

Cern Presentation 2020

COLOR CONFINEMENT



CPHI-2026
CERN

and the

STRONG CONJECTURE



den sivers

Portland Phys. Inst.

STRONG CONJECTURE

The confinement mechanism for QCD involves a domain wall of topological (cp-odd) charge separating the interior volume of hadrons from an exterior volume.

This requirement can be illustrated in spherically symmetric SU(2) by translating the topological structure of 1+1 dim. abelian higgs model into spherical coordinates in 3+1 dim.

Jaffe-Witten problem - Clay Inst. Math.

CONFIRMATION

John Ralston Riet Mulders

Stan Brodsky Jian-Wei Qiu

Gary Goldstein Leonard Gamberg

EXPLAINED

SU(2) GAUGE THEORY WITH
SPHERICAL SYMMETRY

SOUNDS LIKE A BAG MODEL

IS IT ?

NO. IT IS A THEORETICAL CONJECTURE

about

QCD

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about

QCD

TOPOLOGICAL CHARGE

- CP-odd condensate $E_i^a B_i^a(r, t)$

forms in regions with different values of constant $a(r, t)$ as a consequence of Yang-Mills Maxwell equations $-E_L + \frac{\partial}{\partial r}(arE_s) + \frac{\partial}{\partial t}(arB_\phi) = g^2 r^2 E_i^a B_i^a$

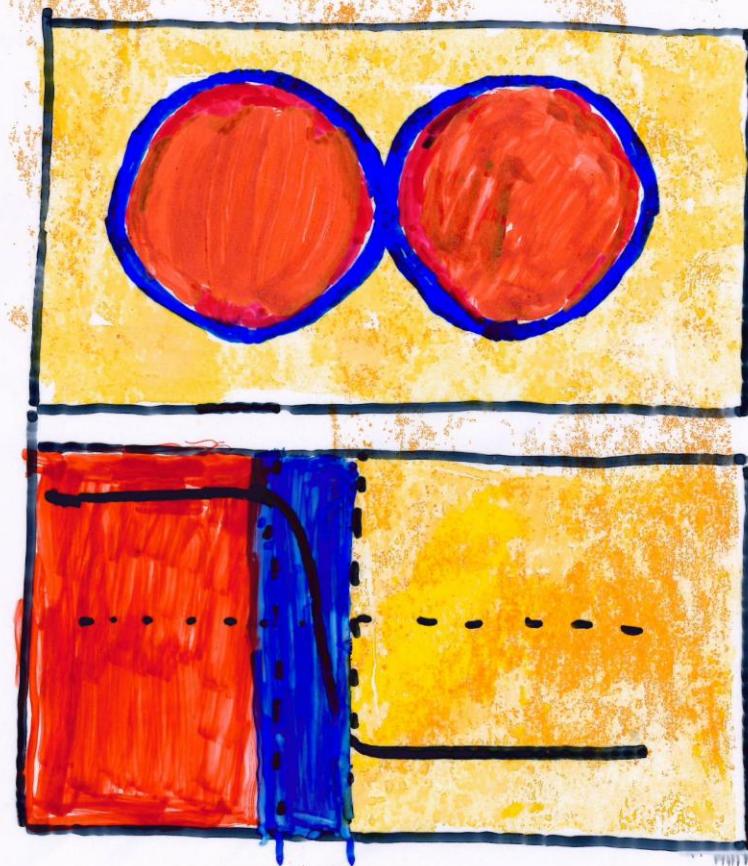
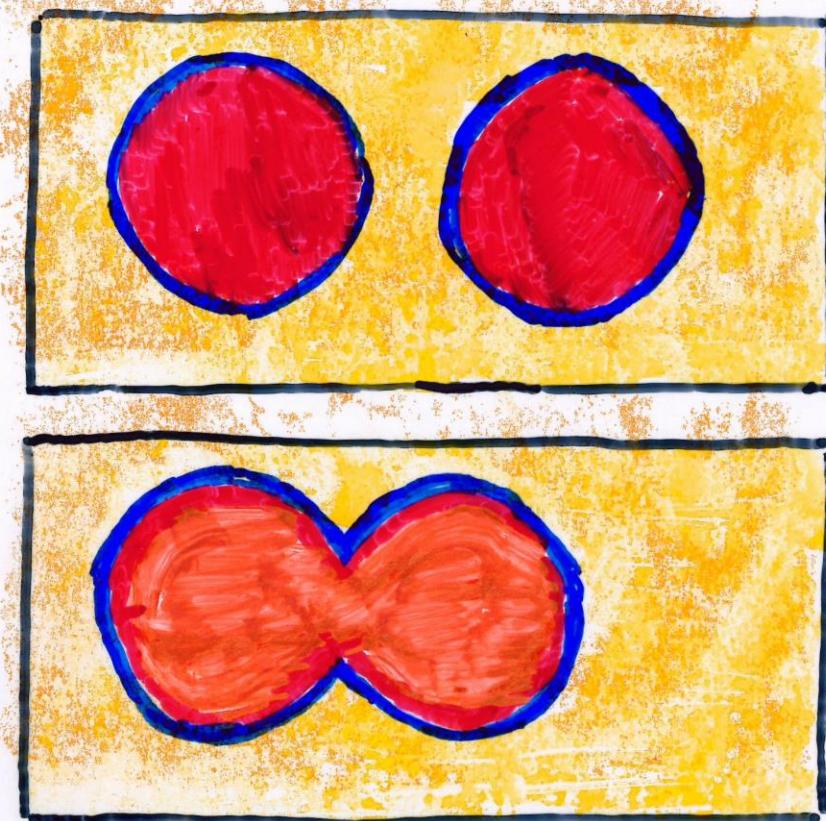


- not a conserved charge

Disappears after EW hadronic decays (ex. $\pi^0 \rightarrow \gamma\gamma$)

Instantons : Merong ..

