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Universality breaking effects in e^+e^- hadroproduction

Correlations in Partonic and Hadronic Interactions - 2020 (CPHI-2020)

CSS Factorization: general features

1. Process with HARD SCALE Q

2. Find and weigh
   IR singular contributions

   - Landau equations
   - Power Counting

3. **Sum over**
   all the regions and all the graphs

   Validity to **every order** in perturbation theory

4. Use Gauge Invariance (Ward Identities)

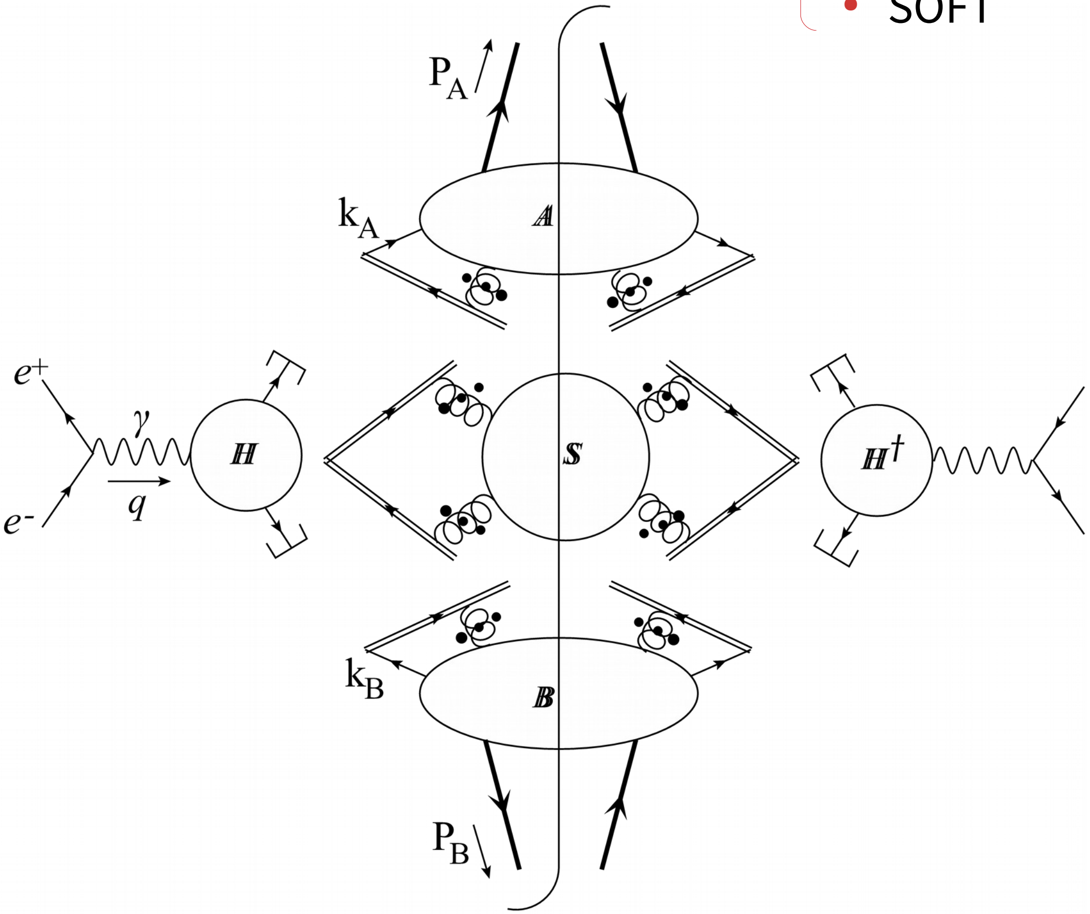
5. **FACTORIZATION** of
   - HARD
   - COLLINEAR(S)
   - SOFT

   contributions.

