Beam Dynamics in MBA Lattices with Different Chromaticity Correction Schemes

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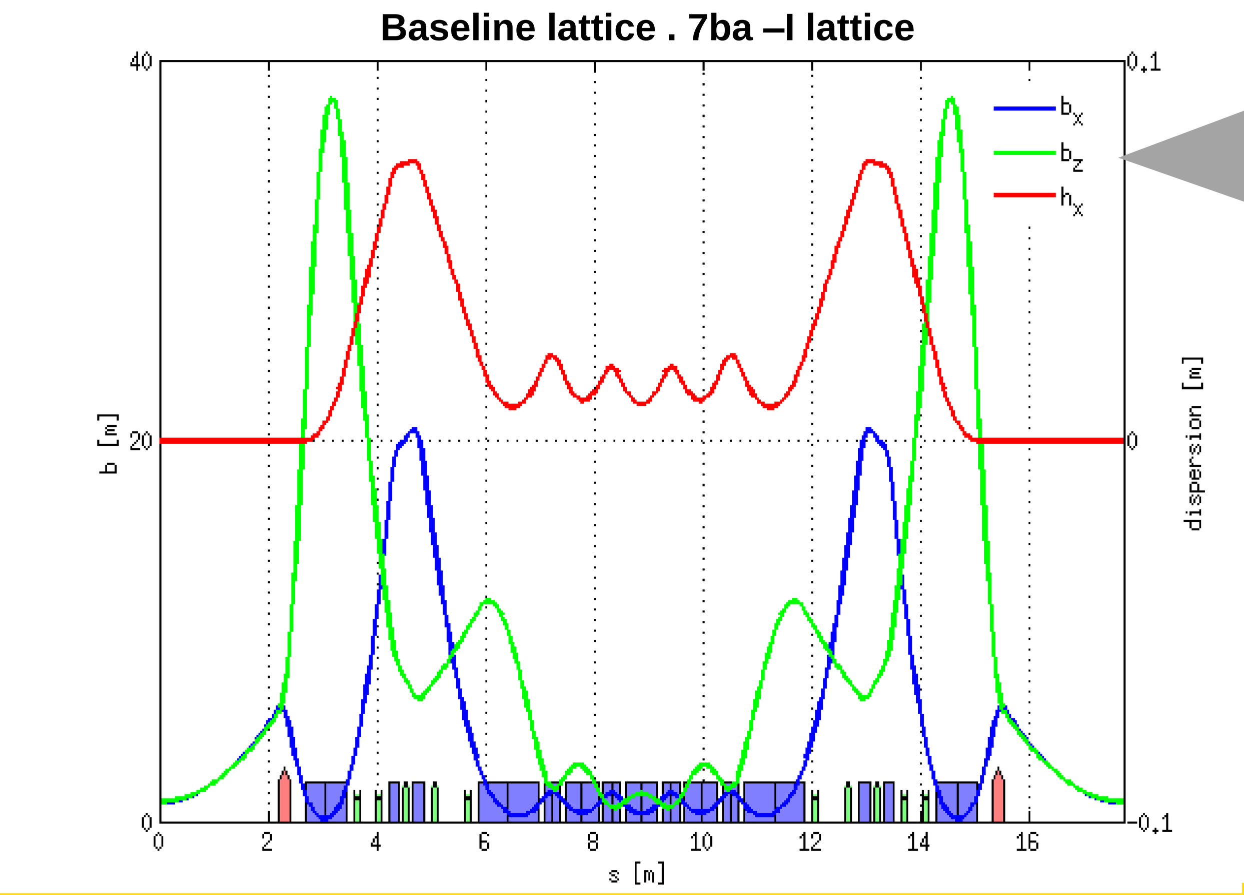
Abstract

Ultra-low emittance lattices are being studied for the future upgrade of the 2.75 GeV SOLEIL storage ring. A first lattice was inspired by the ESRF-EBS-type Multi-Bend-Achromat (MBA) lattice, introducing a –I transformation to compensate the nonlinear impact of sextupoles, with a tight control of the betatron phase advance between them. Though it provides a large on-momentum transverse dynamic aperture in 4D, its off-momentum performance is rather limited. 6D studies reveal intrinsic off-momentum transverse oscillations which are likely to result from an inhomogeneous sextupole distribution in the –I scheme. While an earlier study attempted to employ first-order canonical perturbation theory to describe this phenomenon, a numerical minimisation process of the concerned pathlength is presented in the present contribution.

