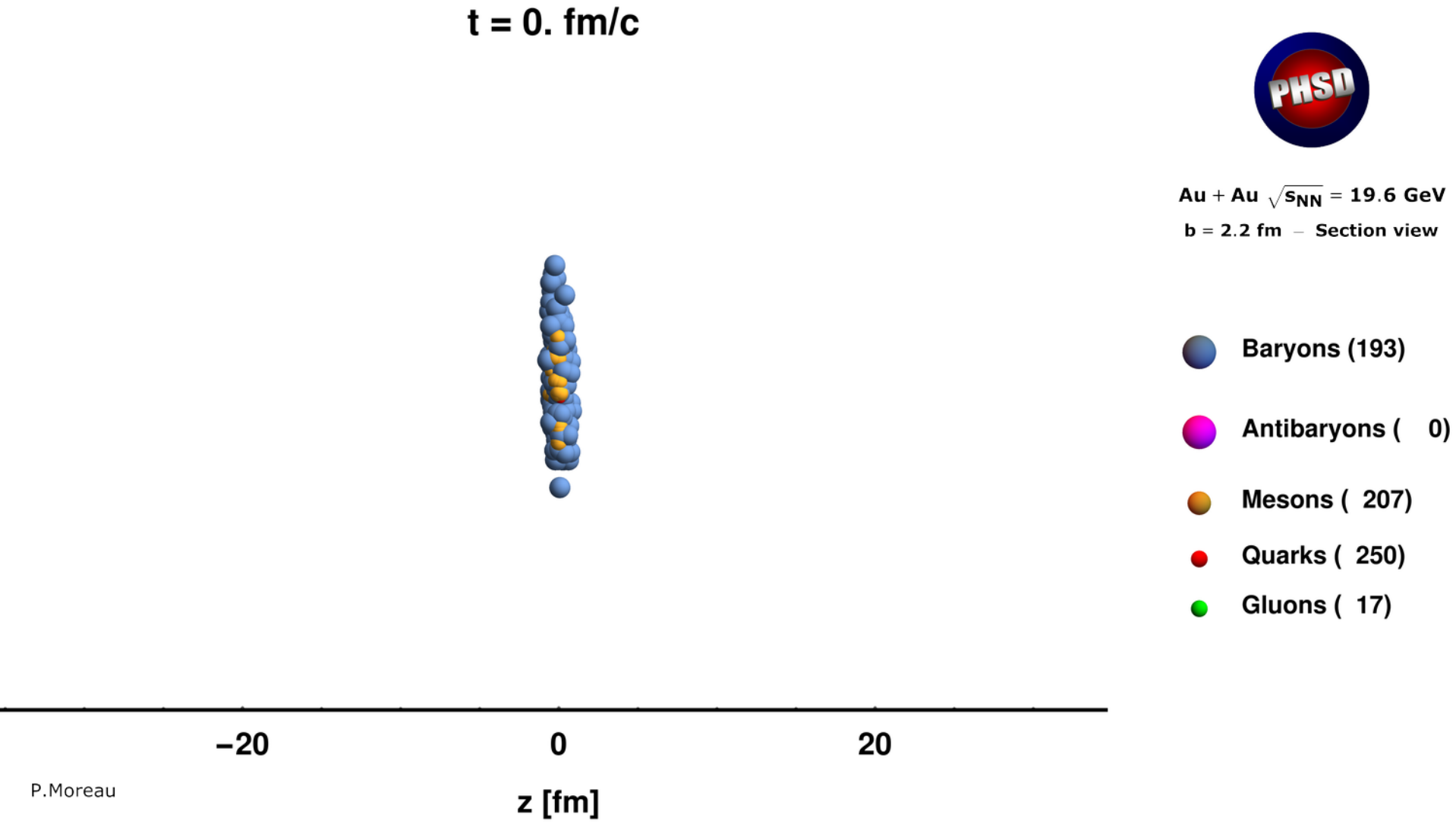

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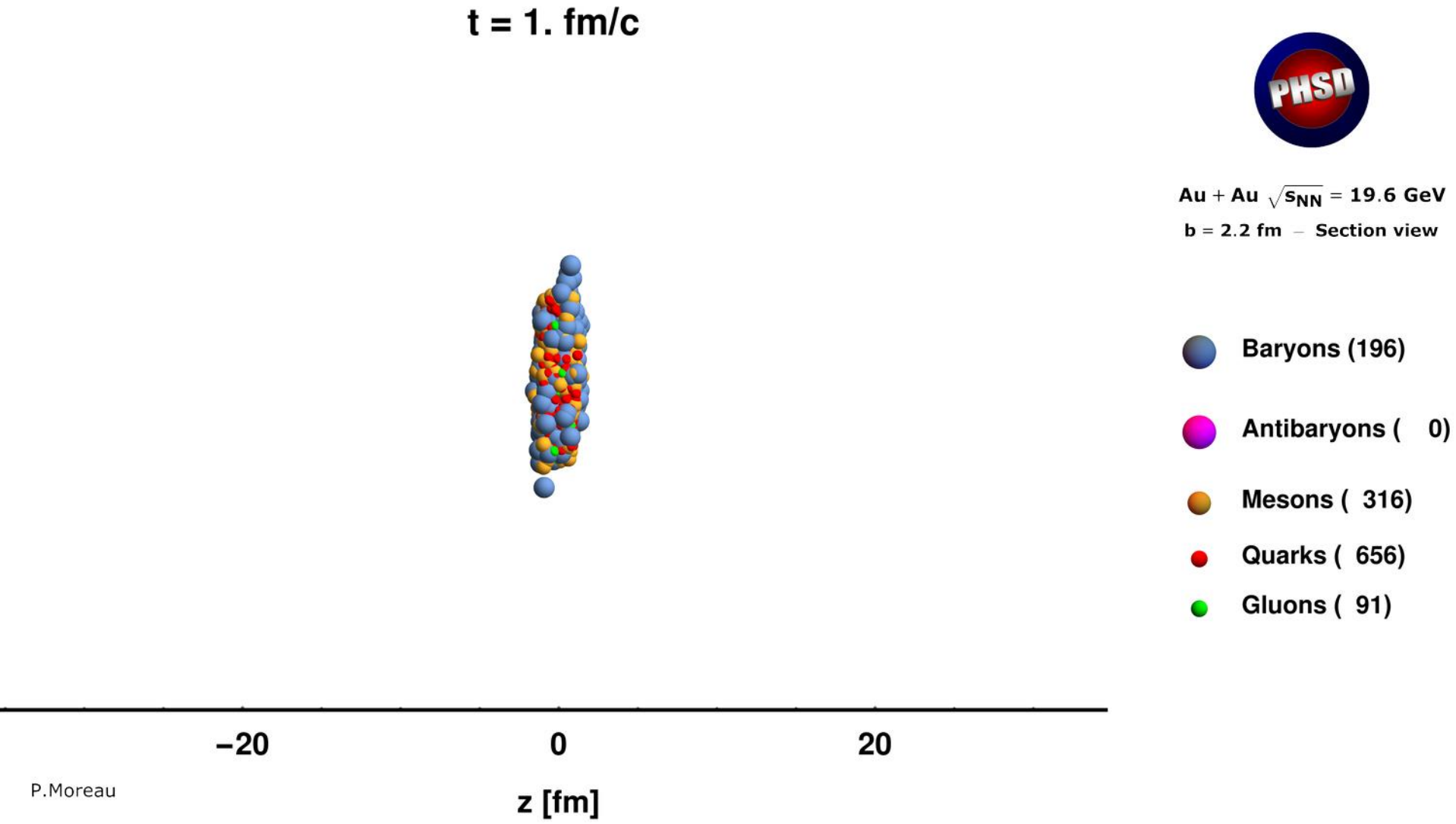

