Probing Gluon Sivers Function in J/Ψ Production at EIC

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Gluon Sivers Function (GSF)

Very little is known about GSF apart from a positivity bound

GSF : two gauge links ; process dependence more involved

GSF for a process can in general be written in terms of two independent Sivers functions, one having a C-even operator structure (f –type)   and the other C-odd (d-type)

Bomhof and Mulders, JHEP 02, 029 (2007),
Buffing, AM, Mulders, PRD 88, 054027 (2013)

Burkardt's sum rule still leaves some room for GSF, moreover d type GSF is not constrained by it. Also Sea quark Sivers function is not well known

J/ψ production and J/Ψ+ jet production at EIC are effective ways to probe the GSF

Recent results from COMASS on GSF

COMPASS Collaboration, J. Phys. Conf. Ser 678, 012050 (2016)

Sivers Asymmetry in J/ψ Production

Due to final state interactions, in ep collision, SSA in heavy quarkonium production is non-zero when the heavy quark pair is produced in color octet state

F. Yuan, PRD 78, 014024 (2008)

Consider the process $$e(l) + p^{\uparrow}(P) \to e(l') + J/\psi(P_h) + X,$$

Differential cross section for unpol process

$$d\sigma = \frac{1}{2s}\frac{d^3l'}{(2\pi)^3 2E_l'}\frac{d^3P_h}{(2\pi)^3 2E_h}\int dx d^2k_\perp (2\pi)^4\delta^4(q+k-P_h) \times \frac{1}{Q^4} L^{\nu\nu'}(l,q)\Phi_g^{\mu\mu'}(x,k_\perp)\mathcal{M}_{\mu\nu}^{\gamma^* g\to J/\psi}\mathcal{M}_{\mu'\nu'}^{*\,\gamma^* g\to J/\psi}$$

$$\Phi_g^{\mu\mu'}(x,k_\perp) = \frac{1}{2x}\left\{-g_T^{\mu\mu'} f_1^g(x,k_\perp^2) + \left(\frac{k_\perp^\mu k_\perp^{\mu'}}{M_p^2} + g_T^{\mu\mu'}\frac{k_\perp^2}{2M_p^2}\right) h_1^{\perp g}(x,k_\perp^2)\right\},$$

$$q = l - l'$$
$$Q^2 = -q^2$$

Transversely polarized target $$\Phi_g^{T\mu\mu'}(x,k_\perp) = -\frac{1}{2x} g_T^{\mu\mu'} \frac{\epsilon_T^{\rho\sigma} k_{\perp\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x,k_\perp^2)$$

Sivers Asymmetry in J/ψ Production

Numerator and denominator of the SSA

$$\frac{d\sigma^{\uparrow}}{dydx_B d^2P_{hT}} - \frac{d\sigma^{\downarrow}}{dydx_B d^2P_{hT}} = \frac{\alpha}{8sxQ^4}\left[A_0 + A_1\cos\phi_h\right]\Delta^N f_{g/p^{\uparrow}}(x, P_{hT}).$$

$$\frac{d\sigma^{\uparrow}}{dydx_B d^2P_{hT}} + \frac{d\sigma^{\downarrow}}{dydx_B d^2P_{hT}} = \frac{2\alpha}{8sxQ^4}\left[A_0 + A_1\cos\phi_h\right] f_{g/p}(x, P_{hT}^2),$$

$$\Delta^N f_{g/p^{\uparrow}}(x, P_{hT}, Q_f) = -2 f_{1T}^{\perp g}(x, P_{hT}, Q_f)\frac{(\hat{P}\times P_{hT}).S}{M_p}.$$

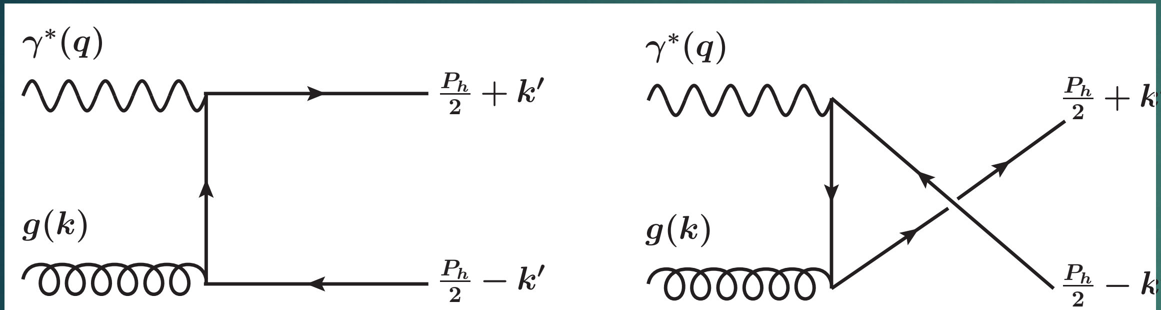
Trento convention

$$z = P.P_h/P.q; \quad x_B = \frac{Q^2}{2P.q}, \quad y = \frac{P.q}{P.l}$$

A_0 and A_1 : calculated in color octet model

AM, S. Rajesh, EPJC 77, 854 (2017)