CSS Factorization: e^+e^- to 2 back-to-back hadrons

1. HARD SCALE Q is the CM energy

5. **FACTORIZATION** of
   - HARD
   - COLLINEAR-to-A and COLLINEAR-to-B contributions.
   - SOFT



KINEMATICS:
In *CM-frame* the two detected hadrons are (almost) **back-to-back**

In the *hadron-frame* the transverse momentum q_{hT} of the photon is small compared to Q:

$$\frac{q_{h,T}}{Q} << 1$$

CSS Factorization: e^+e^- to 2 back-to-back hadrons

The hadronic part of the cross section, differential in q_{hT}, is given by:

Hard Part
specific of the process

$$W^{\mu\nu}(p_A, p_B, q) = \frac{8\pi^3 z_A z_B}{Q} \sum_f \boxed{\mathrm{Tr}_D \left[k^+_{A,\gamma} \gamma^- H^\nu_f(Q)\, k^-_{B,\gamma} \gamma^+ \overline{H}^\mu_f(Q) \right]}$$

$$\int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{-i\vec{q}_{h,T}\cdot\vec{b}_T}\, \boxed{\widetilde{\mathbb{S}}(b_T)}\, \boxed{\widetilde{D}_{1,h_A/f}(z_A, b_T)\, \widetilde{D}_{1,h_B/\overline{f}}(z_B, b_T)} + \text{polarized terms}$$

[John Collins. *Foundations of perturbative QCD*, 2011, Cambridge University Press.]

Unpolarized TMD FFs
relative to the hadrons A and B

Soft Factor

- Contains non-perturbative effects → NOT COMPUTABLE
- Always appears multiplied by collinear objects → NOT EXTRACTABLE FROM DATA

Classification of processes

Each collinear part is always associated to a hadron

- final state → • FFs
- initial state → • PDFs

HADRON CLASS

A process with N hadrons (N collinear parts) belongs to the **N-h class** of processes.

1-hadron class	
$(1, 0)$	DIS
$(0, 1)$	$e^+e^- \to H\,X$

2-hadron class	
$(2, 0)$	Drell-Yan
$(1, 1)$	SIDIS
$(0, 2)$	$e^+e^- \to H_A\,H_B\,X$

Classification in terms of:

$$(\rho,\ \lambda) \quad \text{with } \rho + \lambda = N$$

Number of hadrons in initial state

Number of hadrons in final state

A closer look to the Soft Factor

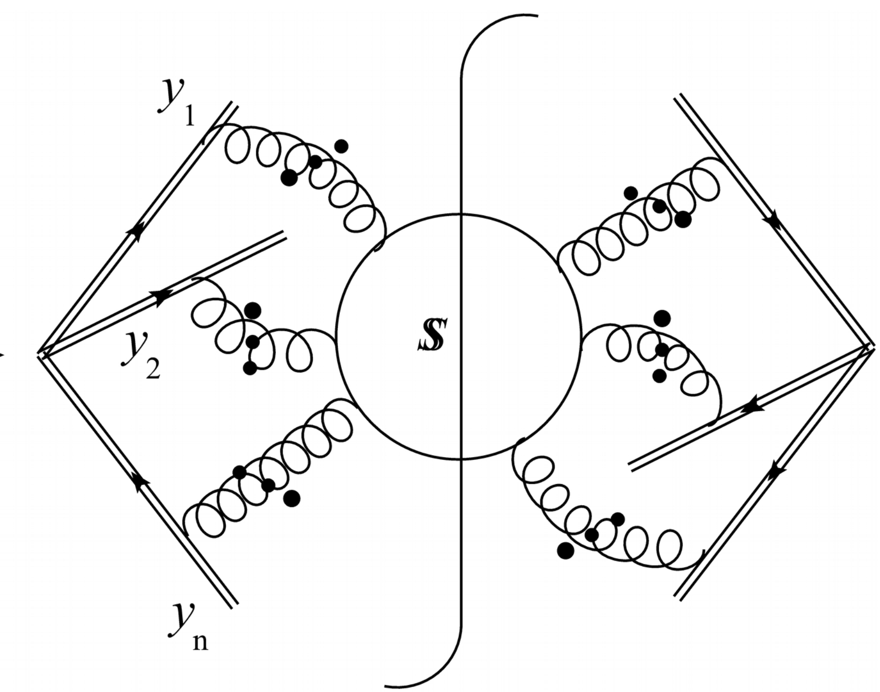
When kinematics allow for TMD-factorization, the various collinear parts can exchange information ONLY through soft gluons

A non-trivial Soft Factor appears in the cross section.

$\mathbb{S}$ depends on:

- The total transverse soft momentum $k_{S,T}$.
- The RG scale μ.
- **The number of collinear parts (N)** of the process. Each of them is replaced in $\mathbb{S}$ by a Wilson Line slightly-space-like thanks to a rapidity cut-off y_k

$$\mathbb{S}_{\text{N-h}}\left(\vec{k}_{S,T};\, \mu,\, \underbrace{\{y_1,\, y_2 \ldots y_N\}}\right) \longleftrightarrow k_S \longrightarrow$$



Lorentz-invariant combination of the rapidity cut-offs.

