Heavy quarks embedded in glasma

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Heavy quarks

Heavy quarks - charm and beauty - due to their large masses are produced with high p_T at the earliest stage of HIC through hard interactions.

Heavy quarks propagate through the QCD medium and test its evolution at all stages.

Heavy quark in a weakly interacting QCD thermal medium (kinetic theory) - a Brownian particle undergoing random kicks from fast-moving particles of the medium (changes of movement, energy loss due to collisions and due to radiation)

What are the effects of pre-equilibrium phase on heavy quarks?

Heavy quarks: $m,\ p^\mu,\ E_{\mathbf{p}},\ \mathbf{v} = \frac{\mathbf{p}}{E_{\mathbf{p}}}$ - mass, four-momentum, energy and velocity of HQ

$Q(t, \mathbf{r}, \mathbf{p})$ - distribution function of heavy quarks in the medium

Soft classical field (because of the large occupation numbers): $A^\mu(x)$

Interaction: $\mathbf{F}(t, \mathbf{r}) \equiv g\big(\mathbf{E}(t, \mathbf{r}) + \mathbf{v} \times \mathbf{B}(t, \mathbf{r})\big)$ - color Lorentz force

$\mathbf{E}(t, \mathbf{r}),\ \mathbf{B}(t, \mathbf{r})$ - chromoelectric and chromomagnetic fields

Fokker-Planck equation

Evolution equation on the distribution function of heavy quarks: *Mrówczyński, Eur. Phys. J. A54, no 3, 43 (2018)*

$$\Big(D - \nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j - \nabla_p^i Y^i(\mathbf{v})\Big) n(t,\mathbf{x},\mathbf{p}) = 0$$

$$\langle Q(t,\mathbf{r},\mathbf{p})\rangle = n(t,\mathbf{r},\mathbf{p}) \qquad D = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$$

Collision terms:

$$X^{ij}(\mathbf{v}) \equiv \frac{1}{2N_c}\int_0^t dt' \left\langle F_a^i(t,\mathbf{x}) F_a^j\left(t', \mathbf{x} - \mathbf{v}(t-t')\right)\right\rangle$$

$Y^i(\mathbf{v}) = X^{ij}(\mathbf{v})\frac{v^j}{T}$ - postulated so that the distribution function satisfies the Fokker-Planck equation in equilibrium

T - temperature of plasma that has the same energy density as in equilibrium

Collision terms determine energy loss and momentum broadening

Physical meaning: $\frac{\langle \Delta p^i\rangle}{\Delta t} = -Y^i(\mathbf{v})$ $\qquad$ $\frac{\langle \Delta p^i \Delta p^j\rangle}{\Delta t} = X^{ij}(\mathbf{v}) + X^{ji}(\mathbf{v})$

CGC: before the collision

Classical Yang-Mills equations: $\left[D_\mu, F^{\mu\nu}\right] = J^\nu$ $\qquad J^\mu_{1,2}(x^\mp, \vec{x}_\perp) = \delta^{\mu\pm}\rho_{1,2}(x^\mp, \vec{x}_\perp)$

Solutions of CYM equations:

$$A^\pm_{1,2}(x^\pm, \vec{x}_\perp) = 0$$

$$A^i_{1,2}(x^\pm, \vec{x}_\perp) = \theta(x^\mp)A^i_{1,2}(\vec{x}_\perp)$$

$$A^i_{1,2}(\vec{x}_\perp) = -\frac{1}{ig}U_{1,2}(\vec{x}_\perp)\partial^i U^\dagger_{1,2}(\vec{x}_\perp)$$

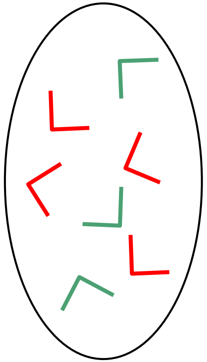
pure gauge transform of vacuum

$U(\vec{x}_\perp) \equiv U[g, \rho, \vec{x}_\perp]$ - the unitary matrix

Chromoelectric and chromomagnetic fields are given by
the respective components of the strength tensor. They are:

$$E^z_{1,2}(x^\pm, \vec{x}_\perp) = 0 \qquad E^i_{1,2}(x^\pm, \vec{x}_\perp) = -\frac{\delta(x^\mp)}{\sqrt{2}}A^i_{1,2}(\vec{x}_\perp)$$

$$B^z_{1,2}(x^\pm, \vec{x}_\perp) = 0 \qquad B^i_{1,2}(x^\pm, \vec{x}_\perp) = \mp\epsilon^{ij}E^j_{1,2}(x^\pm, \vec{x}_\perp)$$



Only transverse components are different from zero.

