Properties of the strongly intensive observable $\Sigma$ in high energy pp collisions in a string fusion model

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Motivation. String fusion effects.

$\text{pp} \rightarrow \text{pA} \rightarrow \text{AA}$ - the increase of the string density in transverse plain leads to the string fusion (color ropes or string cluster formation).


The same in pp collisions with increasing energy and centrality LHC!

$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.
$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).


The same ideas in DIPSY:
Forward-Backward (FB) Rapidity Correlations

Forward-Backward (FB) Rapidity Correlations: \((k_z, \mathbf{k}_\perp) \Rightarrow (\eta, \mathbf{k}_\perp)\)

\[\eta \equiv \frac{1}{2} \ln \frac{k_0 + k_z}{k_0 - k_z}, \quad \eta' \equiv \frac{1}{2} \ln \frac{\mathbf{k} + k_z}{|\mathbf{k}| - k_z} = -\ln \tan \left(\frac{\theta^*}{2}\right)\]

The correlation coefficient:

\[b_{BF} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{\text{cov}(F, B)}{D_F} \quad (1)\]
Definitions

Short- and long-range rapidity correlations

Traditional Observables

Traditional FB correlation:
\[ B, F \Rightarrow n_B, n_F - \text{the extensive variables} \Rightarrow b_{nn} \]


The locality of strong interaction in rapidity \( \Rightarrow \)

Short-Range FB Correlations (SRC),
between particles from a same source (string).
\( z - \eta \) correspondence, X. Artru, Phys.Rept.\textbf{97}(1983)147,

V.V.V., arXiv:0812.0604

Event-by-event variance in the number of cut pomerons (strings) \( \Rightarrow \)

Long-Range FB Correlations (LRC) at large \( \eta_{sep} \)
(the trivial ”volume” fluctuations).

We’ll look for observables, which is not sensitive to the fluctuation in the number of sources (strings), but is sensitive to the fluctuation in the quality of sources (e.g. to the formation of string clusters by string fusion).
Advanced Observables

The string fusion processes under consideration affects both LRC and SRC. The LRC is sensitive to fluctuations in both quantity and type of source. The SRC is sensitive to the modification of the properties of a single string in process of their fusion into string clusters.

Unfortunately for traditional observables the $n_F-n_B$ correlation is strongly influenced by the ”volume” fluctuations.

We can suppress the influence of these trivial ”volume” fluctuations compared to the contribution of string fusion processes:

1) for LRC going from traditional extensive variables $n_F$ and $n_B$ to new intensive ones, e.g. event-mean transverse momenta $p_F$ and $p_B$ of all particles ($n_F$ and $n_B$) in the intervals $\delta \eta_F$ and $\delta \eta_B$ (see e.g. [V.V., EPJ Web of Conf. 125, 04022 (2016)]).

2) for SRC going from $b_{nn}$ to more sophisticated correlation observables, e.g. to the strongly intensive observable $\Sigma(n_F, n_B)$ (see e.g. [E. Andronov, V.V., Eur.Phys.J.A 55(2019)14, V.V., EPJ Web Conf. 191(2018)04011]).
The strongly intensive observable $\Sigma(n_F, n_B)$

The strongly intensive quantities


We define the strongly intensive observable $\Sigma(n_F, n_B)$ between multiplicities in forward ($n_F$) and backward ($n_B$) windows


$$
\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F \rangle + \langle n_B \rangle} \left[ \langle n_F \rangle \omega_{n_B} + \langle n_B \rangle \omega_{n_F} - 2 \text{cov}(n_F, n_B) \right],
$$

(2)

where

$$
\text{cov}(n_F, n_B) \equiv \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle,
$$

(3)

and $\omega_{n_F}$ and $\omega_{n_B}$ are the corresponding scaled variances of the multiplicities:

$$
\omega_n \equiv \frac{D_n}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}.
$$

(4)
The model with identical strings

The fundamental characteristics of a string:

one- and two-particle rapidity distributions from a single string decay:

\[ \lambda(\eta) = \mu_0, \quad \lambda_2(\eta_1, \eta_2) = \lambda_2(\eta_1 - \eta_2) = \lambda_2(\Delta \eta) \]

\( \Lambda(\Delta \eta) \) - two-particle correlation function of a string:

\[ \Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\Delta \eta)}{\mu_0^2} - 1 = \Lambda(\Delta \eta). \]

\( \delta \eta \) - the width of the observation windows (below we suppose \( \delta \eta \ll \eta_{\text{corr}} \)),

