Fluctuations of Energy Density in Ultra-Central Collisions

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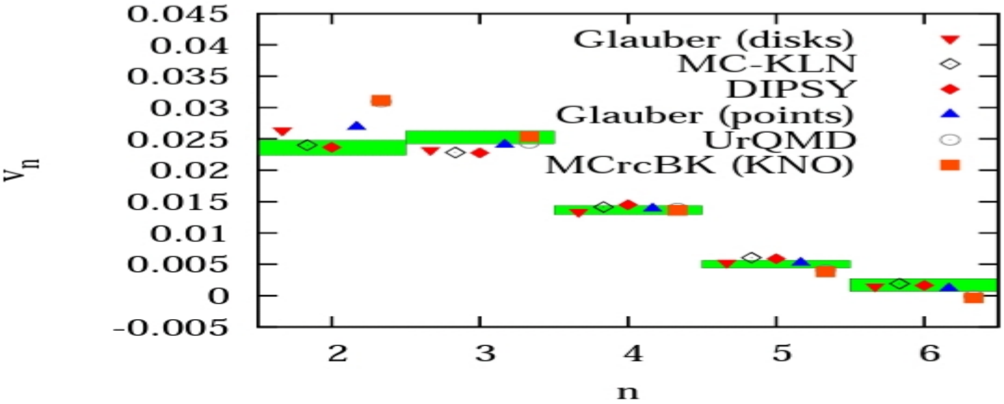
Introduction

- Problem - In ultra-central regime, we have $v_3 \sim v_2$ and there is no one existing model that can calculate $v_n\{2\}$ with a good accuracy.
- Goal - Find what must be the initial conditions in order to get the experimental data in this regime.
- Method - Relate v_n to n-point functions of initial energy density.
- Low density regions - Fluctuations are larger than monte carlo existing models.

Motivation

The existing initial condition models that caluclates the harmonic flows through a hydrodynamic evolution aren't able to generate results with a good accuracy related to experimental data in ultra-central regime.



Harmonic flows from ATLAS collaboration in $0-1\%$ cent. bin. We can see that the results got from existing models are not to able to fit with the experimental data. Image from (*Luzum, M. et. al. Nuclear Physics A 904-905 (2013) pp. 377-380*)

N-point functions and Harmonic flows

- Starting point $\rho(z) = \langle \rho(z) \rangle + \delta\rho(z)$,
  where $\rho(z)$ is the energy density in one event and $\delta\rho(z)$ is the fluctuation of energy density. We have used $z = x + iy$.
  Assumptions
- **Local Fluctuations** - Neglect correlations.
  $\Rightarrow \langle \delta\rho(z_1)\delta\rho(z_2) \rangle = \delta(z_1 - z_2)\kappa_2(z_1)$
  $\Rightarrow \langle \delta\rho(z_1)\delta\rho(z_2)\delta\rho(z_3) \rangle = \delta(z_1 - z_2)\delta(z_1 - z_3)\kappa_3(z_1)$.
- **Small Fluctuations** - The RMS values of ϵ_n are given by $\langle \rho(z) \rangle$ and by 1 and 2-point functions → power series in $\delta\rho$.

In these equations, $\kappa_1(z) = \langle \rho(z) \rangle$, $\kappa_2(z) \propto var(\rho(z))$ are the cumulants per surface unit. These results are from *ArXiv 1604.07230*.