After the collisions: glasma

Glasma fields develop in the forward light-cone region.

Analytical approach to solve CYN for the glasma fields proposed in:

Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015)

CGC looses applicability soon after the collision → proper time of such an evolving system is small

τ -acts as an expansion parameter

$$A^i_\perp(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A^i_{\perp(n)}(\vec{x}_\perp)$$
$$A(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(\vec{x}_\perp)$$

$$|A^i_{\perp(n)}| \sim Q_s^n |A|$$
$$|A_{(n)}| \sim Q_s^{n+1} |A|$$
$$|A| = \sqrt{A^i_1 A^i_1}$$

radius of convergence is set by the only time scale - saturation momentum scale as

$$1/Q_s$$

τQ_s -dimensionless

Boundary conditions connect different light-cone sectors:

$$A^i_\perp(\tau = 0, \vec{x}_\perp) = A^i_1(\vec{x}_\perp) + A^i_2(\vec{x}_\perp)$$
$$A(\tau = 0, \vec{x}_\perp) = -\frac{ig}{2}[A^i_1(\vec{x}_\perp), A^i_2(\vec{x}_\perp)]$$

Using the boundary conditions the system of coupled YM equations can be solved recursively.

Expansion in the proper time

Expansion of chromodynamic fields

$$\mathbf{E} = \mathbf{E}_{(0)} + \tau\mathbf{E}_{(1)} + \tau^2\mathbf{E}_{(2)} + \dots$$
$$\mathbf{B} = \mathbf{B}_{(0)} + \tau\mathbf{B}_{(1)} + \tau^2\mathbf{B}_{(2)} + \dots$$

$$E^z_{(0)} = ig[A^i_1, A^i_2]$$
$$B^z_{(0)} = ig\epsilon^{ij}[A^i_1, A^j_2]$$

0th order fields are purely longitudinal

(superposition of two pure-gauge potentials is not pure gauge due to non-linear character of non-Abelian theory)

$$E^i_{(1)} = -\frac{1}{2}\left(\sinh\eta\left[D^i_{(0)}, E_0\right] + \cosh\eta\epsilon^{ij}\left[D^j_{(0)}, B_0\right]\right)$$
$$B^i_{(1)} = \frac{1}{2}\left(\cosh\eta\epsilon^{ij}\left[D^j_{(0)}, E_0\right] - \sinh\eta\left[D^i_{(0)}, B_0\right]\right)$$

1st order fields are purely transverse

(induced by the decrease of longitudinal fields after a short time)

To compute the energy loss and momentum broadening we need the correlators of fields:

$$X^{ij}(\mathbf{v}) = \frac{g^2}{2N_c}\int_0^t dt'\Big[\langle E^i_a(t,\mathbf{x})E^j_a(t',\mathbf{x}')\rangle + \epsilon^{jkl}v^k\langle E^i_a(t,\mathbf{x})B^l_a(t',\mathbf{x}')\rangle$$
$$+ \epsilon^{ikl}v^k\langle B^l_a(t,\mathbf{x})E^i_a(t,\mathbf{x}')\rangle + \epsilon^{ikl}\epsilon^{jmn}v^k v^m\langle B^l_a(t,\mathbf{x})B^n_a(t',\mathbf{x}')\rangle\Big]$$

Correlators of gauge potentials

Potentials of different nuclei are uncorrelated: $\langle A^i_{1a} A^j_{2b} \rangle = 0$

Potentials of the same nuclei are correlated with:

$$\langle \rho_a(x^\mp, \vec{x}_\perp)\rho_b(y^\mp, \vec{y}_\perp)\rangle = \frac{g^2}{N_c^2-1}\delta_{ab}\lambda(x^\mp, \vec{x}_\perp)\delta(x^\mp - y^\mp)\delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

$$\int dx^\mp \lambda(x^\mp, \vec{x}_\perp) = \mu(\vec{x}_\perp)$$

volume density of sources (λ) — area density of sources (μ)

