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POLYELECTRONS*

BY

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SUMMARY

Theoretical evidence for the existence of entities composed entirely of electrons and positrons is presented in the following article, together with a discussion of their properties.[1] The simplest of these entities consists of one electron and one positron, bound together in a structure similar to that of the hydrogen atom. The next higher entity is composed of two positrons and one electron or of two electrons and one positron. The bi-electron system is stable by 6.77 ev against dissociation. Against annihilation, it has a life time of 1.24×10^{-10} sec., when the spins of the two particles are parallel, and a life several orders of magnitude greater, when the spins are antiparallel. The tri-electron system also has a radioactive mean life of the order of 10^{-10} sec., and is calculated to be stable by at least 0.19 ev against dissociation into a bi-electron and a free electron or positron. The production of a bi-electron, by interaction of an energetic gamma ray with the field of force of an atomic nucleus, is calculated to occur with a probability about 10^{-4} times less than that for production of an electron-positron pair. The possibilities are discussed for observing atomic and molecular spectra in which the positron plays the role of an especially light hydrogen ion. An experiment is suggested which can be used to check the theory of the perpendicular polarization of the gamma rays given off in the annihilation process. The similarities and distinct differences between polyelectrons and cosmic ray mesons are discussed.

THE THEORY OF ELECTRON-POSITRON PAIRS

The discovery by Anderson,[2] in 1932, of the creation of pairs of electrons and positrons by electromagnetic radiation, and the subsequent interpretation of this observation, in the light of Dirac's[3] already existing relativistic theory of the spinning electron, initiated a fruitful branch of physics which is now often known under the name of "*pair theory.*"

Pair theory is still in the course of development. On the fundamental side, it does not, at this time, possess the well defined concepts and clarity of formulation of electromagnetism, quantum mechanics, or relativity theory, in spite of important advances due to Dirac,[4] Heisenberg,[5] Weisskopf,[6] and Bohr.[7] For example, the state of disturbance of the electron-positron field in the neighborhood of an atomic nucleus is still imperfectly understood. Likewise, the properties of the higher entities of the type described here remain completely unstudied. Obviously, it will be an important task for physicists, in the next few years, so to systematize our knowledge of pair theory and so to correlate it with observation that its now unsuspected implications will be uncovered.

APPLICATIONS OF PAIR THEORY TO THE STUDY OF COSMIC RADIATION

Fortunately, pair theory has evolved far enough both to have allowed major applications to the interpretation of cosmic ray phenomena, in the past, and to permit the analysis of the new entities treated below.

These two developments of the theory are not independent. The increase in knowledge of cosmic ray problems which has come about through past applications of the electron-positron theory, is responsible for an appreciable interest, at the present time, in the properties of short-lived particles whose mass is intermediate between that of the electron and that of the proton.

The growth of understanding which led to this interest began when pair theory, in the hands of Oppenheimer and Plesset,[8] Dirac,[9] Sauter,[10] and Bethe and Heitler,[11] yielded a satisfactory prescription from which to calculate the probability of elementary collision phenomena. Among these, the most important were the conversion of a quantum of radiation into an electron-positron pair, the annihilation of a positron by an atomic electron, and the deceleration of electrons and positrons with emission of radiation. In terms of these collision probabilities, Oppenheimer and Carlson,[12] in the United States, and Bhabha and Heitler,[13] in England, independently, in 1936, successfully explained the portion of the cosmic radiation which is easily absorbable in lead. This so-called soft component of the radiation they interpreted as a mixture of electrons, positrons, and quanta, undergoing, in its passage downward through the atmosphere, a rapid multiplication in number, *via* formation of pairs, radiation by the particles of the pairs of new photons, thence more pair formation, and so on. With this increase in number, their picture associates an equally rapid degeneration in energy, until, in the lower layers of matter, effective multiplication comes to a halt. It became clear that, as a consequence of this degeneration, the strength of the soft component passed its peak in the upper levels of the atmosphere and that it accounted, at sea level, only for about twenty per cent of the total cosmic ray intensity. This clarification of the role of the soft component brought into light, especially by way of the experimental investigations of Anderson and Neddermeyer,[14] the true character of the hard component, which constituted by far the larger part of the radiation at sea level. To explain the properties of this component, it was necessary to assume the existence of singly charged particles, or mesons (from the Greek *meso*, intermediate, and *on*, ending used for nouns; hence, entity of intermediate mass), both positive and negative, with masses of the order of two hundred times the electronic mass. It was further necessary to attribute to these mesons an instability against break-up into a positively or negatively charged electron, plus some type of non-ionizing radiation, an instability associated with a mean life of the order of a microsecond.[15]