Domain Zones



■ interior volume

■ exterior volume

acr) R_0 \xrightarrow{r}
topological charge

A. The Yang-Mills Millennium Prize*

(* Clay Mathematics Institute .. \$1G each)

posed by A. Jaffe & E. Witten (2000)

$$S_{YM}^G = \frac{1}{4g^2} \int Tr F \wedge {}^*F$$

* Hodge Dual
G compact group

F curvature, $F = dA + A \wedge A$, of G-bundle connection A

Prove \nexists QFT in 4-dim. Space time

- 1.) "mass gap", $\Delta > 0$
- 2.) confinement (quarks & gluons)
- 3.) chiral symmetry breaking

Streater Wightman Axioms

Gauge theories \models

Cluster Decomposition

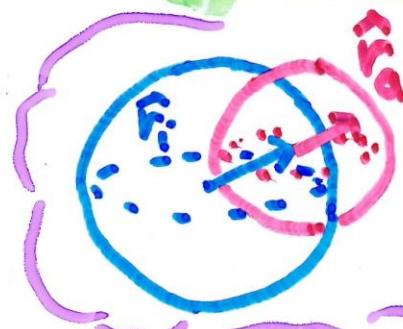
non-Abelian gauge theories accommodate:

(A) area law for Wilson loops

(B) emergent structures such as
topological domain walls

local fields \Rightarrow non-local correlations

I. Bars-Witten ansatz for spher-sym. SU(2) Ralston-Sivers field strength formulation



gauge connection: $gA_0 = A_0(r,t) \hat{r}_a$

$$gA_i^a(r,t) = A_1(r,t) \delta_{ia} + \frac{\alpha(r,t) \sin\omega(r,t)}{r} \epsilon_{ia}^T + \frac{\alpha(r,t) \cos\omega(r,t) - 1}{r} \epsilon_{ia}^T$$

$$\delta_{ia} = \hat{r}_i \cdot \hat{r}_a$$

$$\delta_{ia}^T = \delta_{ia} - \delta_{ia} = (\hat{\theta}, \hat{\theta}_a + \hat{\phi}, \hat{\phi}_a)$$

$$\epsilon_{ia}^T = \epsilon_{ia} \cdot \hat{r}_i = (\hat{\phi}_a \hat{\theta}_a - \hat{\theta}_i \hat{\phi}_a)$$

spherical coordinates
for both vector spaces
in bundle

gauge

derivatives

$$D_i^{ab} \hat{r}_b = \frac{\alpha(r,t)}{r} \epsilon_{ia}^S [\omega(r,t)] = \frac{\alpha}{r} [\delta_{ia}^T \cos\omega - \epsilon_{ia}^T \sin\omega]$$

$$-i[\hat{r}, D_i \hat{r}]^a = \frac{\alpha(r,t)}{r} \epsilon_{ia}^A [\alpha(r,t)] = \frac{\alpha}{r} [\delta_{ia}^T \sin\omega + \epsilon_{ia}^T \cos\omega]$$

In terms of the gauge-covariant field strengths

$$gE_i^a = E_L(r,t)\delta_{ia} + E_S(r,t) \overset{S}{\epsilon}_{ia}(\omega(r,b)) + E_A(r,t) \overset{A}{\epsilon}_{ia}(\omega(r,t))$$

$$gB_i^a = B_L(r,t)\delta_{ia} + B_S(r,t) \overset{S}{\epsilon}_{ia}(\omega(r,t)) + B_A(r,t) \overset{A}{\epsilon}_{ia}(\omega(r,t))$$

with

$$E_L(r,t) = -\frac{\partial}{\partial t} A_g(r,t) + \frac{\partial}{\partial r} A_b(r,t) \quad B_L(r,t) = \frac{a^2(r,t)-1}{r^2}$$

$$E_S(r,t) = \frac{a(r,t)}{r} [-A_b(r,t) + \frac{\partial}{\partial t} \omega(r,t)] \quad B_S(r,t) = \frac{1}{r} \frac{\partial a(r,t)}{\partial r}$$

$$E_A(r,t) = -\frac{1}{r} \frac{\partial a(r,t)}{\partial t} \quad B_A(r,t) = \frac{a(r,t)}{r} [A_g(r,t) + \frac{\partial}{\partial r} \omega(r,t)]$$

nonlinearities only from $B_L(r,t) = \frac{a^2(r,t)-1}{r^2}$
coefficients gauge invariant

Yang-Mills Maxwell: $(D^\mu G_{\mu\nu})^a = J_\nu^a(r, t)$

$$J_0^a(r, t) = \frac{1}{r^2} J_0(r, t) \hat{r}_a \quad J_i^a(r, t) = \frac{1}{r^2} J_i(r, t) \delta_{ia} + j_{ia}(r, t) E_{ia}(w) + j_{ik}(r, t) E_{ik}(w)$$

classical adjoint currents

$$-\frac{\partial}{\partial r} (r^2 E_L) + 2ar E_S = J_b(r, t)$$

$$-\frac{\partial}{\partial r} (r^2 E_L) + 2ar B_A = J_y(r, t)$$

$$-\frac{\partial}{\partial t} (ar E_S) + \frac{\partial}{\partial r} (ar B_A) = ar j_a(r, t)$$

$$\alpha(-\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial t^2}) a - r^2(E_S^2 - B_A^2) - \frac{\alpha^2(a^2 - 1)}{r^2} = ar j_a(r, t)$$