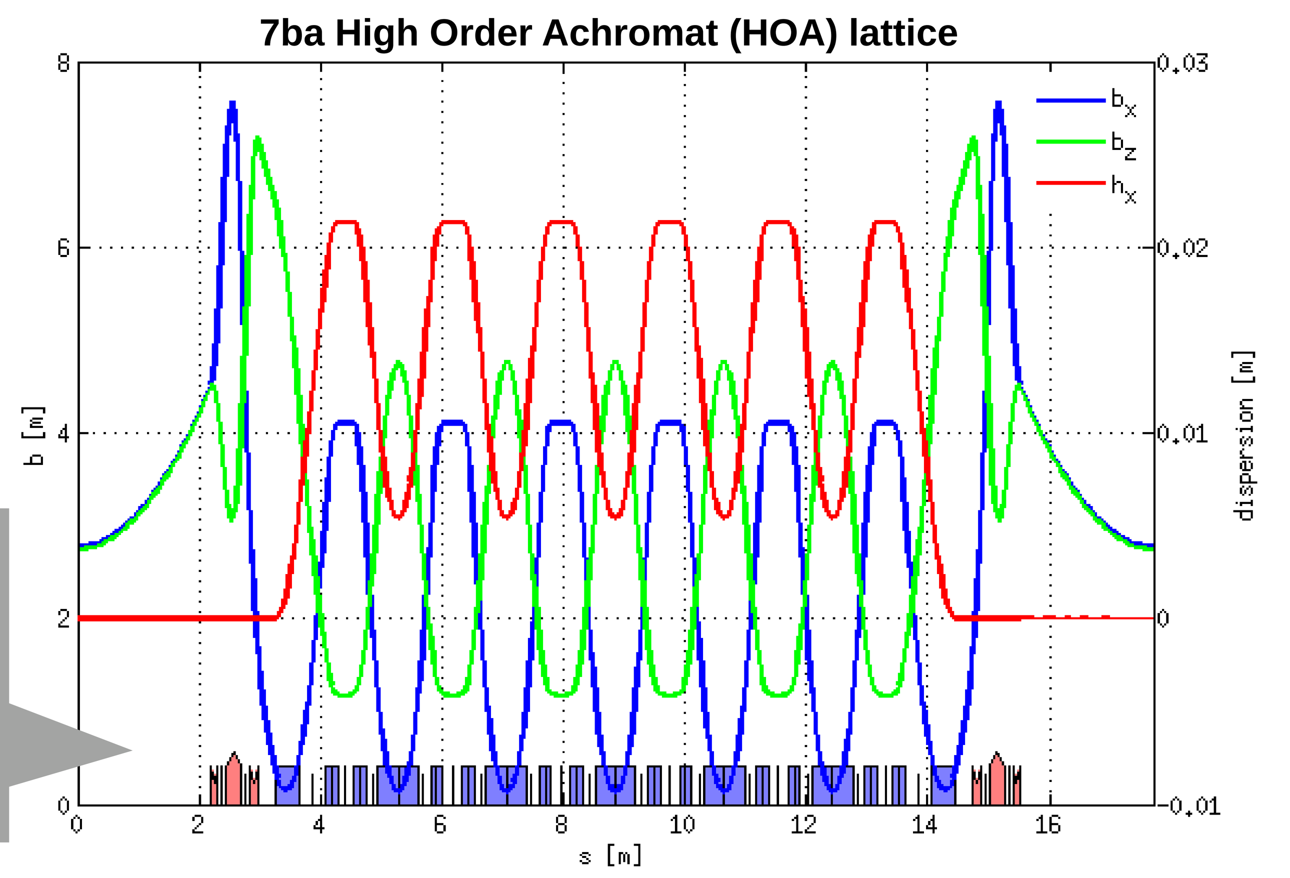


The hybrid lattice [1] uses the *-I* transformation, setting a $((2k+1)\pi, n\pi)$ phase advance between two sextupoles (with $k, n \in \mathbb{N}$) for:

- kick cancellation
- optimised on-momentum acceptance

+ dispersion bumps at the location of the sextupoles for:

- increased efficiency
- global chromaticity correction.