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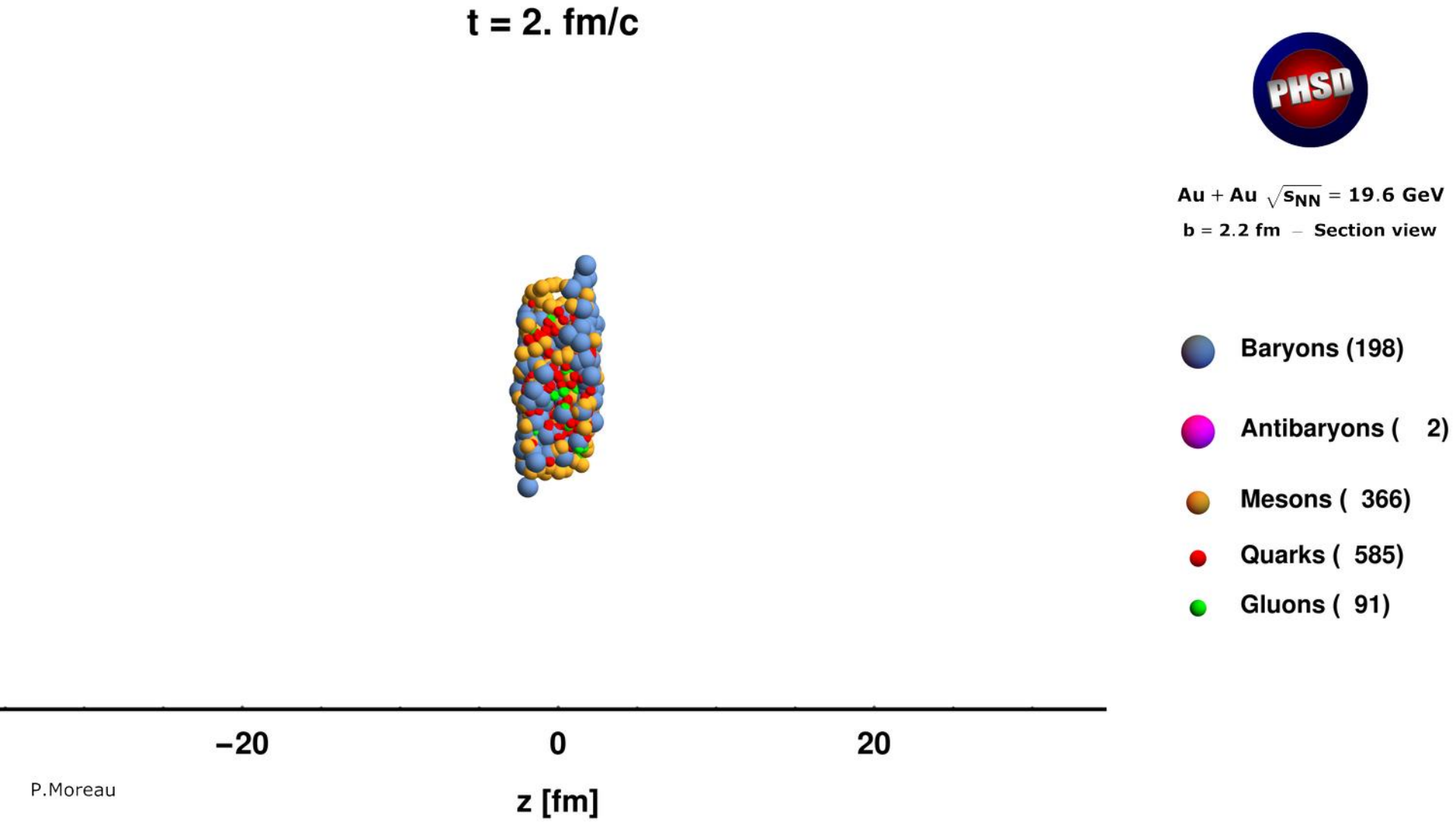

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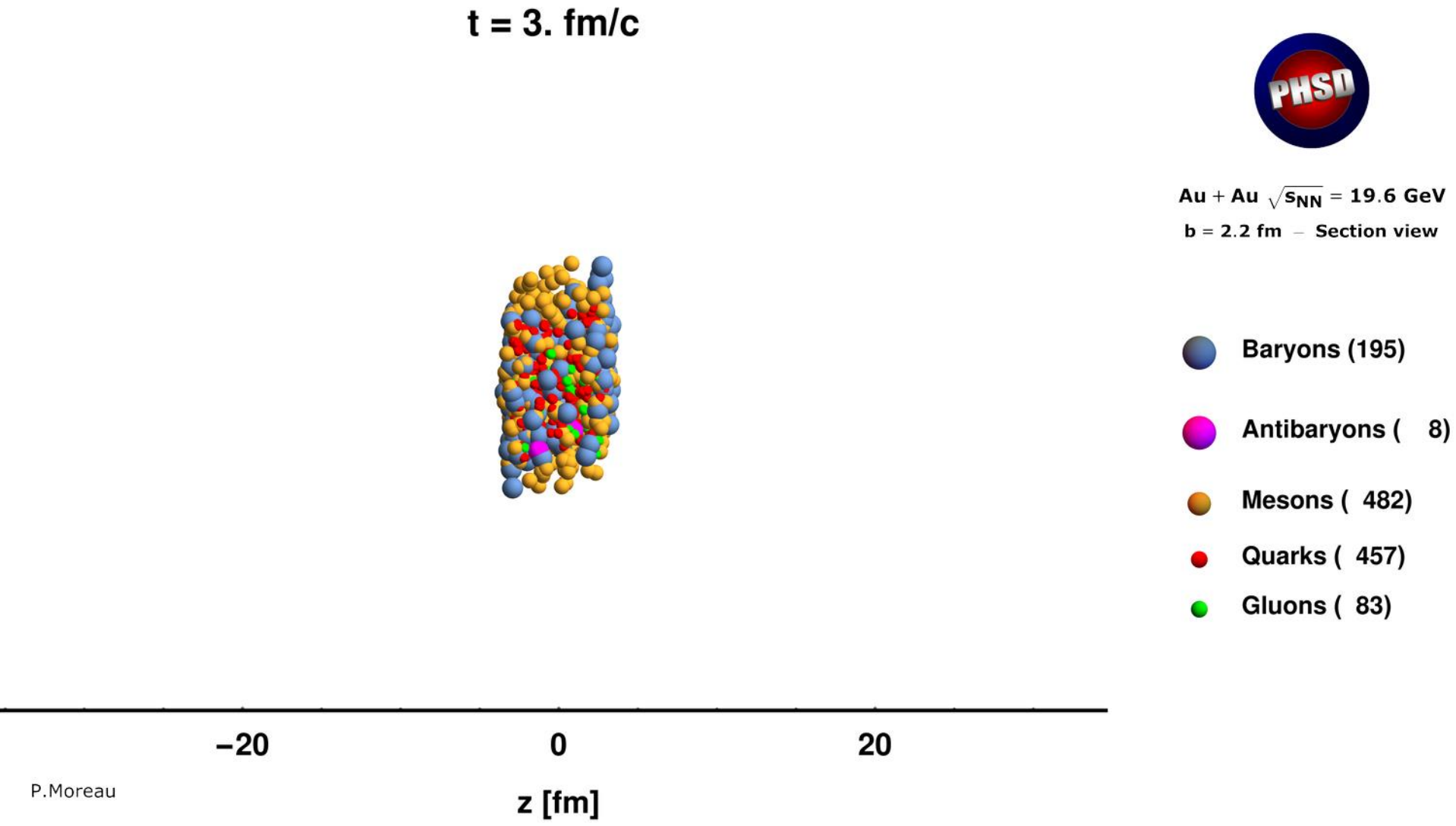