A closer look to TMDs

Each TMD is associated to a certain collinear region.
They are equipped with subtractions in order to avoid double counting and to cancel soft sub-divergences.

TMDs **depend only on their own collinear part** variables:

- The light-cone fraction of momentum z
- The total transverse momentum k_T of the (fragmenting) parton.
- The RG scale μ.
- The rapidity cut-off y_1 introduced by the subtraction procedure.

The resulting definition comes directly from the factorization procedure and hence I will call it **Factorization Definition (F Def.)**:

$$\widetilde{D}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_1\Big) = \lim_{y_{u_2} \to -\infty} \frac{\widetilde{D}^{\text{unsub}}_{H/f}\Big(z,\, b_T;\, \mu,\, y_P - y_{u_2}\Big)}{\widetilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_1 - y_{u_2}\Big)}$$

A Hierarchy of Universality

There is a **hierarchy** of universality for the objects appearing in the cross section:

UNIVERSAL

TMDs belong to this category.
They are sensitive **ONLY** to their own collinear part variables.
The rest of the process is replaced (before subtractions) by a (light-like) Wilson Line in the opposite direction.

HADRON-CLASS UNIVERSAL

The **Soft Factor** (of the process) belongs to this category.
It is blind to the rest of the process **APART FROM** the number of collinear parts involved into it, each one appearing as a slightly space-like Wilson Line.

PROCESS DEPENDENT

The **Hard Factor** belongs to this category.
It is totally process dependent, but it can be computed in perturbation theory at the desired accuracy.

2-h class of processes

The 2-h class of process has a **very special property**:

The Soft Factor in the cross section **is the same object** that appears in the subtraction mechanism implemented inside the Factorization Definition of the TMDs.

$$\widetilde{D}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_A - y_1\Big)\, \widetilde{D}_{H_B/\overline{f}}\Big(z_B,\, b_T;\, \mu,\, y_2 - y_B\Big)\, \widetilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_1 - y_2\Big) =$$

$$= \lim_{\substack{y_{u_1}\to+\infty \\ y_{u_2}\to-\infty}} \frac{\widetilde{D}^{\text{unsub}}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_P - y_{u_2}\Big)\, \widetilde{D}^{\text{unsub}}_{H_B/\overline{f}}\Big(z_B,\, b_T;\, \mu,\, y_{u_1} - y_B\Big)\, \widetilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_1 - y_2\Big)}{\widetilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_1 - y_{u_2}\Big)\, \widetilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_{u_1} - y_2\Big)}$$

In the 2-h class it is possible to **reorganize the soft factors** inside the TMDs definitions

2-h class of processes: Square Root Definition

The standard definition of TMDs in the 2-h class is the **Square Root Definition (SQRT Def)**:

$$\tilde{D}^{\text{sqrt}}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_A - y_n\Big) =$$
$$= \lim_{\substack{y_{u_1}\to+\infty \\ y_{u_2}\to-\infty}} \tilde{D}^{\text{unsub}}_{H_A/f}\Big(z_A,\, b_T;\, \mu,\, y_P - y_{u_2}\Big) \sqrt{\frac{\tilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_1 - y_n\Big)}{\tilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_1 - y_2\Big)\, \tilde{\mathbb{S}}_{\text{2-h}}\Big(b_T;\, \mu,\, y_n - y_2\Big)}}$$

[John Collins. *Foundations of perturbative QCD*, 2011, Cambridge University Press.]

Now the cross section is "**Parton Model-like**":

$$d\sigma \sim \mathbb{H}\, \tilde{D}_{H_A/f}\Big(y_A - y_1\Big)\, \tilde{D}_{H_B/\overline{f}}\Big(y_2 - y_B\Big)\, \tilde{\mathbb{S}}_{\text{2-h}}\Big(y_1 - y_2\Big) =$$
$$= \mathbb{H}\, \boxed{\tilde{D}^{\text{sqrt}}_{H_A/f}\Big(y_A - y_n\Big)\, \tilde{D}^{\text{sqrt}}_{H_B/\overline{f}}\Big(y_n - y_B\Big)}$$

These are the objects that we usually extract from data.

The Soft Factor has disappeared from the cross section!