Pair theory, though it opened the way to the discovery of mesons, did nothing to explain their structure. In the beginning, it was, in fact, assumed by many theoretical physicists that these particles possessed a certain characteristic mass; that they constituted a single new type of elementary particle quite distinct from electrons and positrons; and that they played a role in binding together the neutrons and protons in an atomic nucleus. Subsequent attempts to develop a consistent theory of mesons on the basis of these assumptions have, so far, been unsuccessful, particularly in attempting to account for the mass difference between neutrons and protons, the magnetic moment of these nucleons, and the rate of beta-ray disintegration. The whole theoretical picture is in a state of uncertainty. On this account, it has a special interest, at this time, to turn back to a theory as well founded as the theory of electrons and positrons and to find that it predicts the existence of entities which have masses very nearly integral multiples of the electronic mass and which spontaneously decompose with certain characteristic mean lives.

The present investigation deals only with the properties of those electron-positron systems which have two and three times the electronic mass. It leads to the conclusion that these entities are, in some respects, strikingly like the mesons observed in the cosmic radiation, and, in other ways, definitely different. It has to leave unanswered, for the present, the question whether electron-positron systems of considerably higher mass can exist.[16]

For the name of the new entities of mass 2 and 3, it is proposed to use the word *polyelectron*—this name indicating their purely electronic character—and the symbols, P^{+-}, P^{++-}, and P^{+--}. The present investigation is believed to give good theoretical reason for the existence of these particles and for the conclusion that they play a certain, very small, but finite role in cosmic radiation.

STRUCTURE OF THE LIGHTEST POLYELECTRON

The discussion of systems composed of electrons and positrons is most conveniently divided under three heads: structure, means of formation, and modes of decay. Fortunately, this division is well justified. The life time of such a system against electron-positron annihilation is several orders of magnitude longer than the time which would be required, on the basis of classical mechanics, for the entity to execute one vibration or rotation. In the discussion of structure, we can, therefore, to a high degree of approximation, overlook the possibility of annihilation, and treat the electrons and positrons as two distinct and permanent types of particles. Furthermore, the velocities of the electrons and positrons are as small, relative to the speed of light, as the velocities within atoms and molecules. Consequently, it is sufficient to apply non-relativistic quantum mechanics to obtain a good account of the structure of the polyelectrons in question.

The most important issue, with regard to structure, is stability of the system against dissociation, either into free electrons and positrons, or into polyelectrons of lower mass. If such dissociation is energetically possible, the entity will be expected to break up in a time of the same order as the interval required for one orbital revolution; that is, a time of the order of 10^{-15} second. In this case, the life of the system is so limited that, for practical purposes, the entity may be said never to have existed.

The bi-electron P^{+-} is easily seen to be energetically stable against dissociation. Its energy levels are most easily derived by comparison with the well known values for an idealized hydrogen atom with infinitely great nuclear mass. In both cases, the potential energy of the system is expressed by the same function: $-e^2/r$, of the distance r, between the positive and the negative charge. The kinetic energy, however, for the same time rate of change of r, is only half as great in the bi-electron as in the hydrogen atom. Consequently, the effective, or so-called reduced, mass of the system is cut by a factor two, in the case of the electron-positron system. Referring to the expression for the energy, E_n, of the nth quantum state of the hydrogen atom, with respect to the energy of two charges at an infinite distance from one another,

$$E_n(H_\infty) = -(m/2)(e^2/n\hbar)^2, \quad (1)$$

we therefore conclude that the corresponding expression for the energy of the bi-electron P^{+-} is:

$$E_n(P^{+-}) = -(m/4)(e^2/n\hbar)^2. \quad (2)$$

Thus, the entity in its lowest state, $n = 1$, is stable against dissociation by 6.77 ev.

The wave function of the two-particle system in its lowest state is found by the same procedure of comparison with the hydrogen atom to be represented by the expression:

$$\psi = \pi^{-1/2}(me^2/2\hbar^2)^{3/2} exp(-me^2 r/2\hbar^2), \quad (3)$$

a result which will later be of use.