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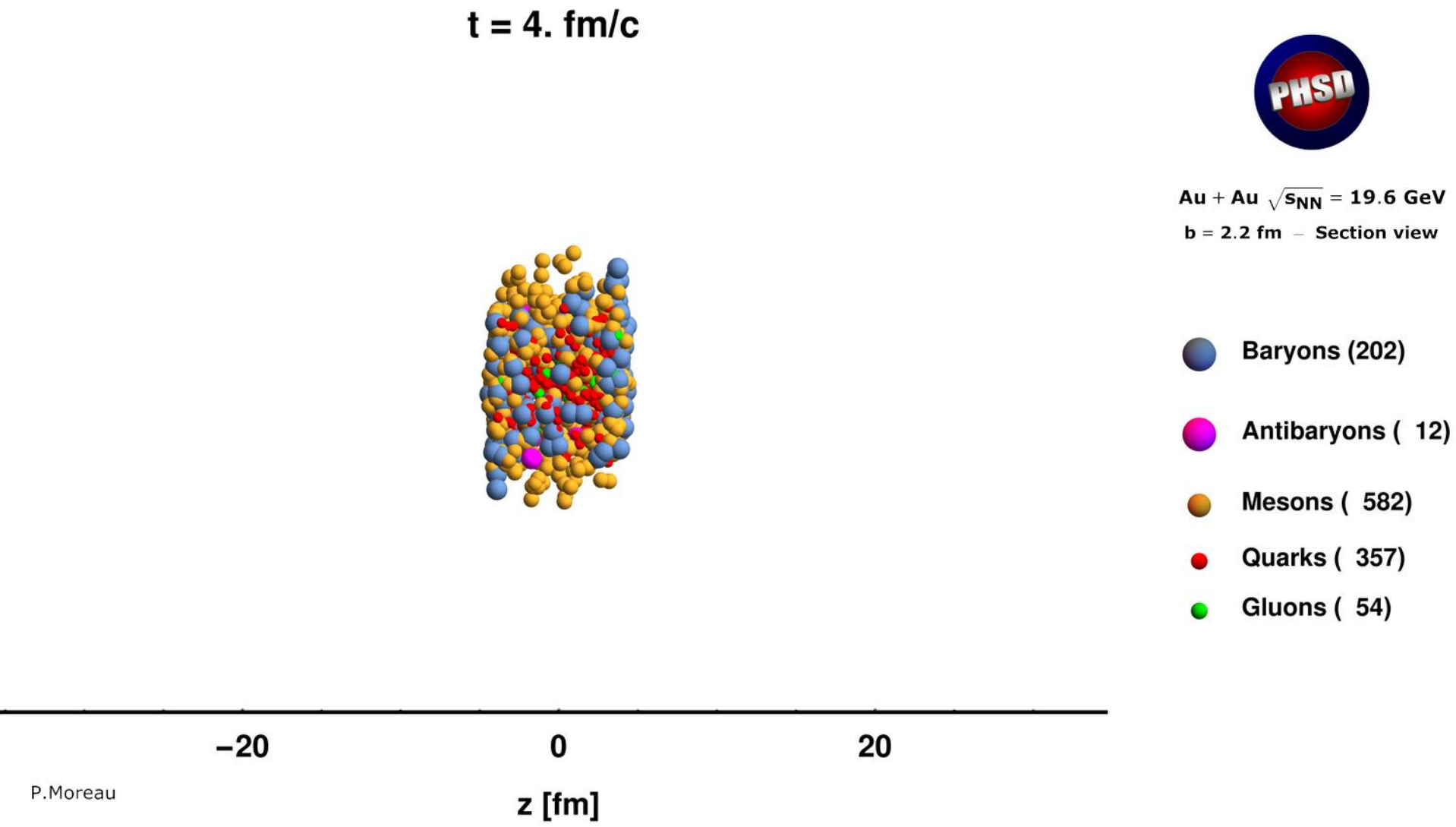

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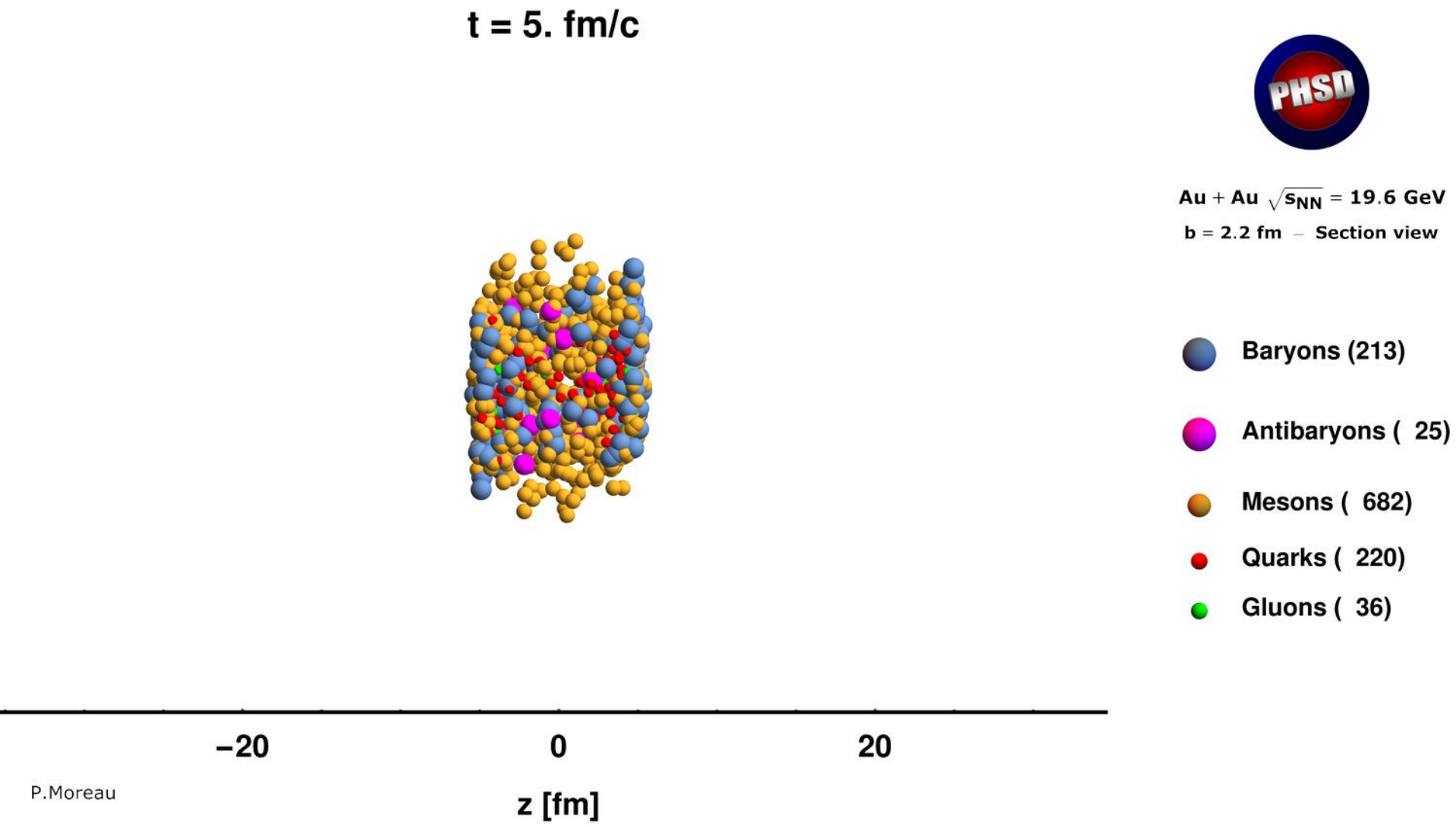