LO Amplitude of $\gamma^* g \to J/\Psi$



Approach of Boer, Pisano, PRD 86, 094007 (2012); Baier, Ruckl, Z. Phys. C 19, 251 (1983)

Factorized form of the cross section. : initial state partons form a heavy quark pair with definite color and angular momentum quantum numbers, and a non-perturbative matrix element through which the pair forms J/ψ

$$M^{\mu\nu}\left(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J{}^{(1,8a)}]\right) = \sum_{L_z S_z} \int \frac{d^3k'}{(2\pi)^3} \Psi_{LL_z}(k') \langle LL_z; SS_z | JJ_z \rangle Tr[O^{\mu\nu}(q,k,P_h,k') \quad P_{SS_z}(P_h,k')]$$

Eigenfunction of OAM L

Spin projection operator

k' : relative momentum of the heavy quark pair. Taylor expansion about k'=0 gives S wave and P wave amplitudes

Small x TMDs : Bacchetta, Boer, Pisano, Taels, arXiv: 1809. 02056 [hep-ph]

LO Amplitude of $\gamma^* g \to J/\Psi$

$$O^{\mu\nu}(q,k,P_h,k') = \sum_{ij} \langle 3i;3j|8a\rangle g_s(ee_c)\left\{\gamma^\nu \frac{\not{P}_h/2 + \not{k}' - \not{q} + m_c}{(P_h/2+k'-q)^2 - m_c^2}\gamma^\mu (T^b)^{ji} + \gamma^\mu (T^b)^{ji} \frac{\not{P}_h/2 + \not{k}' - \not{k} + m_c}{(P_h/2+k'-k)^2 - m_c^2}\gamma^\nu\right\}$$

SU(3) CG coeff : projects out the color state of heavy quark pair, color singlet or color octet

Spin projection operator : projects out spin singlet and triplet

Example : S wave amplitude :

$$\mathcal{M}^{\mu\nu}[{}^1S_0{}^{(8a)}] = 2i\frac{\sqrt{2}g_s(ee_c)\delta^{ab}}{\sqrt{\pi M}(Q^2+M^2)} R_0(0)\epsilon^{\mu\nu\rho\sigma}k_\rho P_{h\sigma}$$

Radial wave function at the origin is related to long distance matrix elements (LDME)

$$\langle 0 \mid \mathcal{O}_8^{J/\psi}({}^1S_J) \mid 0\rangle = \frac{2}{\pi}(2J+1)|R_0(0)|^2$$

Ko, Lee, Song, PRD 54, 4312 (1996)

SSA in color octet model

Contribution to the Sivers asymmetry comes from

$$A_0 = [1+(1-y)^2]\frac{NQ^2}{y^2 M}\Bigg\{\langle 0 \mid O_8^{J/\psi}(^1S_0) \mid 0\rangle + \frac{4}{3M^2}\frac{(3M^2+Q^2)^2}{(M^2+Q^2)^2}\langle 0 \mid O_8^{J/\psi}(^3P_0) \mid 0\rangle$$
$$+ \frac{8Q^2}{3M^2(M^2+Q^2)^2}\left(\frac{4M^2(1-y)}{1+(1-y)^2}+Q^2\right)\langle 0 \mid O_8^{J/\psi}(^3P_1) \mid 0\rangle$$
$$+ \frac{8}{15M^2\,(M^2+Q^2)^2}\left(6M^4+Q^4+12M^2Q^2\frac{1-y}{1+(1-y)^2}\right)\langle 0 \mid O_8^{J/\psi}(^3P_2) \mid 0\rangle\Bigg\}$$

M : mass of J/ψ

$$Q^2 = x_B y s$$

LDMEs : Ma and Venugopalan , PRL 113, 192301 (2014); Chao et al, PRL 108, 242004 (2012); Sharma and Vitev, PRC 87, 044905 (2013).

