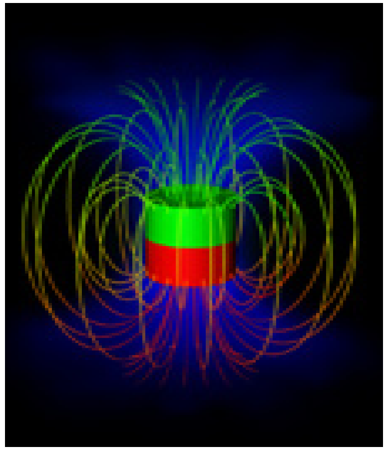
Magnetic Moments

AVA School on Precision Studies

Andreas Mooser - MPIK

Fundamental particles behave like a small magnet

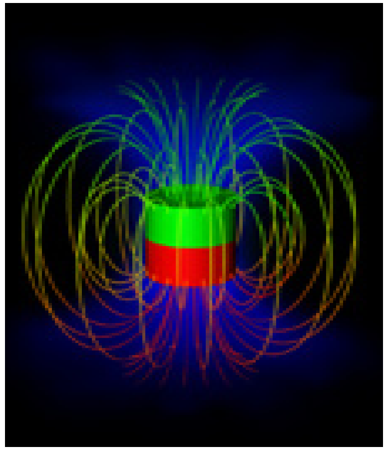


The magnetic moment

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

Every spin caring particle has a magnetic moment

What to learn from magnetic moments?



Test of fundamental physical laws

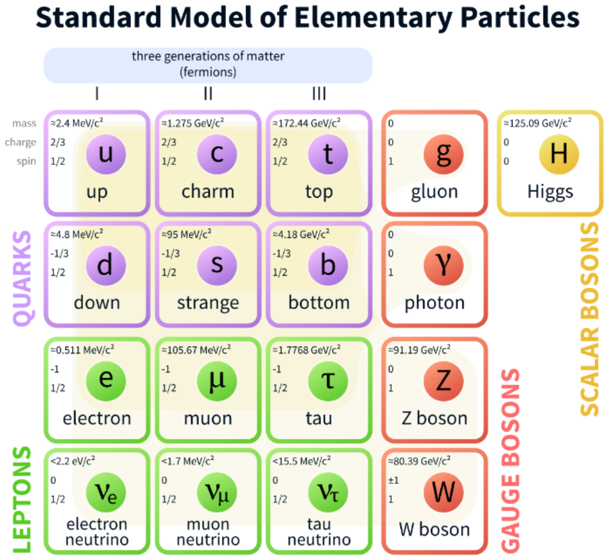
Searches for physics beyond the Standard Model

Determination of fundamental constants

What to learn from magnetic moments?

Test of fundamental laws

- QED
- Bound-state QED
- QCD
- Electro-weak

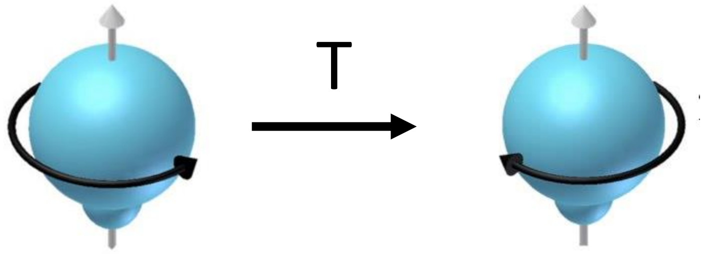


Determination of Fundamental constants

- Magnetic moment
- Finestructure constant
- Rydberg constant
- Electron mass
- Charge radii

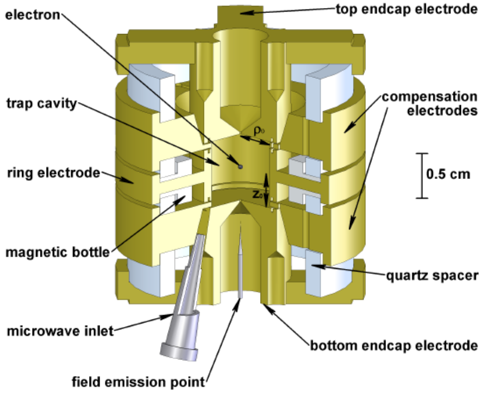
Searches for physics beyond the Standard Model

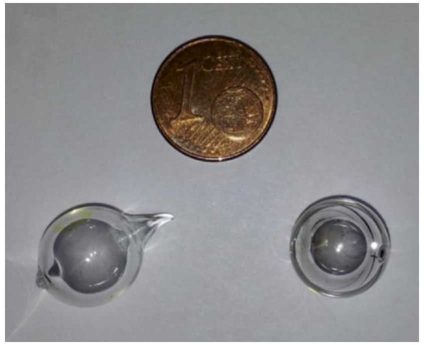
- CPT-invariance
- Searches for EDM
- Fifth Forces

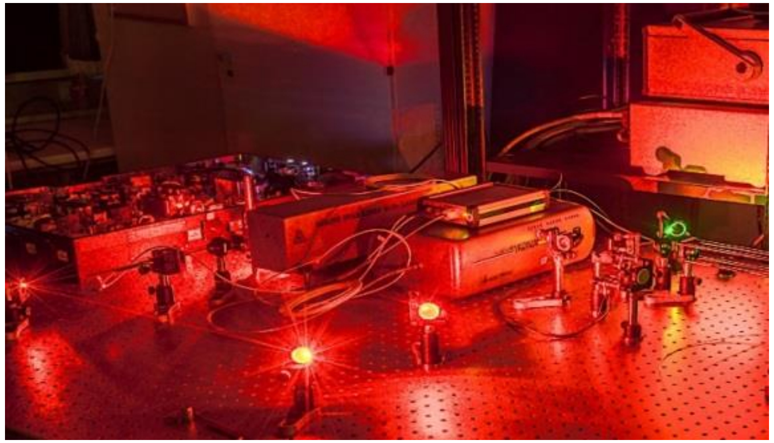


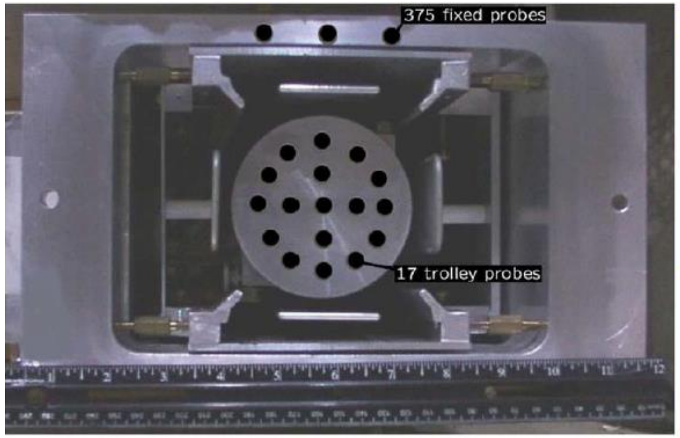
Outline

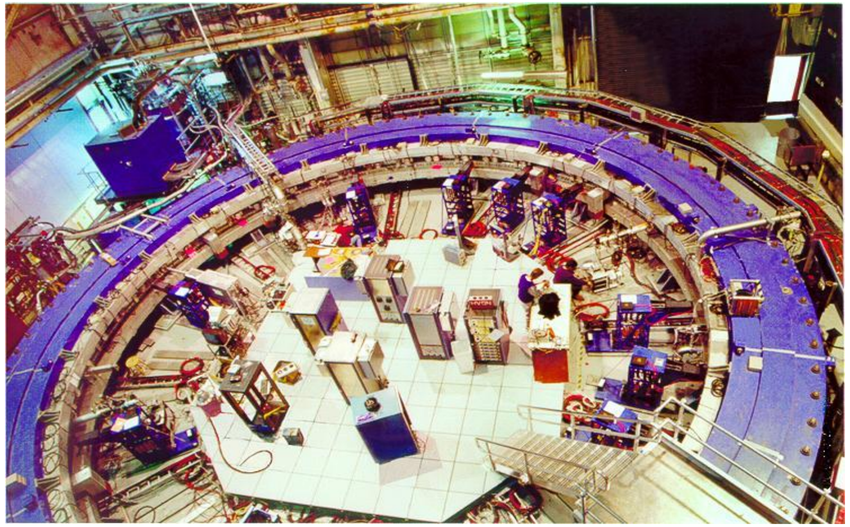
- Electron and positron
    - Muon and antimuon
        - Helium-3









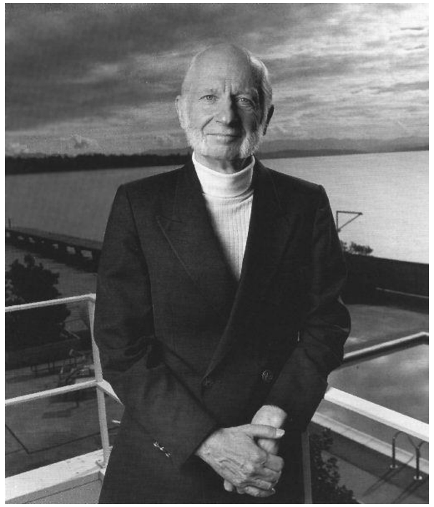


Electron and Positron g-Faktor

Precise comparison of the magnetic moment of the electron and the positron

- First high precision experiment performed in a Penning trap
- First high precision experiment performed with trapped Antimatter
- Most precise test of Quantum-Electro-Dynamics
- Until recently most precise measurement of the fine structure constant

Hans Dehmelt
Nobel price 1989

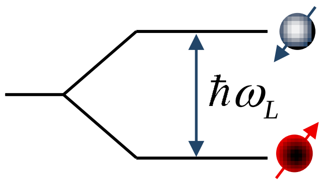


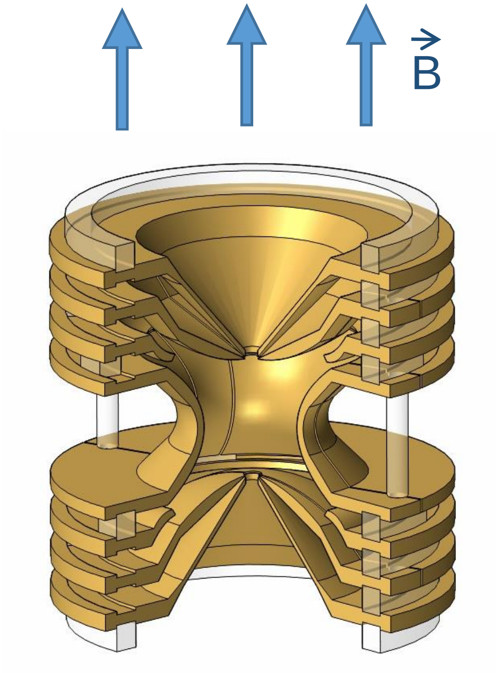
Contuinued by
Gerald Gabrielse

Basic Principle for Penning traps

Determination of Larmor frequency in a given magnetic field

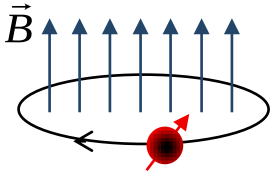
$$\boxed{\omega_L = \frac{g}{2}\frac{e}{m}B}$$





Monitoring magnetic field via simultaneous measurement of the free cyclotron frequency

$$\boxed{\omega_c = \frac{e}{m}B}$$

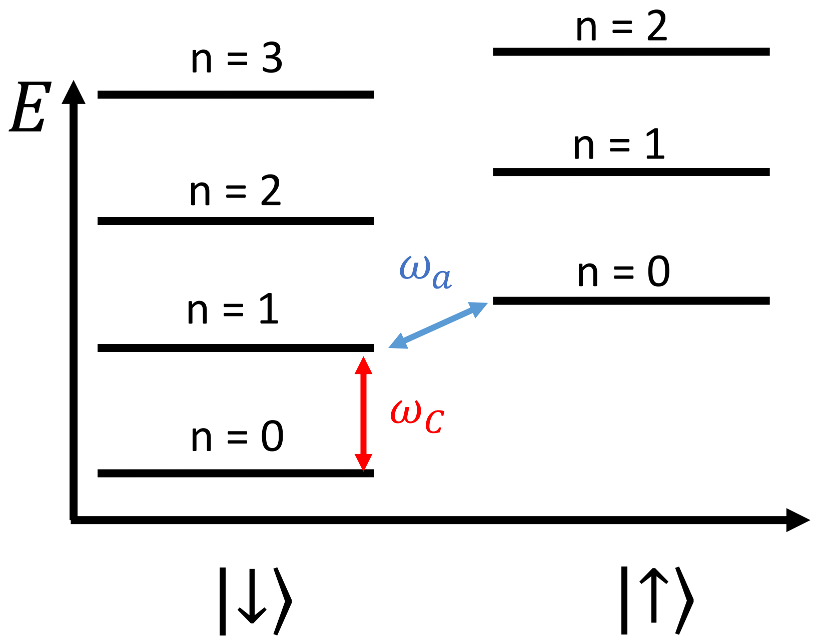


$$\boxed{g = 2\frac{\omega_L}{\omega_c} = 2\frac{\nu_L}{\nu_c}}$$

Special Case for Electron

- Don‘t measure Larmor but so-called anomaly frequency:

$$\frac{(\omega_L - \omega_C)}{\omega_C} = \frac{\omega_a}{\omega_C} = \frac{g - 2}{2} = a_e$$