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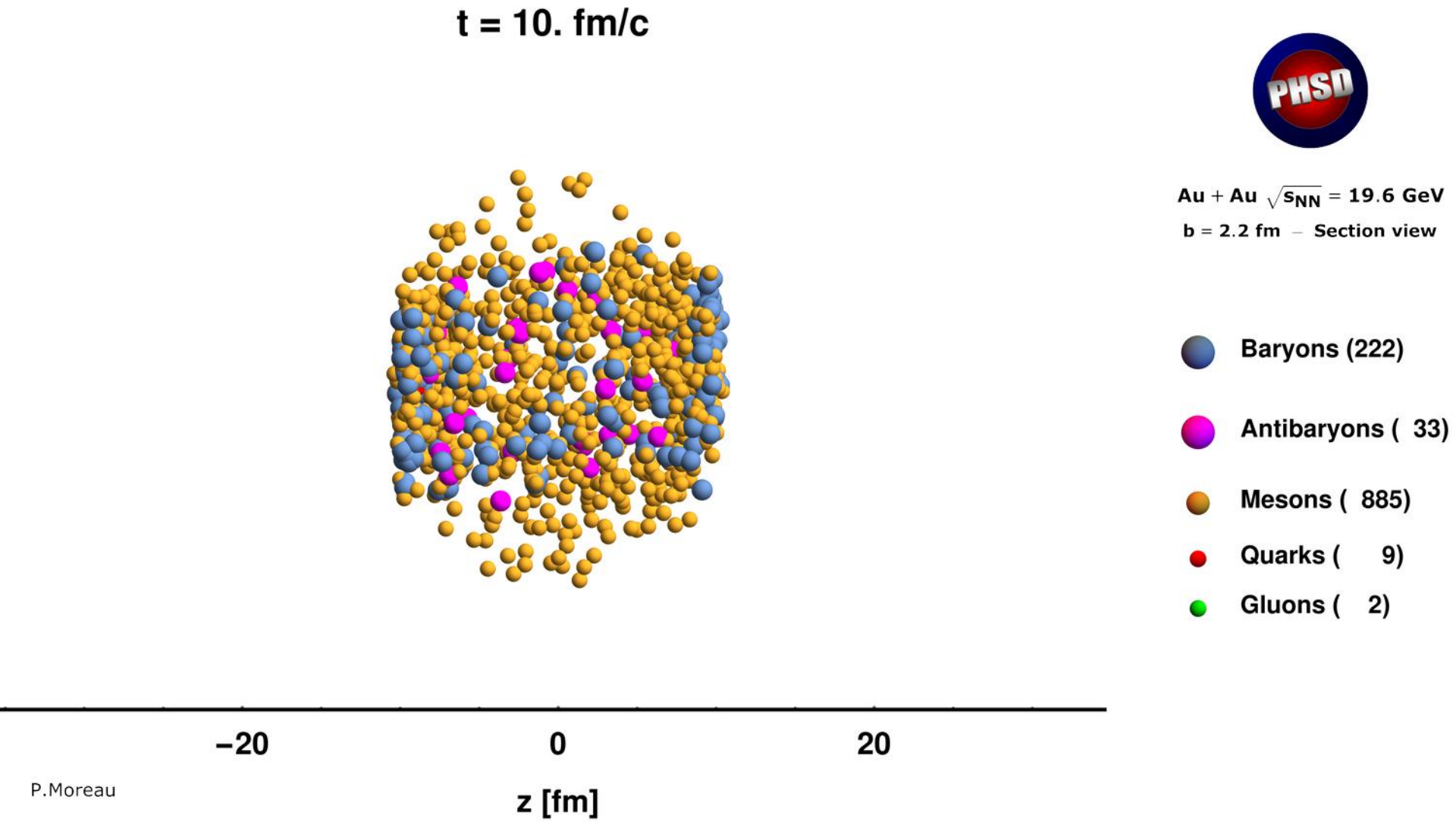

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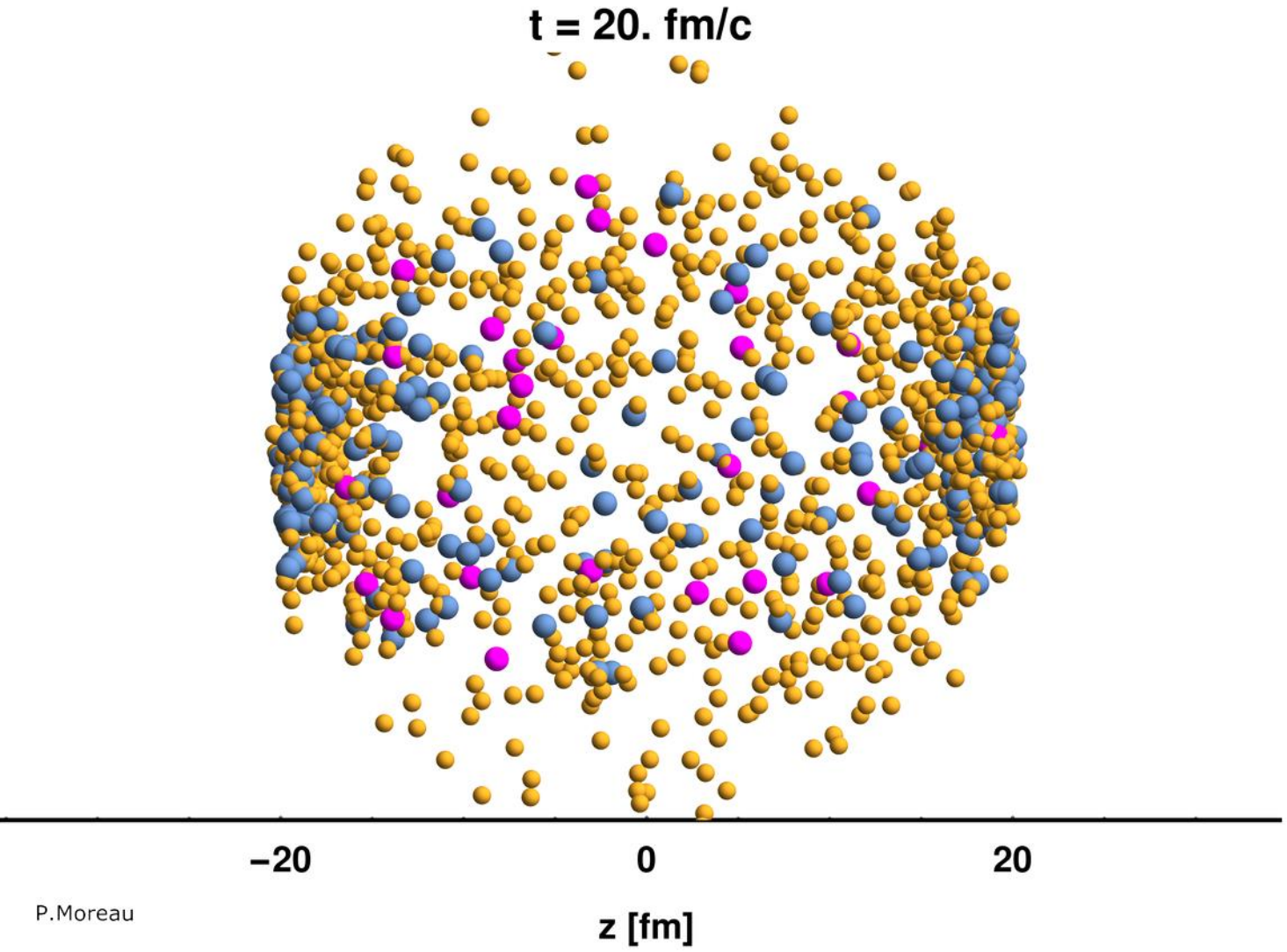