\( \Delta \eta = \eta_{\text{sep}} \) - the distance between the observation windows.

\[ \Sigma(\Delta \eta) = 1 + \mu_0 \delta \eta[\Lambda(0) - \Lambda(\Delta \eta)] \]

Vechernin V, Andronov E 2019 Universe 5 15
Properties of $\Sigma$ in model with independent identical strings

- We see that in the model with identical strings the $\Sigma(\Delta \eta)$ is a really strongly intensive quantity. It does not depend nor on the mean number of strings $\langle N \rangle$, nor on their event-by-event fluctuations $\omega_N \equiv D_N/\langle N \rangle$. It depends ONLY on string parameters: $\mu_0$ and $\Lambda(\Delta \eta)$.

\[
\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B) = \Sigma(\Delta \eta) \quad \text{vs e.g.} \quad C_2(\Delta \eta, \Delta \phi) = \frac{\omega_N + \Lambda(\Delta \eta, \Delta \phi)}{\langle N \rangle}
\]

- The $\Sigma(0) = 1$ and increases with the gap between windows, $\Delta \eta$, as the $\Lambda(\Delta \eta)$ decrease to 0 with $\Delta \eta$, since the correlations in a string go off with increase of $\Delta \eta$.

- The rate of the $\Sigma(\Delta \eta)$ growth with $\Delta \eta$ is proportional to the width of the observation window $\delta \eta$ and $\mu_0$ - the multiplicity produced from one string.

- The model predicts saturation of the $\Sigma(\Delta \eta)$ on the level

\[
\Sigma(\Delta \eta) = 1 + \mu_0 \delta \eta \Lambda(0) = \omega_\mu = D_\mu/\langle \mu \rangle
\]

at large $\Delta \eta$, since $\Lambda(\Delta \eta) \rightarrow 0$ at the $\Delta \eta \gg \eta_{\text{corr}}$, where the $\eta_{\text{corr}}$ is a string correlation length.
The predictions for the $\Sigma(n_F, n_B)$ in the model with independent identical strings


Using the $\Lambda(\Delta \eta, \Delta \phi)$, extracted in V. Vechernin, Nucl. Phys. A939 (2015) 21 from the ALICE pp data on FB correlations in small acceptance windows, separated in azimuth and rapidity [ALICE collab., JHEP05(2015)097]

The string parameters occur dependent on initial energy (!?)

The hint on the increase of the string cluster contribution to $\Sigma(n_F, n_B)$ with collision energy in pp collisions at LHC energies.
The model with identical strings

The ALICE data on $b_{nn}$ in pp

**ALICE collab., JHEP 05(2015)097**

\[
b_{nn} = \frac{\mu_0 \delta \eta \left[ \omega_N + \Lambda(\Delta \eta, \Delta \phi) \right]}{1 + \mu_0 \delta \eta \left[ \omega_N + \Lambda(0,0) \right]} \Rightarrow \omega_N, \ \Lambda(\Delta \eta, \Delta \phi) \Rightarrow \Lambda(\Delta \eta)
\]

The model with identical strings

\(\Sigma(n_F, n_B)\) in windows separated in azimuth and rapidity

For small observation windows:

\[
\Sigma(\Delta \eta, \Delta \phi) = 1 + \frac{\delta \eta \delta \phi}{2\pi} \mu_0 \ [\Lambda(0, 0) - \Lambda(\Delta \eta, \Delta \phi)]
\]

\(\Delta \eta \equiv \eta_{\text{sep}}, \quad \Delta \phi \equiv \phi_{\text{sep}}\)

For observation windows of an arbitrary width \(\delta \eta_F \delta \phi_F\) and \(\delta \eta_B \delta \phi_B\):

\[
\Lambda(\Delta \eta, \Delta \phi) \to J_{FB}(\Delta \eta, \Delta \phi) = \frac{1}{\delta \eta_F \delta \phi_F \delta \eta_B \delta \phi_B} \times \\
\times \int_{\delta \eta_F \delta \phi_F} d\eta_1 \ d\phi_1 \int_{\delta \eta_B \delta \phi_B} d\eta_2 \ d\phi_2 \ \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2),
\]

\(\Lambda(0, 0) \to J_{FF} = \frac{1}{(\delta \eta_F \delta \phi_F)^2} \int_{\delta \eta_F \delta \phi_F} d\eta_1 \ d\phi_1 \int_{\delta \eta_F \delta \phi_F} d\eta_2 \ d\phi_2 \ \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2).\)