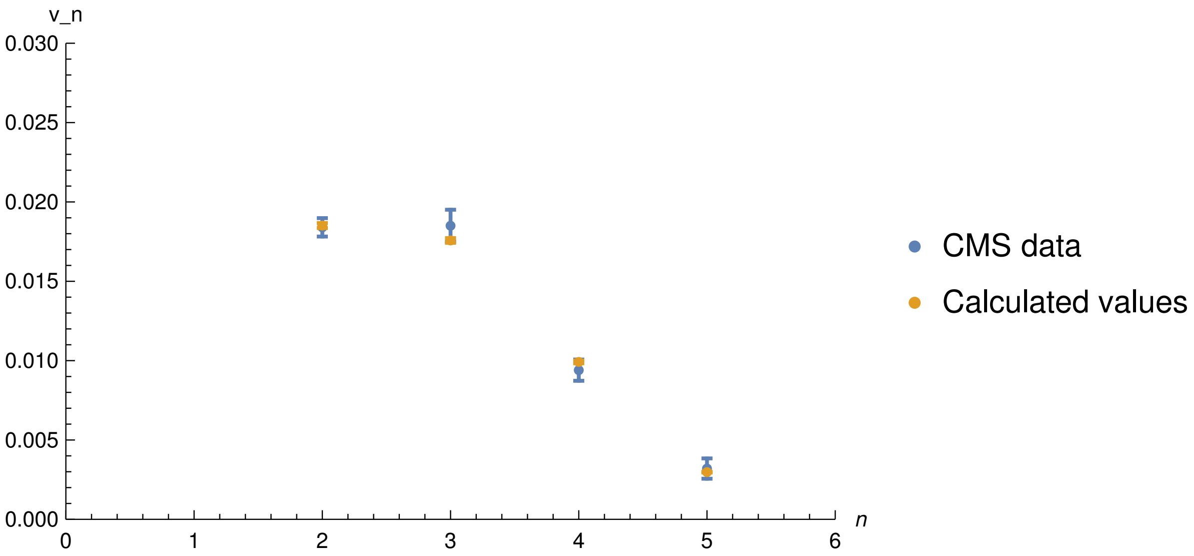
N-point functions and Harmonic flows

- κ_1 It's the mean of energy density for each point over several events → CGC, TRENTO, etc.
- κ_2 Means the fluctuations of each spacepoint. We have parameterized the fluctuations in terms of the mean:

$$\kappa_2 = C^2 \cdot \kappa_1^{2p} \tag{1}$$

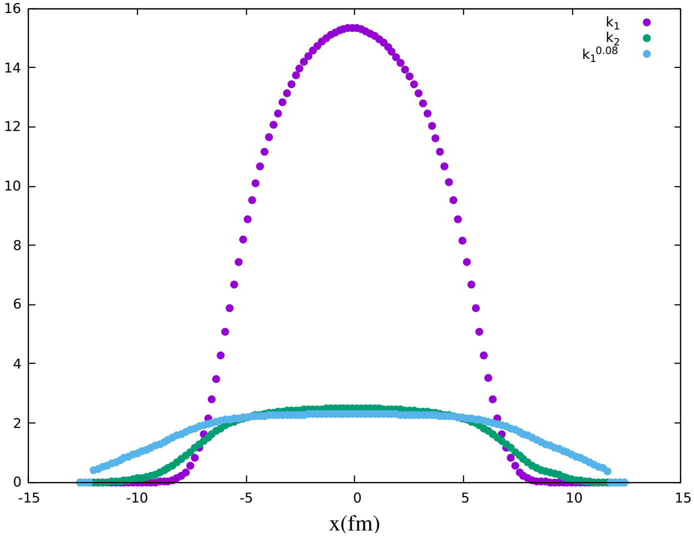
- **Response coefficient** $K_n \equiv \frac{v_n}{\epsilon_n}$ - MUSIC simulations, assuming $K_n \equiv K_n(\frac{\eta}{s})$, where $\frac{\eta}{s}$ is the shear viscosity.

Results



This plot compares the harmonic flows from CMS $0 - 0.02\%$ collaboration and the values got from the power parameterization of the mean. One can see that this parameterization generates harmonic flows with a reasonable accuracy.

Results - Initial conditions



Green dots are the variance per unit surface from TRENTO results and blue dots shows $\kappa_2(z)$ got from exponential law with $p = 0.08$. Purple points are the mean per surface unit from TRENTO initial conditions. We can see that the fluctuations in peripheral zone are larger with the power parameterization than we consider TRENTO results.

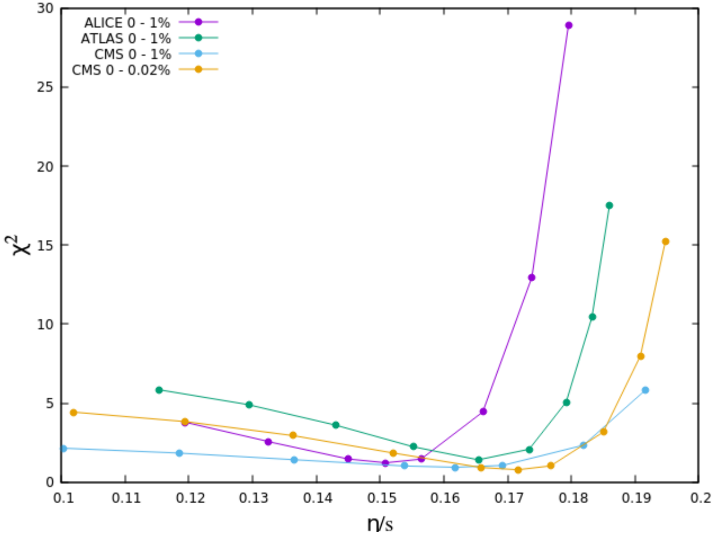
Thank you!