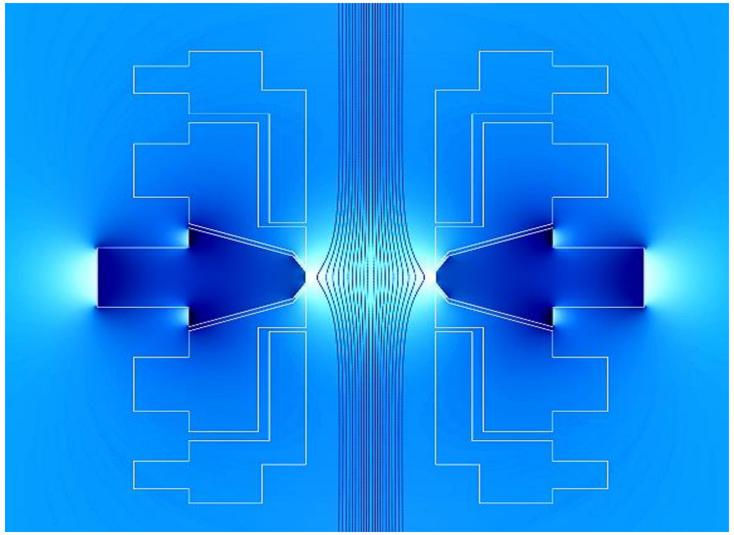
- Direct measurement of QED corrections

$$\frac{g - 2}{2} = 0{,}00115965218113$$

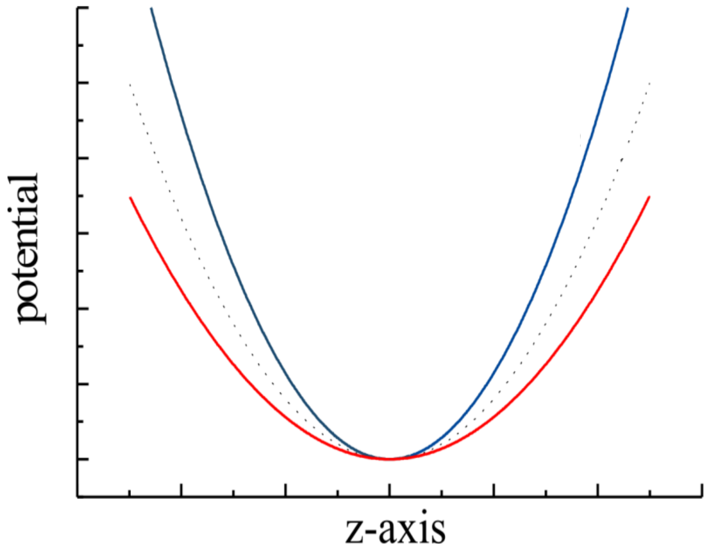
- Gain 3 orders of precision in *g* for free
- Measurement reduces to detection of cyclotron quantum state

How to detect cyclotron quantum jump?

Introduce magnetic inhomogeneity, the magnetic bottle



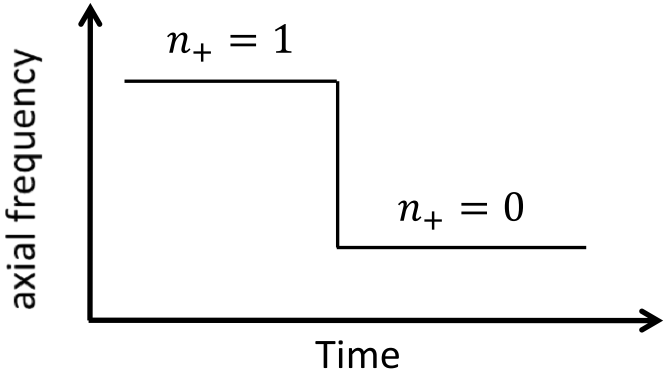
$$B_z = B_0 + B_2\left(z^2 - \frac{\rho^2}{2}\right)$$



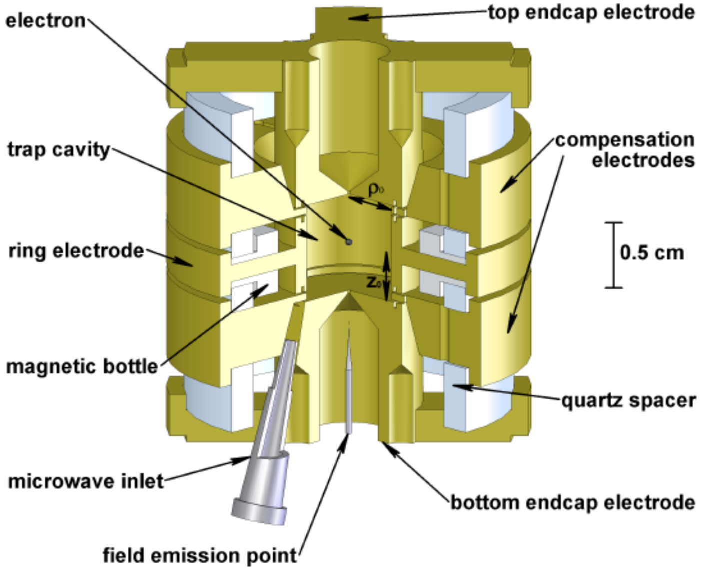
$$\Phi_z \sim n_+ B_z$$

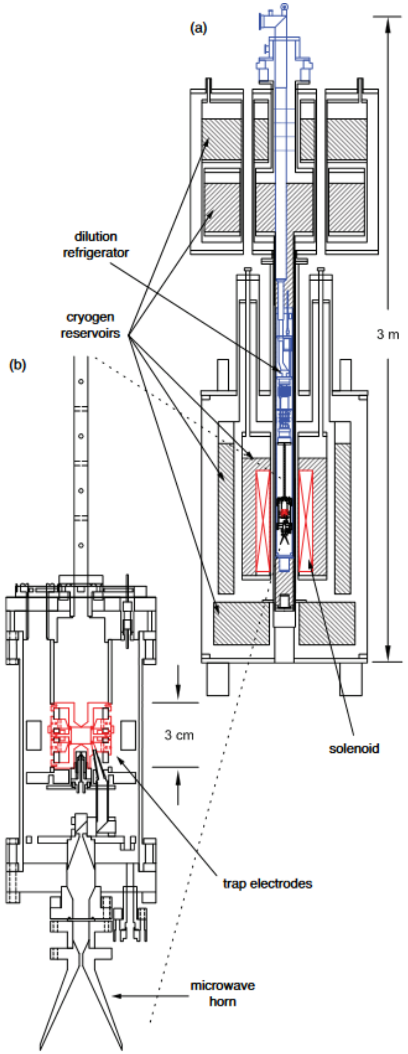
Spin flip results in shift of the axial frequency

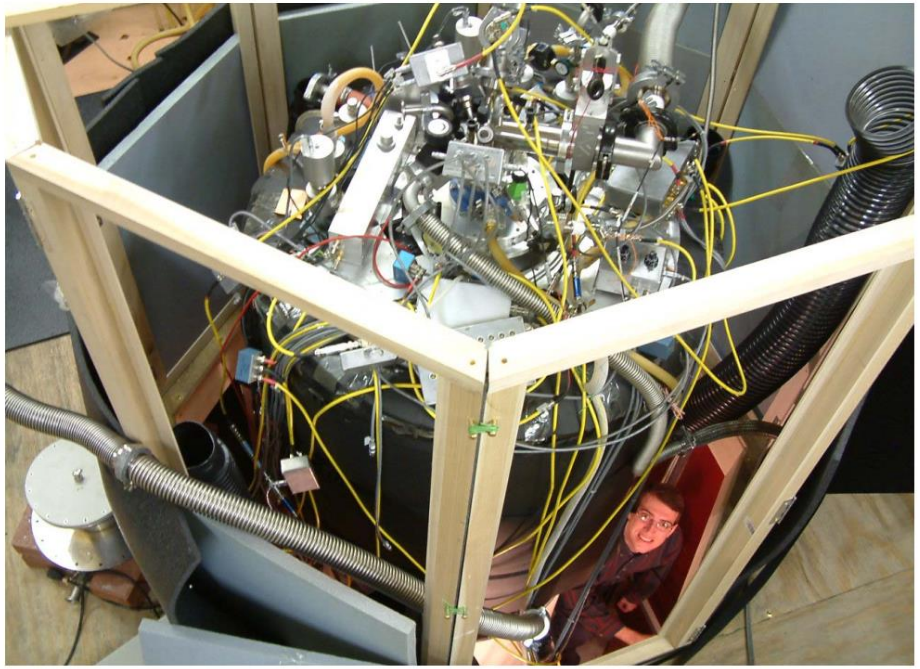
$$\nu_z \sim n_+ B_z$$



Setup


electron
top endcap electrode
trap cavity
compensation electrodes
ring electrode
0.5 cm
magnetic bottle
quartz spacer
microwave inlet
bottom endcap electrode
field emission point


(a)
dilution refrigerator
cryogen reservoirs
3 m
(b)
3 cm
solenoid
trap electrodes
microwave horn



Observation of quantum jumps

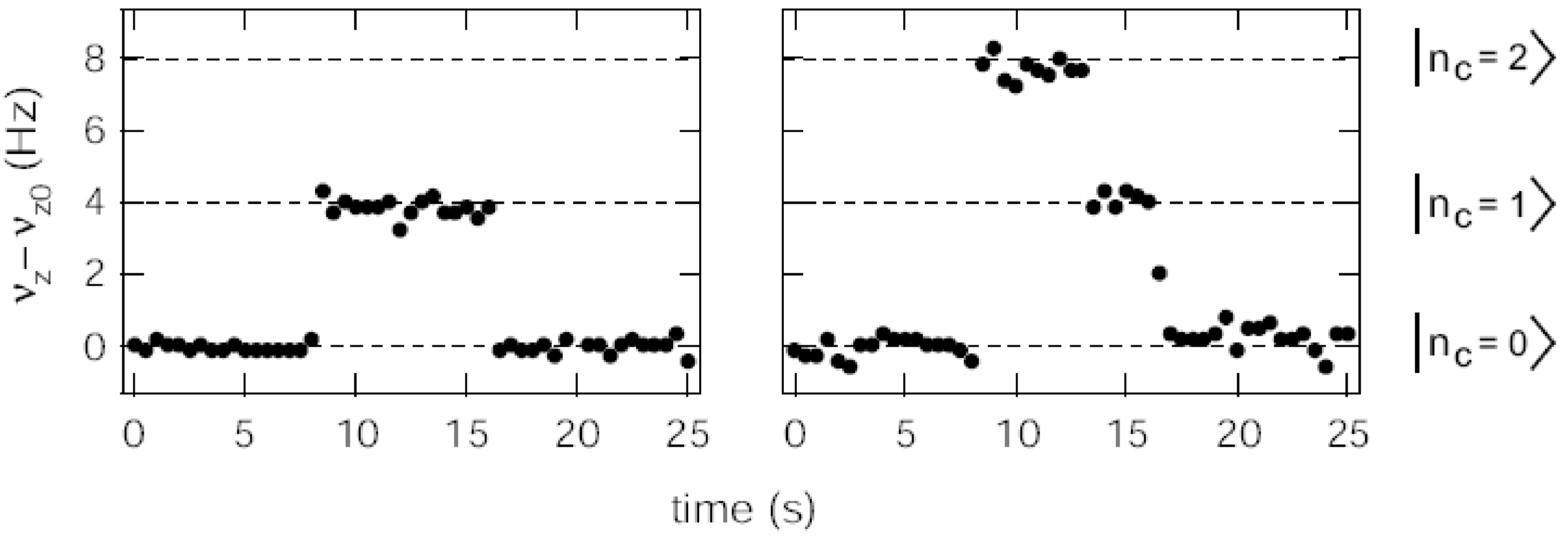


Figure 2.14: Axial frequency shift (with $\nu_z \approx 200$ MHz) caused by quantum cyclotron transitions of a single electron between the ground and first excited state (left) and between the ground and first two excited states (right).