The so-called HOA lattice [2] is built in several identical small cells, where phase advance is fixed in both planes to cancel geometric resonances over each cell. ensure the cancellation of resonances up to the third order. Each cell owns its pair of sextupoles for local chromaticity correction.

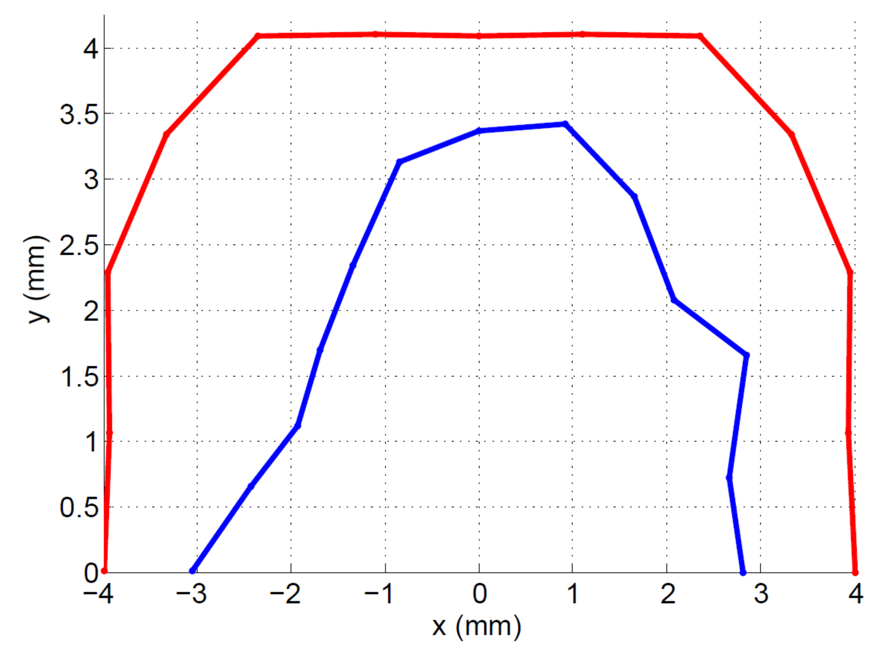
PATH LENGTHENING EFFECT AND MINIMISATION PROCESSES

PATH LENGTHENING EFFECT

A strong coupling between the longitudinal and the transverse planes in the *–I* lattice makes a particle go off-energy each turn - by increased path length, and falls out of the off-momentum acceptance, reducing the transverse dynamic aperture (see below).

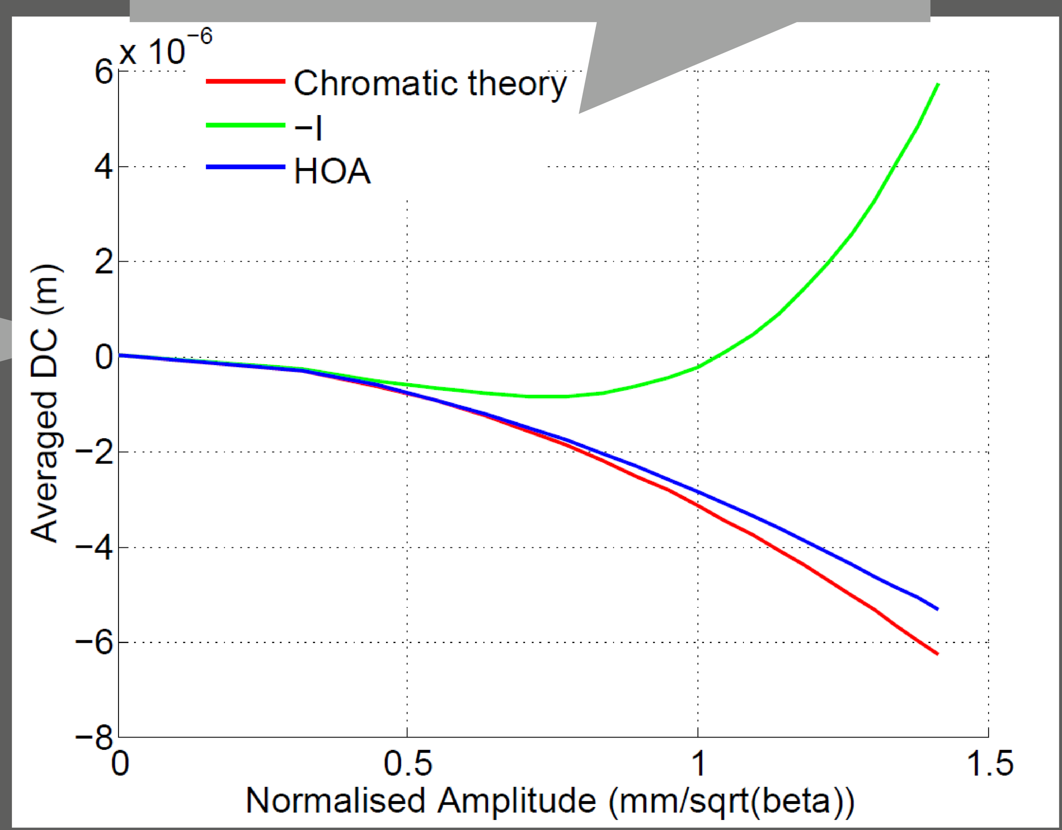
The path lengthening due to large amplitude betatron motions is studied in both lattices. The concerned effect depends on the chromaticity (ξ_x, ξ_y) in both planes :