For the energy of the tri-electron P^{++-}, no exact calculation is possible. A lower limit to the stability of this entity may, however, be derived from the well-known variational procedure of Ritz. It is only necessary to assume an approximate expression for the wave function of the system, and to calculate for this wave function the expectation value of the energy, in order to have the desired upper limit on the algebraic value of the energy of the system in its ground state. We follow the treatment of the essentially identical problem of the helium atom which has been given by Hylleraas[17] and briefly summarized by Bethe.[18]

We write the trial wave function in the form:

$$exp(-kme^2 s/2\hbar^2)[1 + a(kme^2 u/\hbar^2) + b(kme^2 t/\hbar^2)^2], \quad (4)$$

where s represents the sum of the distances of the unlike particle from the two like particles; t represents the difference between these two distances; and u represents the distance between the two like particles. We obtain for the expectation value of the energy of the system the low value, $-0.257036\ me^4/\hbar^2 = -6.96$ ev, when we give the constants in the wave function the values:

$$a = 0.1203$$
$$b = 0.05719$$
$$k = 0.72102.$$

In comparison, the energy of the bi-electron P^{+-}, together with one additional free particle, is only $-0.25\ me^4/\hbar^2 = -6.77$ ev. Consequently, we conclude that the three-particle polyelectron is stable by at least $0.007036\ (me^4/\hbar^2) = 0.1906$ ev against dissociation of any kind.

For the stability of the four-particle system, P^{++--}, no proof has, so far, been found. One variational calculation was carried out, assuming for wave function the Gaussian expression:

$$exp - (me^2/\hbar^2)^2[\alpha(r_{1a}^2 + r_{2a}^2 + r_{1b}^2 + r_{2b}^2) + \beta(r_{12}^2 + r_{ab}^2)]. \quad (5)$$

The corresponding expectation value for the energy, E, with respect to four free particles at rest is found to be given by the relation:

$$(\hbar^2/me^4)E = 3(2\alpha + \beta) \quad \textit{(from kinetic energy)}$$
$$+4\pi^{-1/2}(\alpha + \beta)^{1/2} \quad \textit{(from repulsions)}$$
$$-16\pi^{-1/2}\alpha^{1/2}(\alpha + \beta)^{1/2}(3\alpha + \beta)^{-1/2} \quad \textit{(from attractions).}$$

The minimum value of the right hand side is approximately -0.367.

The polyelectron in question is, therefore, stable by at least $0.367 \times 27.09 = 9.93$ ev against dissociation into four free particles.

The energy of one free particle and three particles bound into a tri-electron of the type P^{++-} is seen, from the results above, to be approximately 6.96 ev below that of four free particles. Consequently, the four-particle entity is stable by an amount at least 3 ev against dissociation into a tri-electron. However, the crude variational calculation just described lacks 3.6 ev of being able to guarantee against division of the system P^{++--} into two bi-electrons, the combined energy of which is -2×6.77 ev $= -13.54$ ev. Because the question of the stability of the four-particle system has not been settled, we shall limit the subsequent discussion to the two- and three-particle entities.

For a general picture of the structure of the lightest electron-positron systems, we may supplement the foregoing discussion of energies with some discussion of spin and spatial extension. In the bi-electron P^{+-}, the spins of the individual particles may be either parallel or anti-parallel. Consequently, when the system is in its ground state and has no orbital angular momentum, the total spin is either 0 or 1 quantum unit, these two possibilities having the relative statistical weights 1 and 3. In the ground state of the tri-electrons P^{++-} and P^{+--}, the wave function is symmetrical with respect to the positions of the two like particles, and anti-symmetrical with respect to their spins. Therefore, the total spin of the system is one half quantum unit.

The extension of both two- and three-particle polyelectrons is of the same order as that of a hydrogen atom, roughly 10^{-8} cm. The two-particle system admits of excited states which are also stable against dissociation, and for which the spatial spread of the wave function is proportional to the square of the quantum number of the state.

PROBABILITY OF DECAY BY ANNIHILATION

The life time of a positron in one of the systems P^{+-} or P^{++-} against annihilation by an electron, will be expected to be roughly of the same order as the life time of a positron in solid matter, or $\sim 10^{-10}$ second, for the number of electrons per unit volume of space occupied by the positron is also roughly of the same order in the two cases.

A more accurate calculation of the mean life against annihilation may be given for the system P^{+-}. For this purpose, it is sufficient to recall that Dirac[9] has shown that an electron at rest presents to a positron moving towards it with a velocity, v, low in comparison with the speed of light, c, a cross-section for the annihilation process given by the expression:

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$$\pi(e^2/mc^2)^2(c/v). \tag{6}$$