The model with identical strings

Σ for δη δφ windows separated in azimuth and rapidity

**0.9 TeV, δη=0.2, δφ=0.25π**

![Graph for 0.9 TeV, δη=0.2, δφ=0.25π](image)

**2.76 TeV, δη=0.2, δφ=0.25π**

![Graph for 2.76 TeV, δη=0.2, δφ=0.25π](image)

**7 TeV, δη=0.2, δφ=0.25π**

![Graph for 7 TeV, δη=0.2, δφ=0.25π](image)

The model with string clusters

**Σ(\(n_F, n_B\)) in the model with string fusion**

In the model with string fusion on transverse grid we find


\[
Σ(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \sum_k(\mu_F, \mu_B) , \quad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} , \quad (5)
\]

where \(k\) is a degree of string overlapping and \(\langle n^{(k)} \rangle\) is a mean number of particles produced from areas with such overlapping. \(\sum \alpha_k = 1\).

Here \(\sum_k(\mu_F, \mu_B)\) is the variable \(\Sigma\) for the cluster formed by \(k\) strings:

\[
\sum_k(\mu_F, \mu_B) = \sum_k(\Delta \eta) = 1 + \mu_0^{(k)} \delta \eta[\Lambda_k(0) - \Lambda_k(\Delta \eta)] ,
\]

where \(\mu_0^{(k)}\) and \(\Lambda_k(\Delta \eta)\) are the corresponding parameters of the string cluster.

\[
\Lambda_k(\Delta \eta) = \Lambda_0^{(k)} \exp[-|\Delta \eta|/\eta_{corr}^{(k)}]
\]
The model with string clusters

Σ(n_F, n_B) in the model with string fusion

For such string cluster, formed by k fused strings, we expect, basing on the string decay picture


1) larger multiplicity from one string, \( \mu_0^{(k)} > \mu_0 \),
2) smaller correlation length, \( \eta_{corr}^{(k)} < \eta_{corr} \).

This corresponds to the analysis of the net-charge fluctuations in the framework of the string model for pp and AA collisions

[A. Titov, V.V., PoS(Baldin ISHEPP XXI)047(2012)].

Both factors lead to the steeper increase of \( \Sigma_k(\Delta \eta) \) with \( \Delta \eta \) and its saturation at a higher level

That is in accordance with the energy dependence obtained above for \( \Sigma(n_F, n_B) \) from the ALICE pp data.
The model with string clusters

$\Sigma(n_F, n_B)$ in the model with string fusion


\[
\mu_0^{(k)} = \mu_0^{(1)} \sqrt{k} , \quad \Lambda_0^{(k)} = \Lambda_0^{(1)} = \text{const} , \quad \eta_{\text{corr}}^{(k)} = \eta_{\text{corr}}^{(1)} / \sqrt{k} ,
\]

which is instructive to compare with

\[
\mu_0^{(k)} = \mu_0^{(1)} k , \quad \Lambda_0^{(k)} = \Lambda_0^{(1)} / k , \quad \eta_{\text{corr}}^{(k)} = \eta_{\text{corr}}^{(1)} = \text{const} .
\]

for the case without string fusion in a given transverse cell.

(In last case $\Sigma(n_F, n_B) = \Sigma_1(\mu_F, \mu_B)$ and does not depends on $\alpha_k$.)

The values of the parameters $\Lambda_0^{(1)} = 0.8$ and $\eta_{\text{corr}}^{(1)} = 2.7$ were chosen so that to obtain a correspondence with the values of the $\Sigma(n_F, n_B)$ obtained in [Vechernin V 2018 Eur.Phys.J.:Web of Conf. 191 04011].

Note that in that paper the $\Sigma(n_F, n_B)$ was calculated on the base of the string pair correlation function, $\Lambda(\Delta \eta)$, extracted in [V.Vechernin, Nucl.Phys.A939(2015)21] from the ALICE data on the FB correlations [ALICE collab.,JHEP05(2015)097] in the approx. of IDENTICAL strings.
MC calculations of $\Sigma(n_F, n_B)$ in the model with string clusters formation

- Modelling the initial string distribution in the impact parameter plane of pp collisions for different initial energies to take into account string fusion processes. Like in [V. Vechernin, I. Lakomov. Proceedings of Science (Baldin ISHEPP XXI) (2013) 072].
- Monte Carlo simulations of string configurations and calculation of weighting factors $\alpha_k$ as a function of centrality and initial energy of pp collision.