Correlators of gauge fields:

$$\langle A^i_a(\mathbf{x}_\perp) A^j_b(\mathbf{x}'_\perp)\rangle = \delta^{ab}\left(\delta^{ij}_\perp C_1(r) - \hat{r}^i \hat{r}^j C_2(r)\right)$$

$$C_1(r) \equiv \frac{m^2 K_0(mr)}{g^2 N_c (mrK_1(mr)-1)}\left[e^{\frac{g^4 N_c \mu (mrK_1(mr)-1)}{4\pi m^2 (N_c^2-1)}} - 1\right]$$

$$C_2(r) \equiv \frac{m^3 r K_1(mr)}{g^2 N_c (mrK_1(mr)-1)}\left[e^{\frac{g^4 N_c \mu (mrK_1(mr)-1)}{4\pi m^2 (N_c^2-1)}} - 1\right]$$

$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{x}'_\perp, \quad r \equiv |\mathbf{r}|, \quad \hat{r}^i \equiv r^i / r$

$K_0, \ K_1$ MacDonald functions

$m \approx \Lambda_{\rm QCD} \approx 200 \ \rm MeV$ infrared regulator

$\mu = g^{-4}\,(N_c^2 - 1)Q_s^2$ charge density per unit transverse area

Correlators of electric and magnetic fields

0th order correlators

$$M_E(r) \equiv 2C_1^2(r) - 2C_1(r)\,C_2(r) + C_2^2(r)$$
$$M_B(r) \equiv 2C_1^2(r) - 2C_1(r)\,C_2(r)$$

$$\langle E_a^z(\mathbf{x}_\perp)\,E_b^z(\mathbf{x}'_\perp)\rangle = g^2 N_c \delta^{ab} M_E(r)$$
$$\langle B_a^z(\mathbf{x}_\perp)\,B_b^z(\mathbf{x}'_\perp)\rangle = g^2 N_c \delta^{ab} M_B(r)$$
$$\langle E_a^z(\mathbf{x}_\perp)\,B_b^z(\mathbf{x}'_\perp)\rangle = 0$$

1st order correlators

$$M'_E(r),\; M'_B(r)$$

$$\tau' \langle E_a^0(\mathbf{x}_\perp)\,E^i_{(1)b}(t',\mathbf{x}'_\perp,z')\rangle = -\frac{g^2}{2} N_c \delta^{ab}\,\hat{r}^i z'\,M'_E(r)$$
$$\cdots$$

(correlators of three gluon fields vanish)

all possible combinations of $E^0,\ B^0,\ E^i_{(1)},\ B^i_{(1)}$ at order of τ

Energy loss and momentum broadening

$$X^{ij}(\mathbf{v}) = \frac{g^2}{2N_c}\int_0^t dt' \Big[\big\langle E_a^i(t,\mathbf{x})E_a^j(t',\mathbf{x}')\big\rangle + \epsilon^{jkl}v^k\big\langle E_a^i(t,\mathbf{x})B_a^l(t',\mathbf{x}')\big\rangle$$
$$+ \epsilon^{ikl}v^k\big\langle B_a^l(t,\mathbf{x})E_a^i(t,\mathbf{x}')\big\rangle + \epsilon^{ikl}\epsilon^{jmn}v^k v^m\big\langle B_a^l(t,\mathbf{x})B_a^n(t',\mathbf{x}')\big\rangle\Big]$$

We were able to obtain an explicit analytic form of the tensor:

$$X^{ij}(\mathbf{v}) = \frac{g^4(N_c^2-1)}{4}\int_0^t dt' \Big\{2n^i n^j M_E(r) - \Big(n^i\hat{r}^j z' - n^j\hat{r}^i z\Big)M'_E(r)$$
$$+ \epsilon^{jkl}v^k\Big(n^i n^n \epsilon^{lmn}\hat{r}^m t' + n^l n^n \epsilon^{imn}\hat{r}^m t\Big)M'_E(r)$$
$$+ \epsilon^{ikl}v^k\Big[\epsilon^{jmn}v^m\Big(2n^l n^n M_B(r) - (n^l\hat{r}^n z' - n^n\hat{r}^l z)\,M'_B(r)\Big)$$
$$- \Big(n^l n^n \epsilon^{jmn}\hat{r}^m t' + n^j n^n \epsilon^{lmn}\hat{r}^m t\Big)M'_B(r)\Big]\Big\}.$$

$$\mathbf{n} = (0,0,1)$$

$$\frac{dE}{dx} = -\frac{v}{T}\frac{v^i v^j}{\mathbf{v}^2}X^{ij}(\mathbf{v}) \qquad \hat{q} = \frac{2}{v}\left(\delta^{ij} - \frac{v^i v^j}{\mathbf{v}^2}\right)X^{ij}(\mathbf{v})$$