Consequently, the probability per unit time for disappearance of the electron is equal to the product of the factor,

$$\pi(e^2/mc^2)^2c, \tag{7}$$

and the probability that a positron lies in a unit volume in the immediate neighborhood of the electron. This probability is given for the ground state of the bi-electron by the square of the wave function of EQUATION 3, evaluated at the point $r = 0$, and, for excited states of zero orbital angular momentum, is smaller by the factor n^3. We conclude that the probability per second of annihilation is given by the expression:

$$(e^2/mc^2)^2c(me^2/2n\hbar^2)^3. \tag{8}$$

However, this result represents an average over the four possible spin states of the two-particle system. A closer examination indicates that annihilation occurs, in the first approximation, only for the singlet state which has zero spin angular momentum. Consequently, for this state, the decay constant with respect to annihilation is four times the number just given. The mean life of the singlet state is, therefore:

$$T = (2\hbar/m)(137n)^3(\hbar/e^2)^2 = 1.24 \times 10^{-10}n^3 \text{ sec.} \tag{9}$$

Decay of the three triplet states is forbidden in the first approximation and, in the next approximation, should probably be expected to occur at a rate slower than that for the singlet state by a factor of the order (velocity of particle/velocity of light)^2 or of the order of $(1/137)^2$. If this expectation is correct, the life of the three triplet states should be of the order of 10^{-6} seconds.

The existence of higher excited states offers the possibility for various decay chains which terminate in annihilation of the bi-electron. About these possibilities, it need only be said that essentially radiative transitions alone are possible out of states whose orbital angular momentum is different from zero, while, for states with zero orbital angular momentum, not only is direct annihilation always possible, but also, out of such states with quantum numbers $n = 2$ or higher, radiative transitions are possible, which lead by way of p-states down to the lowest s-states, with the highest annihilative decay probability.

Regardless of whether the bi-electron $P+-$ is excited or not, and, if excited, regardless of whether the entity gives off part of its energy in the visual spectrum or not, it ends its life by the emission of two photons with energy totaling nearly $2mc^2$ or 1.02 Mev. These photons split the energy approximately equally and go off in nearly opposite directions. The lack of precise equality and opposition arises partly from the Doppler effect associated with unavoidable thermal agitation, and partly from the natural uncertainty which is conditioned by the finite life of the polyelectron.

For the three-particle entities P^{++-} and P^{+--}, we must expect annihilation governed by the short time scale, of the order of 10^{-10} seconds, which holds for two-particle polyelectrons in singlet states of zero orbital angular momentum. In other words, the spin of one or the other of the two like particles will certainly be oriented properly to annihilate the third particle.

The probability of two-quantum annihilation is so great, in the case of the three-particle polyelectron, that we can neglect beside it the chance for single quantum annihilation, a mechanism which was excluded, in the case of the entity P^{+-}, by the laws of conservation of momentum and energy. The existence of a third particle provides, in principle, a means to take up the momentum of recoil, in case a single photon comes off. However, the likelihood is small that this third particle, at the moment of their annihilation, will be interacting with the other two sufficiently closely to take up the surplus momentum. Consequently, we expect, in the majority of cases, that decay will lead from a three-particle system to a single particle, moving in very nearly the old direction with very nearly the old velocity, and that it will be accompanied by the emission of two approximately equal and opposite photons.

FORMATION OF POLYELECTRONS

The formation of a polyelectron of the type P^{+-} by the collision of two photons is the opposite of the process in which such an entity disappears by double quantum emission. To calculate the cross-section, σ, for the process of formation, we therefore combine the principle of microscopic reversibility with our knowledge of mean life, T, for the process of annihilation.

We consider a large volume, V. In this volume, the number of singlet polyelectron states with momenta in the ranges dP_x, dP_y, dP_z about the values P_x, P_y, and P_z will be:

$$V\,dP_x\,dP_y\,dP_z/h^3. \tag{10}$$

These states will undergo transitions, *via* annihilation, at the rate,

$$V\,dP_x\,dP_y\,dP_z/h^3T \text{ per second}, \tag{11}$$

to states in which there is present a pair of photons and no polyelectron. The number of such states in which the momentum of one photon lies in the range $dp'_x\,dp'_y\,dp'_z$, and the other lies in the range $dp''_x\,dp''_y\,dp''_z$, is:

$$(2V\,dp'_x\,dp'_y\,dp'_z/h^3)(2V\,dp''_x\,dp''_y\,dp''_z/h^3). \qquad (12)$$

The number of formation processes taking place per second will be obtained by multiplying this expression with the quantity,

$$\sigma c/V, \qquad (13)$$

which contains the cross-section, velocity, and density of the photons. The principle of microscopic reversibility states that, in a given time interval, the number of transitions in the one sense equals the number of transitions in the other sense. Consequently, we arrive at the relation:

$$(1/T) = (4c/h^3)\int\sigma(dp_x'dp_y'dp_z'dp_x''dp_y''dp_z''/dP_x\,dP_y\,dP_z). \qquad (14)$$