Contribution from unpolarized gluons only considered in the denominator

AM, S. Rajesh, EPJC 77, 854 (2017)

Gluon Sivers Function

$$\Delta^N f_{g/p^\uparrow}(x, k_\perp) = 2\mathcal{N}_g(x) f_{g/p}(x, \mu) h(k_\perp) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

D'Alesio, Murgia, Pisano , JHEP 09, 119 (2015)

$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}.$$

$$h(k_\perp) = \sqrt{2e} \frac{k_\perp}{M_1} e^{-k_\perp^2/M_1^2}$$

Best fit parameter sets : SIDIS1 And SIDIS2

$$(a) \quad \mathcal{N}_g(x) = (\mathcal{N}_u(x) + \mathcal{N}_d(x))/2$$ BV-a

$$(b) \quad \mathcal{N}_g(x) = \mathcal{N}_d(x)$$ BV-b

Boer and Vogelsang, PRD 69, 094025 (2004)

Best fit parameters for u and d quark Sivers function from

Anselmino et al, JHEP 04, 046 (2017)

Non-universality :

$$\Delta^N_{\rm DY} f_{g/p^\uparrow}(x, k_\perp) = -\Delta^N_{\rm SIDIS} f_{g/p^\uparrow}(x, k_\perp)$$

TMD Evolution

$$f_{1T}^{\perp g}(x, \boldsymbol{P}_{hT}, Q_f) = -\frac{1}{2\pi P_{hT}} \int_0^\infty db_\perp b_\perp J_1(P_{hT} b_\perp) f_{1T}^{\prime\perp g}(x, b_\perp, Q_f)$$

$$f_{g/p}(x, P_{hT}, Q_f) = \frac{1}{2\pi} \int_0^\infty db_\perp b_\perp J_0(P_{hT} b_\perp) f_{g/p}(x, b_\perp, Q_f)$$

Aybat, Rogers, PRD 83, 114042 (2011); Aybat, Collins, Qiu, Rogers, PRD 85, 034043 (2012)

Derivative of Sivers function obeys same evolution equation as unpol TMD

$$f_{1T}^{\prime\perp g}(x, b_\perp, Q_f) = f_{1T}^{\prime\perp g}(x, b_\perp, Q_i) \exp\left\{ -\int_{c/b_*}^{Q_f} \frac{d\mu}{\mu} \left(A \log\left(\frac{Q_f^2}{\mu^2}\right) + B \right) \right\} \times \exp\left\{ -\left[g_1^{\text{sivers}} + \frac{g_2}{2} \log\frac{Q_f}{Q_0} \right] b_\perp^2 \right\}$$

Perturbative expansion for A and B

Initial scale of TMDs $Q_i = c/b_*(b_\perp)$ $c = 2e^{-\gamma_E}$

Final scale $Q_f = M$

$$b_*(b_\perp) = \frac{b_\perp}{\sqrt{1 + \left(\frac{b_\perp}{b_{max}}\right)^2}} \approx b_{max}; \ (b_\perp \to \infty)$$

$$b_*(b_\perp) \approx b_\perp; \quad (b_\perp \to 0)$$

Best fit parameters from Echevarria et al, PRD 89, 074013 (2014)

TMD Evolution

$$f_1^g(x,b_\perp,Q_i) = \sum_{i=g,q} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{i/g}(x/\hat{x}, b_\perp, \alpha_s, Q_i) f_{i/p}(\hat{x}, c/b_*) + \mathcal{O}(b_\perp \Lambda_{QCD}),$$

TMDs at initial scale : Coefficient function is calculated perturbatively for each TMD

