# Non-equilibrium attractor in high-temperature QCD plasmas

Dekrayat Almaalol

Collaborators: Michael Strickland and Aleksi Kurkela

Phys. Rev. Lett. 125, 122302

Initial Stages 2021 (online)

Jan 10, 2021

# Non-equilibrium effects and Hydrodynamization

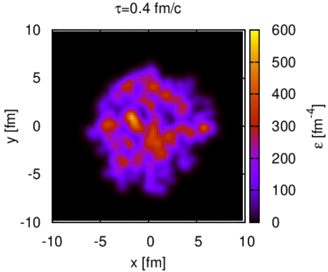



figure: $\epsilon_0$ profile for a hydrodynamics simulation

B.Schenke, S.Jeon, C.Gale PRL.106,042301

- Large order hydrodynamic gradient expansion is divergent! M.Heller and M. Spalinski, PRL. 115,072501 (2015)
- System lives most of its life-time in a state of out of equilibrium $\Rightarrow$ Large Knudsen number prior hydro as well as at freeze out H.Niemi, G.Denicol arxiv.1404.7327

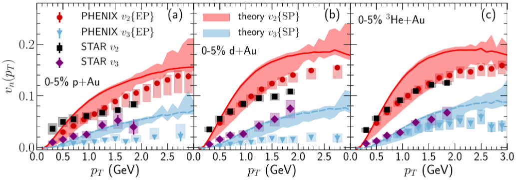
$$\mathrm{Kn} = \frac{\text{microscopic scale}}{\text{macroscopic scale}}$$

Specially true for small systems!

- Hydrodynamic constitutive equations was proven applicable in Ads/CFT holography

  P. Chesler, L. Yaffe PRD 82, 026006 (2010)

- Systems with different sizes develope a collective behaviour

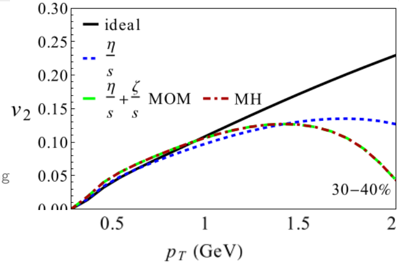



B.Schenke. S, Chun . T, Prithwish PRB 803 (2020) 135322

Uncertainty in our understanding of the regime applicability of fluid dynamics.

# Non-equilibrium effects at freezeout

- Large $\delta f$ corrections at freezeout directly affect the anisotropic flow $v_2(p_T)$ (figure: J.Noronha-Hostler, J.Noronha, F.Grassi PRC 90 (2014) 3, 034907
- $\delta f$ can be computed for a particular form of $C[f]$ (K. Dusling G.Moore, and D. Teaney Phys. Rev. C 81, 034907 (2008))




The *quadratic ansatz* ($\alpha = 0$)

$$\frac{\delta f_{(i)}}{f_{\text{eq}}(1+f_{\text{eq}})} = \frac{3\overline{\Pi}}{16T^2}(p^2 - 3p_z^2)$$

$$\overline{\Pi} = \Pi/\epsilon = 1/3 - T^{zz}/\epsilon$$

The *LPM ansatz* ($\alpha = 0.5$)

$$\frac{\delta f_{(ii)}}{f_{\text{eq}}(1+f_{\text{eq}})} = \frac{16\overline{\Pi}}{21\sqrt{\pi}\,T^{3/2}}\left(p^{3/2} - \frac{3p_z^2}{\sqrt{p}}\right)$$

$$\overline{\Pi} = \Pi/\epsilon = 1/3 - T^{zz}/\epsilon$$

The *aHydro freeze-out ansatz*

$$f(p) = f_{\text{Bose}}(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda)$$