We introduce new variables of integration through six equations of the form:

$$p_x' = \tfrac{1}{2}P_x + p_x; \qquad p_x'' = \tfrac{1}{2}P_x - p_x. \qquad (15)$$

Then the volume element in EQUATION 14 reduces to $dp_x\,dp_y\,dp_z$. We now specialize to the case in which the two photons have momenta which are approximately opposite in direction and which are, therefore, also approximately equal to mc in magnitude. Then, integration over all solid angles, with due allowance for the identical role of the two photons, gives for the volume element $2\pi(mc)^2dp$. If the energy of one of the photons is fixed, and the energy of the other varies by the amount dE, then the momentum change dp is equal to $dE/2c$. The principle of microscopic reversibility takes the form:

$$1/T = (4\pi m^2c^2/h^3)\int\sigma\,dE. \qquad (16)$$

We know, of course, that the cross-section, σ, possesses a sharp resonance, when the energy, E, of the second quantum lies in the neighborhood of mc^2. We can even deduce the detailed dependence of cross-section upon energy, by applying the familiar theory of resonance, according to which the quantity, σ, is represented as a function of E by an expression of the form,

$$\sigma = \text{constant}/(1 + (2\delta ET/\hbar)^2), \qquad (17)$$

where the constant is determinable from EQUATION 16. However, we are not primarily interested in the obviously improbable possibility that two gamma rays of just the right energy shall collide head on to form a polyelectron of the type P^{+-}. Our real purpose, in studying the mechanism of formation from two photons, is to deduce the cross-section for creation of the polyelectron by the more feasible mechanism of impact of a single photon upon an atomic nucleus.

The probability for photoelectric creation of the entity P^{+-} in the field of force of a nucleus can be deduced, in a reasonable approximation for quanta of high energy, from the cross-section for formation from two photons, by means of an expedient due to Williams[19] and Weizsäcker.[20] We consider the process of creation in a frame of reference moving with the velocity which the polyelectron possesses after its formation. In this frame of reference, the initial photon travels in one direction with an energy close to mc^2, and the nucleus travels in a nearly opposite direction, with a speed close to that of light. An electron which is at rest in this frame of reference and which is subject to the field of the passing nucleus experiences a field of force very nearly equivalent to that in a pulse of radiation. Thus, the electron may be treated with considerable accuracy as if it were subject to the action of two oppositely directed beams of radiation. Consequently, the unknown action of the nucleus can be expressed in terms of the already known effect of a second photon.

We denote the energy of the incident quantum, in the laboratory frame of reference, by W. Then, the sum of the rest and kinetic energies of the polyelectron in this frame is also equal to W. The ratio of this quantity to the rest energy of the polyelectron, $W/2mc^2$, determines the velocity of the polyelectron and, hence, the speed of the moving frame of reference. We suppose that this ratio is large compared to unity. Then, in this moving frame, the action of a nucleus of charge Ze may be described, according to Williams,[19] as approximately equivalent to that of a beam of photons, of which the number, dN, in the energy interval, dE, is given by the equation:

$$dN = (2/\pi)(Z^2/137)(dE/E)\,ln\,(W/2E). \tag{18}$$

We multiply this number by the cross-section, σ, for formation of a polyelectron by collision of one quantum of energy mc^2 and another of energy E. We integrate the product with respect to energy and use EQUATION 16 for the integrated cross-section. We arrive at the result:

(number of polyelectrons produced per cm.^{3} and per sec.)
= (number of nuclei per cm.^{3}) . (number of photons per cm.^{3}) . (velocity of light).

$$(Z^2/137)\,(h^3/2\pi^2m^3c^4T)\,ln\,(W/2mc^2), \tag{19}$$

where the quantities expressed in words are all understood to be measured in the moving frame of reference.