Energy loss and momentum broadening

$$\frac{dE}{dx} = -\frac{v_\parallel^2}{vT}\left[f_E^0(v_\perp) + v_\perp f_E^1(v_\perp)\right]$$

$$\hat{q} = \frac{2v_\perp^2}{v}\left[\frac{f_E^0(v_\perp)}{v^2} + f_B^0(v_\perp) + \frac{v_\perp}{v^2} f_E^1(v_\perp) + \frac{(1-v_\parallel^2)}{v_\perp} f_B^1(v_\perp)\right]$$

$$f_{E,B}^0(v_\perp) \equiv \frac{g^4(N_c^2-1)}{2} \int_0^t dt'\, M_{E,B}(r)$$

$$f_{E,B}^1(v_\perp) \equiv \frac{g^4(N_c^2-1)}{4} \int_0^t dt'\,(t-t')M'_{E,B}(r)$$

$$r = v_\perp(t-t')$$

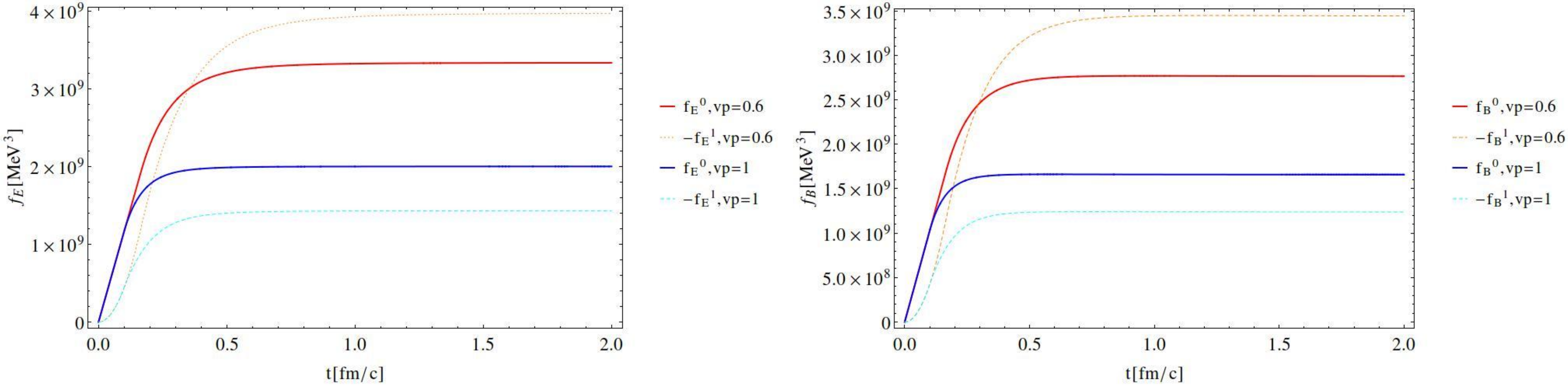
$M_{E,B}(r)$ diverges when $r \to 0$ **(CGC breaks down at small distances.)**

regularization procedure: $M^{\text{reg}}(r) \equiv \Theta(r_s - r)\, M(r_s) + \Theta(r - r_s)\, M(r)$

$$r_s = Q_s^{-1}$$

Functions $f_{E,B}$

$$m = 200\text{MeV},\ N_c = 3,\ g = 1,\ Q_s = 2\text{GeV}$$



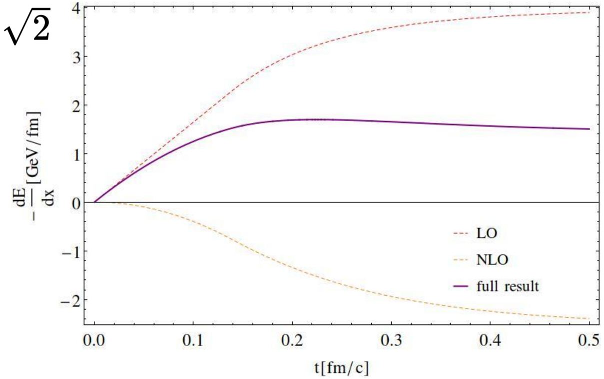
Saturation is reached before t=1fm/c.