Extra material

Using the power parameterization, we have the results below

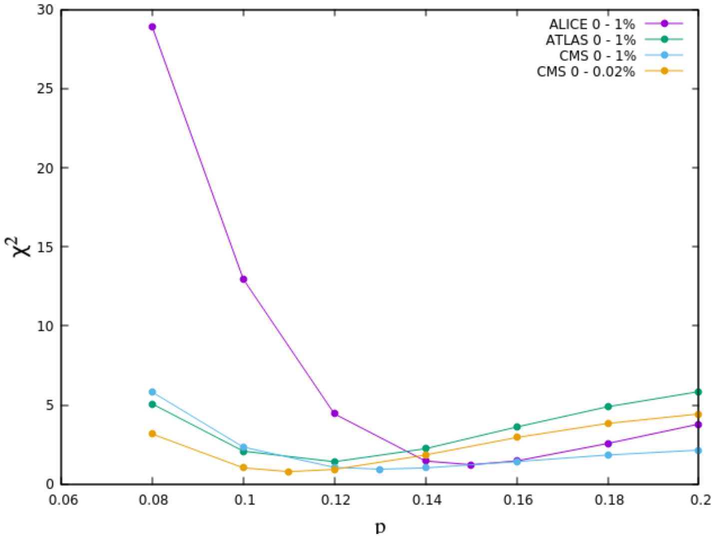
κ_1	Collaboration	p	C	η/s	χ^2
CGC	ALICE $0-1\%$	0.15	12.64	0.15	1.23
CGC	ATLAS $0-1\%$	0.12	10.58	0.16	1.42
CGC	CMS $0-1\%$	0.13	10.70	0.16	0.94
CGC	CMS $0-0.02\%$	0.11	09.18	0.17	0.80
TRENTO	ALICE $0-1\%$	0.12	3.38	0.14	0.78
TRENTO	ATLAS $0-1\%$	0.05	2.20	0.16	0.72
TRENTO	CMS $0-1\%$	0.08	2.55	0.16	0.58
TRENTO	CMS $0-0.02\%$	0.04	1.85	0.16	0.37

Extra material



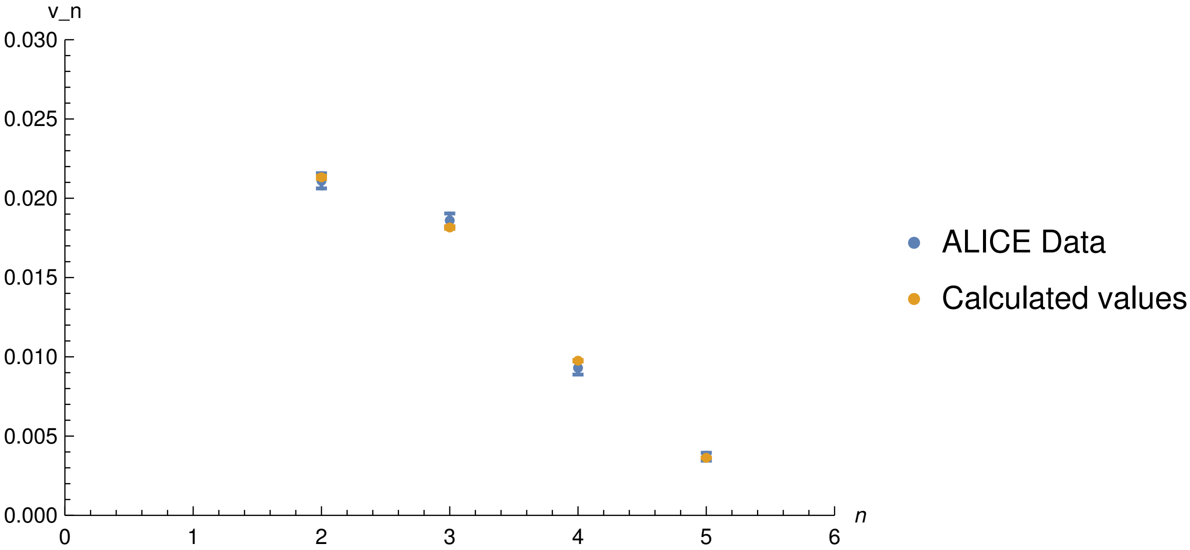
The shear viscosity values and the corresponding χ^2 got from power parameterization and the least square.