To transform EQUATION 19 to the laboratory frame of reference, we note, first of all, that the velocity of light and the number of polyelectrons produced per cm.^{3} and per second are described by the same numbers in both frames of reference. Next, we express the ratio $W/2mc^2$ in the form, cosh χ, and write the relativistic equations for transformation of density, ρ, and flux, J, in the direction of the meson in the form:

$$\rho_{mov} = \rho_{lab} \cosh \chi - (J_{lab}/c) \sinh \chi$$
$$(J_{mov}/c) = -\rho_{lab} \sinh \chi + (J_{lab}/c) \cosh \chi. \quad (20)$$

For the photons, $J_{lab} = c\rho_{lab}$. Insertion of this relation in the first of the two equations above leads to the result:

$$\rho_{mov} = \rho_{lab}\, exp\, (-\chi) \qquad (photons).$$

For the nuclei, $J'_{lab} = 0$ and, consequently,

$$\rho'_{mov} = \rho'_{lab} \cosh \chi \qquad (nuclei).$$

We have assumed that cosh χ is large, in comparison with unity. Consequently, this function can be represented with sufficient approximation in the form, $(\frac{1}{2})\, exp\, \chi$. Thus, for the product of photonic and nuclear density, we have the product:

$$\rho_{mov}\rho'_{mov} \doteq \tfrac{1}{2}\, \rho_{lab}\, \rho'_{lab}. \quad (21)$$

We insert this result into EQUATION 19, for the rate of production of polyelectrons. We conclude that the cross-section for creation of the entity, P^{+-}, in the field of force of a nucleus of charge, Ze, by a photon whose energy, W, is large in comparison with the rest energy, mc^2, of an electron, is given approximately by the expression:

$$(Z^2/137)(h^3/4\pi^2 m^3 c^4 T)\, ln\, (W/2mc^2)$$
$$= (Z^2/(137)^4 n^3)\pi(e^2/mc^2)^2\, ln\, (W/2mc^2), \quad (22)$$

where n is the quantum number of the state in which the polyelectron is formed.

To assess the size of the cross-section for production of a pair of electrons and positrons which stick together, we may compare it with the cross-section, given by Bethe and Heitler,[11] for production of a pair of any kind by an energetic quantum:

$$(28/9)(Z^2/137)(e^2/mc^2)^2\, ln\, \ldots \quad (23)$$

Here, the dots indicate that the argument of the logarithm does not differ enough from the argument of the logarithm in EQUATION 22 to make any significant difference in the comparison. We conclude that polyelectrons of the type P^{+-} are produced, relative to the usual pairs, in an abundance ratio approximately equal to:

$$(9\pi/28)(1/137)^3 \sim 10^{-6}. \quad (24)$$

From the smallness of the cross-section for their formation by photoelectric effect, it appears likely that the simplest entities composed of electrons and positrons play a very minor role indeed in the cosmic radiation. It is, nevertheless, interesting to learn from pair theory that such particles exist and that they are formed by a mechanism susceptible, in principle, to laboratory confirmation.

The smallness of the probability (EQUATION 24) that positron and electron will go off bound to each other, may be given a simple interpretation. In a frame of reference in which their total momentum is zero, the two particles have, on the average, a momentum, one relative to the other, of the order of mc. On the other hand, the relative momenta of the two particles in a system of the type P^{+-} is of the order $me^2/\hbar$. Consequently, that fraction of the accessible part of momentum space which leads to the desired type of particle is only of the order, $[(me^2/\hbar)/mc]^3$ or $(1/137)^3$, in agreement with our calculations.

For the formation of an entity of the type P^{+--}, the most reasonable mechanism appears to be the interaction of a photon with an atomic electron. The process of the formation of an electron-positron pair in the field of an atomic electron has already been analyzed by Wheeler and Lamb.[21] For all three particles to remain bound together, after the act of formation, the conditions are still more special, however, than those encountered in the case of the polyelectron P^{+-}. In order for their relative momenta to be of the required order, $me^2/\hbar$, it is necessary that the energy of the incident photon should lie within a small fraction of an electron volt of the critical energy $4mc^2 = 2.04$ Mev. Consequently, we conclude that the systems, P^{+--}, probably play an even smaller role in the cosmic radiation than those entities, P^{+-}, which have twice the electronic mass.

BREAK-UP OF POLYELECTRONS IN PASSING THROUGH MATTER

In studying the decay of polyelectrons, we took account of the possibility of mutual annihilation of pairs of particles, but we made no allowance for the destruction of the system *via* external disturbances. The entities, P^{+-} and P^{+--}, have, however, an essentially atomic extension. In passing through matter, they will, therefore, be susceptible to losing and recapturing electrons after the manner of an alpha particle. If the system is moving at very high speed through a medium of significant density, there will be an overwhelming probability of its consisting of an isolated positron or electron. On the other hand, a positron which is undergoing moderation to energies small in comparison with its rest energy will have a continually increasing probability of picking up an electron and of forming a system of the type P^{+-}.