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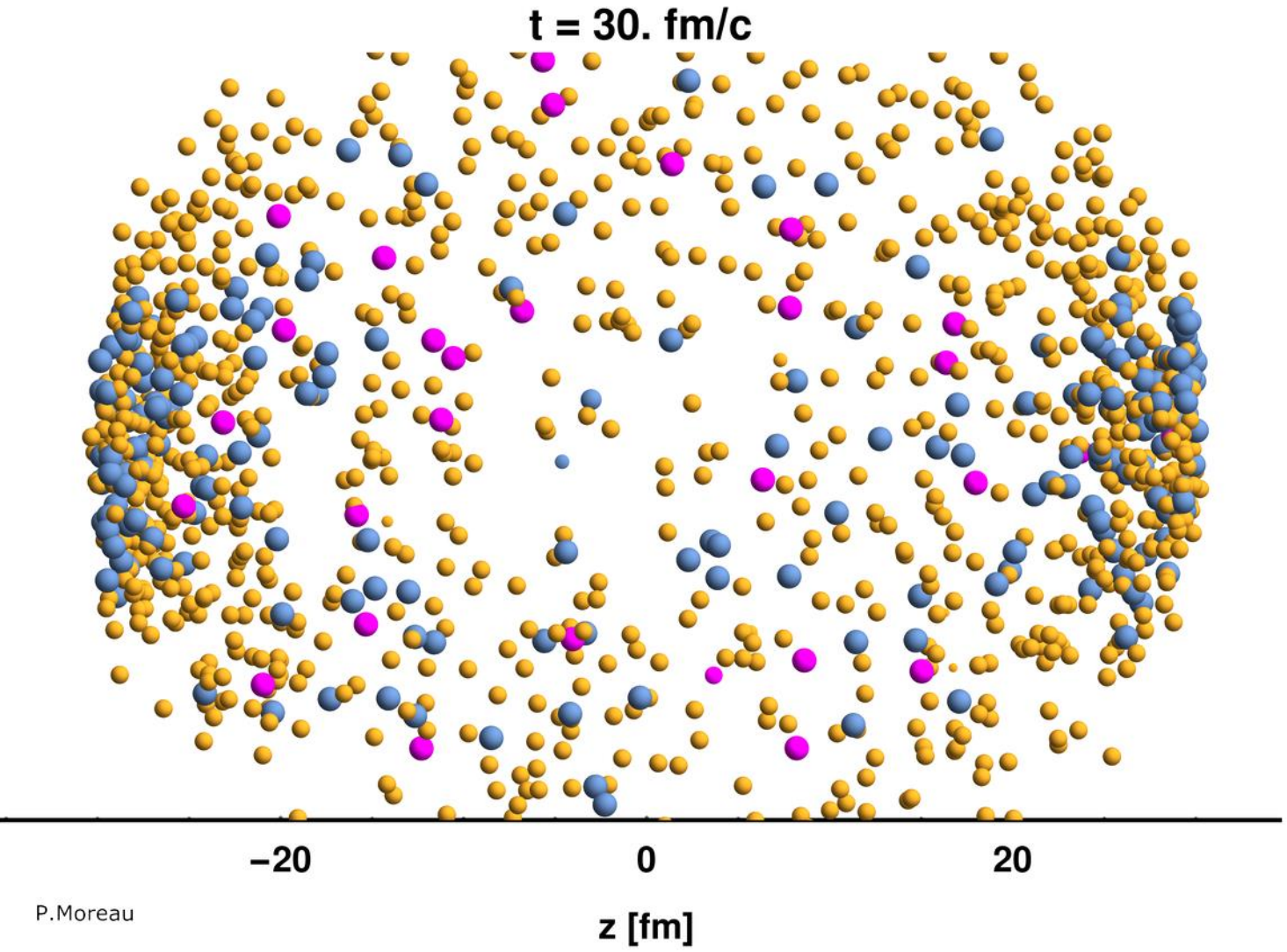
# Stages of a collision in PHSD



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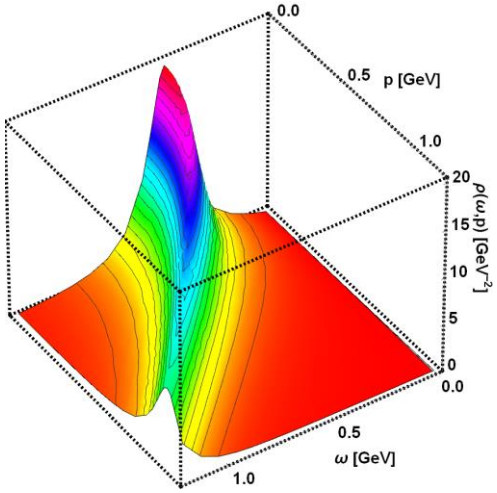
# Dynamical QuasiParticle Model (DQPM)

- The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions

$$\rho_j(\omega,\mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega-\tilde{E}_j)^2+\gamma_j^2} - \frac{1}{(\omega+\tilde{E}_j)^2+\gamma_j^2}\right)$$
$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2-\mathbf{p}^2-M_j^2\right)^2+4\gamma_j^2\omega^2}$$




- Resummed properties of the quasiparticles are specified by scalar complex self-energies:

$$\Pi = M^2 - 2i\gamma\omega$$

  - Real part of the self-energy: thermal mass ($M_g, M_q$)
  - Imaginary part of the self-energy: interaction width of partons ($\gamma_g, \gamma_q$)

- Propagator: $\Delta = (\omega^2 - p^2 - \Pi)^{-1}$

Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

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# DQPM - Parton properties

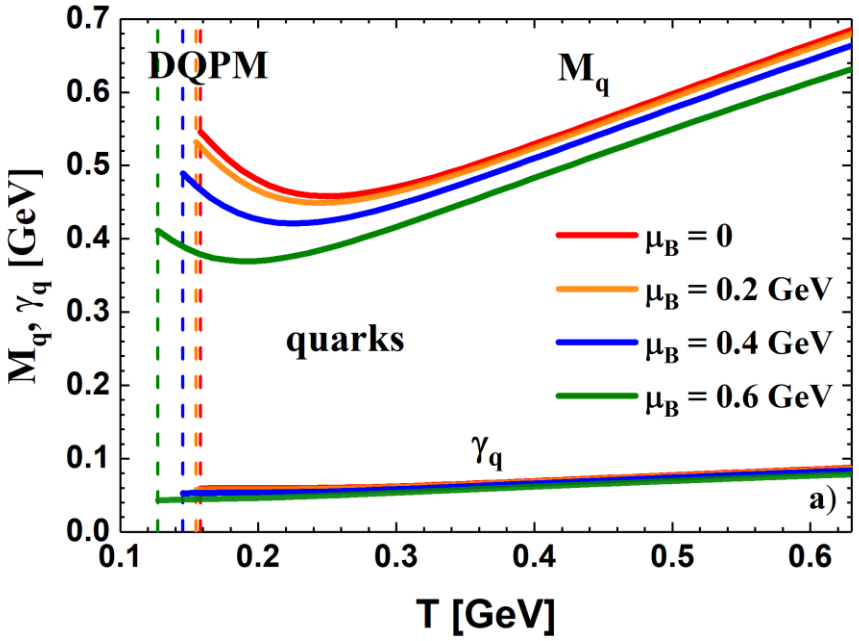
- **Modeling of the quark/gluon masses and widths (inspired by HTL calculations)**