Brian Carl Odom, PhD thesis (2014)

Observation of quantum jumps

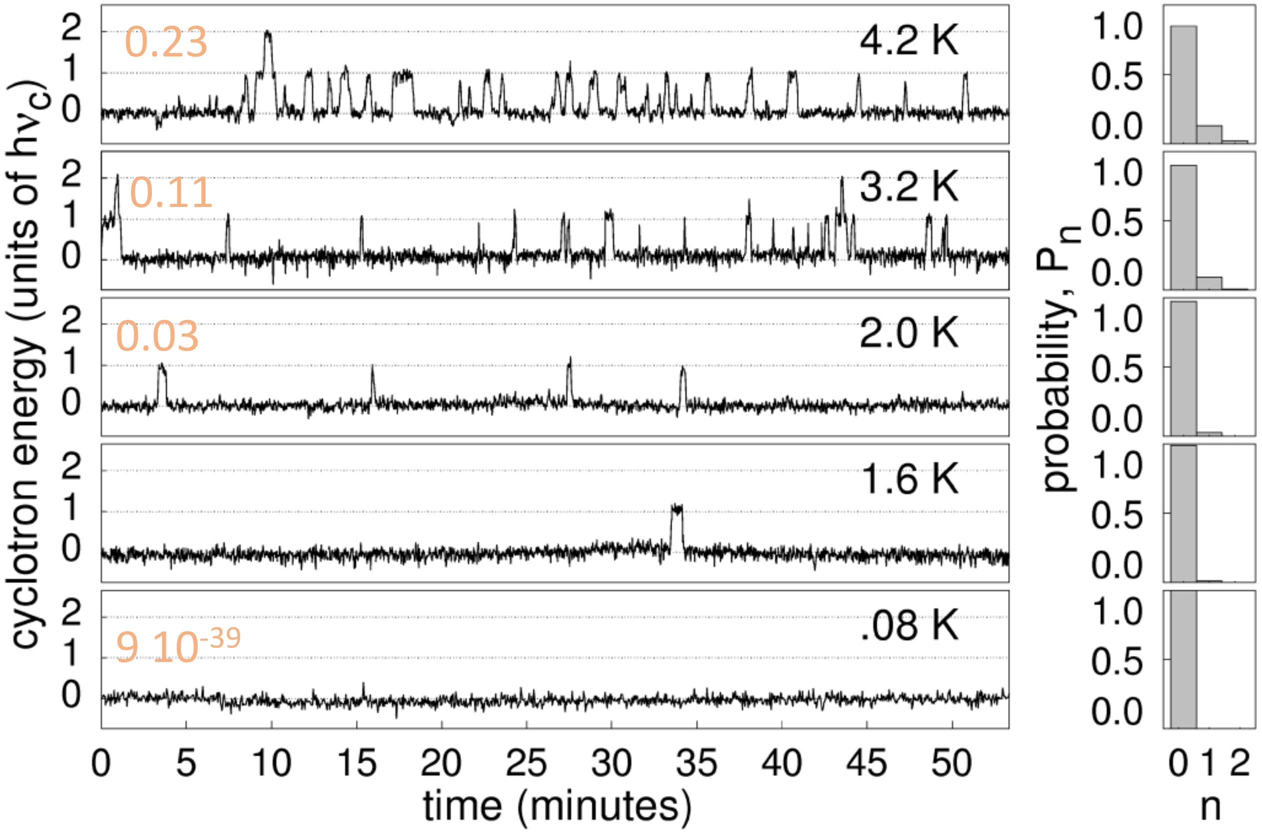
Electron cools radiativly to temperature of surrounding

Excited by thermal photons

Temperature reduction: lower thermal photon density

Effectively measured: Axial frequency as a function of time

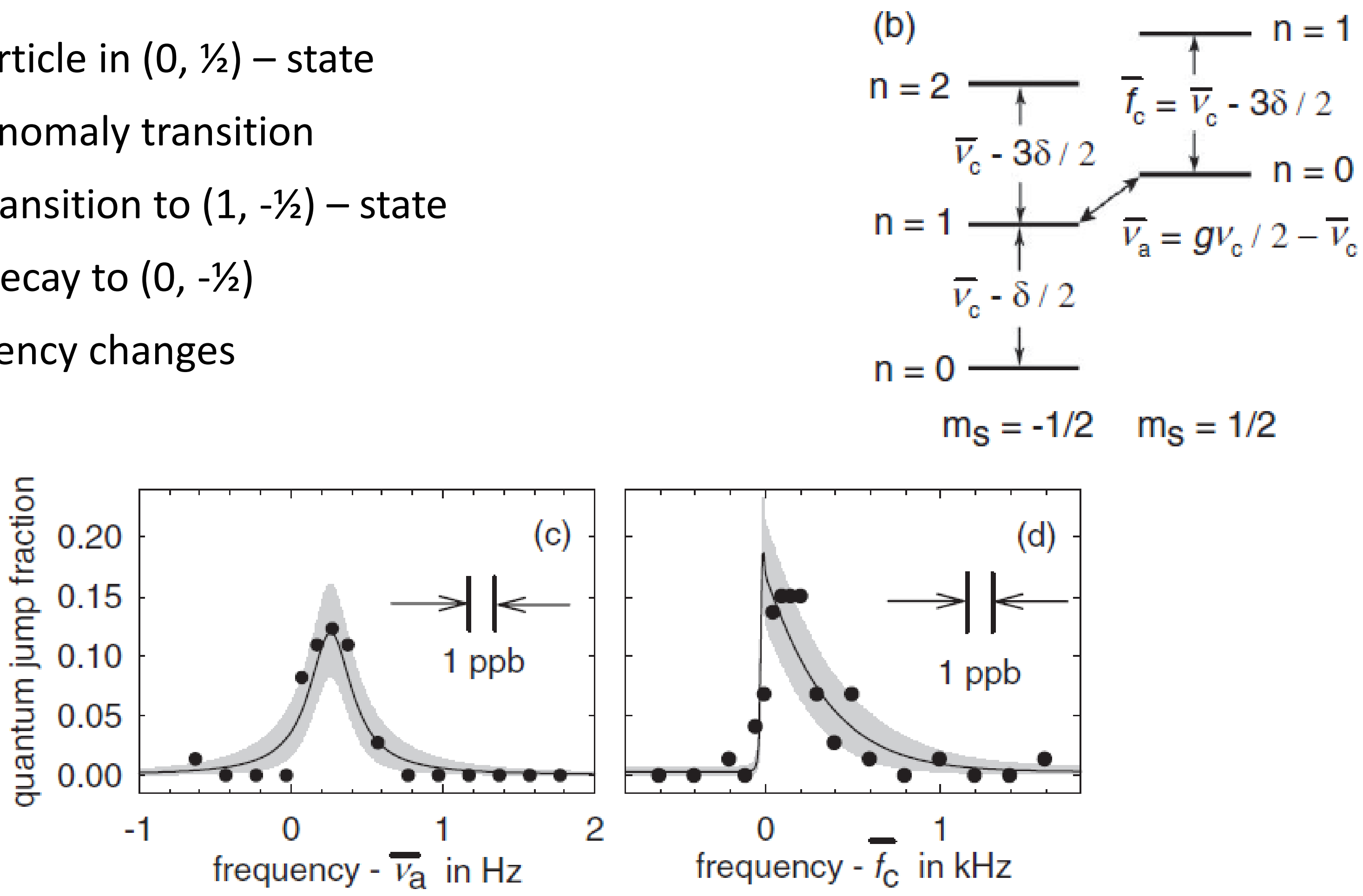
Avg. number of thermal photons



Peil, S. & Gabrielse, G. *Phys. Rev. Lett.* **83** , 1287–1290 (1999).

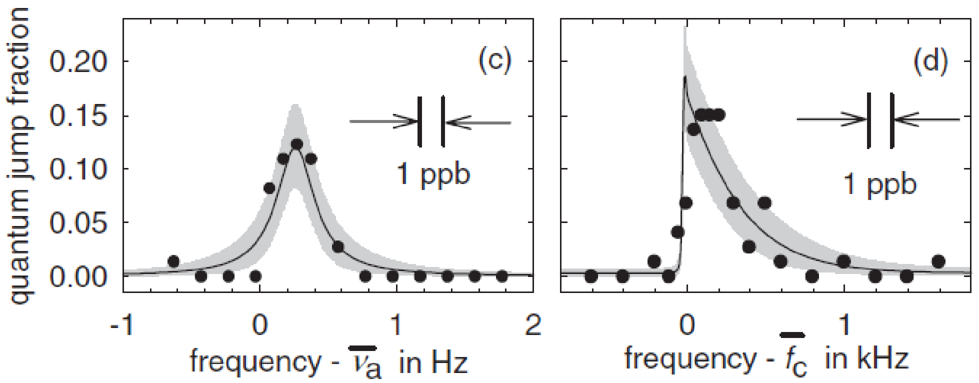
Measurement Sequence

1. Prepare particle in (0, ½) – state
2. Drive the anomaly transition
3. Anomaly transition to (1, -½) – state
4. Radiative decay to (0, -½)
5. Axial frequency changes

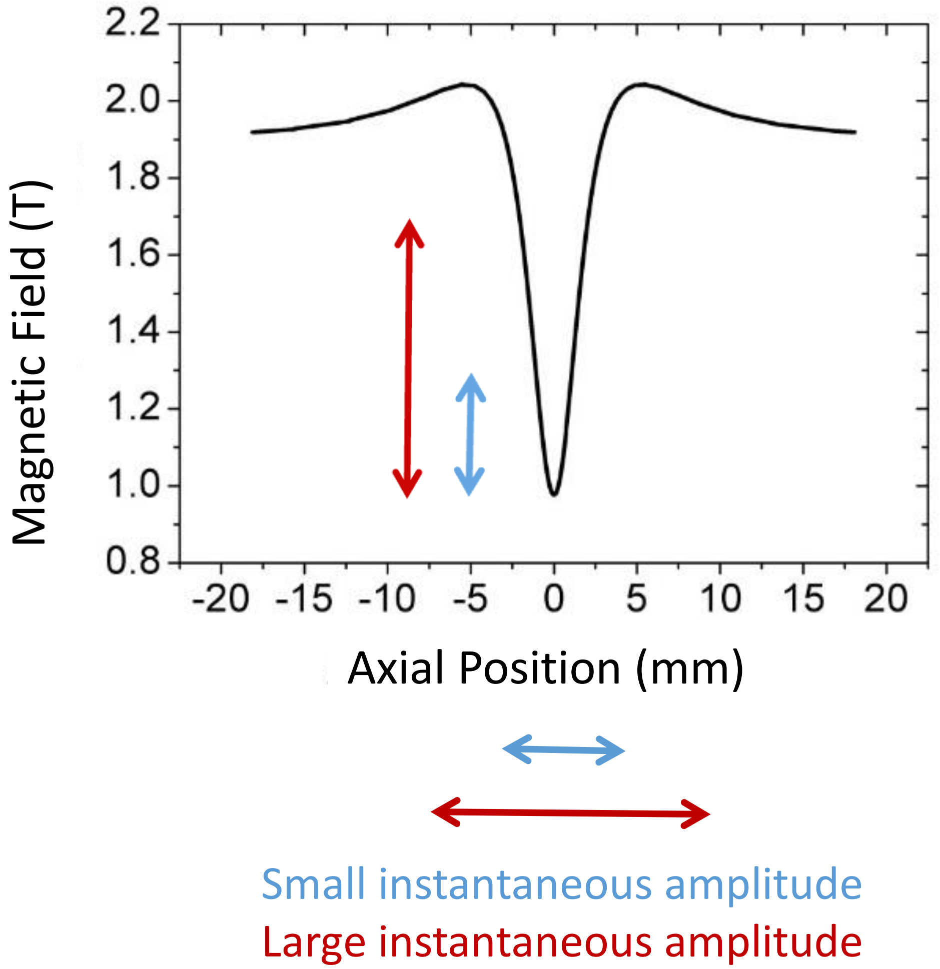


B. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse Phys. Rev. Lett. 97, 030801 (2006)

Lineshape



- Particle oscillates at finite axial amplitude
- Axial amplitude varies due to coupling to thermal bath (detection system)
- Axial amplitude follows Boltzmann distribution of thermal bath
- Average magnetic field seen in inhomogeneous magnetic field is also Boltzmann distributed

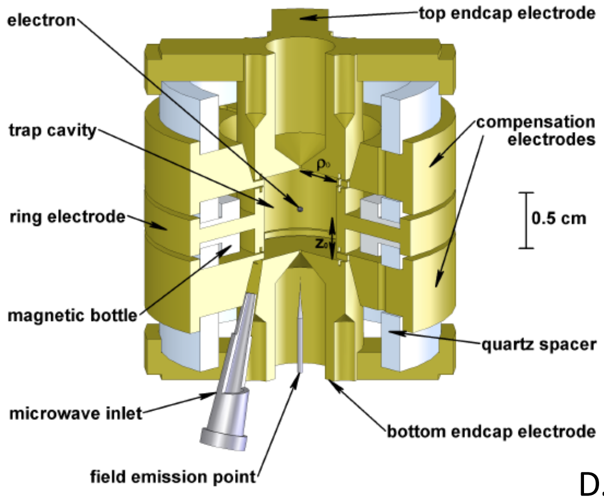


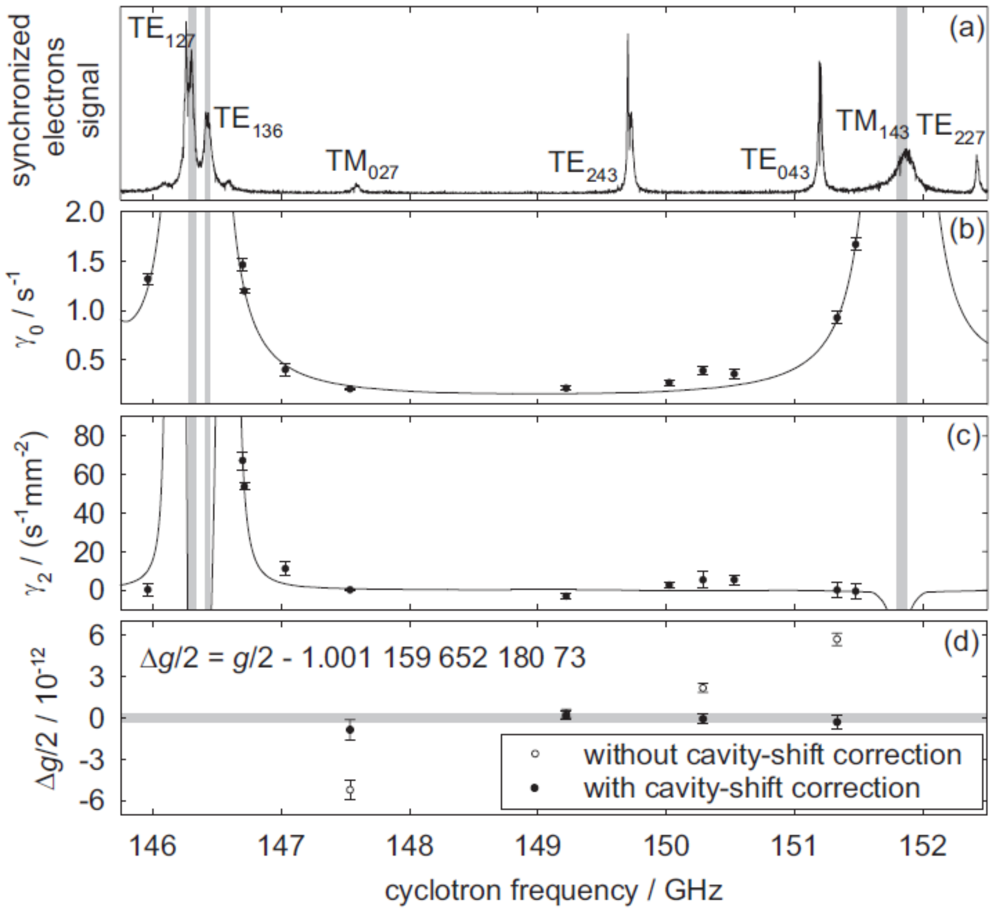
Dominant systematic effect

- Metal electrodes from a resonant microwave cavity – resonant radiation modes
- Modes can couple to the electron cyclotron motion, altering its damping rate and shifting its frequency

$$\bar{\omega}_c = \omega_c \left(1 + \frac{\Delta\omega_c}{\omega_c}\right)$$

- Tune cyclotron frequency out of cavity modes by changing the magnetic field
- And compare to theory