At leading order

$$f_1^g(x,b_\perp,Q_i) = f_{g/p}(x, c/b_*) + \mathcal{O}(\alpha_s),$$

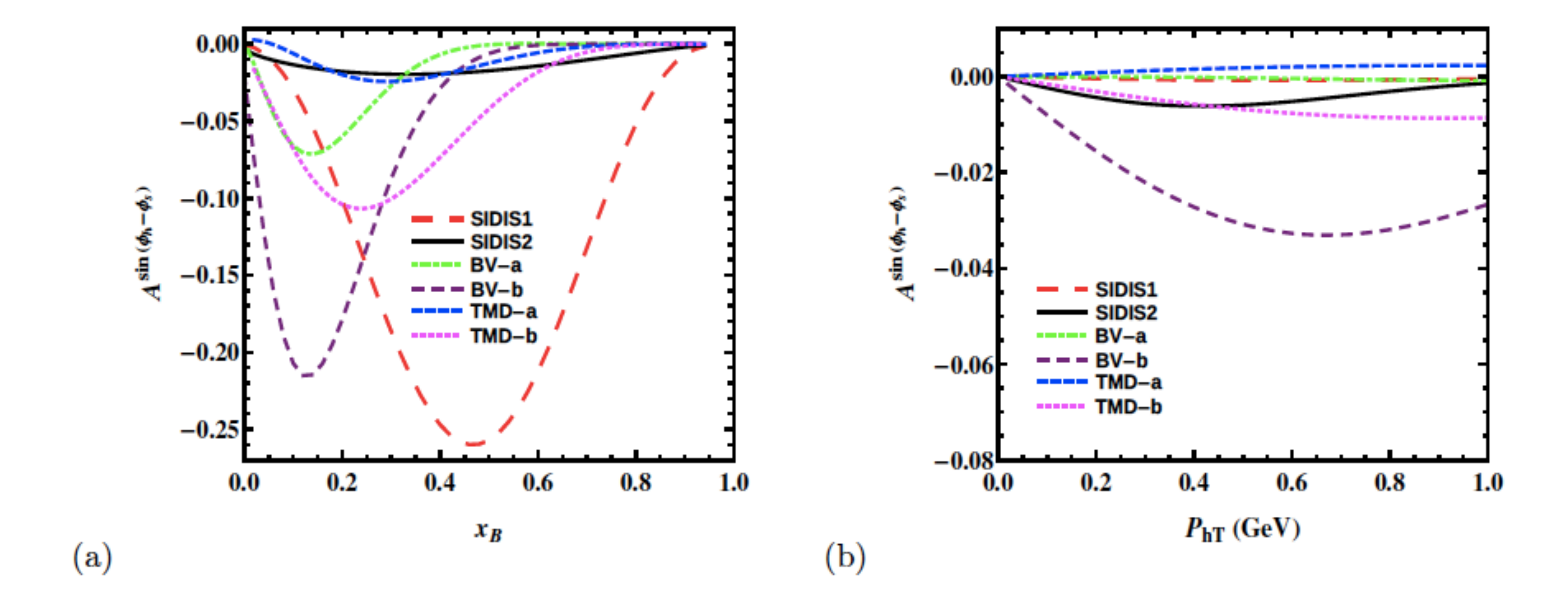
$$f_{1T}^{\prime\perp g}(x,b_\perp,Q_i) \simeq \frac{M_p b_\perp}{2} T_{g,F}(x,x,Q_i)$$