2-h class of processes: Square Root Definition

Square Root Definition

Advantages:
- It is compatible with Factorization.
- It solves the problem of the Soft Factor in the 2-h class.
- The new TMDs depend only on one rapidity cut-off y_n (symmetry in the evolution equations).
- Perturbative computations are easier.
- Gauge invariance is more explicit.

Disadvantage:
- It reduces the universality of TMDs.
  The new TMDs are universal ONLY inside the 2-h class.

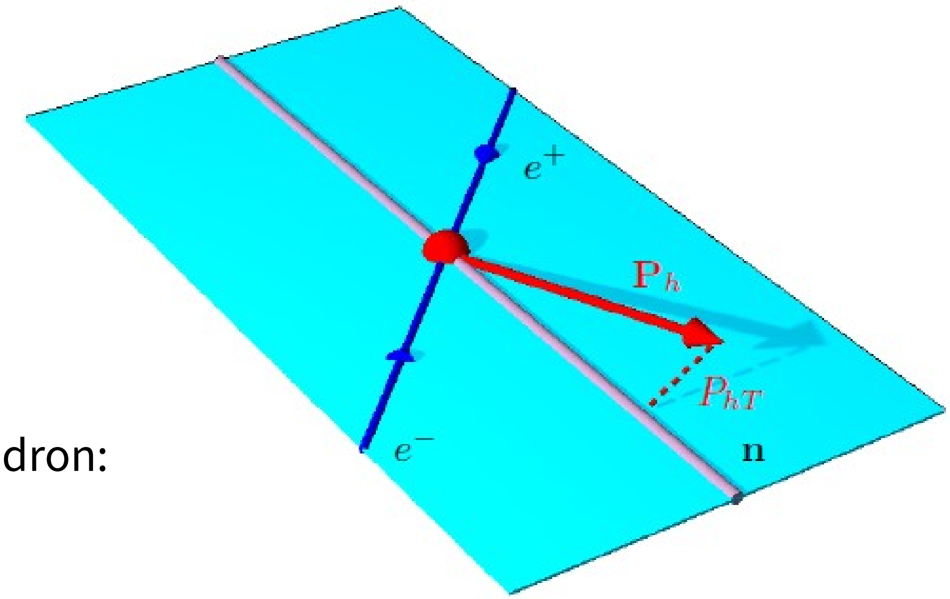
The SQRT Def. is OPTIMAL for the 2-h class

However, it becomes problematic if one has to deal with N-h class processes, where $N \neq 2$.

e^+e^- in 1 hadron jet: BELLE data

$$e^+e^- \to HX$$

Belle Collaboration provides the cross section differential in:



[The Belle Collaboration. *Phys.Rev. D99 (2019) no.11, 112006*]