Extra material



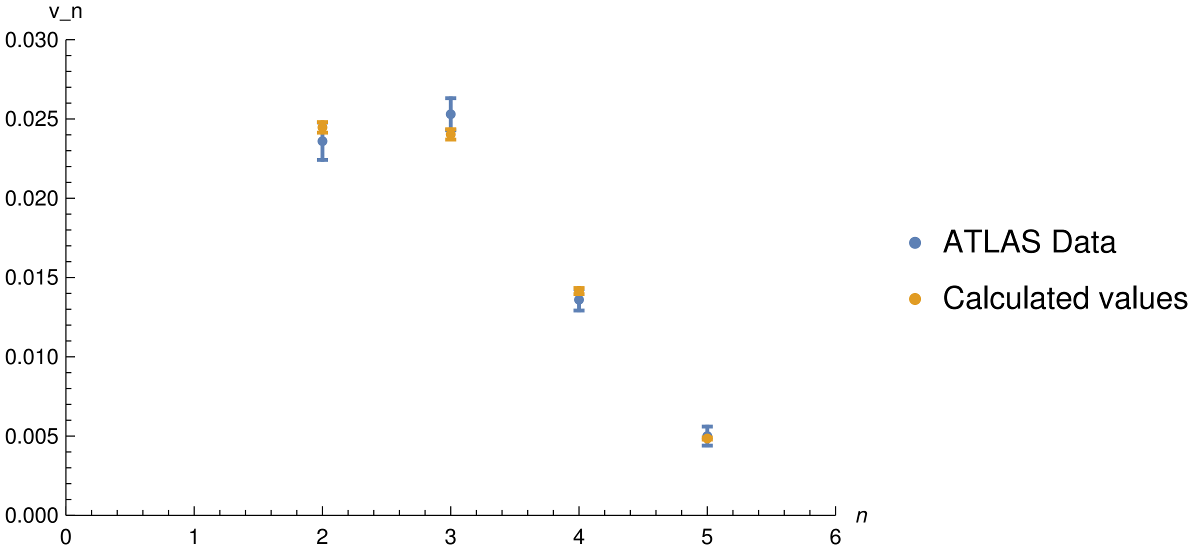
The free parameter p values and the corresponding χ^2 got from power parameterization and the least square.

Extra material



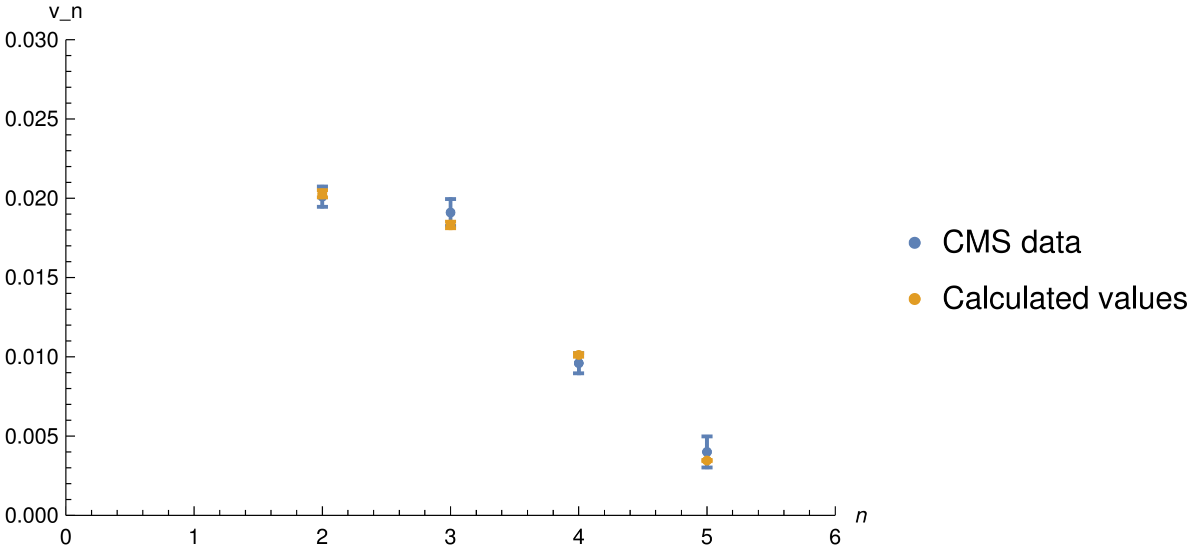
This plot compares the harmonic flows from ALICE $0-1\%$ collaboration and the values got from the power parameterization of the mean. One can see that this parameterization generates harmonic flows with a reasonable accuracy.