$$\Delta C = -2\pi(J_x \xi_x + J_y \xi_y)$$

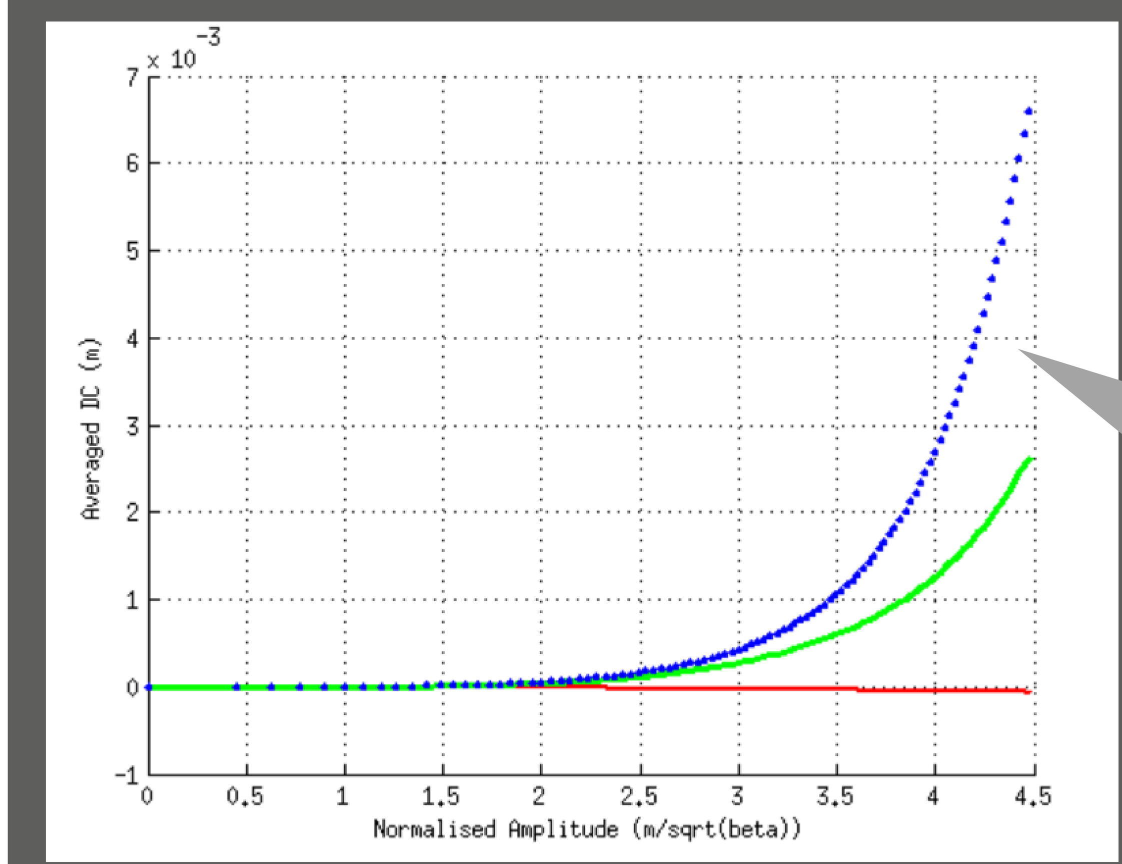


Transverse on-momentum dynamic aperture of a *-I* cell, with natural chromaticities. In red, the lattice without RF, in blue, RF system added, with a voltage of 1.1 MV.