Qiu-Sterman function

$$T_{g,F}(x,x,Q_i) = \mathcal{N}_g(x) f_{g/p}(x,Q_i)$$

Two choices of N_g (Same as before): TMD –a and TMD –b parametrizations

Numerical results

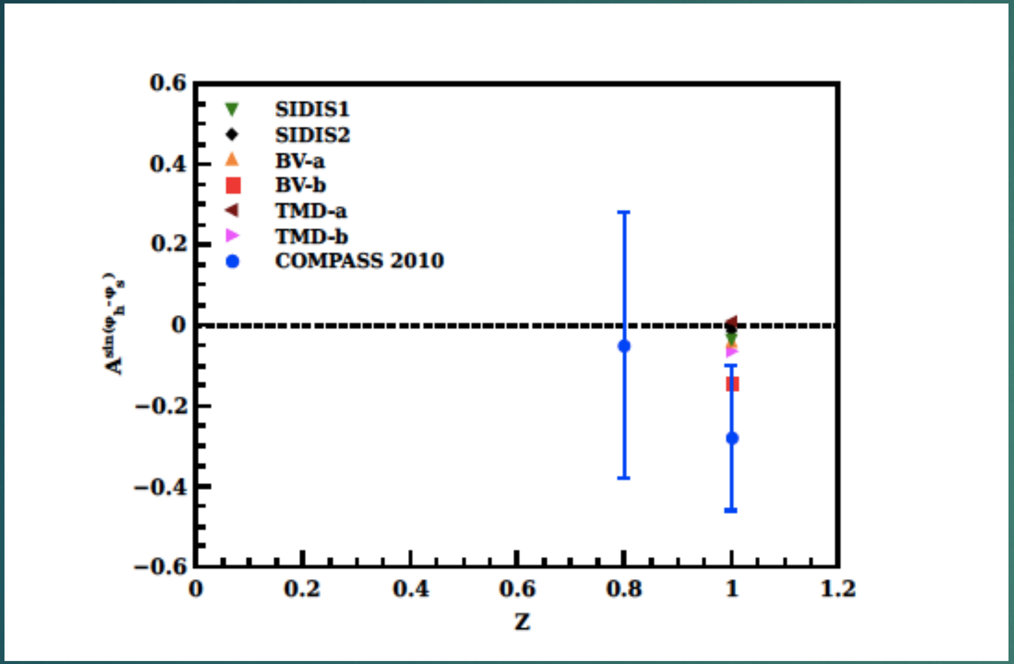


Sivers asymmetry for EIC $\sqrt{s} = 45\ GeV$ using different parametrizations

Integration ranges are $0 < P_{hT} < 1\ GeV; 0.1 < y < 0.9; 0.001 < x_B < 0.9$

AM, S. Rajesh, EPJC 77, 854 (2017)

Numerical Results



Sivers asymmetry compared with COMPASS data

Data from J. Phys. Conf. Ser. 678, 012050 (2016) (COMPASS)

$$\sqrt{s} = 17.2\ GeV; 0 < P_{hT} < 1\ GeV;$$
$$0.1 < y < 0.9; 0.0001 < x_B < 0.65$$

All parametrizations give negative asymmetry

AM, S. Rajesh, EPJC 77, 854 (2017)

BV-b gives results within the error bar of the experiment

Gluon Sivers function in Inclusive Photoproduction of J/ψ

Consider inclusive process $e(l) + p^{\uparrow}(P) \to J/\psi(P_h) + X$

In the kinematical limit when the photon is almost real (forward scattering)

Dominating subprocess is photon-gluon fusion $\gamma(q) + g(k) \to J/\psi(P_h) + g(p_g)$

Two types of contributions (1) Direct : photon interacts electromagnetically with partons in the proton (2) Resolved : photon acts as a source of partons and they interact strongly with the partons in the proton

We consider only direct photoproduction , resolved photo production mainly contributes in low z region

$$z = \frac{P.P_h}{P.q}$$

Can be determined using Jacquet-Blondel Method without detecting final lepton

LO photon-gluon fusion $\gamma + g \to J/\psi$ Contributes at z=1: removed using cut on z

Gluon Sivers function in inclusive photoproduction of J/ψ

Diffractive process contributes at $z \approx 1; \quad P_{hT} \approx 0$

Ryskin, Z. Phys. C 57, 89 (1993)