D. Hanneke, S. Fogwell, and G. Gabrielse Phys. Rev. Lett. 100, 120801 (2008)

Developments on the way

20 years

- Resolve lowest cyclotron and spin states
- Quantum jump spectroscopy
- Cavity-controlled spontaneous emission (linewidth reduction)
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons probe cavity radiation modes
- Elimination of nuclear paramagnetism (silver electrodes)
- One-particle self-excited oscillator

After 25 years of development

PRL **100,** 120801 (2008) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2008

New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

D. Hanneke, S. Fogwell, and G. Gabrielse*

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 4 January 2008; published 26 March 2008)

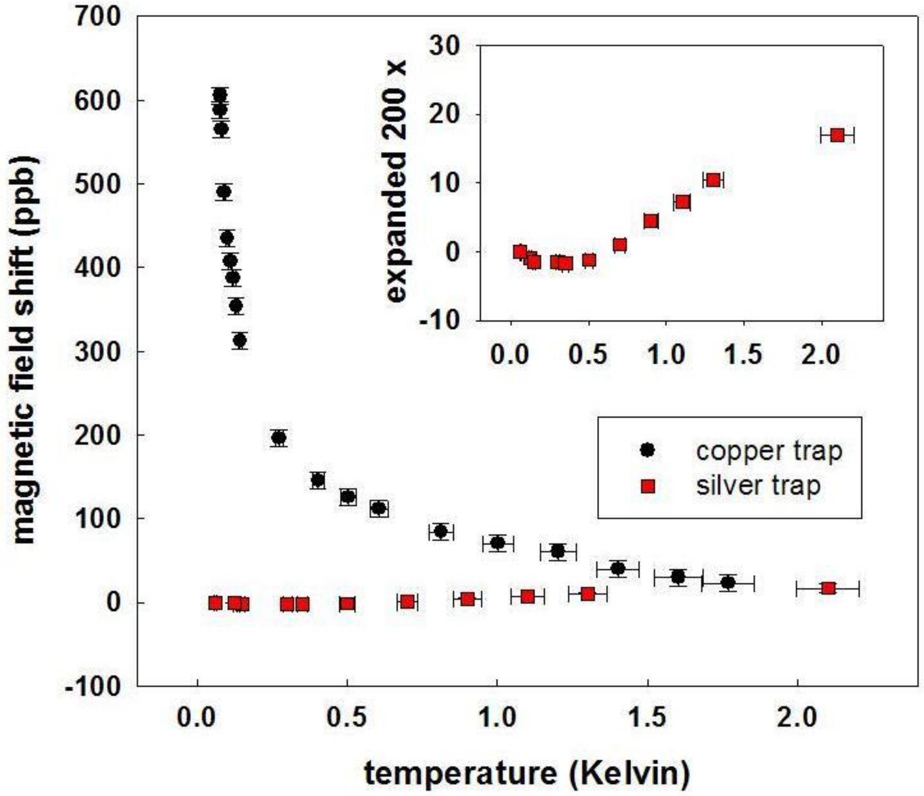
A measurement using a one-electron quantum cyclotron gives the electron magnetic moment in Bohr magnetons, $g/2 = 1.001\,159\,652\,180\,73\,(28)$ [0.28 ppt], with an uncertainty 2.7 and 15 times smaller than for previous measurements in 2006 and 1987. The electron is used as a magnetometer to allow line shape statistics to accumulate, and its spontaneous emission rate determines the correction for its interaction with a cylindrical trap cavity. The new measurement and QED theory determine the fine structure constant, with $\alpha^{-1} = 137.035\,999\,084\,(51)$ [0.37 ppb], and an uncertainty 20 times smaller than for any independent determination of α.

DOI: 10.1103/PhysRevLett.100.120801 PACS numbers: 06.20.Jr, 12.20.Fv, 13.40.Em, 14.60.Cd

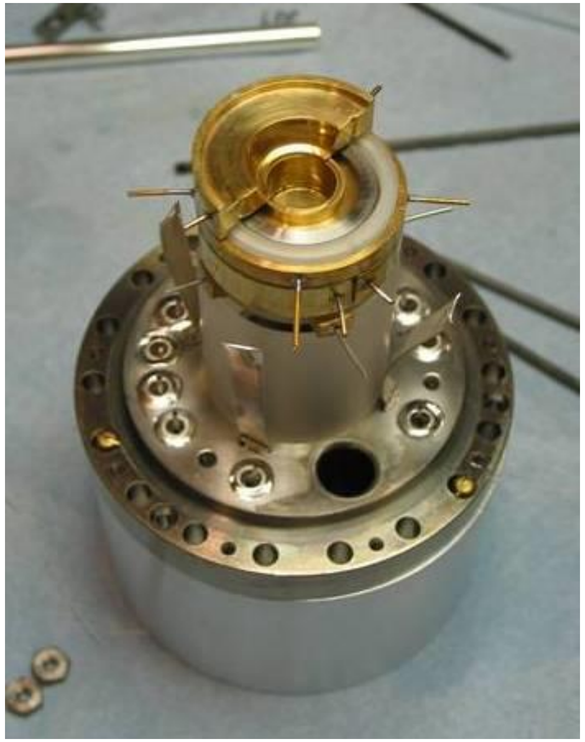
$$g/2 = 1.001\,159\,652\,180\,73\,(28)\ \underline{[0.28\,\mathrm{ppt}]}$$

Nuclear Paramagnetism

Magnetism of copper at lowest temperatures



- Build new trap out of silver
- New vacuum enclosure out of titanium



Electron and Positron

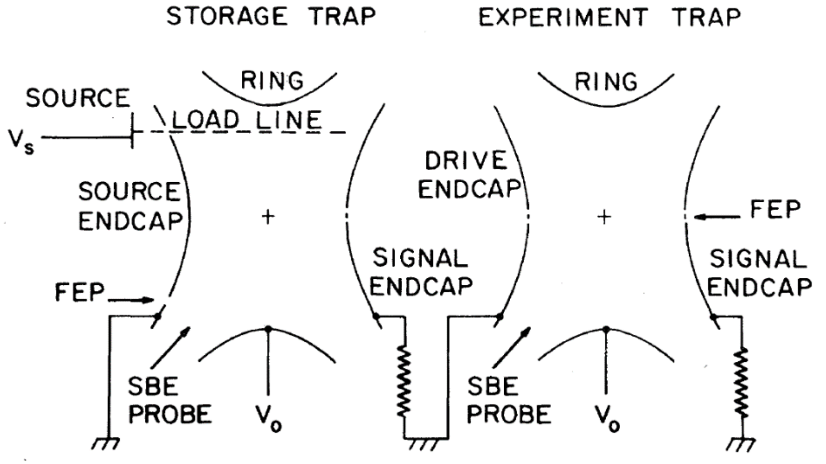
VOLUME 59, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JULY 1987

New High-Precision Comparison of Electron and Positron *g* Factors

Robert S. Van Dyck, Jr., Paul B. Schwinberg, and Hans G. Dehmelt
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 23 March 1987)

Single electrons and positrons have been alternately isolated in the same compensated Penning trap in order to form the geonium pseudoatom under nearly identical conditions. For each, the g-factor anomaly is obtained by measurement of both the spin-cyclotron difference frequency and the cyclotron frequency. A search for systematic effects uncovered a small (but common) residual shift due to the cyclotron excitation field. Extrapolation to zero power yields e^+ and e^- g factors with a smaller statistical error and a new particle-antiparticle comparison: $g(e^-)/g(e^+) = 1 + (0.5 \pm 2.1) \times 10^{-12}$.

PACS numbers: 14.60.Cd, 06.30.Lz, 12.20.Fv, 32.30.Bv



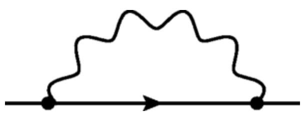
P. B. Schwinberg, R. S. Van Dyck, Jr., and H. G. Dehmelt
Phys. Rev. Lett. 47, 1679 (1981)

- Same method used for positron – currently known to 2 ppt
- Best CPT test for leptons $|E^-_{0,-1} - E^+_{0,1}|/m_0c^2 = |\Delta a|\hbar\omega_c/2m_0c^2 = |3 \pm 12| \times 10^{-22}$
- Redo measurement with positron in improved setup – cavity shift
- Within error bounds no diurnal variations observed

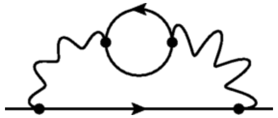
Electron g-Faktor and QED

-Dirac equation gives g =2 without construction

-Background field fluctuates due to minimum energy of harmonic oscillator vacuum states.



Self Energy 1st order



Vacuum polarization

- Effects described by Swinger series

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$
$$+ a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

$$\boxed{a_e(theo) = \frac{g-2}{2} = 0{,}00115965218113\ (84)}$$

C_2	0,5
C4	-0,328478965579
C6	1,181241456587
C8	-1,9144(35)
$a_{\mu,\tau}$	2,720919(3) 10^{-12}
$a_{hadronic}$	1,682(20) 10^{-12}
a_{weak}	0,0297(5) 10^{-12}

5th order 12 672 diagrams calculated

Most precise test of QED

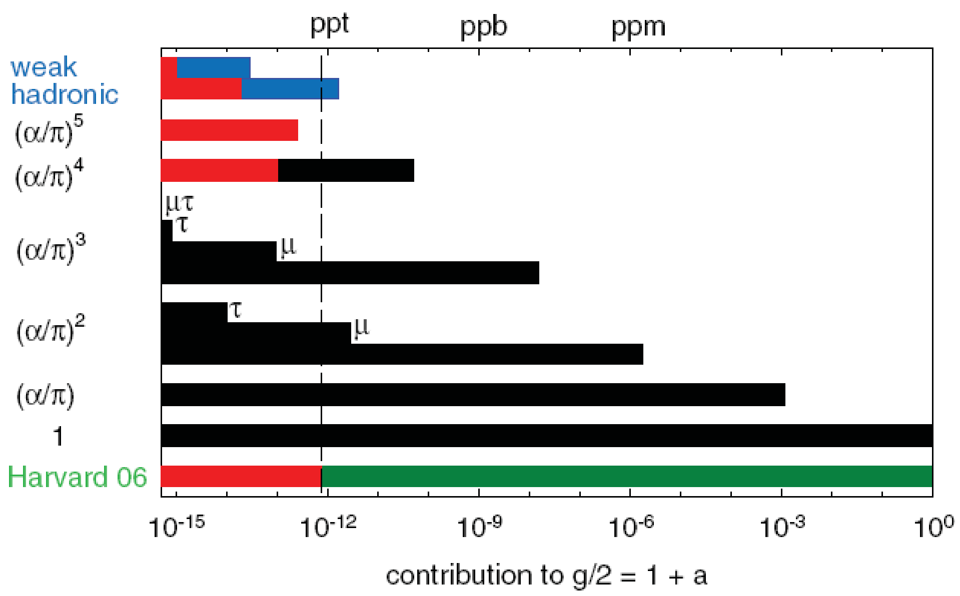


FIG. 2 (color). Contributions to $g/2$ for the experiment (green), terms in the QED series (black), and from short-distance physics (blue). Uncertainties are in red. The μ, τ, and $\mu\tau$ indicate terms dependent on mass ratios m_e/m_μ, m_e/m_τ and the two ratios, m_e/m_μ and m_e/m_τ, respectively.