Bianchi constraints

$$\left. \begin{aligned} \frac{\partial}{\partial r} (ar E_A) - \frac{\partial}{\partial t} (ar B_S) &= 0 \\ -E_L + \frac{\partial}{\partial r} (ar E_S) + \frac{\partial}{\partial t} (ar B_A) &= g^2 r^2 E_i^a B_i^a \end{aligned} \right\}$$

covariant current conservation

$$\left. \begin{aligned} -\frac{\partial}{\partial t} J_0 + \frac{\partial}{\partial r} J_y &= 2ar j_S = -2\frac{\partial}{\partial t} (ar E_S) + 2\frac{\partial}{\partial r} (ar B_A) \end{aligned} \right\}$$

Extension to SU(3)

$a=1-3 \text{ SU}(2) \Rightarrow a=1-8 \text{ SU}(3)$

$$\hat{\Sigma}_a \Rightarrow \hat{\Sigma} |h\rangle_a \quad |h\rangle_a = (h_3, h_8)$$

t_3, t_8 diagonal

$t_1-t_2, t_4-t_3 \Rightarrow 3 \text{ SU}(2)$ subgroups

O(r y) G(y b) P(br) with \pm charge

$$[t_3, Q_O^\pm] = \pm Q_O^\pm \quad [t_3, Q_G^\pm] = \mp \frac{1}{2} Q_G^\pm \quad [t_3, Q_P^\pm] = \mp \frac{1}{2} Q_P^\pm$$

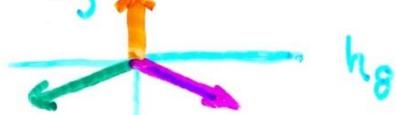
$$[t_8, Q_O] = 0$$

$$a = (1, 2)$$

$$[t_8, Q_G^\pm] = \mp \frac{\sqrt{3}}{2} Q_G^\pm$$

$$[t_8, Q_P^\pm] = \pm \frac{\sqrt{3}}{2} Q_P^\pm$$

$$\xi_{ia} \Rightarrow \xi_{i\bar{a}} \hat{\Sigma}_i \hat{\Sigma} |h\rangle_a$$



$$\sum_c |w_c\rangle = (0, 0)$$

$$|w_O\rangle = (1, 0) \quad |w_G\rangle = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$|w_P\rangle = \left(-\frac{1}{2}, +\frac{\sqrt{3}}{2}\right)$$

"Hawser Laid"

off-diagonal gluons carry charge

CP-ODD GAUGE STRUCTURES

consider vectors v_i in 3-space

consider vectors v_a in adjoint rep of $SU(N)$

$$\begin{array}{l} \text{Parity OP } P(v_i, v_a) P^{-1} \rightarrow (-v_i, v_a) \\ \text{C-Conj OP } C(v_i, v_a) C^{-1} \Rightarrow (v_i, -v_a) \end{array}$$

Decomposing the direct product $v_i v_a$ into longitudinal and transverse structures $L_{ia} \delta_{ia}^T \epsilon_{ia}^T$

$$P(L_{ia} \delta_{ia}^T \epsilon_{ia}^T) P^{-1} = \pm (-L_{ia} \delta_{ia}^T \epsilon_{ia}^T)$$

$$C(L_i, \delta_i^T, \epsilon_i^T) C^{-1} = \pm (-L_i \delta_i^T, \epsilon_i^T)$$

$$CP(L_i, \delta_i^T, \epsilon_i^T) P^{-1} C^{-1} = (\pm)(L_{ia} \delta_{ia}^T \epsilon_{ia}^T)$$

Spherical coord.

Cylindrical coord

$$v_i = v_r \hat{r}_i + v_\theta \hat{\theta}_i + v_\phi \hat{\phi}_i \quad v_a = v_r \hat{r}_a + v_\theta \hat{\theta}_a + v_\phi \hat{\phi}_a$$

$$v_i = v_z \hat{z}_i + v_\rho \hat{r}_i + v_\phi \hat{\phi}_i \quad v_a = v_z \hat{z}_a + v_\rho \hat{r}_a + v_\phi \hat{\phi}_a$$

CP odd \neq CP viol can be CP sep.

Define an interior volume $r \leq R_0 - \Delta$

a transition volume $R_0 - \Delta \leq r \leq R_0 + \Delta$

an exterior volume $r \geq R_0 + \Delta$

choose a simple solution in the interior volume

$$A_0(r) = C_E r \quad A_1(r) = C_M r \quad a(r) = 1 \quad \omega(r) = 0$$

$$E_L(r) = C_E \quad E_S(r) = C_E \quad E_A(r) = 0$$

$$B_L(r) = 0 \quad B_S(r) = 0 \quad B_A(r) = C_M$$

$$E_i^a E_i^a - B_i^a B_i^a = 3C_E^2 - 2C_M^2 \quad E_i^a B_i^a = 0$$

$$J_0(r) = 0 \quad J_1(r) = 2rC_M \quad r j_S(r) = C_M \quad r j_A(r) = \frac{2rC_M^2}{r} \\ = r^2(C_E^2 - C_M^2)$$