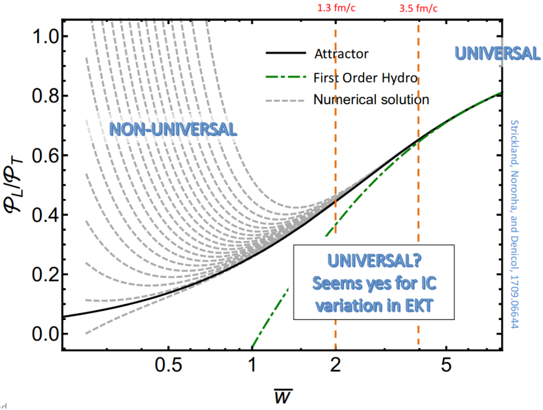
$$\overline{\mathcal{M}}^{nm}_{\text{aHydro}}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{\mathcal{H}^{nm}(\alpha)}{[\mathcal{H}^{20}(\alpha)]^{(n+2m+2)/4}}$$

# The attractor concept

A set of points in the phase space of the dynamical variables to which a family of solutions of an evolution equation merge at sufficiently late times.

- Memory loss of initial conditions
- Hydrodynamics as a universal attractor

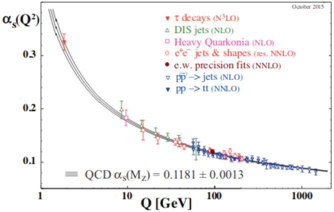
  Michal P. Heller Phys. Rev. Lett. 115, 072501
- Characterized by the competition between the expansion and interaction rate
- Microscopic model dependent




$$\overline{\mathrm{W}} = \frac{\tau}{\tau_R} = \frac{\tau \mathrm{T}}{4\pi\,\bar{\eta}} = \mathrm{K_n}^{-1}$$

# QCD medium at high temperature

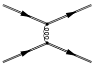
- $\alpha_s = g^2/4\pi \Rightarrow \sqrt{s} \to \infty \Rightarrow$ weakly coupled QCD
  $g^2 N_c f \ll 1 \Rightarrow$ **Effective kinetic theory**
- Scales and Degrees of freedom: hard momentum particles with $p \sim 2\pi T$ in a background of soft particles $p \sim gT$.
- Dynamics of typical ultrarelativistic excitations (quarks and gluons) are given by Arnold,Moore,Yaffe JHEP 0301 (2003) 030



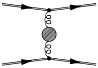

$$P^\mu \partial_\mu f_{q,g}(\mathbf{p}) = -C[f(\mathbf{p})], \qquad f_{q,g} \propto \frac{dN_{g,q}}{d^3x d^3p}$$

$$\frac{df_{q,g}(\mathbf{p})}{d\tau} - \frac{p_z}{\tau}\partial_{p_z} f_{q,g}(\mathbf{p}) = -\mathcal{C}_{\mathbf{2}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] - \mathcal{C}_{\mathbf{1}\leftrightarrow\mathbf{2}}[f_{q,g}(\mathbf{p})] \qquad \underline{0+1\text{d Bjorken}}$$

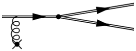
- At LO , transport processes at different momentum scales in $C[f]$



Hard elastic scattering $\sim T$



soft elastic scattering $m_D \sim gT$



collinear splitting
LPM included

# Initial distribution $-\frac{d\mathbf{f}_{\mathbf{p}}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[\mathbf{f}_{\mathbf{p}}] + \mathcal{C}_{2\leftrightarrow 2}[\mathbf{f}_{\mathbf{p}}] + \mathcal{C}_{\exp}[\mathbf{f}_{\mathbf{p}}].$

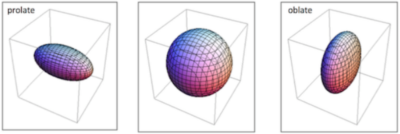
- Romatschke-Strickland thermal

$$\mathrm{f}_{0,\mathrm{RS}}(\mathbf{p}) = f_{\mathrm{Bose}}\left(\sqrt{\mathbf{p}^2 + \xi_0 p_z^2}/\Lambda_0\right)$$

anisotropy parameter $(-1 < \xi_0 < \infty)$

$\Lambda_0$ is set by Landau matching

Strickland, Phys. Rev. Lett.115, 182301(2015)