Extra material



This plot compares the harmonic flows from ATLAS $0-1\%$ collaboration and the values got from the power parameterization of the mean. One can see that this parameterization generates harmonic flows with a reasonable accuracy.

Extra material



This plot compares the harmonic flows from CMS $0-1\%$ collaboration and the values got from the power parameterization of the mean. One can see that this parameterization generates harmonic flows with a reasonable accuracy.

Fluctuations of Energy Density in Ultra-Central Collisions

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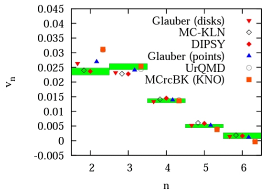
Introduction

The existing initial conditions models for heavy ion collisions give the flow coefficients v_n with a good accuracy for the most of cases. However, in ultra-central events, i.e. centrality bin $0-1\%$ or more central, there is no model for initial conditions that provide all the v_n in a satisfactory way, as you can see on fig.(1). Thus, It is *sine qua non* to understand what properties the system must have under these centralities in order to the flow coefficients are compatible with the experimental data, remembering that the fluctuations play an important role in these cases, since the participant is almost symmetric.

In this work, we have made a parameterization of energy density and its fluctuations via 1 and 2 point functions, assuming that ones are small and local. Besides, we constrain them by using experimental data for $v_n\{2\}$ in conjuction with hydodynamic simulations performed on MUSIC, which is a C++ code that simulates all system evolution. We believe that it is possible to describe the experimental data with the requirement that the fluctuations are larger than mean in regions of low density. We comment on the implications of this finding, as well as the limitations of this initial analysis and how it can be improved in the future



Harmonic flows from ATLAS colaboration in centrality bin $0-1\%$. We can see that no initial condition model used could calculated all v_n with reasonable accuracy. Image from[4].

N-Point Functions and Harmonic flows

The starting point is to write energy density as $\rho(z) = \langle\rho(z)\rangle + \delta\rho(z)$, where $\delta\rho(z)$ is the fluctuation of energy density and z is a shorthand notation to (x, y). Besides this, if we consider that the fluctuations are small and the correlations are local, we find a constrain between the fluctuations and the correlations:

$$\langle\delta\rho(z_1)\delta\rho(z_2)\rangle = \kappa_2(z_1)\delta(z_1 - z_2) \qquad (1)$$

where $\kappa_2(z)$ is κ_2 per unit surface. Combining this equation with the definition of eccentricites:

$$\epsilon_n = \frac{\int_z r^n \rho(z)}{\int_z |r|^n \rho(z)} \qquad (2)$$

We get a expression for rms value of eccentricites $\epsilon_n\{2\}$[1]:

$$\epsilon_n\{2\} \equiv \langle|\epsilon_n|^2\rangle^{\frac{1}{2}} \approx a = \sqrt{\frac{\int_z r^{2n}\kappa_2(z)}{(\int_z r^n \kappa_1(z))^2}} \qquad (3)$$

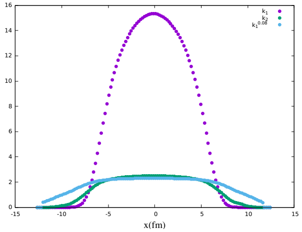
$$\Rightarrow v_n\{2\} = K_n * \sqrt{\frac{\int_z r^{2n}\kappa_2(z)}{(\int_z r^n \kappa_1(z))^2}} \qquad (4)$$