Gluon and heavy quark fragmentation for larger values of P_{hT}

To choose inelastic process we have used the kinematical cut $0.3 < z < 0.9$

Contribution from photon-quark fusion negligible compared to photon-gluon fusion

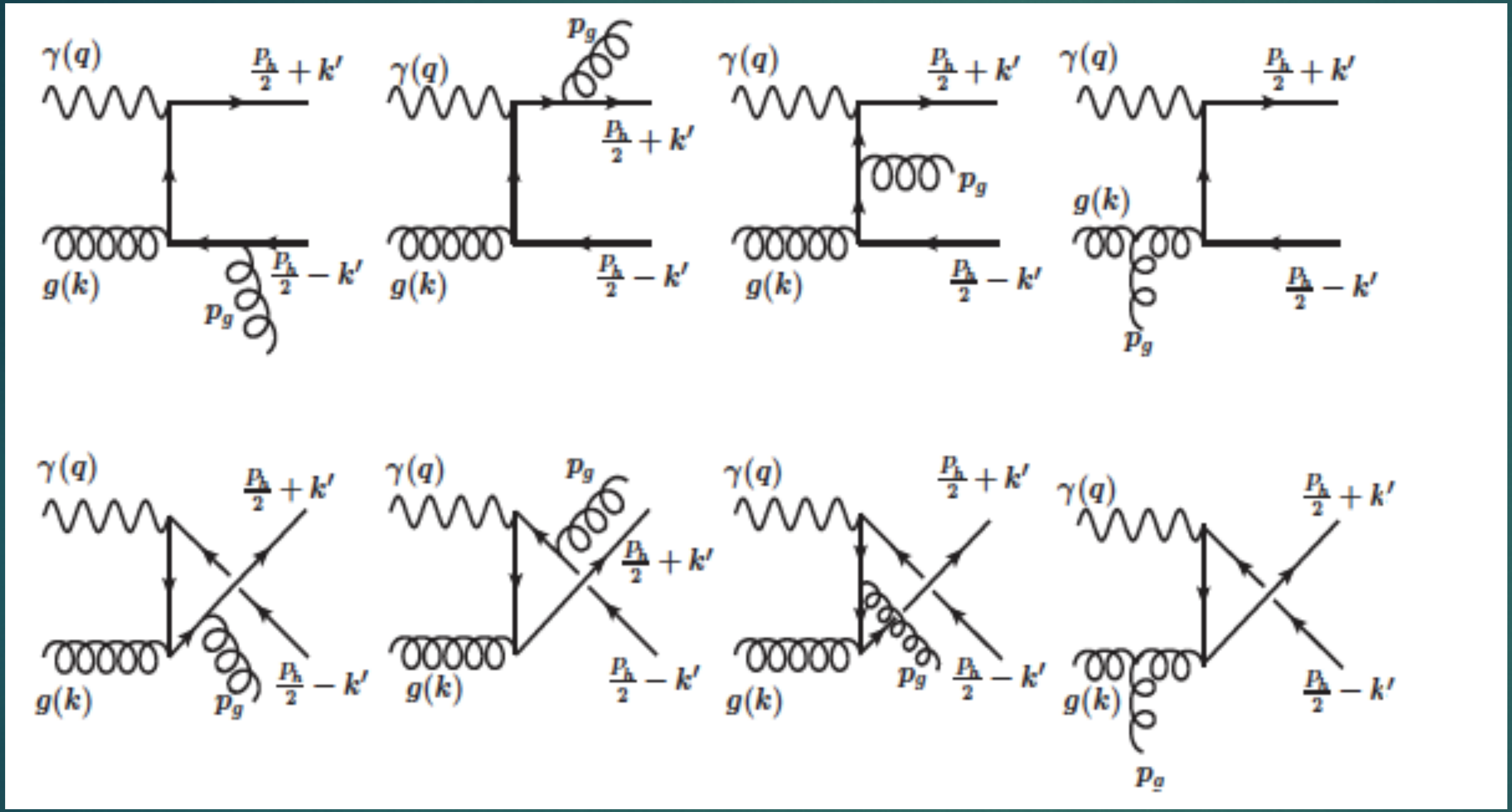
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

Final state heavy quark pair is produced unpolarized in photon-gluon fusion ; contribution to the numerator of the asymmetry comes mainly from gluon Sivers function

Linearly polarized gluons do not contribute to the denominator as long as the lepton is unpolarized

D'Alesio, Flore, Murgia, PRD 95, 094002 (2017); Anselmino et al, PRD 70, 074025 (2004)

Inclusive photoproduction of J/ψ



We follow the same approach as before to calculate the amplitude for heavy quarkonium production in color octet model

Virtual diagrams contribute at z=1

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PRD 98, 014007 (2018)

Inclusive photoproduction of J/Ψ

Numerator of the asymmetry

Amplitude calculated in NRQCD

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = \frac{d\sigma^{ep^{\uparrow}\to J/\psi X}}{dz d^2 \boldsymbol{P}_T} - \frac{d\sigma^{ep^{\downarrow}\to J/\psi X}}{dz d^2 \boldsymbol{P}_T}$$
$$= \frac{1}{2z(2\pi)^2}\int dx_\gamma dx_g d^2\boldsymbol{k}_{\perp g} f_{\gamma/e}(x_\gamma)\Delta^N f_{g/p^{\uparrow}}(x_g, \boldsymbol{k}_{\perp g})$$
$$\times\, \delta(\hat{s} + \hat{t} + \hat{u} - M^2)\frac{1}{2\hat{s}}|\mathcal{M}_{\gamma+g\to J/\psi+g}|^2,$$

Weizsacker-Williams distribution for photons inside an electron

Frixione et al, PLB, 319, 339 (1993)

$$f_{\gamma/e}(x_\gamma) = \frac{\alpha}{2\pi}\left[2m_e^2 x_\gamma\left(\frac{1}{Q^2_{min}} - \frac{1}{Q^2_{max}}\right) + \frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2_{max}}{Q^2_{min}}\right]$$

$$Q^2_{min} = m_e^2\frac{x_\gamma^2}{1-x_\gamma}$$

Same parametrizations for Sivers function and unpolarized TMD as before

Inclusive photoproduction of J/Ψ

Amplitude can be written as

$$\mathcal{M}\left(\gamma g \to Q\bar{Q}[^{2S+1}L_J{}^{(1,8)}](P_h)+g\right)=\sum_{L_zS_z}\int\frac{d^3\boldsymbol{k}'}{(2\pi)^3}\Psi_{LL_z}(\boldsymbol{k}')\langle LL_z;SS_z|JJ_z\rangle \times\mathrm{Tr}[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')],$$