Yang-Mills Maxwell
equations apply
throughout
these
regions

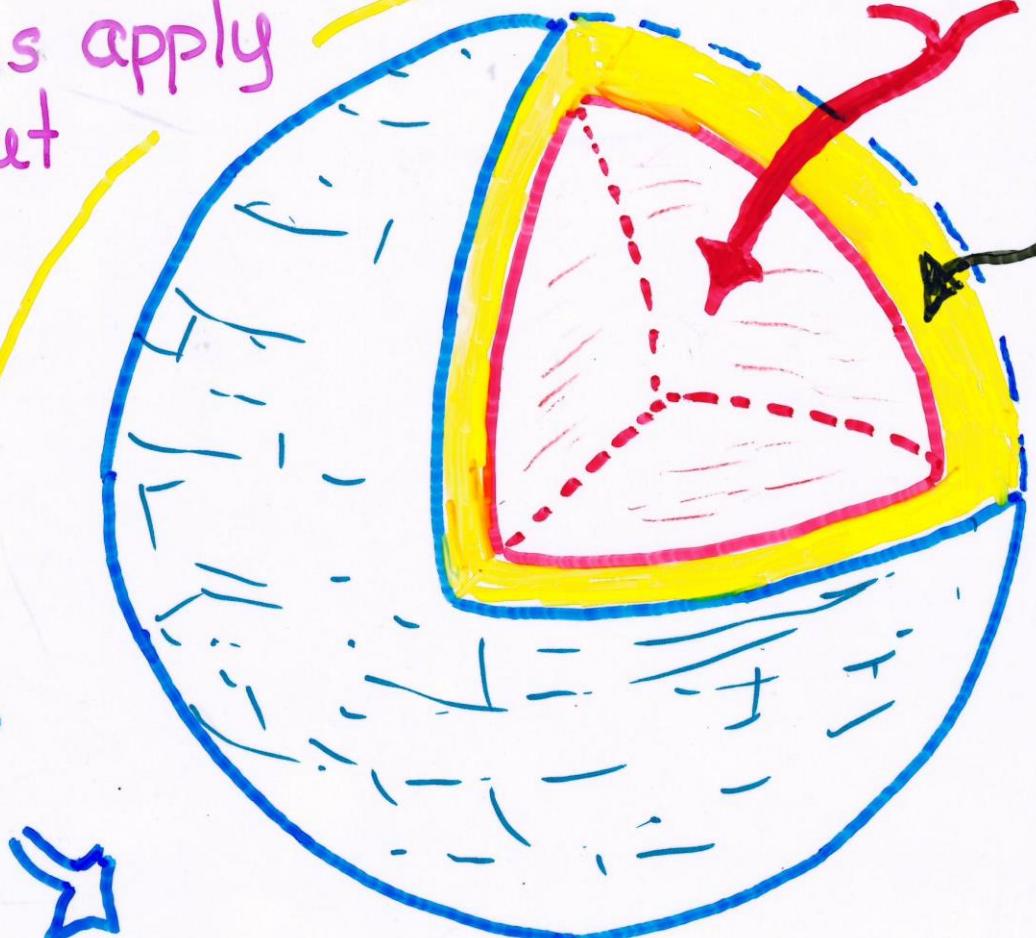
Interior vol. $r < R_0 - \Delta$

Transition
volume

$$R_0 - \Delta \leq r \leq R_0 + \Delta$$



Exterior
volume
 $r > R_0 + \Delta$



$$2R_0$$

$$2\Delta$$

DIMENSIONAL COMPRESSION

= Landau-Ginsburg-Higgs -

Transverse gauge fields act as charged
Pseudoscalars

r^2 -dependent metric

$$r > R$$

$r^2(F_{gm}F^{lm})$ enhanced exterior

$$r < R$$

$\frac{(1-\frac{r^2}{R^2}-1)^2}{r^2}$ enhanced interior

dimensional compression also for SU(3)

II. TOPOLOGICAL CHARGE

in Spherically symmetric $SU(2)$

$$\partial^i K_1(r,t) = g^2 r^2 E_i^a B_i^a(r,t) \quad \text{topological current}$$

$$K_0(r,t) = (\alpha^2 - 1) A_1(r,t) + \alpha^2 \frac{\partial}{\partial r} \omega(r,t)$$

$$K_1(r,t) = -(\alpha^2 - 1) A_0(r,t) + \alpha^2 \frac{\partial}{\partial t} \omega(r,t)$$

$$r\alpha(r,t) E_S(r,t) = (K_1 - A_0)(r,t) \quad r\alpha(r,t) B_A(r,t) = (K_0 + A_1)(r,t)$$

when $\alpha(r,t) = \pm 1$

$$K_0(r,t) = \frac{\partial}{\partial r} \omega(r,t) \quad K_1(r,t) = \frac{\partial}{\partial t} \omega(r,t)$$

$$\text{so that } \partial^i K_1 \Big|_{\alpha=\pm 1} = 0$$

Classification of condensates in SU(2) chromostatics with spherical symmetry

$a(r) = \pm 1$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a = 0$ color electric

$a(r) = \pm 1$ $E_i^a E_i^a = 0$ $B_i^a B_i^a \neq 0$ color magnetic

$a(r) = \pm 1$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a \neq 0$ color glass

$a(r) = \pm 1$ $E_i^a E_i^a = 0$ $B_i^a B_i^a = 0$ sterile vacuum

$a(r) = 0$ $E_i^a E_i^a = 0$ $B_L B_L = \frac{(\pm)}{r} +$ 't Hooft Polyakov

$a(r) = c \neq \pm 1, 0$ $E_i^a E_i^a \neq 0$ $B_i^a B_i^a \neq 0$ topological
or dyonic

A domain wall is a region where $a'(r) \neq 0$
that separates other condensates
and also carries topological charge

Derrick's Theorem (1964)

$$\mathcal{L} = -\frac{1}{2} \text{tr} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} (\partial^\mu \phi)^\dagger (\partial_\mu \phi) - V(\phi)$$

$$S_{\text{energy}}^{\text{static}} = I_G(A) + I_K(A, \phi) + I_V(\phi)$$

given a static (soliton) $\bar{A}_j, \bar{\phi}$

define scaled $f_\lambda(x) = \bar{\phi}(\lambda x)$ $g_{j\lambda}(x) = \lambda \bar{A}_j(\lambda x)$

$$I_G = \frac{1}{2} \int d^D x \text{tr}(G_{ij})$$

$$I_K = \frac{1}{2} \int d^D x (\partial^\mu \phi)(\partial_\mu \phi)$$

$$I_V = \int d^D x V(\phi)$$

$$S_{\lambda}^{\text{static}} = I_G(g_\lambda) + I_K(g_\lambda, f_\lambda) + I_V(f_\lambda)$$

$$= \lambda^{4-D} I_G(\bar{A}) + \lambda^{2-D} I_K(\bar{A}, \bar{\phi}) + \lambda^{-D} I_V(\bar{\phi})$$

this is stationary at $\lambda=1$ if

$$0 = (D-4) I_G(\bar{A}) + (D-2) I_K(\bar{A}, \bar{\phi}) + D I_V(\bar{\phi})$$

This allows soliton solutions in $D=2$ and $D=3$

Only solution $D=4$ soliton in pure gauge sector
no scalars and I_G scale invariant