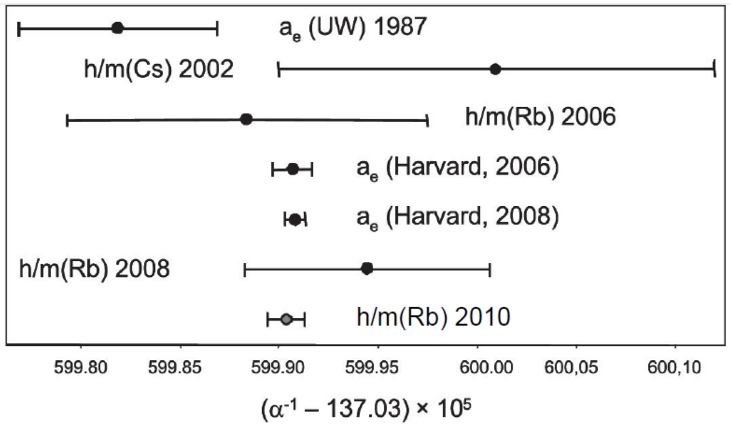
G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom Phys. Rev. Lett. 97, 030802 (2006)

Determination of finestructure constant

- Take measurement and compare to theory to extract finestructure constant

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$
$$+ a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

$$\alpha^{-1} = 137.035\,999\,710\,(90)\,(33)\,[0.66\text{ ppb}][0.24\text{ ppb}],$$
$$= 137.035\,999\,710\,(96)\,[0.70\text{ ppb}].$$

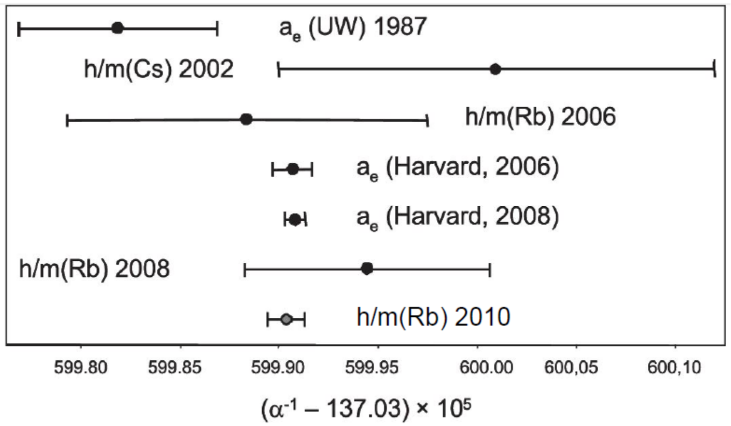


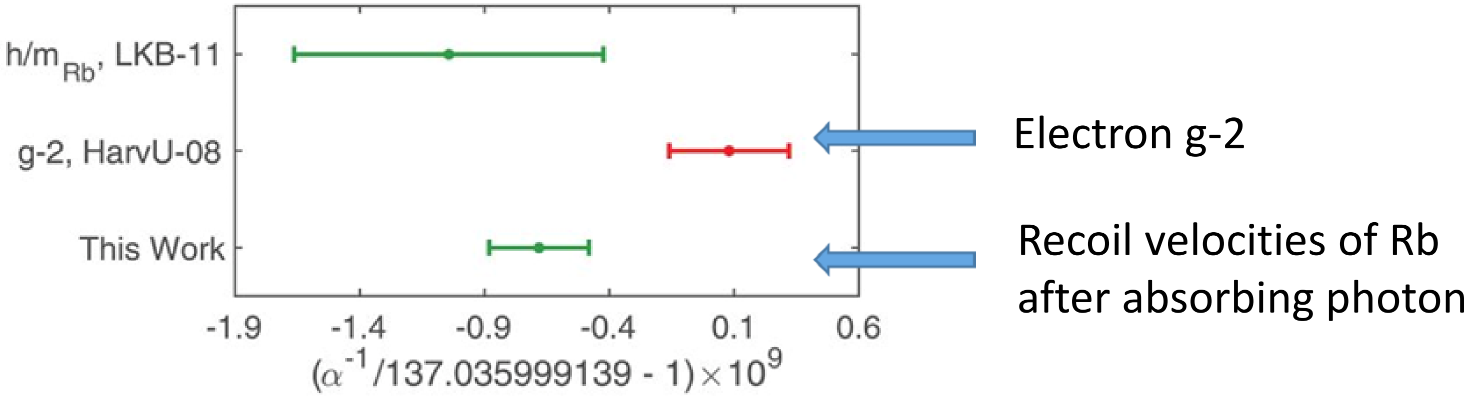
Determination of finestructure constant

- Take measurement and compare to theory to extract finestructure constant

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \ldots$$
$$+ a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

$$\alpha^{-1} = 137.035\,999\,710\,(90)\,(33)\,[0.66\text{ ppb}][0.24\text{ ppb}],$$
$$= 137.035\,999\,710\,(96)\,[0.70\text{ ppb}].$$





Parkeret al.,Science 360, 191–195 (2018)

Electron precisely measured – Why measure the Muon/Antimuon?

- Perturbative contributions to magnetic moment scale with mass

$$g = 2(1 + a_\mu) \qquad a_\mu(QED) \propto \left(\frac{m_\mu}{m_e}\right)^2 a_e(QED)$$

a_e(had) = 1,682(20) 10^{-12}
a_e(weak) = 0,0297(5) 10^{-12}

a_μ(had) = 709.6 (7.) 10^{-10}
a_μ(weak) = 15.4 (0.3) 10^{-10}

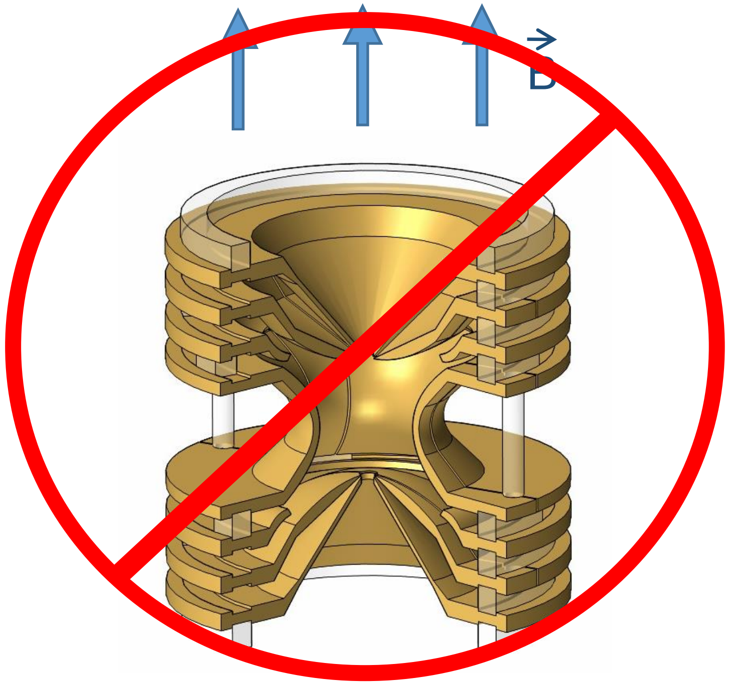
All effects, also beyond SM, are enhanced by a factor of 200^2

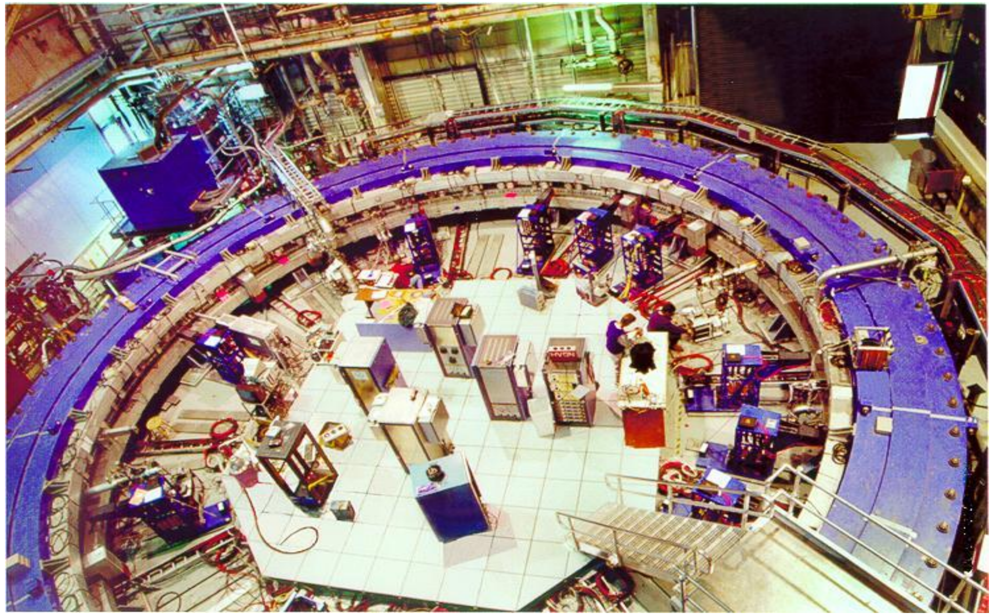
However….

electron lifetime: …..

muon lifetime: $2.20*10^{-6}$ s

tauon lifetime: $2.96*10^{-13}$ s





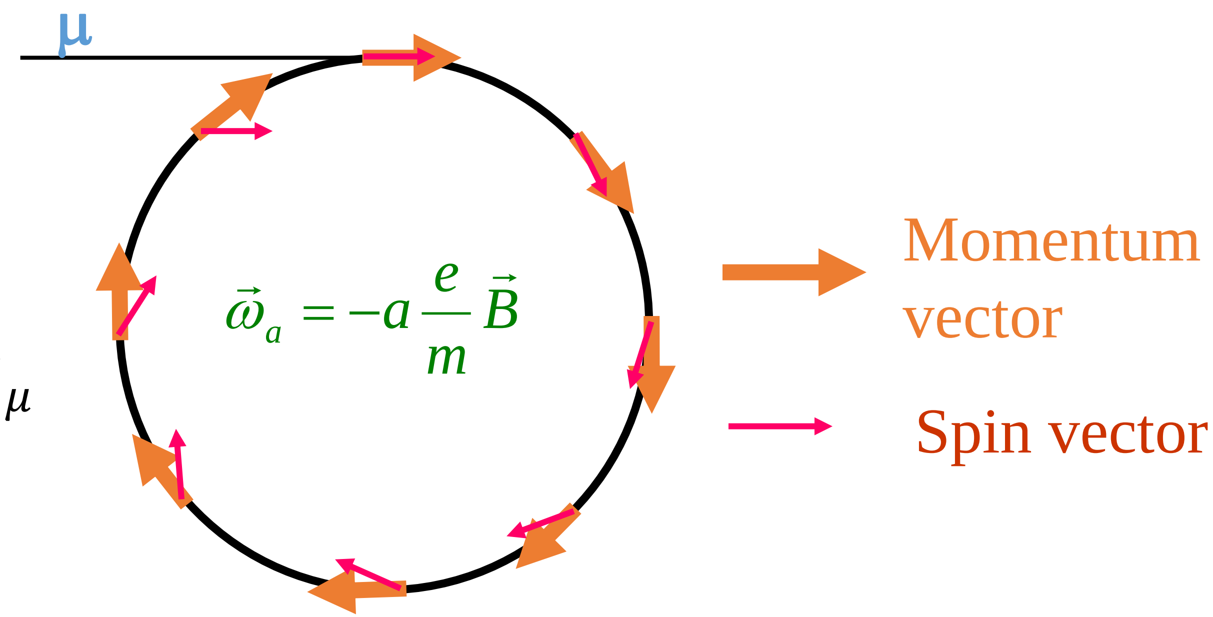
How to measure muon *g*?

- Same principle as for electron

$$\omega_c = \frac{eB}{m}$$

$$\omega_s = \frac{g}{2}\frac{eB}{m}$$

$$\Rightarrow \quad \frac{\omega_s - \omega_c}{\omega_c} = \frac{\omega_a}{\omega_c} = \left(\frac{g-2}{2}\right) = a_\mu$$



However

- Magentic field of storage ring stores only in horizontal plane
- Need vertical focussing to store beam - electrostatic quadrupole fields
- For a relativistic particle this modifies the frequencies

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]$$

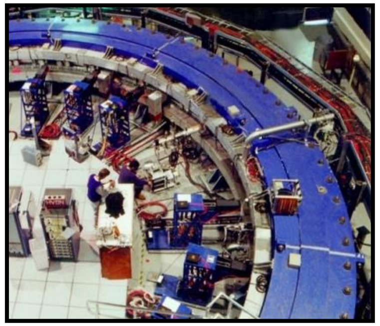
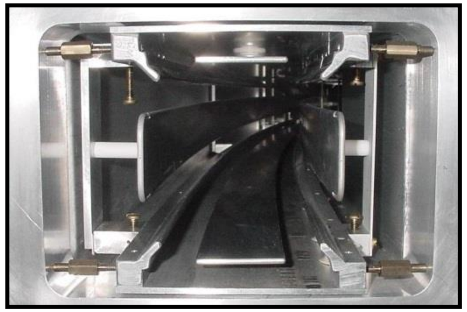
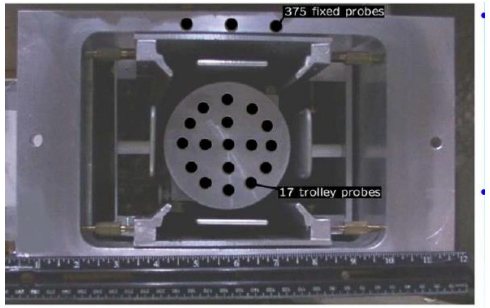
$$\vec{\omega}_c = \frac{e}{mc}\left[\frac{\vec{B}}{\gamma} - \frac{\gamma}{\gamma^2 - 1}\vec{\beta} \times \vec{E}\right]$$