$$\alpha_k = \frac{\langle n^{(k)} \rangle}{\sum_{k=1}^{\infty} \langle n^{(k)} \rangle} = \frac{\langle m^{(k)} \rangle \mu_0^{(k)} \delta \eta}{\sum_{k=1}^{\infty} \langle m^{(k)} \rangle \mu_0^{(k)} \delta \eta} = \frac{\langle m^{(k)} \rangle \sqrt{k}}{\sum_{k=1}^{\infty} \langle m^{(k)} \rangle \sqrt{k}},$$

where the $\langle m^{(k)} \rangle$ is the mean number of clusters with $k$ fused strings, which we take from our MC simulations of the string configurations.
- Calculation the $\Sigma(n_F, n_B)$ for different centralities of pp collision at few LHC energies using the relation (5).
Figure: The strongly intensive observable $\Sigma(n_F, n_B)$ for pp collisions as a function of the rapidity distance $\Delta \eta = \Delta y$ between the centers of the FB observation windows, for two widths of windows: $\delta \eta=0.2$ (left panel) and $\delta \eta=0.4$ (right panel), and for two initial energies: 0.9 TeV (dashed lines) and 7 TeV (solid lines), calculated for particles with transverse momenta in the interval 0.3-1.5 GeV/c, as in the experimental analysis in [ALICE collab., JHEP05(2015)097].

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with energy is caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.
The strongly intensive variable $\Sigma(n_F, n_B)$ at different centralities as a function of the rapidity distance between the observation windows $\Delta \eta = \Delta y$ for $pp$ collisions at energies 900 and 13000 GeV for the width of the observation windows $\delta \eta = 0.4$.

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with the collision centrality is also caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.
Comparing the $\Sigma(n_F, n_B)$ with preliminary ALICE data

For the comparison of the string model predictions with preliminary ALICE data for the $\Sigma(n_F, n_B)$ in pp collisions at energies 0.9 - 7 TeV see the poster of Andrey Erokhin (for the ALICE Collaboration) "Forward-backward multiplicity correlations with strongly intensive observables in pp collisions", IS2021, Contribution ID:187, (Tuesday, 12 January 2021):

The model with independent identical strings

- In this version of the model the variable $\Sigma(n_F, n_B)$ depends only on the individual characteristics of a string and is independent of both the mean number of strings and its fluctuation, which reflects its strongly intensive character.

- So the studies of this observable enable to extract from the experimental data these fundamental characteristics of an individual string - a mean number of particles per unit of rapidity, $\mu_0$, and the pair correlation function, $\Lambda(\Delta \eta, \Delta \phi)$, for particles produced from a fragmentation of a single string.

- However in this version of the model the string parameters occur dependent on collision energy. This fact can be considered as a signal that with increasing of the initial energy of a pp collision due to the string fusion the formation of the sources with new properties - the string clusters - takes place.
The model with string fusion and string clusters formation

- In this case the observable $\Sigma(n_F, n_B)$ is equal to a weighted average of its values for different string clusters, $\Sigma_k(\mu_F, \mu_B)$, with weight factors, $\alpha_k$, which are proportional to the mean number of the particles, produced from all clusters formed by the $k$ fused strings.
- The $\Sigma(n_F, n_B)$, through these weight factors, $\alpha_k$ becomes dependent on collision conditions - its energy and centrality.
- Analyzing these dependencies of the $\Sigma(n_F, n_B)$ we can extract from the experimental data the information on the individual characteristics of the string clusters - the multiplicity density, $\mu_0^{(k)}$, and the pair correlation function, $\Lambda_k(\Delta\eta)$, for particles, produced from a decay of a given cluster.
- In the framework of this approach it was shown that the overall increase of the $\Sigma(n_F, n_B)$ in pp collisions with collision energy and centrality can be explained by the formation of string clusters with new properties.

The research was supported by the Russian Foundation for Basic Research grant (No. 18-02-40075)
Backup slides
Fitting the parameters of the initial string distribution in the impact parameter plane of pp collisions

\[ \sigma_{MC \ simulations}^{ND} = \frac{n_{sim}(N=0)}{n_{sim}(N>0)} S_b \]

\[ \mu_0^{(k)} = \mu_0^{(1)} \sqrt{k} \text{ with } \mu_0^{(1)} = 0.7 \]
The parametrization of the single correlation function

The parametrization for the pair correlation function \( \Lambda(\eta, \phi) \) of a single string (reflecting the Schwinger mechanism of a string decay, was suggested in [V.V., Nucl. Phys. A939(2015)21]:

\[
\Lambda(\eta, \phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\phi^2}{\varphi_1^2}} + \Lambda_2 \left( e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}}. \tag{6}
\]