$-1 < \xi < 0$ $\quad$ $\xi = 0$ $\quad$ $\xi > 0$

- CGC non-thermal

$$\mathrm{f}_{0,\mathrm{CGC}}(\mathbf{p}) = \frac{2A}{\lambda}\frac{\tilde{\Lambda}_0}{\sqrt{\mathbf{p^2} + \xi_0 p_z^2}}\exp^{-\frac{2}{3}\left(\mathbf{p}^2 + \xi_0 \hat{p}_z^2\right)/\tilde{\Lambda}_0^2}$$

The initial scale $\tilde{\Lambda}_0$ is related to the saturation scale $\tilde{\Lambda}_0 = \langle \mathrm{p_T} \rangle_0 \approx 1.8\,\mathrm{Q_s}$

$A$ is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation

A. Kurkela and Y. Zhu, Phys. Rev. Lett.115, 182301(2015)

T. Lappi, Phys. Lett.B703, 325-330 (2011)

# Numerical calculations

- Momentum-discretized Boltzmann equation Kurkela and Zhu PRL 115, 182301 (2015))

$$n_{ij}(t+\Delta t) = n_{ij}(t) - \Delta t \left( C_{ij}^{\exp} + C_{ij}^{2\leftrightarrow 2} + C_{ij}^{1\leftrightarrow 2} \right)$$
$$n_{x_p,p} = 4\pi p^2/(2\pi)^3 f_{x_p,p}$$

- 2D grid $\{x_i, p_j\}$ for $p = |\mathbf{p}|$ and $x_p = \cos(\hat{\mathbf{p}} \cdot \hat{\mathbf{z}})$ with $250 \times 2000$ grid points

Ohio Supercomputer Center
An OH·TECH Consortium Member

- The different integral moments read

$$n = \nu \sum_{ij} n_{ij}, \quad \epsilon = \nu \sum_{ij} p_j n_{ij}$$

# Higher moments of the distribution function

M. Strickland, JHEP2018, 128; 1809.01200.

- A general moment of the distribution function is defined by

$$\mathcal{M}^{nm}[f] \equiv \int dP\,(p.u)^n\,(p.z)^{2m}\,f(x,p)$$

$$\mathcal{M}^{10} = n = \int dP\,(P.u)\,f\,. \quad \text{(number density)}$$

$$\mathcal{M}^{20} = \varepsilon = \int dP\,(p\cdot u)^2\,f\,, \quad \text{(energy density)}$$

$$\mathcal{M}^{01} = P_L = \int dP\,(p\cdot z)^2\,f\,. \quad \text{(longitudinal pressure)}$$

- The corresponding equilibrium values using a Bose distribution,

$$\mathcal{M}^{nm}_{\rm eq} = \frac{T^{n+2m+2}\Gamma(n+2m+2)\zeta(n+2m+2)}{2\pi^2(2m+1)}\,.$$