The path length of the *–I* lattice (green) follows a rule different from the known chromatic dependence (red). The effect is less prominent in the HOA lattice (blue).



Path length as a function of amplitude after 1 turn, averaged on the input phase, for both *–I* and HOA lattices, at a corrected chromaticity of (1,1).



Averaged path lengthening considering the three described methods -- linear formula of ΔC in red, 6D tracking of the *-I* lattice in green and perturbed path length using the minimisation script – first step, in blue dots.

FIRST-ORDER PERTURBATION THEORY

Using the first-order perturbation theory described by M. Takao in [3-4], the distortion of the averaged trajectory coordinates (x, x') can be derived according to dipolar and sextupolar gradients. The perturbed Hamiltonian considered is :

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}(k_x^2 + g_0)x^2 - \frac{1}{2}g_0 y^2 + \frac{g_1}{3!}(x^3 - 3xy^2) + \frac{1}{2}k_x x(p_x^2 + p_y^2),$$

Distorted averaged trajectory coordinates are :

$$\begin{aligned}\langle x(s)\rangle_{\phi_x} = &-\frac{J_x\sqrt{\beta_x}}{4\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\,(g_1\beta_x + k_x\gamma_x)\cos(\overline{\psi}(s',s))\\ &-\frac{J_y\sqrt{\beta_x}}{4\sin(\pi\nu_x)}\int_s^{s+C} ds'\sqrt{\beta_x}\,(-g_1\beta_y + k_x\gamma_y)\cos(\overline{\psi}(s',s))\\ &-\frac{J_x\sqrt{\beta_x}}{2\sin(\pi\nu_x)}\int_s^{s+C} ds' k_x\beta_x^{-1/2}\alpha_x\left(\sin(\overline{\psi}(s',s)) + \alpha_x\cos(\overline{\psi}(s',s))\right)\\ &+\frac{J_x\sqrt{\beta_x}}{4\sin(\pi\nu_x)}\int_s^{s+C} ds' k_x\beta_x^{-1/2}\cos(\overline{\psi}(s',s))\end{aligned}$$

Higher-order terms will be required to describe the tracked path length of the low emittance lattices under study, as the first-order theory remains close to the general chromatic theory. The second order in perturbation, neglected in this contribution for the sake of simplicity, appears to be responsible for the path length effect in the low emittance lattices.

NUMERICAL MINIMISATION

Using the sextupoles already present in the lattice, a minimisation script has been written in Accelerator Toolbox (Matlab based). A first round of calculations proved the inherent pathlength could be modified, yet larger and destroying the on-momentum properties of the hybrid lattice.

A second round of minimisation will try to conserve the on-momentum dynamic aperture by only using secondary sextupoles, which should have less effects on the –I transformation.

PERSPECTIVES

Higher-order theory, if obtained, will be used to reduce or even cancel the path lengthening effect observed in the *-I* lattice. This optimisation may be done, either analytically or numerically using sextupoles, under the constraint of correcting the chromaticity, in order to restore its on-momentum performance.

[1] A. Loulergue *et al.*, "Baseline Lattice for the Upgrade of SOLEIL", in *Proc. IPAC'18*, Vancouver, Canada, Apr.-May 2018.
[2] R. Nagaoka *et al.*, "Study of Higher-Order Achromat (HOA) Lattice as an Alternative Option for the SOLEIL Storage Ring Upgrade", presented at the 10th International Particle Accelerator Conf. (IPAC'19), Melbourne, Australia, May 2019, paper MOPRB005, this conference.
[3] M. Takao, "Impact of Betatron Motion on Path Lengthening and Momentum Aperture in a Storage Ring", in *Proc. EPAC'08*, Genoa, Italy.
[4] M. Takao, Phys. Rev. E 72 (2005), 046502.

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