The coefficients K_n were calculated by MUSIC, which is a C++ code that calculates all hydro evolution of the system, from initial conditions to kinetic freeze-out e generates the spectra of final stage. This coefficient is defined by the ratio between the harmonic flow v_n and the eccentricity ϵ_n. According to hydro description, the harmonic flows are strongly related to the shear viscosity to entropy ratio η/s. Thus, we perform simulations changing the shear viscosity to get the expressions for K_n via linear fit.

Also, we use the fact that the eccentricites are small in ultra-central collisions. In other centralities bins, the relation between v_n and ϵ_n has terms as $\epsilon_n * \epsilon_m$. This assumption proved to be essential to the model to work. In the fig.(2), we show the values of fluctuations obtained by TRENTO, which is a code that generates fluctuating initial conditions, and the values of fluctuations using a exponential relation between the mean $\kappa_1(z)$ and the variance $\kappa_2(z)$. TRENTO model doesn't work in ultra-central events whereas, the exponential relation

$$\kappa_2(z) = C^2 * \kappa_1(z)^{2p} \qquad (5)$$

where p is a parameter put *ad hoc*, works. Thus, we have thought that the fluctuations must be small and larger than one calculated by Monte-Carlo existing models in regions of low density.



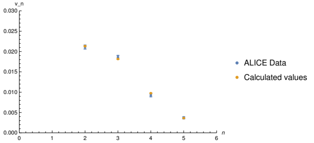
Comparison between the mean and the fluctuations obtained by TRENTO (green dots) and by exponential law (blue dots). X-axis means the position related to CM.

Results

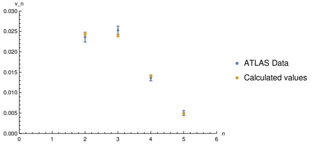
In order to calculate the harmonic flows, we found a relation between κ_1 and κ_2 so that we can apply the eq.(4) to calculate the harmonic flows. Then, we have applied the least square method using the experimental data. Thus, we have found the "fitted values" of harmonic flows, and the value of the shear viscosity. In the table below, we see the values that was obtained with this ansatz and in fig(5) we can see that the calculated values of v_n and experimental data from ALICE, ATLAS and CMS collaborations[2] in centrality bin $0-1\%$ have a reasonable accuracy.

Collaboration	p	C	η/s	χ^2_{dof}	Cent. bin
ALICE	0.15	12.64	0.15	1.23	$0-1\%$
ATLAS	0.12	10.58	0.16	1.42	$0-1\%$
CMS	0.13	10.70	0.16	0.94	$0-1\%$
CMS	0.11	9.18	0.17	0.80	$0-0.02\%$

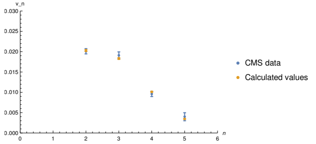
p, C, η/s e χ^2 for each collaboration with centrality bin $0-1\%$.



Experimental $v_n\{2\}$ in $0-1\%$ from ALICE collaboration and calculated values with this mapping.



Experimental $v_n\{2\}$ in $0-1\%$ from ATLAS collaboration and calculated values with this mapping.



Experimental $v_n\{2\}$ in $0-1\%$ from CMS collaboration and calculated values with this mapping.

The table below shows the extracted values of shear viscosity for

each collaboration and the corresponding χ^2

Final Considerations

In ultra-central events, the triangular flow $v_3\{2\}$ is very close, and bigger than in some cases, to elliptic flow $v_3\{2\}$ and this one doesn't occurs in less central events. In order to describe these results, we parameterize the energy density and its fluctuations via 1 and 2 point functions and we have assumed that the fluctuations are bigger than the mean in low density regions. We have combined this ansatz with hydro simulations and we have compared to experimental data through the least square method and we have got results with a good accuracy. Besides, the shear viscosity obtained is close to phenomenological results [4]. Fpr the future works, we think that to improve these results adding other physical parameters, as bulk viscosity to s ratio $(\frac{\zeta}{s})$ to response coefficients, as well to think in asymetry on fluctuations.

References

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