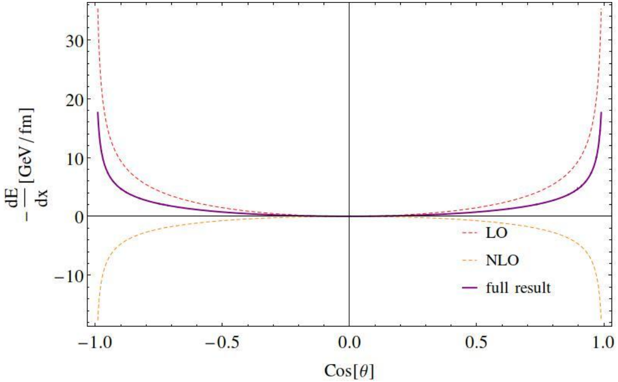
$$f^0_{E,B} > f^1_{E,B} \quad \text{for} \quad v_\perp > 0.73$$

Energy loss and momentum broadening

$$m = 200\text{MeV},\ N_c = 3,\ g = 1,\ Q_s = 2\text{GeV}$$

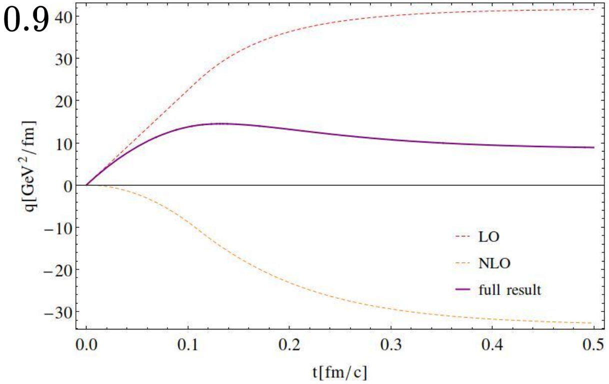
$v_{||} = v_{\perp} = 1/\sqrt{2}$

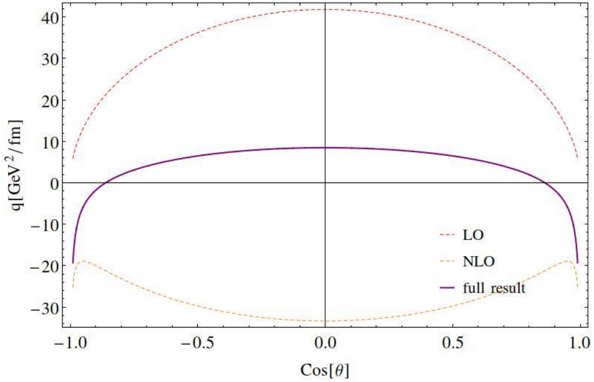




energy loss is minimal when HQ moves perpendicularly to the axis

$v_{||} = 0,\ v_{\perp} = 0.9$





momentum broadening is maximal when HQ moves perpendicularly to the axis

Remarks

saturation value at $v = 0.9, \ \cos\theta = 0$

$$\hat{q}_{\rm LO} = 42{\rm GeV}^2/{\rm fm} \qquad \hat{q}_{\rm LO+NLO} = 8.5{\rm GeV}^2/{\rm fm}$$

momentum broadening inferred from a jet quenching of heavy ion collisions

$$\hat{q}_{\rm JQ}: \ 1.5 - 7.0 \ {\rm GeV}^2/{\rm fm}$$

glasma may provide a significant contribution to jet quenching

- regularization procedure:
  we checked a few possibilities - results are not very sensitive to them
- higher order terms and convergence - validity of CGC - work in progress

Summary and conclusions

- Collision terms of the Fokker-Planck equation for HQ transported through glasma were derived
- Energy loss and momentum broadening of HQ were computed
- Both quantities are strongly directionally dependent
- Energy loss is maximal when the heavy quark moves along the collision axis
- Momentum broadening is maximal when the heavy quark moves perpendicularly to the axis
- The values of both transport coefficients are sizeable so the glasma phase may have a large effect on the jet quenching observed in HIC
- Higher order terms have to be carefully studied to draw a firm conclusion

NOTE: many papers use only LO terms, while here we obtained NLO corrections!