Operate at specific energy „magic gamma“

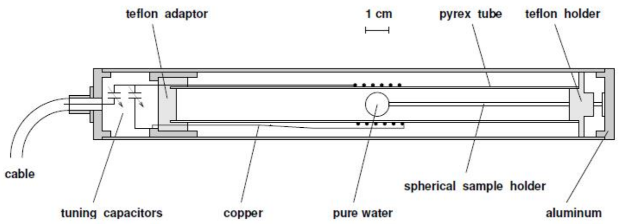
Measure with „external“ B-field sensor

Measurement of Cyclotron Frequency

- Measure magnetic field using array of water NMR probes inside ring

- Relate measured NMR frequencies to absolute standard to determine B-field



(a) Absolute calibration probe

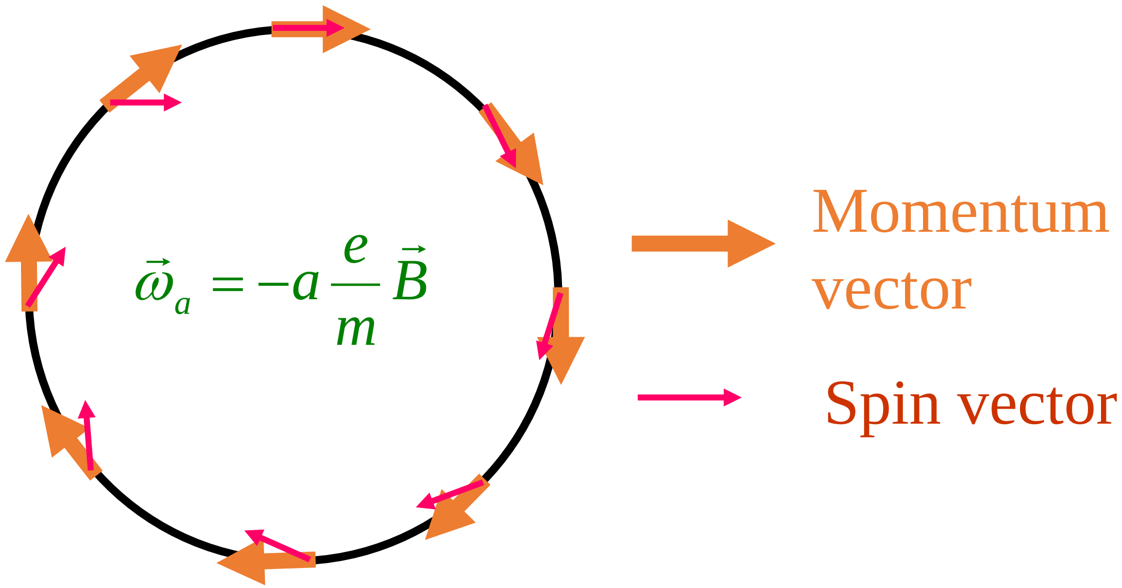
(b) Spherical Pyrex container

$$\omega_{\text{probe}} = (1-\delta_t)\omega_p, \text{ where}$$
$$\delta_t = \sigma(\mathrm{H_2O},\mathrm{T}) + \delta_b + \delta_p + \delta_s.$$

- s : diamegnetic shielding
- $\delta_{b=}$: bulk suscebility (T-dependent)
- $\delta_{p=}$: paramagnetic inpurities in water
- $\delta_{s=}$: para- and diamagnetism of probe

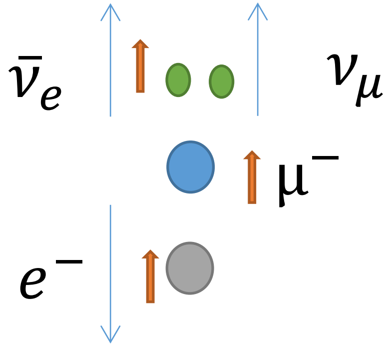
Muon (g-2) Technical Design Report, arXiv:1501.06858

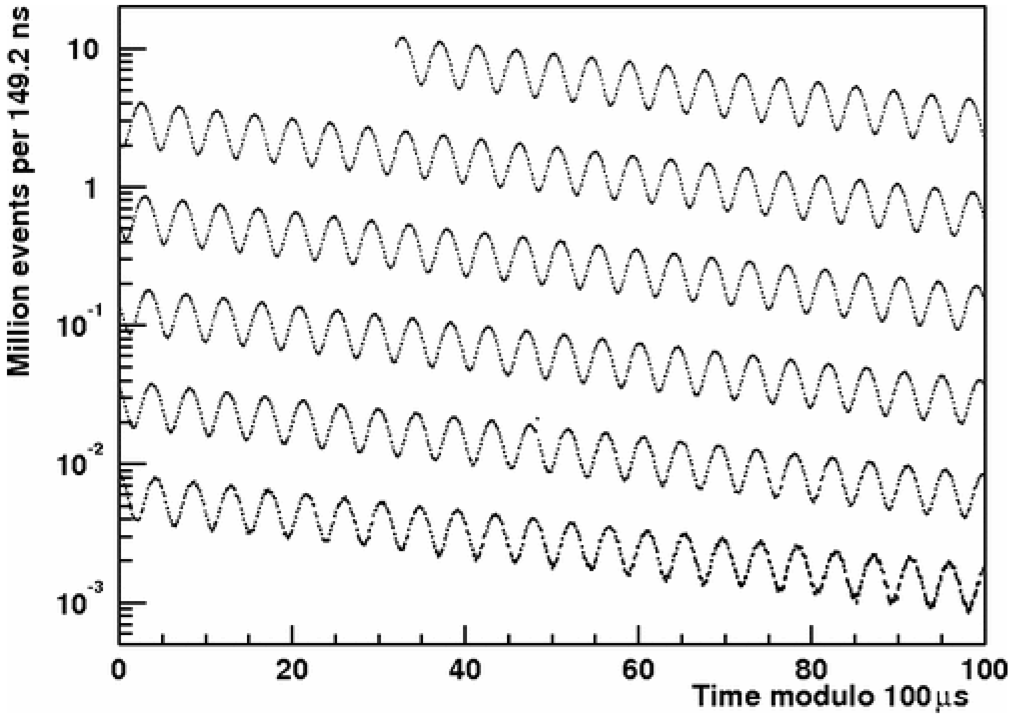
Measurement of Anomalie Frequency



$$N(t) = N_0 e^{-\frac{t}{\gamma\tau}}\left[1 + A\cos(\omega_a t + \phi)\right]$$

In parity violating muon decay the high energy *positron is preferentially emitted against the muon spin direction*

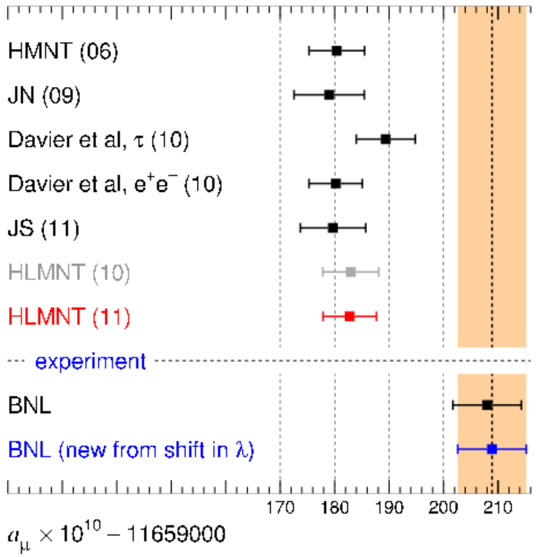




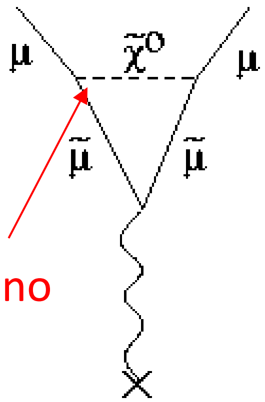
G. W. Bennett *et al.* (Muon g-2 Collaboration) Phys. Rev. D 73, 072003 (2006)

Result

- a_μ measured with precision of 0.5 ppb
- Muon/antimuon are found to agree within ppb
- But 3.6 Sigma discrepancy observed to theory



Ideas for interpretation

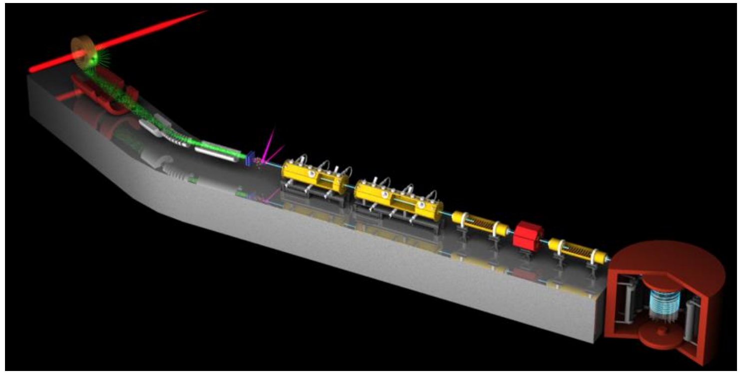


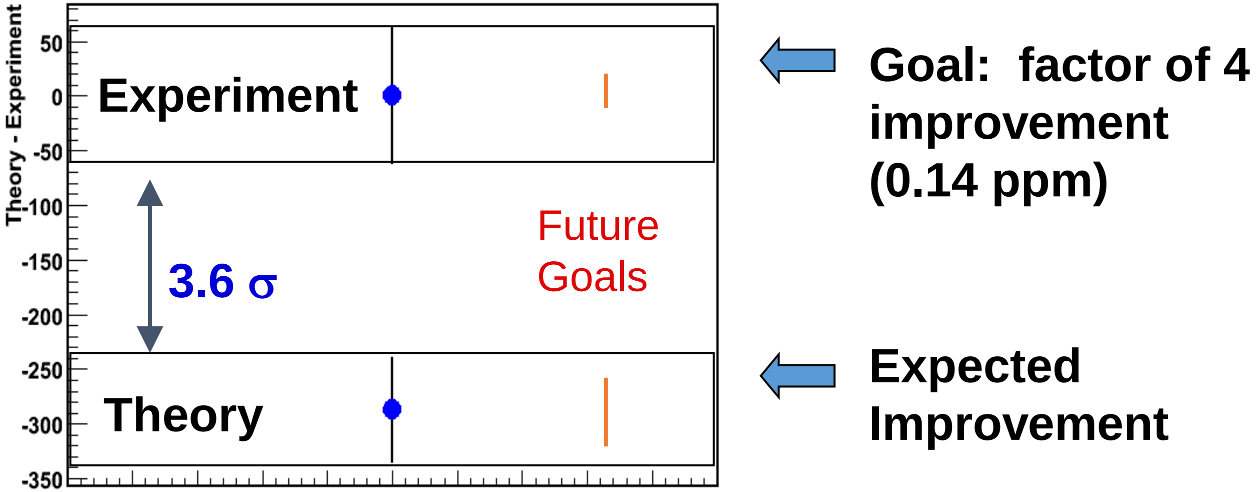
- Superymmetry – every SM particle has partner with same QM numbers exept spin that differs by 1/2
- 5^{th} force mediated by new massive gauge boson (yukawa interaction)

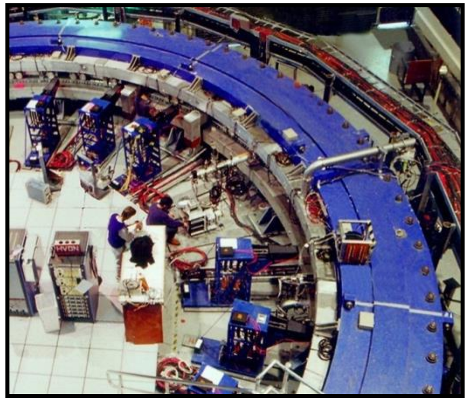
Discrepancy not significant

G. Venanzoni / Nuclear and Particle Physics Proceedings 273–275 (2016) 584–588

Improved measurement palnned at Fermilab and J-PARC







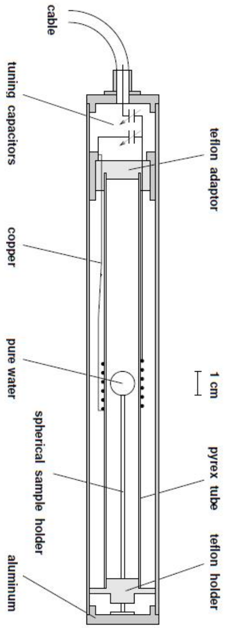
- Higher statistics- precision in anomaly frequency – higher intensity muon beam
- More and improved magnetic field sensors
- Improved accuracy for magnetic field measurement – idea to use ^{3}He as additional probe

Magnetometry

- NMR probes are the standard for absolute B-field measurements
  - Relates magnetic field via fundamental constant to frequency

$$\begin{aligned} \omega_{\text{probe}} &= (1-\delta_t)\omega_p, \ \text{where} \\ \delta_t &= \sigma(\mathrm{H_2O}, \mathrm{T}) + \delta_b + \delta_p + \delta_s . \end{aligned}$$

- **s** : diamegnetic shielding
- $\boldsymbol{\delta}_{b=}$: bulk suscebility (T-dependent)
- $\boldsymbol{\delta}_{p=}$: paramagnetic inpurities in water
- $\boldsymbol{\delta}_{s=}$: para- and diamagnetism of probe



(a) Absolute calibration probe

(b) Spherical Pyrex container

- Challenging systematic effects:
  - Diamagnetic shielding
  - Shape factor (bulk diamagnetism) – spherical probe
  - Susceptibility of probe material
  - Paramagnetic impurities in probe sample
  - Magnetic materials of probe structure
  - Gas in probe samples