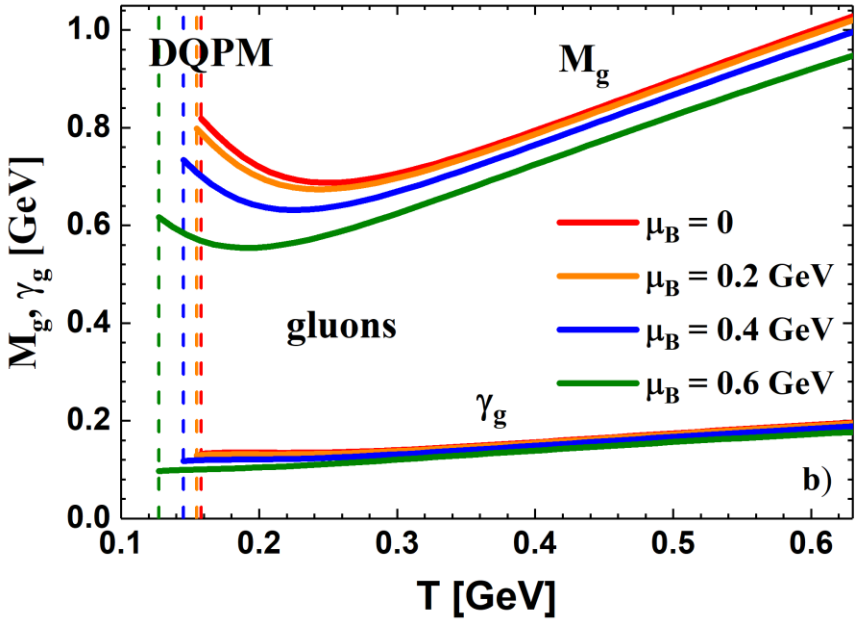
$$M_g^2(T,\mu_B) = \frac{g^2(T,\mu_B)}{6}\left(\left(N_c+\frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$M_{q(\bar{q})}^2(T,\mu_B) = \frac{N_c^2-1}{8N_c} g^2(T,\mu_B)\left(T^2+\frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_{q(\bar{q})}(T,\mu_B) = \frac{1}{3}\frac{N_c^2-1}{2N_c}\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

$$\gamma_g(T,\mu_B) = \frac{1}{3}N_c\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$



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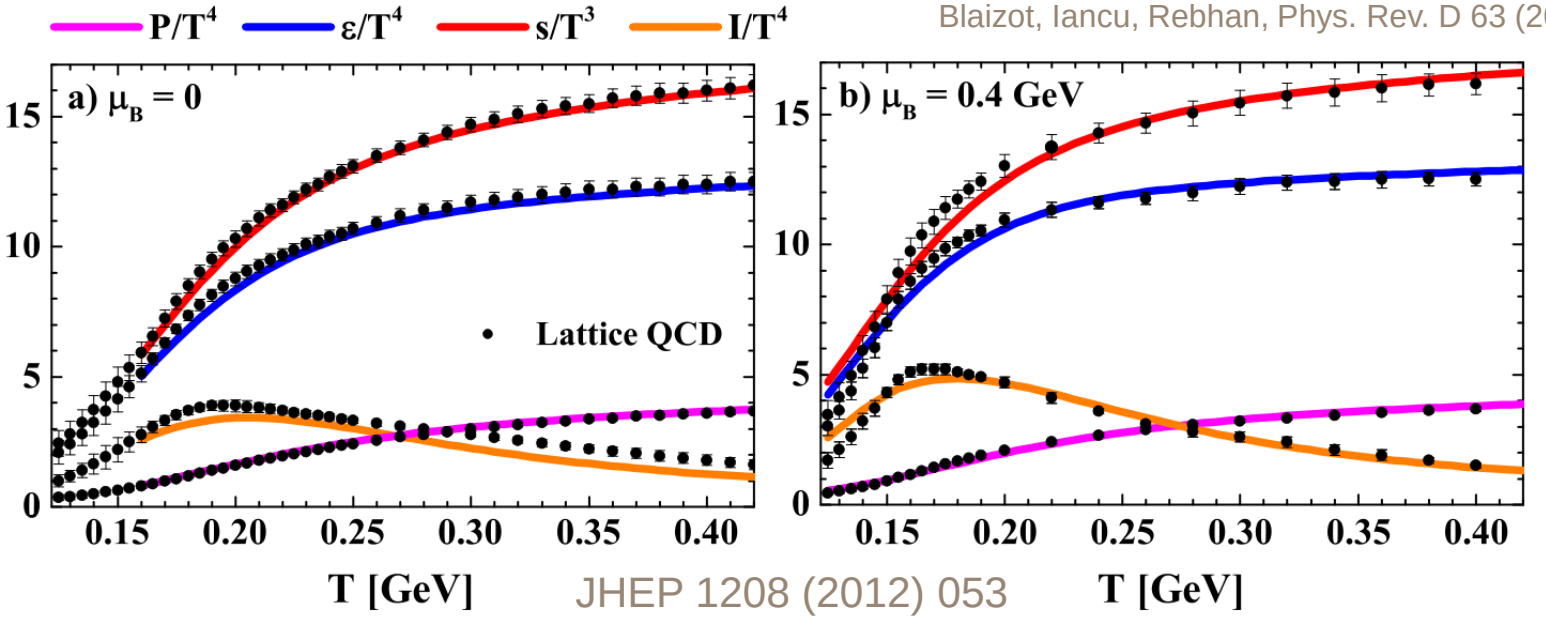
# DQPM Thermodynamics

- **Entropy and baryon density in the quasiparticle limit:**

$$s^{dqp} = -\int \frac{d\omega}{2\pi}\frac{d^3p}{(2\pi)^3}\Big[d_g\,\frac{\partial n_B}{\partial T}\left(\mathrm{Im}(\ln -\Delta^{-1}) + \mathrm{Im}\,\Pi\,\mathrm{Re}\,\Delta\right) + \sum_{q=u,d,s} d_q\,\frac{\partial n_F(\omega-\mu_q)}{\partial T}\left(\mathrm{Im}(\ln -S_q^{-1}) + \mathrm{Im}\,\Sigma_q\,\mathrm{Re}\,S_q\right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}}\,\frac{\partial n_F(\omega+\mu_q)}{\partial T}\left(\mathrm{Im}(\ln -S_{\bar{q}}^{-1}) + \mathrm{Im}\,\Sigma_{\bar{q}}\,\mathrm{Re}\,S_{\bar{q}}\right)\Big]$$

$$n^{dqp} = -\int \frac{d\omega}{2\pi}\frac{d^3p}{(2\pi)^3}\Big[\sum_{q=u,d,s} d_q\,\frac{\partial n_F(\omega-\mu_q)}{\partial \mu_q}\left(\mathrm{Im}(\ln -S_q^{-1}) + \mathrm{Im}\,\Sigma_q\,\mathrm{Re}\,S_q\right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}}\,\frac{\partial n_F(\omega+\mu_q)}{\partial \mu_q}\left(\mathrm{Im}(\ln -S_{\bar{q}}^{-1}) + \mathrm{Im}\,\Sigma_{\bar{q}}\,\mathrm{Re}\,S_{\bar{q}}\right)\Big]$$