# Results (I): QCD non-equilibrium attractor

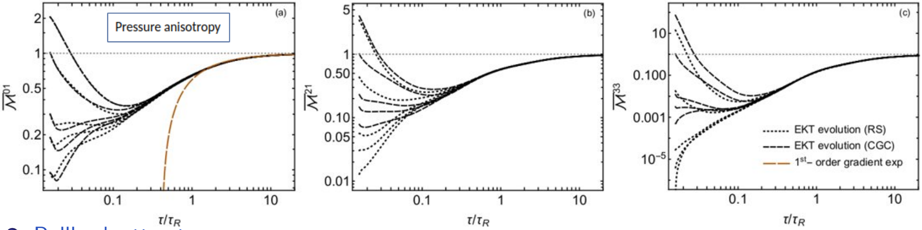
- Forward attractor

$\lambda = 10;\ \bar{\eta} = 0.624$

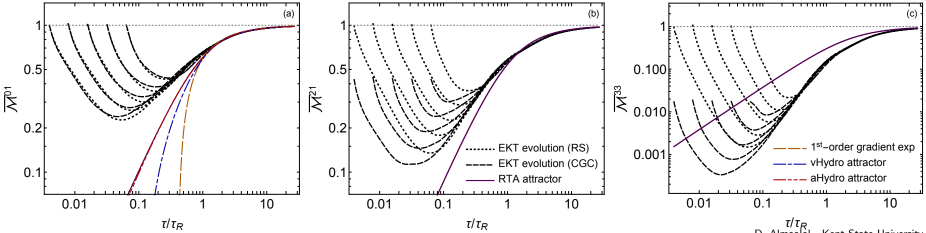
$\xi_0 \in \{-0.84, 0, 5.25, 10.1, 24\},$

$\tau_0 = 0.095\,(\nu/\varepsilon)^{1/4}.$

$$\tau_{\rm R}(\tau) = 4\pi\bar{\eta}/{\rm T}(\tau).$$



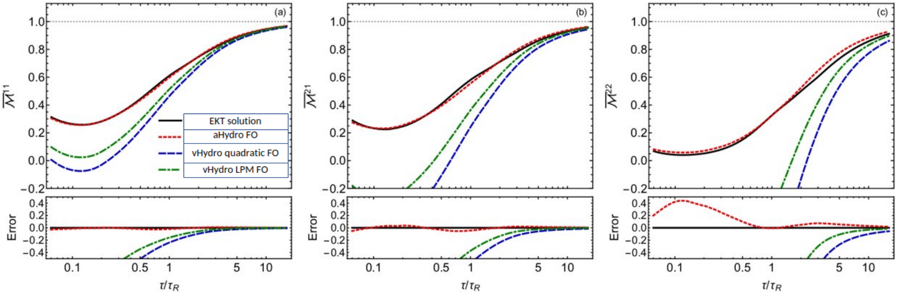

- Pullback attractor



# Results (II): Improve freezeout perscription

| $\tau/\tau_R$ | $\tau$ |
|---|---|
| 0.2 | 0.32 fm/c |
| 0.5 | 0.86 fm/c |
| 1 | 1.88 fm/c |
| 2 | 4.23 fm/c |
| 5 | 14.1 fm/c |
| 10 | 38.5 fm/c |

- Disagreement increases for higher moments and for earlier times.
- Good agreement between aHydro ansatz and EKT at all times