In order to discuss quantitatively the comparative importance of the annihilation of the polyelectron, P^{+-}, and its destruction by loss of its electron, we shall define for each velocity of the entity a certain characteristic density of air for which the two mechanisms are estimated to have equal probability. If the density is higher, electronic loss is the more significant mechanism; if the density is lower than the characteristic

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figure, annihilation is the process more likely to occur. In the following table of characteristic densities, we have assumed, in the absence of better information, that we can use, for the mean free path for loss of an electron in air by a polyelectron, the same figures which were obtained by Rutherford[22] for loss of an electron by a swift neutral helium atom.

TABLE 1

CHARACTERISTIC DENSITIES*

Speed of the particle in cm./sec.	1.81×10^9	1.46×10^9	9.0×10^8	5.6×10^8
Rutherford's figure for mean free path against electronic loss in air of standard density	1.1×10^{-3} cm.	7.8×10^{-4} cm.	5.0×10^{-4} cm.	3×10^{-4} cm.
Mean free path of singlet P^{+-} against annihilation (product of row 1 by mean life of 1.24×10^{-10} sec.)	2.24×10^{-1} cm.	$.81 \times 10^{-1}$ cm.	1.12×10^{-1} cm.	7.0×10^{-2} cm.
Characteristic density for equal probability of annihilation and electronic loss; density expressed in multiples of standard density of air *	0.0049	0.0043	0.0045	0.0043

* The characteristic density for a two-particle polyelectron in a triplet state is probably roughly 10^{-4} times as great as the figures here listed.

It is clear, from the figures in TABLE 1, that the annihilation of polyelectrons of the type P^{+-} with velocities in the given range will take place, with significant probability, only at a level in the atmosphere where the density is less than its value at sea level by a factor of more than 100.

Briefly, the great spatial extension of the lightest polyelectrons and the consequent ease with which they may be torn apart both furnish arguments against their playing any appreciable part in normal cosmic ray phenomena.

POSSIBILITY OF TESTING SOME OF THE PRESENT PREDICTIONS OF PAIR THEORY

Quite apart from the possibly questionable direct bearing of polyelectrons upon cosmic ray phenomena, it is natural to ask if there are any experimental implications which may be examined in the laboratory as new tests of the validity of pair theory itself.

One of the most interesting possibilities for a test is suggested by the existence of excited energy levels in the polyelectron, P^{+-}. Radiative transition of the system between these energy levels generates an optical spectrum which differs from that of hydrogen, in its major features, only through the displacement of all lines to the red by a displacement factor of two. To observe any well-defined spectrum of such a character would, of course, appear to call, in the first place, for a gaseous emitter. In addition, the securing of slow polyelectrons requires that a slow positron should be able easily to detach an electron from an atom of the gas. This condition requires that the first ionization potential of the substance of the gas should be close to 6.7 volts. Finally, the means of observation must be capable of picking up over the background spectral lines which have been considerably broadened by the Doppler effect, inevitable in systems which have only twice the electronic mass and which are in thermal equilibrium near room temperature.

The difficulties about Doppler effect and choice of substance with suitable ionization potential are considerably alleviated by renouncing, in the beginning, the study of the particular entity, P^{+-}, and looking at the problem of the test of pair theory in a broader light. The essential point is to find an atomic or molecular system which contains a positron and which possesses several optically combining energy levels. In looking for such a system, it is only necessary to remember that the positron may be regarded as a superlight isotope of the highly reactive hydrogen ion. Consequently, one can look among such compounds as e^+Cl^- for systems which may possess the desired type of energy levels and which will be free of objectionable Doppler effect, on account of their substantial mass.

The actual experiment would consist in irradiation of a suitable gas with slow positrons, the radiative capture of some of these positrons into excited states of entities having somewhat the character of molecules, the transition of these entities to lower levels with the emission of characteristic spectral lines lying in, or near, the visible region, the observation of this spectrum, and the annihilation of each positron by an electron of the corresponding molecule-like entity. The gamma rays given off in this experiment would be of no direct concern to the problem at issue. The test of the pair theory would come in the comparison of the observed and calculated positions of the spectral lines. Obviously, the experiment, though interesting, is difficult.

A second and somewhat simpler experiment would seem to offer a means to check on one of the details of the annihilation process itself. We have already remarked that by far the dominating type of annihilation is that in which the positron combines with an electron whose spin forms a singlet state with respect to the spin of the positron. Associated with this selection of pairs which have zero relative angular momentum, before the annihilation process, is an analogous polarization phenomenon in the two quanta which are left at the end of the process. According to the pair theory, if one of these photons is linearly polarized in one plane, then the photon which goes off in the opposite direction with equal momentum is linearly polarized in the perpendicular plane.