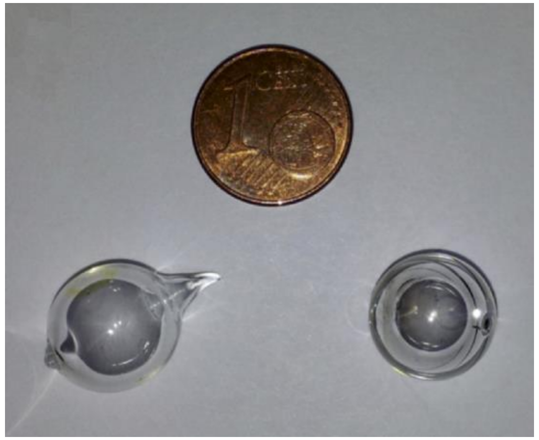
Magnetometry

- Hyper-polarized ^{3}He NMR probes with very different and in cases smaller systematic effects

	Water NMR		^{3}He
Dependence on temperature	1	>	1/100
Dependence on probe shape	1	>	negligible
Susceptibility	1	>	1/1000 (low pressure)
Diamagnetic shielding	1 measured	>	1/10 calculated

- $\Delta B/B = 10^{-12}$ in seconds using hyperpolarized ^{3}He in a 1.5T B-field (13.6mrad after 6.6s @ 48.6MHz)

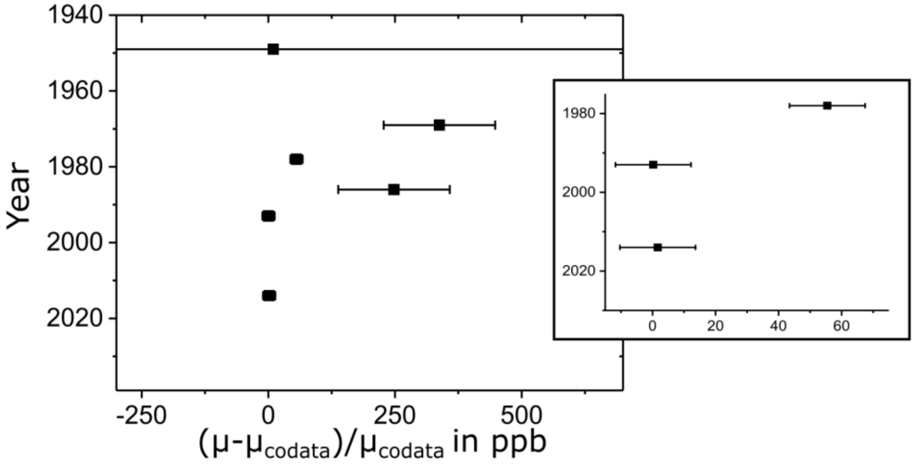
Nikiel A., *et al.* Eur. Phys. J. D **68** 330 (2014)



Nuclear magnetic moment of ^3He

<u>However:</u> μ_{He} only determined indirectly, comparison of NMR probes in same magnetic field

All indirect



Up to now single ^3He to H_2O comparison determines μ_{He} to $1.2*10^{-8}$ only

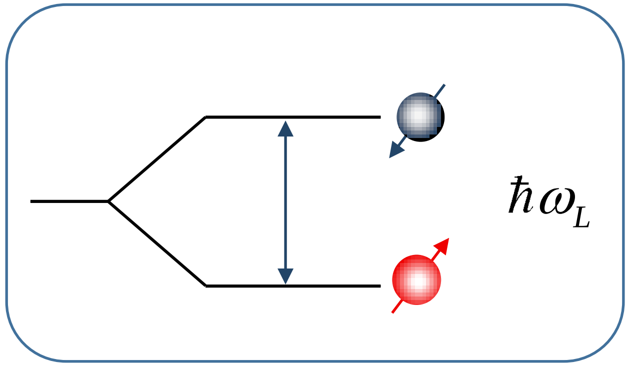
- limited by knowledge of shielded proton magnetic moment

- ^3He probes provide no absolute probe independent of proton NMR probes

  Measurements of same B-field using ^3He and H_2O cannot be considered independent/uncorrelated

^{3}He Magnetic Moment in Penning Trap

Determination of energy splitting between spin-states

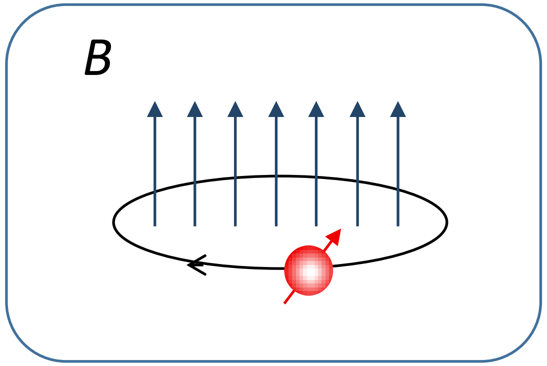


$$\omega_L = 2\frac{\mu_{He}}{\hbar}B$$

$^3He^{2+}$

$$\frac{\frac{\mu_{He}}{e\hbar}}{m_{He}} = \frac{\omega_L}{\omega_c}$$

Simultaneous cyclotron frequency measurement

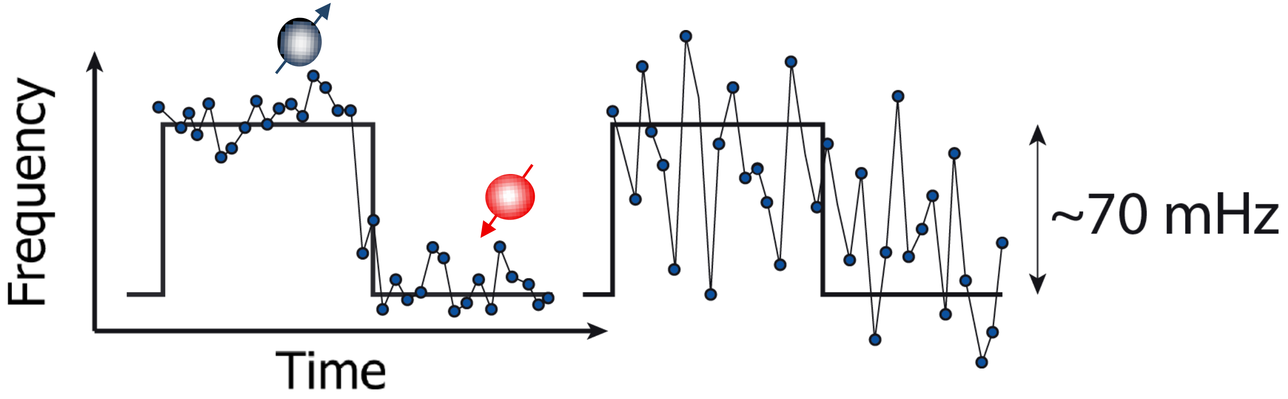


$$\omega_c = \frac{2\,e}{m_{He}}B$$

To determine g-factor of ^{3}He - proton-helion mass ratio needed (known to 30ppt)

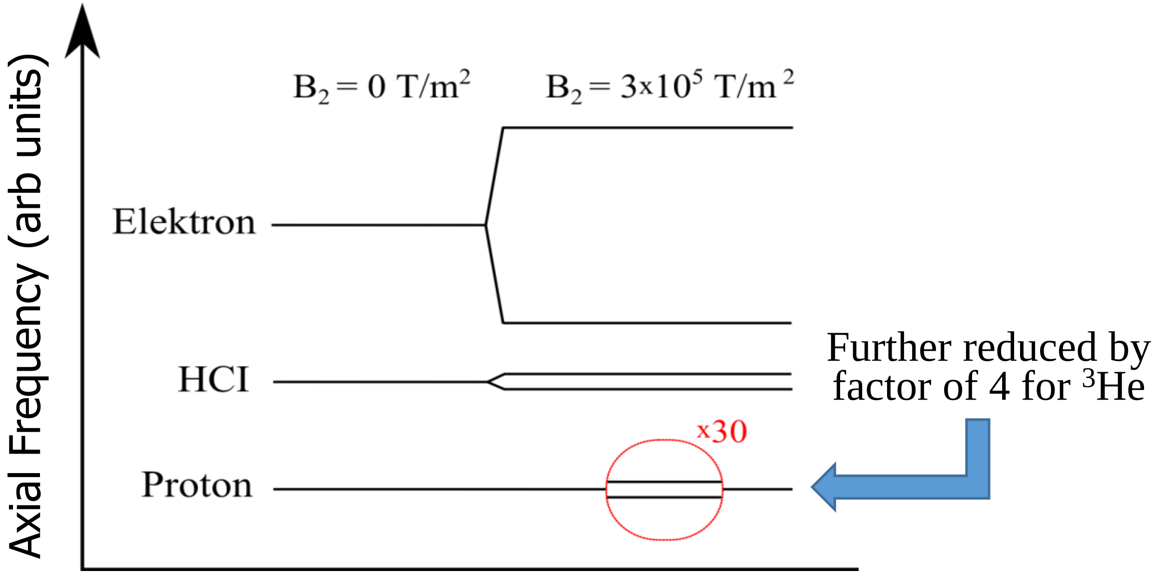
Challenge of Spin-State Detection

Noise on electrodes of some pV/Hz$^{1/2}$ drives random cyclotron quantum transitions.



low energy ~ µeV

high energy ~ meV



Signal Spin Flip:

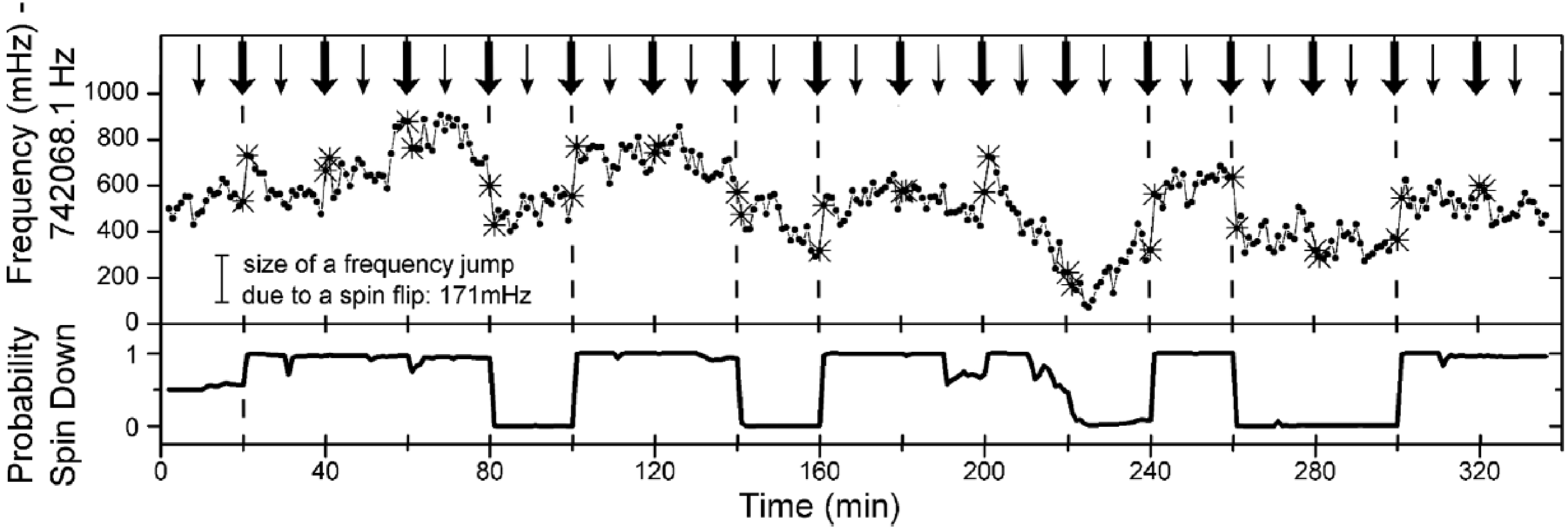
$$\Delta \nu_z \propto \frac{\mu}{m} B_2$$

Noise Spin Flip:

$$\Delta \nu_z \propto q^2 \Delta n_+$$

<u>Low energies particularly important for $^3He^{2+}$</u>

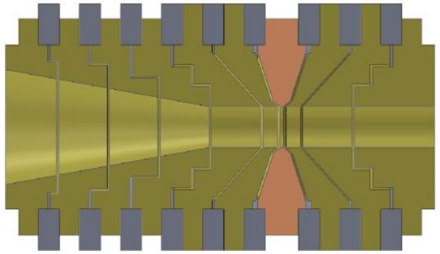
Challenge of Spin-State Detection



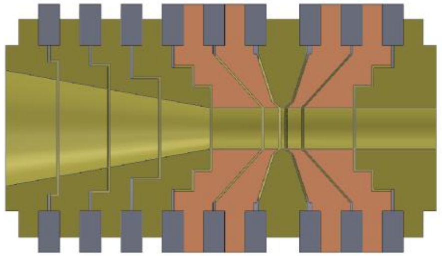
Obtain frequency jump due to spin-transition of $\Delta\nu_z = 70\ mHz \rightarrow \Delta\nu_z/\nu_z = 10^{-7}$