Topology in SU(2) Chromostatics

Ralston - Sivers formalism

$$E_L(r) = A'_0(r)$$

$$E_A(r) = 0$$

$$E_S(r) = \frac{\alpha(r)}{r} A_0(r)$$

$$B_L(r) = \frac{\alpha^2(r)-1}{r^2}$$

$$B_A(r) = \frac{\alpha(r)}{r} [A_y(r) - \omega(r)]$$

$$B_S(r) = -\frac{\alpha'(r)}{r}$$

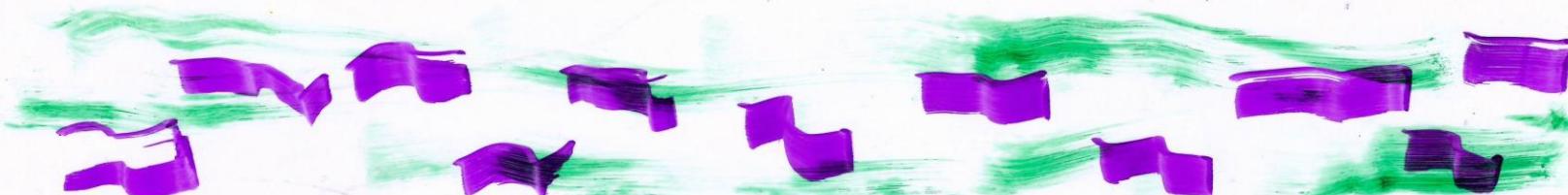
$$K_0(r) = -(\alpha^2(r)-1) A_y(r) + \alpha^2 \omega(r)$$

$$K_1(r) = -(\alpha^2(r)-1) A_0(r)$$

$$\alpha(r) r B_A(r) = \alpha^2(r) [K_0(r) + A_{Lr}(r)]$$

$$\alpha(r) r E_S(r) = \alpha^2(r) [A'_0(r) - K_1(r)]$$

Topological Current \leftrightarrow transverse fields



The t'Hooft-Polyakov monopole is a topological stable solitonic solution to the $SU(2)$ field eqns

Stability involves
a pressure balance

color
glass

$a = \pm$

Internal
pressure

(adiabatic + quantum
fluctuations)

External pressure

from t'Hooft Polyakov condensate

Domain wall

$a=0$

"Shrink wrap" pressure

However, the CP-odd domain wall does not confine radial adjoint charge

$$\text{Energy} \propto \langle A_\theta A_3 \rangle \hat{r}_a e^{i(t-r)}$$

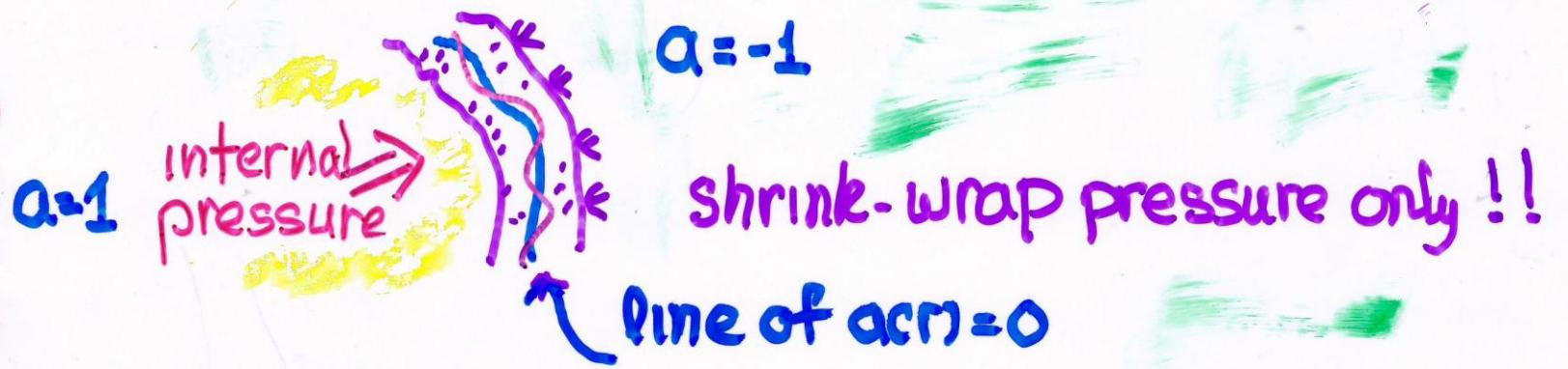
"abelian" gluons can propagate through $a(r)=0$ condensate

unless ...