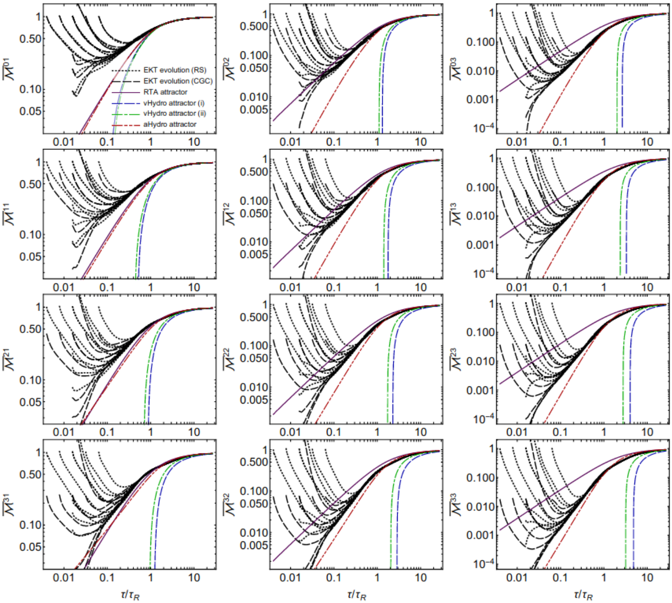



Sensitivity of freeze-out $\delta f$ corrections to the microscopics

# Conclusions

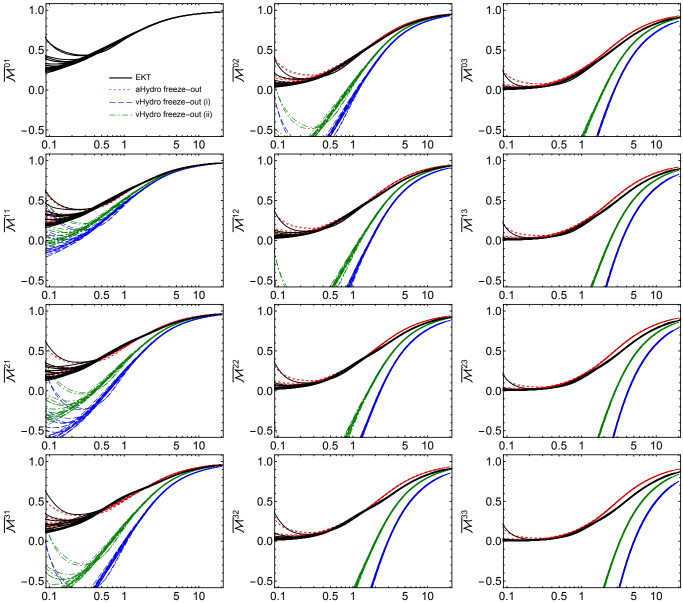
- System experiences early time memory loss of initial conditions
  True for both thermal RS and non-thermal CGC distributions
- Beyond hydrodynamics attractor
  A non-equilibrium attractor for the phase space momentum distribution function
- Different microscopic dynamics affect the non-equilibrium corrections to the phase space distribution function at freezeout
  anisotropic hydrodynamics ansats agrees the best with the pure YangMills EKT
- Next extend the analysis to the full QCD theory by inclusion of quarks

# QCD attractor




- Exists for both IC's
- Extends beyonds hydro to the distribution itself
- Better overall agreement between aHydro & EKT, improved @ higher moments
- Sensitivity of RTA to moments, improves at late time

# Results (II): Improve freezeout perscription



# aHydro ansatz

- The *aHydro freeze-out ansatz*

$$f(p) = f_{\rm Bose}(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda)$$

$$\mathcal{M}^{nm}_{\rm aHydro}(\tau) = \frac{\Gamma(n+2m+2)\Lambda^{n+2m+2}(\tau)}{(2\pi)^2}\mathcal{H}^{nm}(\alpha(\tau))\,,$$

with $\alpha(\tau) = 1/\sqrt{1+\xi(\tau)}$.

$$\overline{\mathcal{M}}^{nm}_{\rm aHydro}(\tau) = 2^{(n+2m-2)/4}(2m+1)\frac{\mathcal{H}^{nm}(\alpha)}{[\mathcal{H}^{20}(\alpha)]^{(n+2m+2)/4}}\,.$$

$\mathcal{H}^{nm}(y) = \frac{2y^{2m+1}}{2m+1}\,{}_2F_1(\frac{1}{2}+m, \frac{1-n}{2}; \frac{3}{2}+m; 1-y^2)\,.$

- aHydro attractor

$$\overline{\Pi} = \frac{1}{3}\left[1 - \frac{\mathcal{R}_L(\xi)}{\mathcal{R}(\xi)}\right],$$

$$\tau\frac{d\log\epsilon}{d\tau} = -\frac{4}{3} + \overline{\Pi}\,,$$

$$\frac{1}{1+\xi}\dot{\xi} - \frac{2}{\tau} + \frac{\mathcal{R}^{5/4}(\xi)}{\tau_{\rm eq}}\xi\sqrt{1+\xi} = 0\,,$$