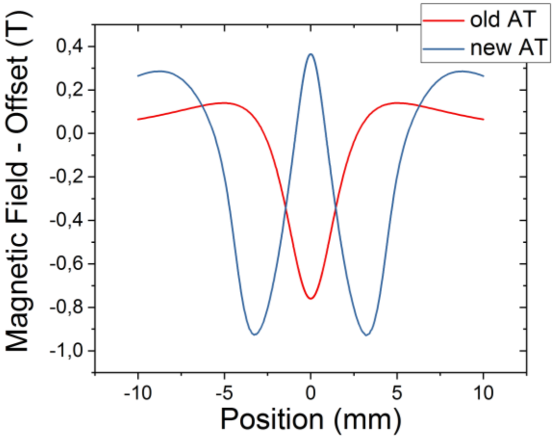
New Developments

old AT



new AT

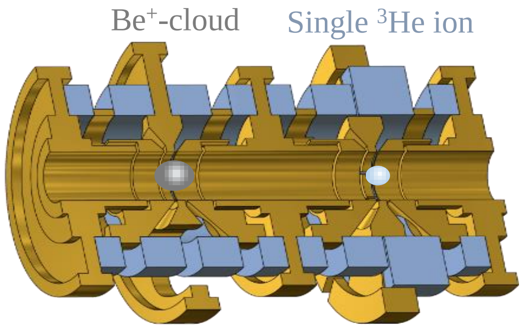


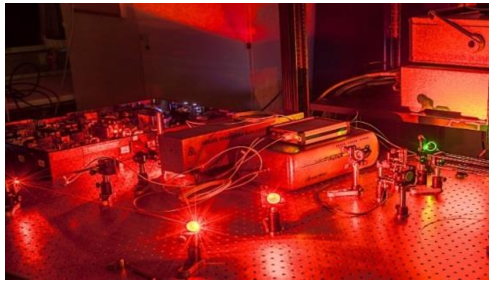


- Larger inhomogeneity
  - 300 T/mm² -> 600 T/mm²

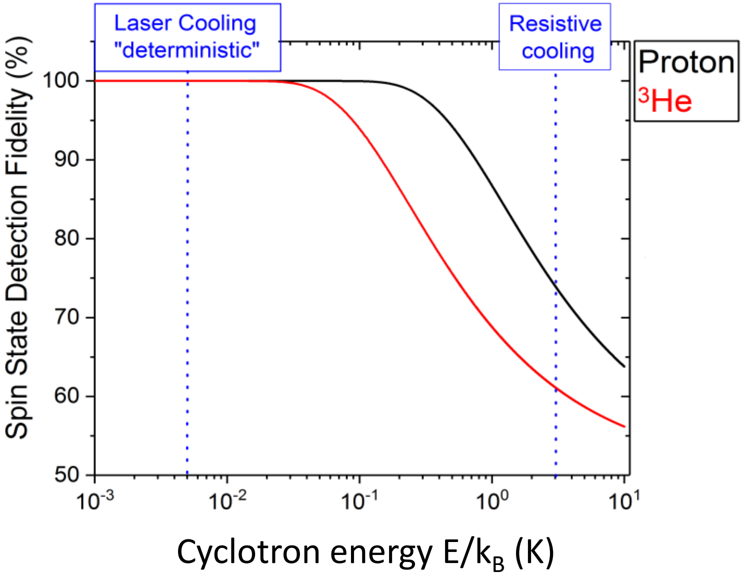
$$\Delta\nu_z \sim \frac{\mu_k B_2}{m\nu_z}$$

- Larger magnetic field
  - Larger energy spacing between cyclotron quantum states
  - Lower rate for random cyclotron quantum transitions



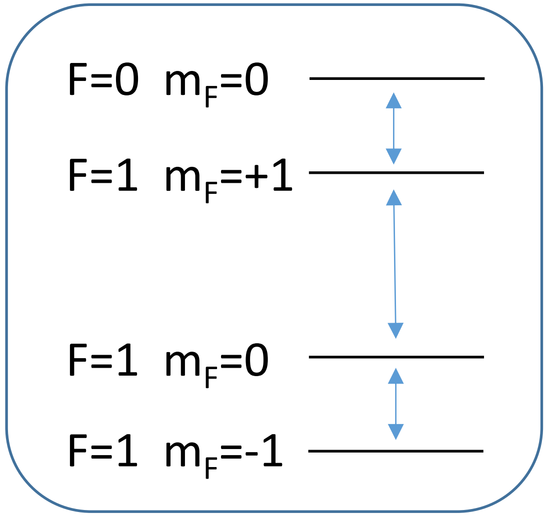


$$\tau_{exc} = 2\,\pi\,\omega\,C_T \frac{\sqrt{m_p m_{Be}}}{q^2} D_{eff} \frac{1}{\sqrt{N_{Be}}}$$



Magnetic Moments in Penning Traps

Determination of energy splitting between spin-states



$$\omega_{F=I\pm\frac{1}{2}}\,(g_I, g_j\,, E_F, B)$$

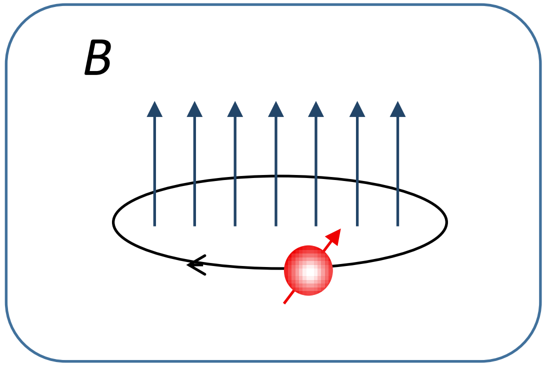
$^3He^+$

$1^2S_{1/2}$

$I=1/2$

$\mu_I<0$

Simultaneous cyclotron frequency measurement

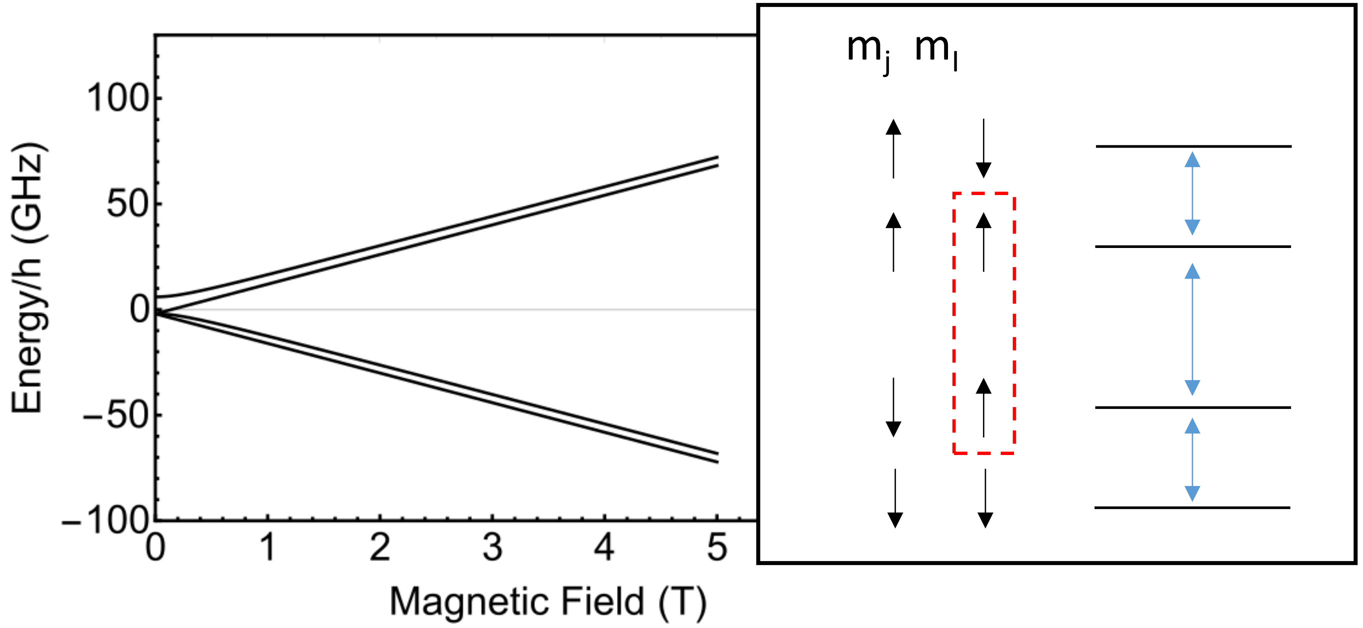


$$\omega_c = \frac{e}{m_{He}}B$$

B-field independent measurement of bound g_I and g_j factors as well as E_F

Spin-State Detection $^3He^+$

Map readout of the nuclear spin-state onto detection of electronic spin-transition



Detect electron spin-transition using cont. Stern-Gerlach effect

$\Delta\nu_z \approx 100Hz$, much easier to detect

Design of new Experiment

Electronics

Alignment

Pre-Vacuum / Cryogenic valve

Penning Trap

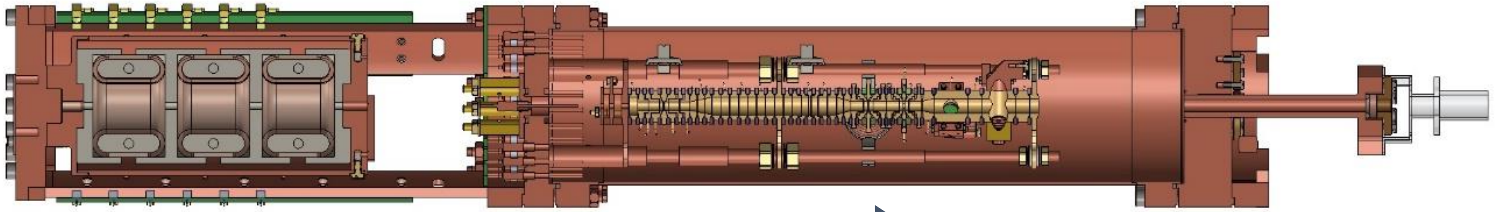
Detection

Ultra-High Vaccum Interface

- Microwave access
- Laser access
- Signal Lines

Laser ports

- 313nm fiber coupled
- 2 x ablation laser free running



Detection Systems

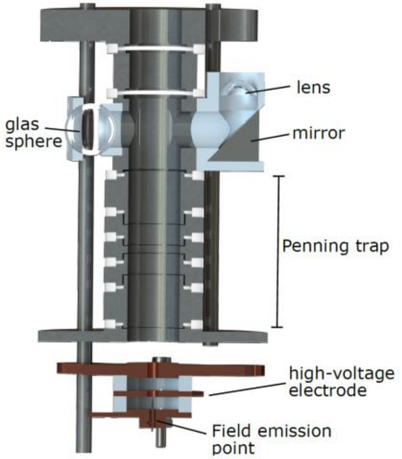
- Superconducting toroids
- Low-noise cryogenic amplifiers

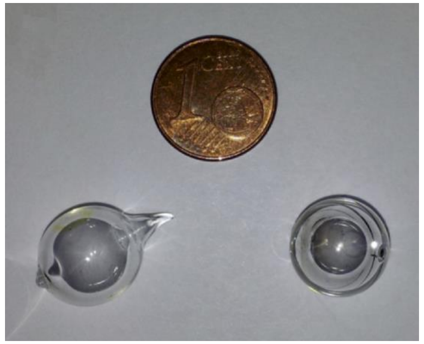
Penning trap

- Four individual Penning traps
- In trap production of ions

First Results

Dedicated Test-Penning trap to investigate ^{3}He source

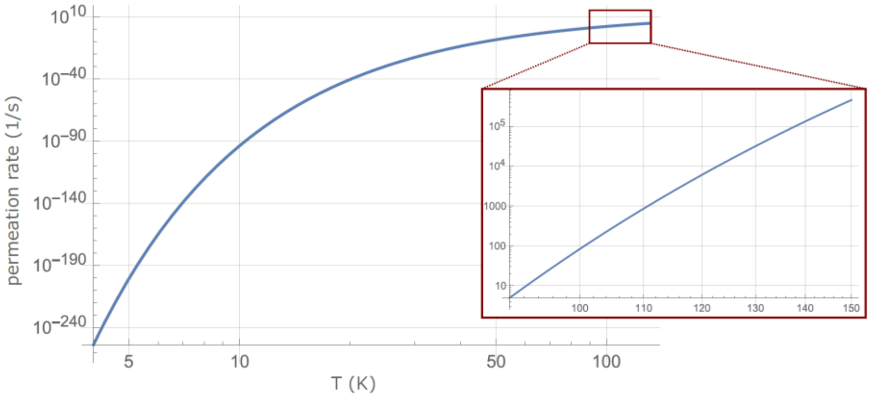


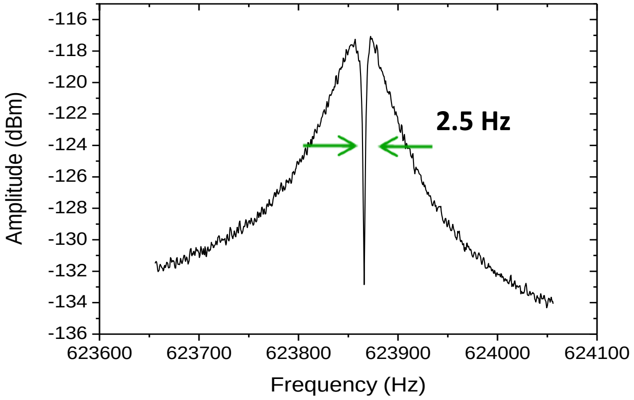


W. Heil – University of Mainz

Heating of ^{3}He filled glas sphere using resistor and/or laser

Utilizes strongly temperature dependent helium permeation through glass





First Signal of Helium-3:

Conclusion

- Magnetic Moments:
  - Test of fundamental physical laws
  - Searches for physics beyond the SM
  - Determination of fundamental constants
- Electron/Positron:
  - Deviations with respect to SM predictions – further measurements
  - Most precise test of CPT invariance with leptons – in agreement
- Muon/Antimuon
  - Deviations with respect to SM predictions – new experiments planned/running
- Helium-3:
  - Standard for absolute magnetometry
  - Test of bound-state QED in nuclear-spin dependent system
  - Investigation of nuclear structure effects