This formula has the nearside peak, characterizing by parameters \( \Lambda_1, \eta_1 \) and \( \varphi_1 \), and the awayside ridge-like structure, characterizing by parameters \( \Lambda_2, \eta_2, \eta_0 \) and \( \varphi_2 \) (two wide overlapping hills shifted by \( \pm \eta_0 \) in rapidity, \( \eta_0 \) - the mean length of a string decay segment). We imply that in formula (6)

\[
|\varphi| \leq \pi. \tag{7}
\]

If \( |\varphi| > \pi \), then we use the replacement \( \varphi \rightarrow \varphi + 2\pi k \), so that (7) was fulfilled. With such completions the \( \Lambda(\eta, \phi) \) meets the following properties

\[
\Lambda(-\eta, \phi) = \Lambda(\eta, \phi) , \quad \Lambda(\eta; -\phi) = \Lambda(\eta, \phi) , \quad \Lambda(\eta, \phi + 2\pi k) = \Lambda(\eta, \phi) \tag{8}
\]
Fitting the model parameters by FBC in small windows

$\Lambda(\eta_{\text{sep}}, \phi_{\text{sep}})$ was fitted by the ALICE $b_{nn}$ pp data with FB windows of small acceptance, $\delta \eta = 0.2$, $\delta \phi = \pi/4$, separated in azimuth and rapidity [ALICE collab., JHEP 05(2015)097]. It gives for the parameters:

<table>
<thead>
<tr>
<th>$\sqrt{s}$, TeV</th>
<th>0.9</th>
<th>2.76</th>
<th>7.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRC $\mu_0 \omega_N$</td>
<td>0.7</td>
<td>1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>SRC $\mu_0 \Lambda_1$</td>
<td>1.5</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.2</td>
<td>1.15</td>
<td>1.1</td>
</tr>
<tr>
<td>$\mu_0 \Lambda_2$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\eta_2$</td>
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<td>2.0</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>1.7</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

$\omega_N = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$ is the e-by-e scaled variance of the number of strings, $\mu_0$ is the average rapidity density of the charged particles from one string, $i = 1$ corresponds to the nearside and $i = 2$ to the away-side contributions, $\eta_0$ is the mean length of a string decay segment.

The string correlation function $\Lambda(\Delta \eta)$

Then we find $\Lambda(\Delta \eta)$ integrating over azimuth:

$$\Lambda(\eta_{sep}) = \frac{1}{\pi} \int_{0}^{\pi} \Lambda(\eta_{sep}, \phi_{sep}) \, d\phi_{sep}.$$
The string correlation function $\Lambda(\Delta \eta)$

The obtained dependencies in this figure for three initial energies are well approximated by the exponent:

$$\Lambda(\Delta \eta) = \Lambda_0 e^{\frac{|\Delta \eta|}{\eta_{corr}}} ,$$  \hspace{1cm} (9)

with the parameters presented in the table:

\[
\begin{array}{|c|c|c|c|}
\hline
\sqrt{s}, \text{TeV} & 0.9 & 2.76 & 7.0 \\
\hline
\mu_0 \Lambda_0 & 0.73 & 0.83 & 0.93 \\
\eta_{corr} & 1.52 & 1.43 & 1.33 \\
\hline
\end{array}
\]

[V.V., EPJ Web Conf. 191(2018)04011]

We see that the correlation length, $\eta_{corr}$, decreases with the increase of collision energy.

This can be interpreted as a signal of an increase with energy of the admixture of strings of a new type - the fused strings in pp collisions.
Various versions of string fusion

**local fusion (overlaps)**


\[ \langle n \rangle_k = \mu_0 \sqrt{k} \frac{S_k}{\sigma_0}, \quad \langle p_t^2 \rangle_k = p_0^2 \sqrt{k}, \quad k = 1, 2, 3, ... \]

**global fusion (clusters)**


\[ \langle p_t^2 \rangle_{cl} = p_0^2 \sqrt{k_{cl}}, \quad \langle n \rangle_{cl} = \mu_0 \sqrt{k_{cl}} S_{cl} / \sigma_0, \quad k_{cl} = k \sigma_0 / S_{cl} \]

the version of SFM with the finite lattice (grid) in transverse plane

**V.V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136**

Domains in transverse area


It leads to the splitting of the transverse area into domains with different, fluctuating values of color field within them. What was also considered in the CGC approach *A.Kovner., M. Lublinsky*, Phys.Rev. D *83*, 034017 (2011)