Blaizot, Iancu, Rebhan, Phys. Rev. D 63 (2001) 065003




JHEP 1208 (2012) 053

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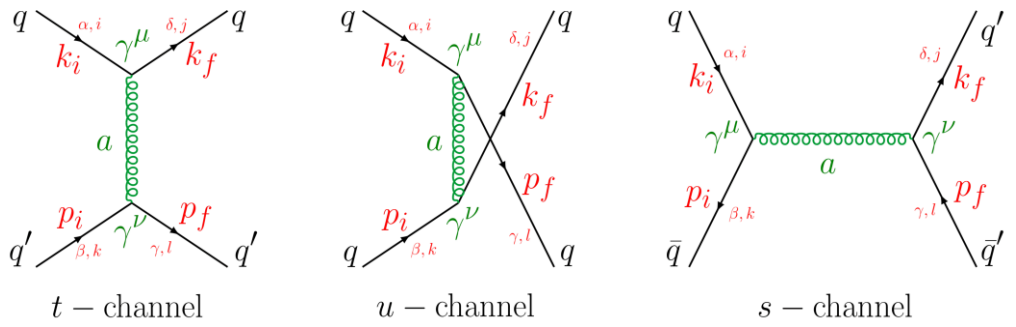
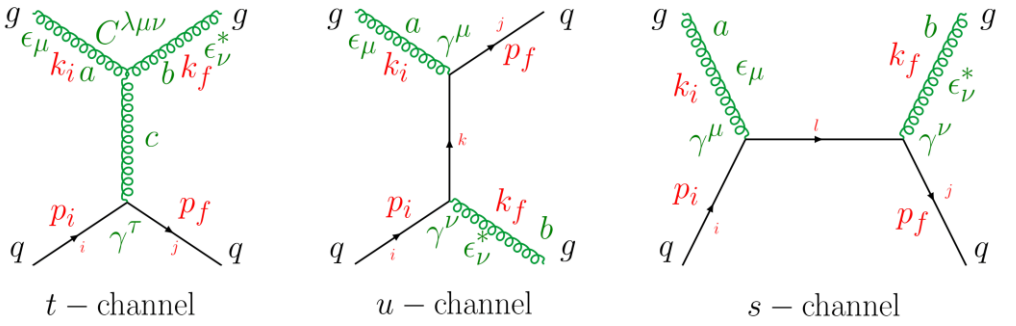
# Partonic interactions: matrix elements

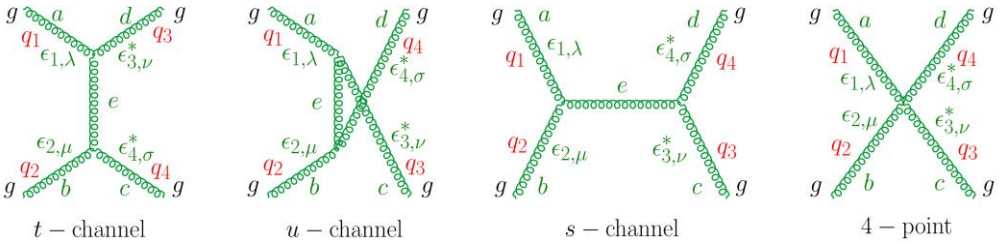
$qq', \; q\bar{q}$




$gq$




$gg$



# Energy-momentum tensor in PHSD

- In each space-time cell of the PHSD, the energy-momentum tensor is calculated by the formula:

$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{E_i}$$

- Diagonalization of the energy-momentum tensor to get the energy density and pressure components expressed in the local rest frame (LRF)

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix} \rightarrow \begin{pmatrix} \epsilon^{LRF} & 0 & 0 & 0 \\ 0 & P_x^{LRF} & 0 & 0 \\ 0 & 0 & P_y^{LRF} & 0 \\ 0 & 0 & 0 & P_z^{LRF} \end{pmatrix}$$

- **Landau-matching condition:**

Xu et al., Phys.Rev. C96 (2017), 024902

$$T^{\mu\nu} u_\nu = \epsilon u^\mu = (\epsilon g^{\mu\nu}) u_\nu$$

# Hydrodynamic decomposition

- **Energy-momentum tensor**

$$T^{\mu\nu} = eu^\mu u^\nu - (P_0 + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

- **Charge currents**

$$J^\mu_{(B,Q,S)} = n_{(B,Q,S)} u^\mu + n^\mu_{(B,Q,S)}$$

- **Reynolds number associated to each dissipative current:**

$$R_\Pi^{-1} = \frac{|\Pi|}{(e + P_0)} \qquad R_\pi^{-1} = \frac{|\pi^{\mu\nu}|}{(e + P_0)} \qquad R^{-1}_{n_{(B,Q,S)}} = \frac{|n^\mu_{(B,Q,S)}|}{n_{(B,Q,S)}}$$

T. Dore, J. Noronha-Hostler, E. McLaughlin, Phys.Rev.D 102 (2020) 7, 074017
G S Denicol 2014 J. Phys. G: Nucl. Part. Phys. 41 124004

- **In PHSD:** $J^\mu_{(B,Q,S)} = \sum_i \frac{p_i^\mu}{E_i}(B_i, Q_i, S_i) \qquad n_{(B,Q,S)} = u_\mu J^\mu_{(B,Q,S)}$

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# HRG & lQCD EoS

- **Hadron resonance gas (HRG) model: equation of state of a noninteracting hadron gas, valid for $T < T_c$ and all $\mu_B$**
  - **Include all mesons and (anti)baryons which are in PHSD**
  - **Particle information (mass, spin, width, quark content, …) from**


https://github.com/scikit-hep/particle

`Particle` : PDG particle data and identification codes

  - **Calculate thermodynamics**

**Start with the partition function in the Grand-Canonical ensemble (GC):**

$$\ln Z^{GC}(T,V,\{\mu_i\}) = \sum_{\text{species } i} \frac{g_i V}{(2\pi)^3} \int d^3p \ln\left(1 \pm e^{-\beta(E_i-\mu_i)}\right)^{\pm 1} \qquad \mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$$

$$P^{GC} = T\frac{\partial \ln Z^{GC}}{\partial V} \qquad N_i^{GC} = T\frac{\partial \ln Z^{GC}}{\partial \mu_i} \qquad S^{GC} = \frac{\partial}{\partial T}\left(T \ln Z^{GC}\right)$$

S. Wheaton et al. / Computer Physics Communications 180 (2009) 84–106

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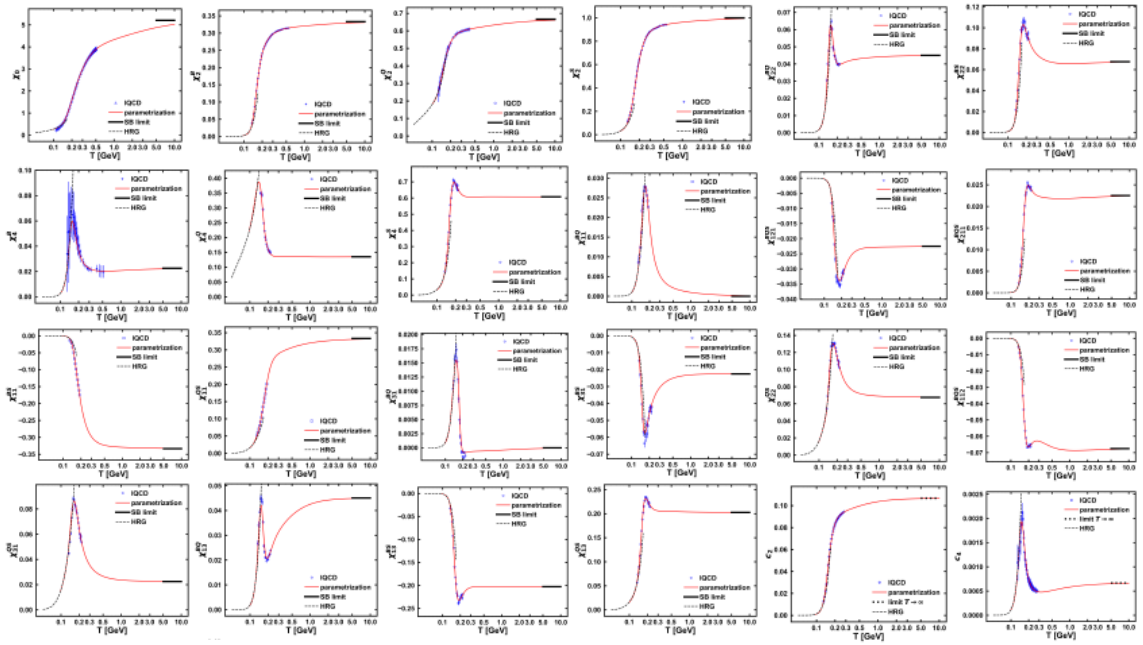
# HRG & lQCD EoS