$$O(q,k,P_h,k')=\sum_{m=1}^{8}\mathcal{C}_m O_m(q,k,P_h,k').$$

Each operator O_m is calculated from individual Feynman diagram, as well as the color factor C_m

$$\mathcal{M}[^{2S+1}S_J{}^{(8)}](P_h,k)=\frac{1}{\sqrt{4\pi}}R_0(0)\mathrm{Tr}[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')]\Big|_{k'=0} =\frac{1}{\sqrt{4\pi}}R_0(0)\mathrm{Tr}[O(0)\mathcal{P}_{SS_z}(0)],$$

Approach of Boer, Pisano, PRD 86, 094007 (2012); Baier, Ruckl, Z. Phys. C 19, 251 (1983)

$$\mathcal{M}[^{2S+1}P_J{}^{(8)}]=-i\sqrt{\frac{3}{4\pi}}R_1'(0)\sum_{L_zS_z}\varepsilon^{\alpha}_{L_z}(P_h)\langle LL_z;SS_z|JJ_z\rangle\frac{\partial}{\partial k'^{\alpha}}\mathrm{Tr}[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')]\Big|_{k'=0} =-i\sqrt{\frac{3}{4\pi}}R_1'(0)\sum_{L_zS_z}\varepsilon^{\alpha}_{L_z}(P_h)\langle LL_z;SS_z|JJ_z\rangle\mathrm{Tr}[O_\alpha(0)\mathcal{P}_{SS_z}(0)+O(0)\mathcal{P}_{SS_z\alpha}(0)]$$

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Amplitude in color octet model

$$s_1 = \hat{s} - M^2, \quad t_1 = \hat{t} - M^2, \quad u_1 = \hat{u} - M^2.$$

$$|\mathcal{M}[{}^3S_1{}^{(8)}]|^2 = \frac{5\pi^3 e_c^2 \alpha_s^2 \alpha}{36M} \langle 0 \mid \mathcal{O}_8^{J/\psi}({}^3S_1) \mid 0 \rangle \frac{512M^2}{s_1^2 t_1^2 u_1^2} \times \left\{ s_1^2(s_1 + M^2)^2 + u_1^2(u_1 + M^2)^2 + t_1^2(t_1 + M^2)^2 \right\}$$

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There are contributions from ${}^3S_1, {}^1S_0, {}^3P_J$ in color octet model

Both color singlet and color octet contributions are included in the denominator

Amplitude squared are calculated using FORM

LDMEs taken from

Chao et al, PRL 108, 242004 (2012); Butenschoen and Kniehl, PRD 84, 051501 (2011) ; Zhang et al, PRL 114, 092006 (2015).