[Dark matter]

P. Rossi
Phys. Rep. 86 (82)

The color-glass/sterile vacuum "kink" soliton with $\Delta a = \pm 2$ is also a topologically stable solution to $SU(N)$ field equations



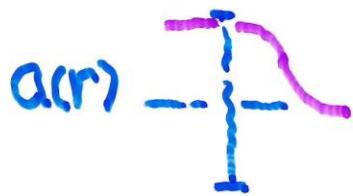
Confines Abelian Gluons - adjoint charge

replaces dual superconducting "QCD vacuum"
condensate with a dual insulator
Translates to cylindrical symmetry -
QCD jets & Collins functions

not yet
studied
extensively

t Hooft Polyakov Soliton

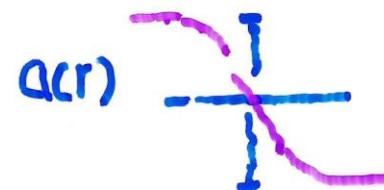
$$CP = -$$



$$\Delta a = -1$$

Chiral Transition Soliton

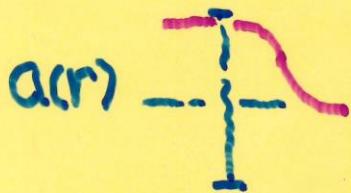
$$a=0$$



$$\Delta a = -2$$

both solitons have domain wall

t Hooft Polyakov Soliton



$$\Delta\alpha = -1$$

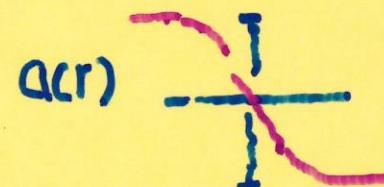
t Hooft Polyakov condensate

$$B_L = -\frac{1}{r^2}$$

- no transverse degrees \Rightarrow no charge
- negative pressure

Chiral Transition Soliton

$$\alpha = 0$$

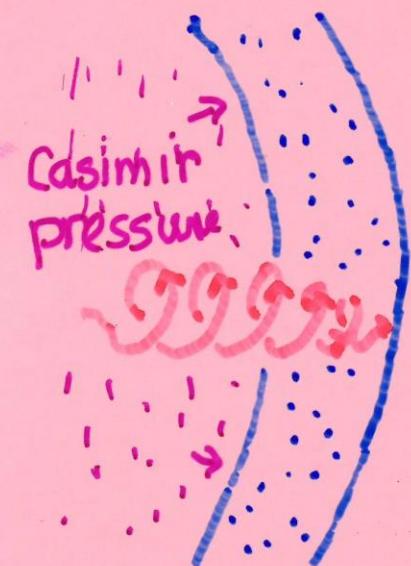


$$\Delta\alpha = -2$$

both solitons have domain wall

$\text{t Hooft Polyakov Soliton}$

$$CP = -$$



$$\Delta a = -1$$

both solitons have domain wall

Chiral Transition Soliton

$$\alpha = \beta$$



$$\Delta a = -2$$

sterile chiral condensate

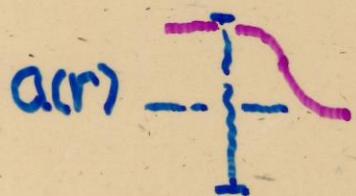
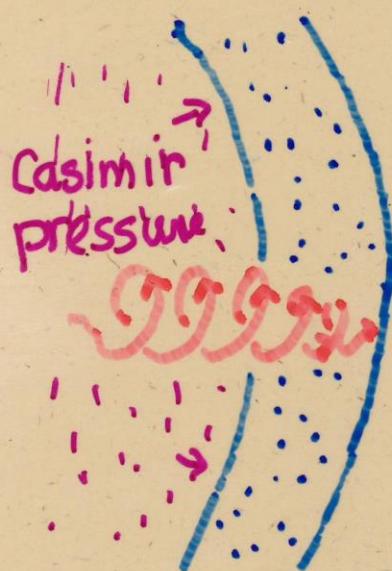
no pressure
but chiral structure