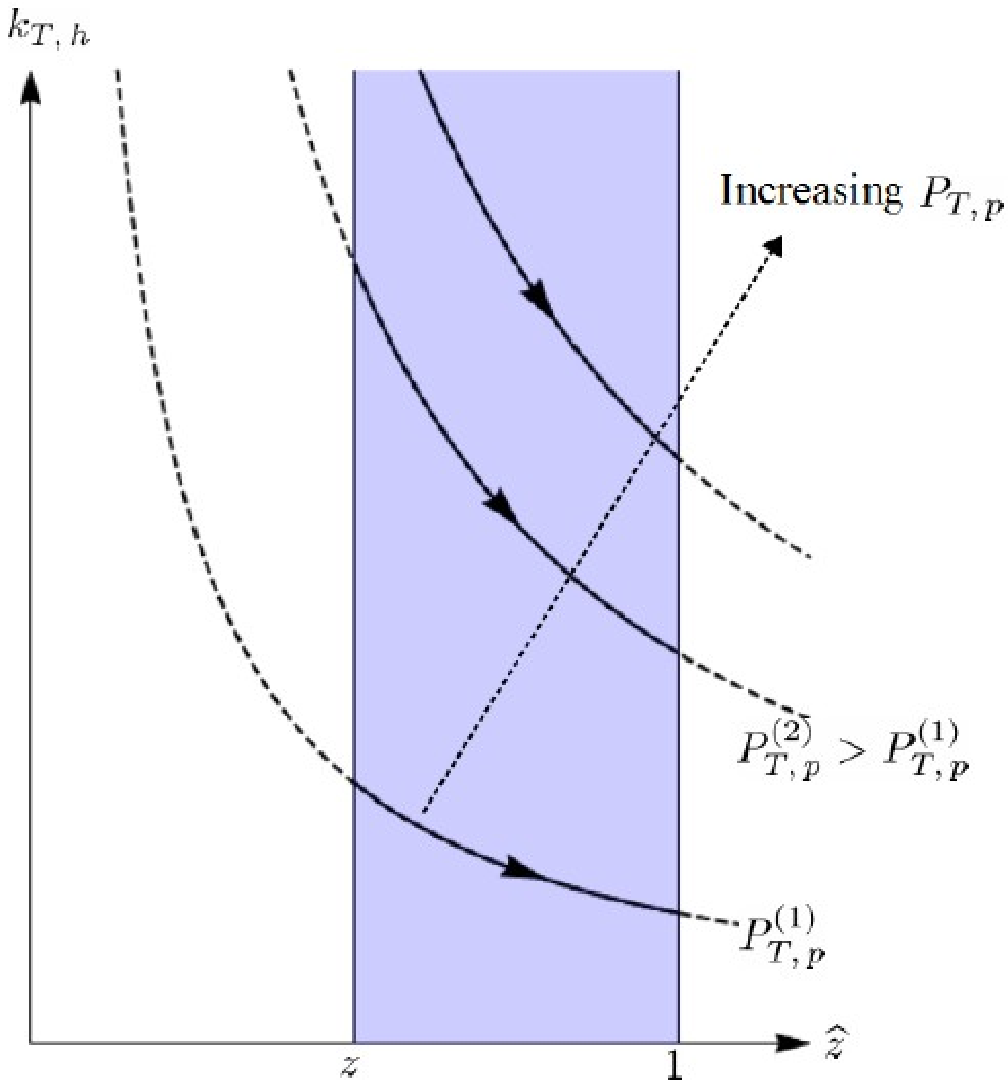
- CM-energy Q
- Fractional energy z of the detected hadron:

$$z = \frac{E_H}{Q/2}$$

- Transverse Momentum P_T of the detected hadron with respect the thrust axis of the jet.

These extra data allow to extract information about TMD (unpolarized) FFs from the collinear factorized cross section…**HOW?**

e^+e^- in 1 hadron jet: cross section



Considering the relation:

$$\vec{k}_{T,h} = -\frac{1}{\hat{z}}\vec{P}_{T,p}\left[1 + \mathcal{O}\left(\frac{P^2_{T,p}}{Q^2}\right)\right]$$

- If P_T is not measured

Sum over **all** the possibilities and the whole set of hyperbole has to be considered.

- If P_T is measured

Just **one** hyperbole has to be considered, that corresponding to the actual value of P_T.

e^+e^- in 1 hadron jet: cross section

For spin-less detected hadrons (pions, kaons) the cross section is:

$$\frac{d\sigma}{dz\,dP_T^2} = \frac{4\,P_T^2}{\pi\,Q^2\,z^2}\int \frac{d^2\vec{b}_T}{(2\pi)^2}\,e^{-i\,\vec{P}_T\cdot\vec{b}_T}\times$$
$$\times\sum_j\int_z^1 \frac{d\widehat{z}}{d\widehat{z}}\,\boxed{\frac{d\widehat{\sigma}_j(z/\widehat{z})}{d(z/\widehat{z})}}\,\boxed{\widetilde{D}_{1,\,H/j}(\widehat{z},\,b_T)}\left[1+\mathcal{O}\left(\frac{P_T^2}{Q^2}\right)\right]$$

Partonic cross section

Unpolarized TMD FF associated to the detected hadron

NOT defined by using the SQRT Def.

This object is NOT the same function appearing in e^+e^- to 2 back-to-back hadrons.

Notice that in this case, **the Soft Factor of the process is unity**, since collinear factorization holds:

$$\mathbb{S}_{(\text{coll.})} = 1 = \mathbb{S}_{\text{1-h}}$$

e^+e^- in 1 hadron jet: cross section

By using **evolution equations for TMDs** we can re-write the cross section as:

$$\frac{d\sigma}{dz\,dP_T^2} = \frac{4\,P_T^2}{\pi\,Q^2\,z^2}\int\frac{d^2\vec{b}_T}{(2\pi)^2}\,e^{-i\,\vec{P}_T\cdot\vec{b}_T}\times$$
$$\times\sum_j\int_z^1\frac{d\hat{z}}{d\hat{z}}\,\frac{d\hat{\sigma}_j(z/\hat{z})}{d(z/\hat{z})}\sum_k\int_{\hat{z}}^1\frac{dw}{w^3}\,d_{H/k}(w,\mu_b)\,\widetilde{C}_j^k(\frac{\hat{z}}{w})\times$$
$$\times\exp\Big\{\frac{1}{4}\,\widetilde{K}(b_T^*,\,\mu_b)\,\log(\frac{Q^2}{\mu_b^2})+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\,[\gamma_D(a(\mu'),\,1)-\frac{1}{4}\gamma_K(a(\mu'))\log(\frac{Q^2}{\mu'^2})]\Big\}\Big]\times$$
$$\times\left[M_j(b_T)\exp\{-\frac{1}{4}\,g_K(b_T)\log(\frac{Q^2\,\hat{z}^2}{M_H^2})\}\right]\left[1+\mathcal{O}\left(\frac{P_T^2}{Q^2}\right)\right]$$

These functions **should be extracted from data (Belle)**.
Everything else is computable in perturbation theory.

Combining e$^+$e$^-$ data from 1-h and 2-h class

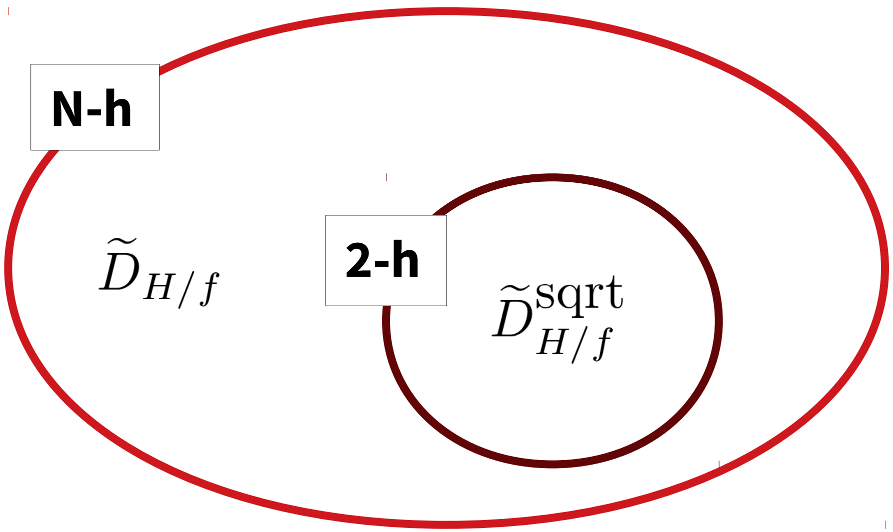
Now, the situation is the following

- From $e^+e^- \to H\,X$: $\longrightarrow$ Extraction of $\widetilde{D}_{H/f}$

The **only possible** definition in 1-h class

- From $e^+e^- \to H_A\,H_B\,X$: $\longrightarrow$ Extraction of $\widetilde{D}^{\text{sqrt}}_{H/f}$

The **optimal** definition in 2-h class



Combining e$^+$e$^-$ in 1-h and 2-h means extracting the same TMD from both the processes.

Since the **F Def.** is more universal then the **SQRT Def.**, we should adopt it also in the 2-h class.

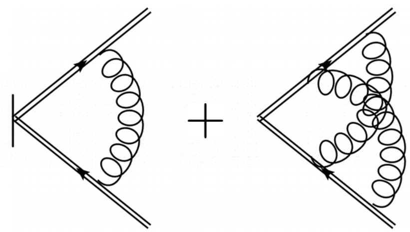
We have to deal directly with the 2-h Soft Factor appearing in the cross section

Comparison between the two TMD definitions

We should look closer at the 2-h Soft Factor: it is not entirely a black box.