To test this prediction, the following experimental arrangement suggests itself: A radioactive source of slow positrons is covered with a foil thick enough to guarantee annihilation of all the positrons. A sphere of lead centered on this source prevents the escape of any of the annihilation quanta, except through a relatively narrow hole drilled through the sphere along one of its diameters. When a photon of energy mc^2 comes out of one end of this channel, we expect a photon also of energy mc^2 to emerge simultaneously from the other end. At each end, a carbon scatterer is placed. Photons scattered by one of these blocks through approximately ninety degrees and into the proper azimuth pass through a gamma ray counter. The scattering process gives a preference to the recording of photons with ~~a selected polarization~~. A similar arrangement applies at the other end of the channel. The relative azimuth of the two counters may be varied at will. Coincidences between the two counters are recorded, (a) when the azimuths of the two counters are identical, (b) when the azimuths differ by a right angle. The observed ratio of (b) to (a) is compared with the computed ratio, as a check on the theory of the annihilation process. The calculated ratio for the case of ideal geometry is 1.080, when the arrangement requires the photons to be scattered through 90°. The theoretically most favorable ratio of 1.100 is obtained when the scattering angle is reduced to 74°30′.

Another possible means of studying this scattering is to use the knock-on electrons, instead of the recoil photons. The polarization is, obviously, as complete for the particles as for the radiation. In case this arrangement is employed, the detecting counters are set to catch electrons knocked on at an angle of about 30°, with respect to the annihilation radiation. The efficiency of counting is increased by this alteration in the plan of the experiment.

Evidently, it is possible, by means of a reasonable experimental procedure, to obtain information bearing most closely upon the problem of the intimate interaction of an electron and a positron.

SUGGESTIONS FROM POLYELECTRON THEORY FOR THE STUDY OF COSMIC RADIATION

We have already seen that we are unlikely to obtain from cosmic ray observations any significant experimental information on the properties of the lightest polyelectrons. However, we can still ask the converse question: What suggestions, however unreasonable, can the theory of polyelectrons bring to the study of cosmic ray mesons? We may be permitted to make three suggestions, in the form of the following trio of questions:

(1) Is there any evidence of a kind of quantization of meson lives into two groups related, by analogy, to the two kinds of polyelectron of mass 2?

(2) Are mesons ever found to undergo the partial decay, two electronic mass units at a time, which is expected of polyelectrons of high mass number (provided that such structures are stable against dissociation)?

(3) Are the masses of cosmic ray mesons approximately equal multiples of the electronic mass?

Obviously, the questions just asked could be stated without any reference at all to the theory of polyelectrons. Obviously, also, much more experimental work must be done, before we are even near the answers to some of these questions. Still, the now-clear theoretical evidence for the existence of the polyelectrons certainly adds some extra interest to the study of these cosmic ray problems.

CONCLUSION

The existence of entities of a new type has been pointed out, and their properties have been studied. The domain of application of the electron-positron pair theory has, in this way, been extended. In particular, a discussion is presented of two experiments possibly suitable for further testing the pair theory. Finally, the properties of the polyelectrons suggest some pertinent questions about the behavior of the already-known cosmic ray mesons.

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1. On October 5, four days after the present paper was submitted to the New York Academy of Sciences, the author learned from Professor **Arthur Ruark** that he had previously envisaged the existence of the particular entity composed of one electron and one positron. Dr. Ruark has discussed the optical spectrum and the life time of this two-particle system in a note dated September 23, 1945, which he intends to submit for publication to the *Physical Review* in the form of a "Letter to the Editor." A reference to unpublished work by **L. Landau** on the properties of the bi-electron has been made by **Alichanian, A., & T. Asatiani.** 1945. J. Phys. U.S.S.R. **9**: 56.
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16. Note added January 7, 1946. Subsequent discussion of the question of stability of large polyelectrons with **E. P. Wigner** and **H. Margenau** has made it clear that the problem of stability of large polyelectrons can be divided into two distinct parts:
    (1) the stability of the system containing two electrons and two positrons; and
    (2) granted that P^{++--} is stable, the further question of stability of a crystal of many such four-particle systems, when account is taken of the balance between the zero-point kinetic energy of these light masses and the potential energy of van der Waals attraction between them. If such a crystal is stable, it will be reasonable to assume that a large polyelectron, however much its state of internal motion may differ from that in a crystal, will also be stable.

    Note added July 4, 1946. According to a kind personal communication from **H. Margenau** and **Aadne Ore,** of results shortly to appear in the *Physical*

Review, further variational calculations of the energy of the polyelectron P^{++--} so far give no more evidence than the calculation in the text for the stability of this four-particle system.

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