Aharanov-Bohm

chiral structure changes sign in down wall

$\text{'t Hooft Polyakov Soliton}$

$$CP = -$$



$$\Delta\alpha = -1$$

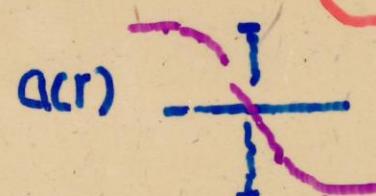
't Hooft Polyakov condensate

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Chiral Transition Soliton

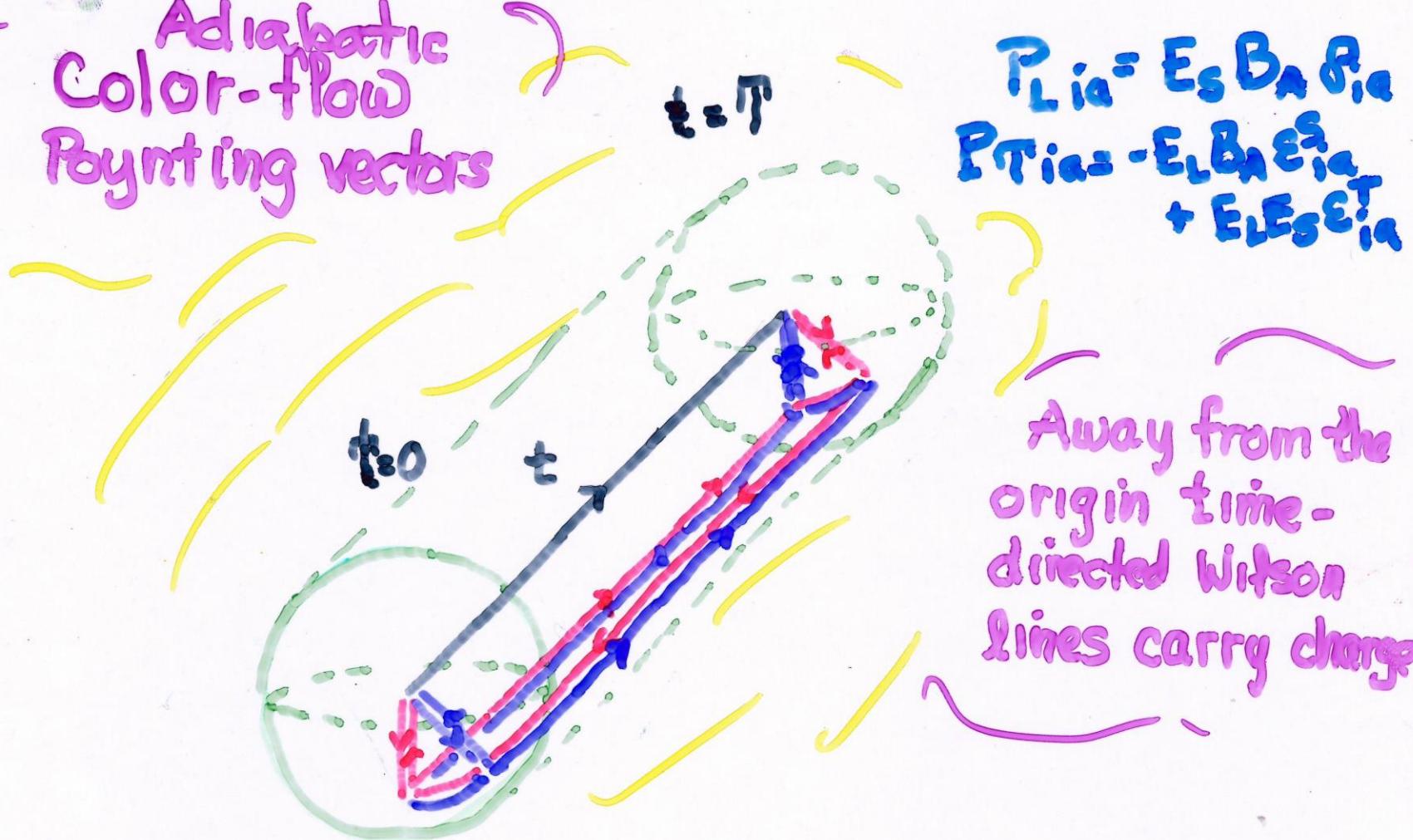
$$Q = \beta$$



sterile chiral condensate
no pressure but chiral structure
Aharonov-Bohm
 $\Delta\alpha = -2$