- We can separate what is computable in pQCD from what is not:

$$\widetilde{\mathbb{S}}_{\text{2-h}}(b_T;\,\mu, y_1 - y_2) = \exp\Big\{ \Big|$$



$$+ \dots \Big|^2 \Big\} \times M_S(b_T;\,\mu, y_1 - y_2)$$

- Its **evolution equation** in the limit of infinite rapidity cut-offs:

$$\widetilde{\mathbb{S}}_{\text{2-h}}(b_T;\,\mu, y_1 - y_2) = \widetilde{\mathbb{S}}_{\text{2-h}}(b_T;\,\mu_0, 0)\exp\Big\{ \frac{y_1 - y_2}{2}\, \widetilde{K}(b_T;\,\mu) \Big\} + \mathcal{O}\Big(e^{-\frac{y_1 - y_2}{2}} \Big)$$

$$\mathbb{S}_{\text{2-had}}(b_T;\,\mu,\, y_1 - y_2) = \boxed{e^{\frac{y_1 - y_2}{2}\, \widetilde{K}(b_T^\star;\,\mu)}}\ \boxed{M_S(b_T)\, e^{-\frac{y_1 - y_2}{2}\, g_K(b_T)}} + \mathcal{O}\Big(e^{-(y_1 - y_2)} \Big)$$

A new non-perturbative object:
the **SOFT MODEL**

Comparison between the two TMD definitions

Combining all information we can **directly compare** the F Def. and the SQRT Def.:

$$\frac{\widetilde{D}^{\text{sqrt}}_{H/f}\left(z,\, b_T;\, \mu,\, y_P - y_n\right)}{\widetilde{D}_{H/f}\left(z,\, b_T;\, \mu,\, y_P - y_1\right)} = \boxed{\exp\left\{\frac{y_1 - y_n}{2}\, \widetilde{K}(b_T^*;\, \mu)\right\}} \times$$

Perturbative Deviation

$$\times \boxed{\sqrt{M_S(b_T)}\, \exp\left\{-\frac{y_1 - y_n}{2}\, g_K(b_T)\right\}}$$

Non-Perturbative Deviation

Setting the cut-offs to the same value we have the simple relation:

$$\widetilde{D}^{\text{sqrt}}_{H/f}\left(z,\, b_T;\, \mu,\, y_P - y_n\right) = \widetilde{D}_{H/f}\left(z,\, b_T;\, \mu,\, y_P - y_n\right) \times \sqrt{M_S(b_T)}$$

The two definitions differ **ONLY IN THEIR NON PERTURBATIVE PART**,
by a square root of the **SOFT MODEL**

Strategy

3 non-perturbative functions should be extracted from data:

- The TMD model $M_j(b_T)$
- The Soft Kernel model (TMD evolution) $g_K(b_T)$

Collinear Physics

- The Soft Model $M_S(b_T)$

Soft Physics

Once they are known, we should be able to describe **both** $e^+e^- \to HX$ and $e^+e^- \to H_A\, H_B\, X$data.

HOW?

Strategy

1. Extract the UNPOLARIZED TMD FFs from 1-h class data, i.e. the functions $M_j(b_T)$ and $g_K(b_T)$:

$$\frac{d\sigma}{dz\, dP^2_{T,\,p}} = \frac{4\, P^2_{T,\,p}}{\pi\, Q^2\, z^2} \sum_j d\widehat{\sigma}_j \otimes \boxed{D_{1,\,H/j}(\,P_{T,\,p})} \left[1 + \mathcal{O}\left(\frac{P^2_{T,\,p}}{Q^2}\right)\right]$$

2. Extract the Soft Model $M_S(b_T)$ from the 2-h class data, since now it is the only remaining unknown function.

$$W^{\mu\nu}_{\rm unp.}(p_A, p_B,\, q) = \frac{8\pi^3 z_A\, z_B}{Q} \sum_f \mathbb{H}_f(Q) \times$$
$$\times \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{-i\vec{q}_{h,\,T}\cdot\vec{b}_T}\, \boxed{M_S(b_T)}\, \widetilde{D}_{1,\,h_A/f}(z_A,\, b_T)\, \widetilde{D}_{1,\,h_B/\overline{f}}(z_B,\, b_T)$$

3. Now everything is known. In principle, we can extract all the TMDs appearing in 1-h and 2-h classes.

Summary

1. The processes can be classified in hadron classes.

2. The Soft Factor that appears in the cross sections involving the TMDs is less universal than TMDs themselves. Furthermore, it is neither totally computable in perturbation theory, nor extractable independently from experimental data.

3. The Square Root Definition solves the Soft Factor problem in the 2-h class.

4. In $e^+e^- \to H\,X$ the TMDs can only be defined through the Factorization Definition.

5. Adopting the Factorization Definition also in $e^+e^- \to H_A\,H_B\,X$ implies that we have to extract another non-perturbative function, the Soft Model.

6. The 1-had and 2-had class data can be combined using a well defined strategy.