- **Hadron resonance gas (HRG) model: equation of state of a noninteracting hadron gas, valid for $T < T_c$ and all $\mu_B$**

- **Lattice QCD EoS at arbitrary $T, \mu_B, \mu_Q, \mu_S$**

  **Parametrization for the susceptibilities $\chi$ inspired from:**

  J. Noronha-Hostler, P. Parotto, C. Ratti, and J. M. Stafford: Phys. Rev. C **100**, 064910



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# HRG & lQCD EoS

- **Hadron resonance gas (HRG) model: equation of state of a noninteracting hadron gas, valid for $T < T_c$ and all $\mu_B$**
- **Lattice QCD EoS at arbitrary $T, \mu_B, \mu_Q, \mu_S$**
- **Match the HRG EoS and the lQCD EoS at the critical temperature $T_c(\mu)$**

$$\frac{P}{T^4} = \frac{1}{2}\left[1 - f(T,\mu_J)\right]\frac{P_{HRG}}{T^4} + \frac{1}{2}\left[1 + f(T,\mu_J)\right]\frac{P_{lQCD}}{T^4}$$

**Matching function:**

$$f(T,\mu_J) = \tanh((T - T_c(\mu_B))/\Delta T_c) \qquad \text{with } J = \{B,Q,S\} \text{ and where } \Delta T_c = 0.1 T_c(0)$$

**Same procedure as in:**

Akihiko Monnai, Björn Schenke, and Chun Shen: Phys. Rev. C 100, 024907

# Extraction of freeze-out parameters

- To compare the PHSD trajectories with the isentropic trajectories, we need to find first the freeze-out parameters
  - Use the particle densities from the HRG EoS as a function of $T, \mu_B, \mu_Q, \mu_S$
  - Take into account decay of unstable particles in order to fit the PHSD final particle yields


https://github.com/scikit-hep/decaylanguage

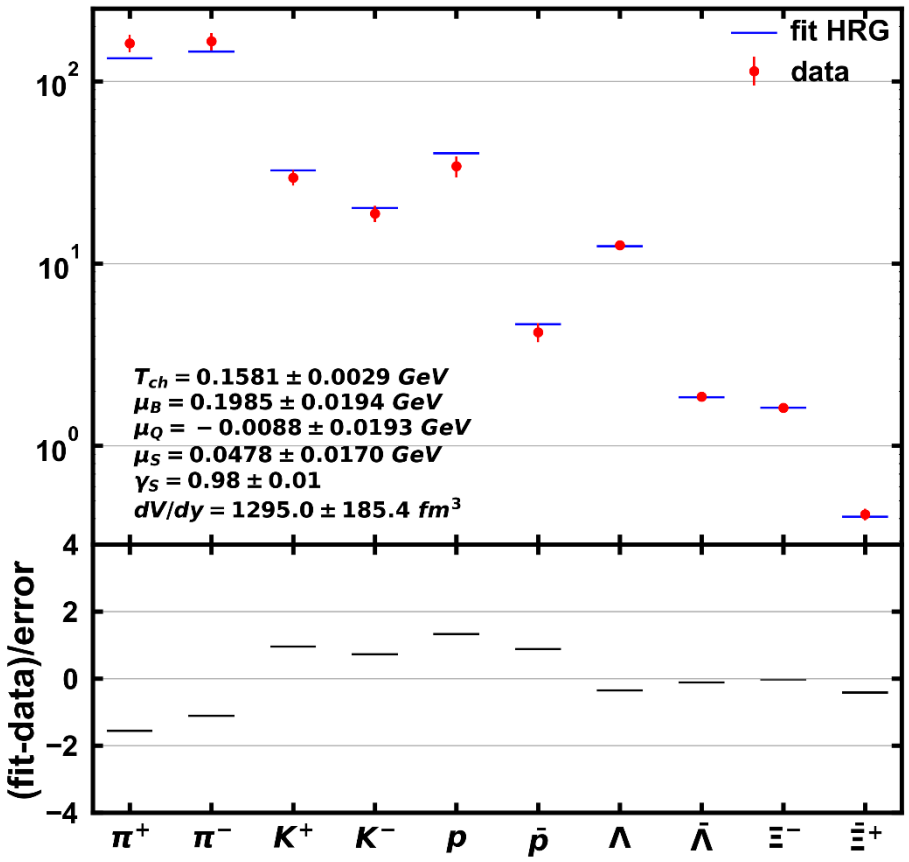
DecayLanguage: describe, manipulate and convert particle decays

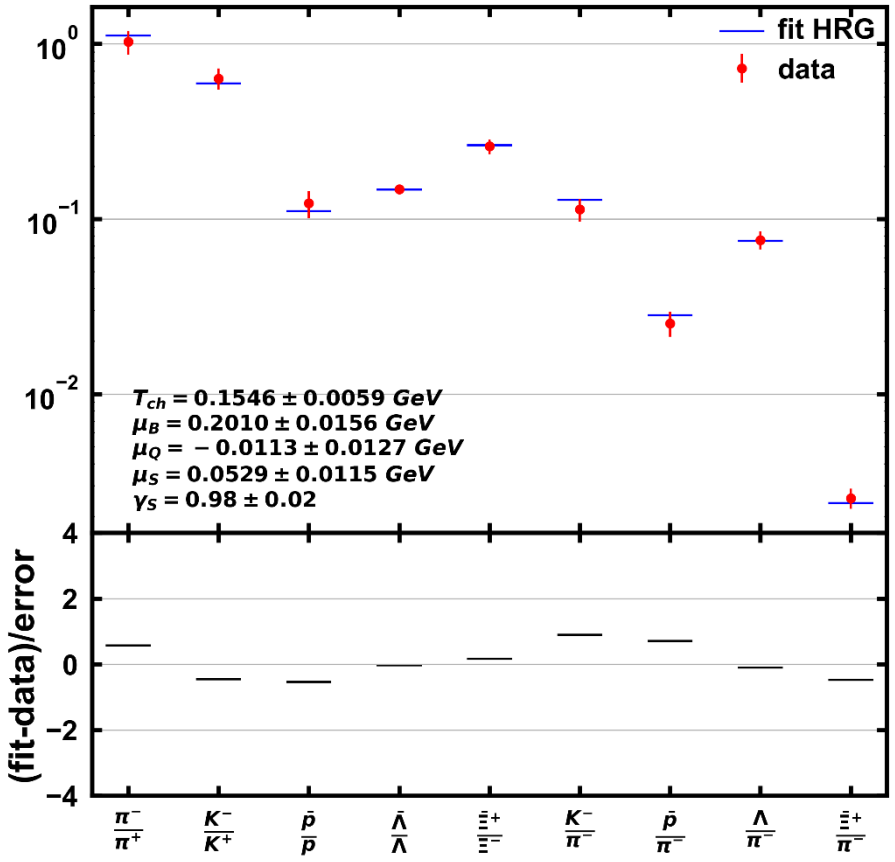
  - Fit model to PHSD data and obtain $T, \mu_B, \mu_Q, \mu_S$ at freeze-out


https://github.com/scikit-hep/iminuit

# Extraction of freeze-out parameters

## Freeze-out parameter fit to PHSD particle yields for Au+Au at 19.6 GeV



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# Other trajectories at $\sqrt{s_{NN}} = 62.4$ GeV