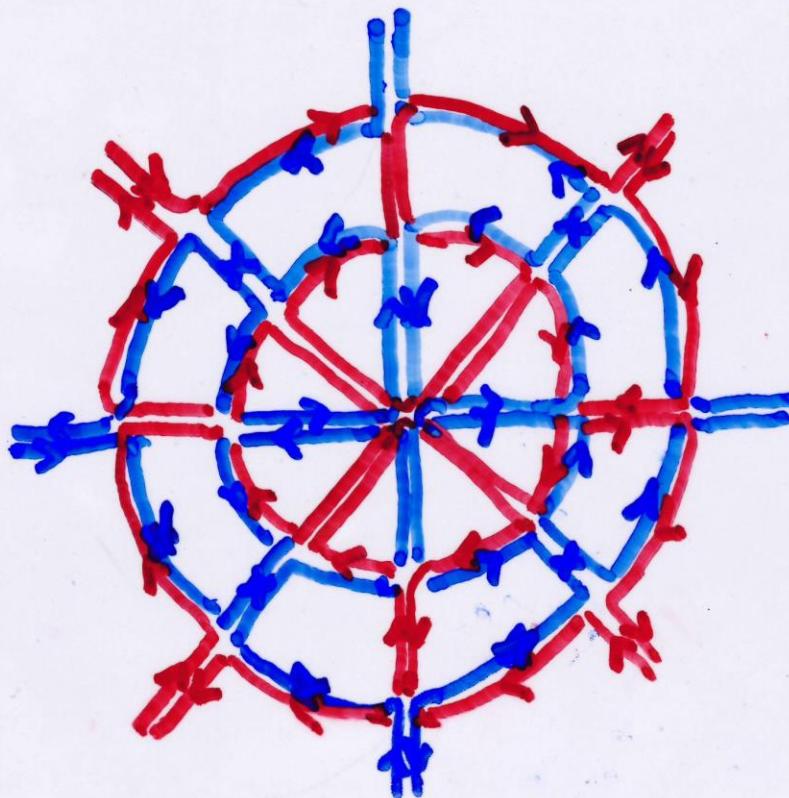
both solitons have domain wall

Adiabatic
Color-flow
Poynting vectors



Adjoint Wilson loops with $A_0(r) = 0$
Adiabatic Evolution Confined Condensate

Here is the sketch showing the chiral nature covariant Derivatives generate



Let's compare putting on another layer with $a=1$ with two alternatives

Here is a sketch showing what a type-1 sol'n could look like

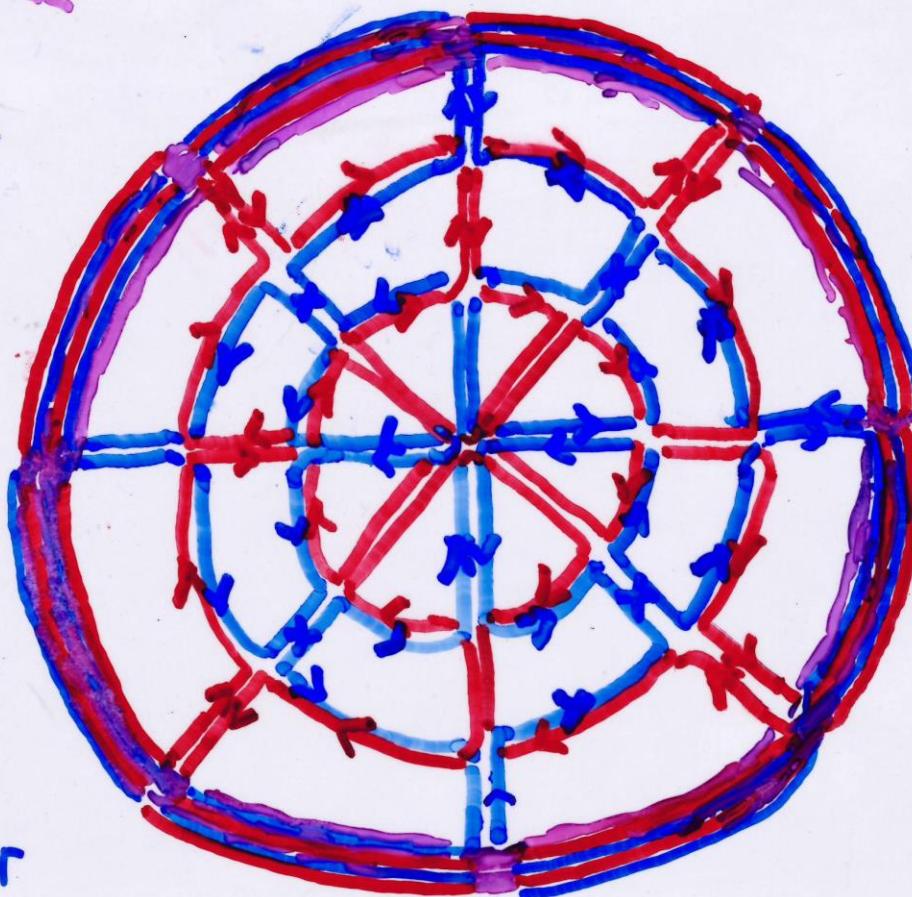
Exterior
region

't Hooft
Polyakov
condensate

$$B_L B_L = \frac{\pm 1}{r^4}$$

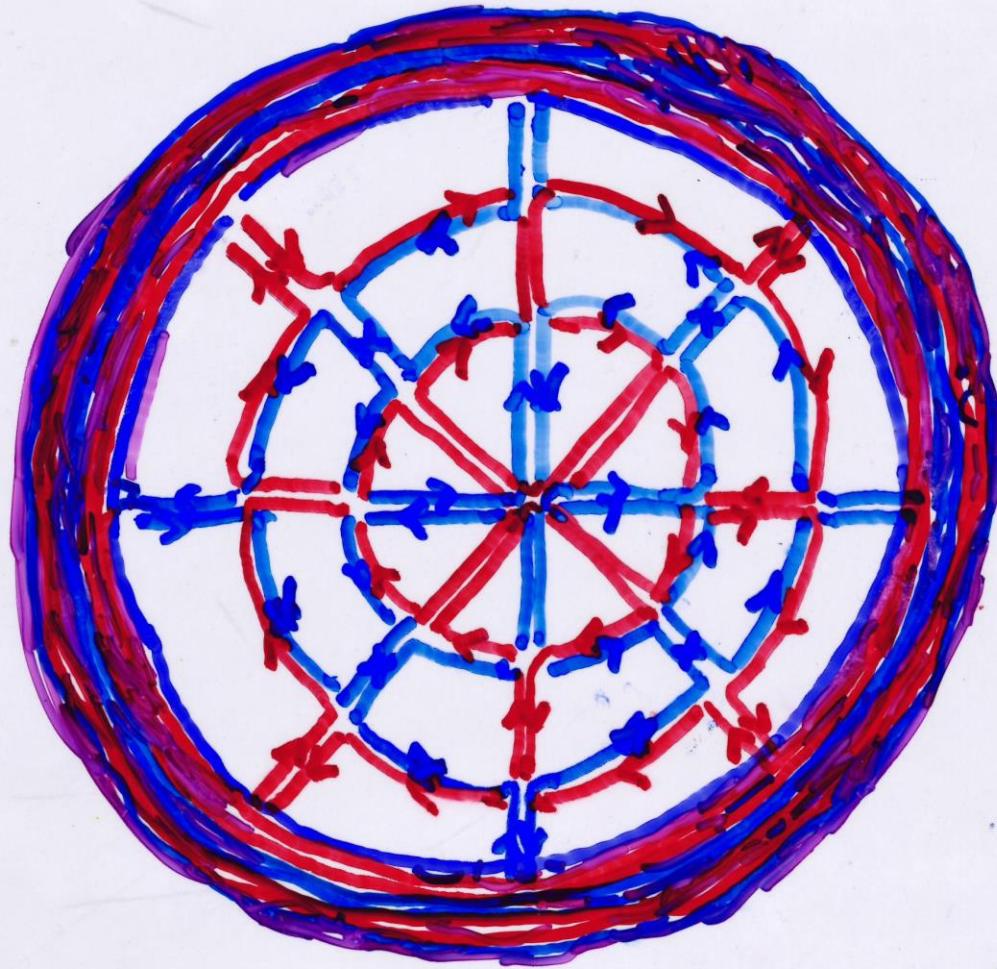
repels all color

$$Q=0$$



topological
 E_i ; B_i
shown in
purple

Here is a sketch showing what a type-2 soln could look like



requires
less topolgs
charge

exterior
region
a sterile
vacuum condensat
with opposite
 $\alpha = -1$ chirality from int

IV. Constructive Field Theory

a. jaffe

A topological stable solution to
the classical (Yang-Mills Maxwell) eq.'ns
provides robust scaffolding



for understanding hadron structure

ORBITAL CHROMODYNAMICS

and the

PION TORNADO

$$\begin{aligned} L(T \rightarrow d \downarrow T^*) & \\ P(T \rightarrow n \downarrow T^*) & \\ \langle L \rangle = 1 & \end{aligned}$$

$$J(J+1) = L(L+1) + S(S+1) + 2L \cdot S$$

den sivers

den sivers



CONTINUUM

John Ralston Piet Mulders

Stan Brodsky Jian-Wei Qiu

Gary Goldstein Leonard Gamberg

EXPLAINED

SU(2) GAUGE THEORY WITH
SPHERICAL SYMMETRY

MODYNAMIC

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glass

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"abelian" gluons can
propagate through $a(r)=0$
condensate

unless ...

[Dark matter]

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