Amplitude squared in color octet model

$$
\begin{aligned}
|\mathcal{M}[{}^3P_1{}^{(8)}]|^2 = & \frac{\pi^3 e_c^2 \alpha_s^2 \alpha}{8M} \langle 0 \mid \mathcal{O}_8^{J/\psi}({}^3P_1) \mid 0 \rangle \frac{2048}{m^2 s_1^4 t_1^4 u_1^4 \, (m^2+u_1)^2} \Big\{ 8 s_1 t_1 u_1 m^{20} + 4 (5 s_1^2 t_1^2 \\
& + (s_1+t_1)^2 u_1^2) m^{18} + 4 u_1 \left(2 \left(s_1^2 - 4 t_1 s_1 + t_1^2 \right) u_1^2 + s_1 t_1 \left(5 s_1^2 + 8 t_1 s_1 + 5 t_1^2 \right) \right) \\
& \times m^{16} + 2 (-2 \left(s_1^2 + 16 t_1 s_1 + t_1^2 \right) u_1^4 + s_1 t_1 \left(12 s_1^2 + 23 t_1 s_1 + 12 t_1^2 \right) u_1^2 + 2 s_1^2 t_1^2 \\
& \times \left(3 s_1^2 - 7 t_1 s_1 + 3 t_1^2 \right)) m^{14} + 2 u_1 (-10 (s_1+t_1)^2 u_1^4 + (2 s_1^4 - 16 t_1 s_1^3 + 71 t_1^2 s_1^2 \\
& - 16 t_1^3 s_1 + 2 t_1^4) u_1^2 + s_1 t_1 \left(-2 s_1^4 + 3 t_1 s_1^3 - 22 t_1^2 s_1^2 + 3 t_1^3 s_1 - 2 t_1^4 \right)) m^{12} \\
& + (- \left(16 s_1^2 + 7 t_1 s_1 + 16 t_1^2 \right) u_1^6 + 2 \left(6 s_1^4 - 31 t_1 s_1^3 + 109 t_1^2 s_1^2 - 31 t_1^3 s_1 + 6 t_1^4 \right) u_1^4 \\
& - s_1 t_1 \left(3 s_1^4 + 16 t_1 s_1^3 + 20 t_1^2 s_1^2 + 16 t_1^3 s_1 + 3 t_1^4 \right) u_1^2 - 2 s_1^2 t_1^2 (2 s_1^4 + 7 t_1 s_1^3 - 10 t_1^2 s_1^2 \\
& + 7 t_1^3 s_1 + 2 t_1^4)) m^{10} + u_1 (\left(-4 s_1^2 + 3 t_1 s_1 - 4 t_1^2 \right) u_1^6 + (12 s_1^4 - 28 t_1 s_1^3 + 159 t_1^2 s_1^2 \\
& - 28 t_1^3 s_1 + 12 t_1^4) u_1^4 + s_1 t_1 \left(5 s_1^4 - 16 t_1 s_1^3 - 45 t_1^2 s_1^2 - 16 t_1^3 s_1 + 5 t_1^4 \right) u_1^2 - s_1^2 t_1^2 \\
& \times \left(3 s_1^4 + 5 t_1 s_1^3 - 12 t_1^2 s_1^2 + 5 t_1^3 s_1 + 3 t_1^4 \right)) m^8 + (3 s_1 t_1 u_1^8 + \left(4 s_1^4 + 55 t_1^2 s_1^2 + 4 t_1^4 \right) \\
& \times u_1^6 + s_1 t_1 \left(3 s_1^4 - 16 t_1 s_1^3 - 87 t_1^2 s_1^2 - 16 t_1^3 s_1 + 3 t_1^4 \right) u_1^4 + s_1 t_1 (2 s_1^6 - t_1 s_1^5 + 21 t_1^2 s_1^4 \\
& - 15 t_1^3 s_1^3 + 21 t_1^4 s_1^2 - t_1^5 s_1 + 2 t_1^6) u_1^2 + s_1^3 (s_1 - t_1)^2 t_1^3 (2 s_1 + t_1)(s_1 + 2 t_1)) m^6 \\
& + s_1 t_1 u_1 (u_1^8 + \left(2 s_1^2 + 3 t_1 s_1 + 2 t_1^2 \right) u_1^6 - \left(s_1^4 + 12 t_1 s_1^3 + 59 t_1^2 s_1^2 + 12 t_1^3 s_1 + t_1^4 \right) u_1^4 \\
& + \left(2 s_1^6 - 7 t_1 s_1^5 + 24 t_1^2 s_1^4 - 7 t_1^3 s_1^3 + 24 t_1^4 s_1^2 - 7 t_1^5 s_1 + 2 t_1^6 \right) u_1^2 - s_1^2 (s_1 - t_1)^2 \\
& \times t_1^2 \left(s_1^2 + t_1 s_1 + t_1^2 \right)) m^4 - s_1^2 t_1^2 u_1^2 (3 u_1^6 + \left(2 s_1^2 + 13 t_1 s_1 + 2 t_1^2 \right) u_1^4 \\
& + \left(5 s_1^4 - 13 t_1 s_1^3 - 7 t_1^2 s_1^2 - 13 t_1^3 s_1 + 5 t_1^4 \right) u_1^2 + s_1^2 (s_1 - t_1)^2 t_1^2) m^2 \\
& + s_1^3 t_1^3 \left(s_1^2 + t_1 s_1 + t_1^2 \right) u_1^3 \left((s_1 - t_1)^2 + 3 u_1^2 \right) \Big\}
\end{aligned}
$$

$$\langle 0 \mid \mathcal{O}_1^{J/\psi}({}^{2S+1}S_J) \mid 0 \rangle = \frac{N_c}{2\pi}(2J+1)|R_0(0)|^2,$$

$$\langle 0 \mid \mathcal{O}_8^{J/\psi}({}^{2S+1}S_J) \mid 0 \rangle = \frac{2}{\pi}(2J+1)|R_0(0)|^2,$$

$$\langle 0 \mid \mathcal{O}_8^{J/\psi}({}^{3}P_J) \mid 0 \rangle = \frac{2N_c}{\pi}(2J+1)|R_1'(0)|^2.$$

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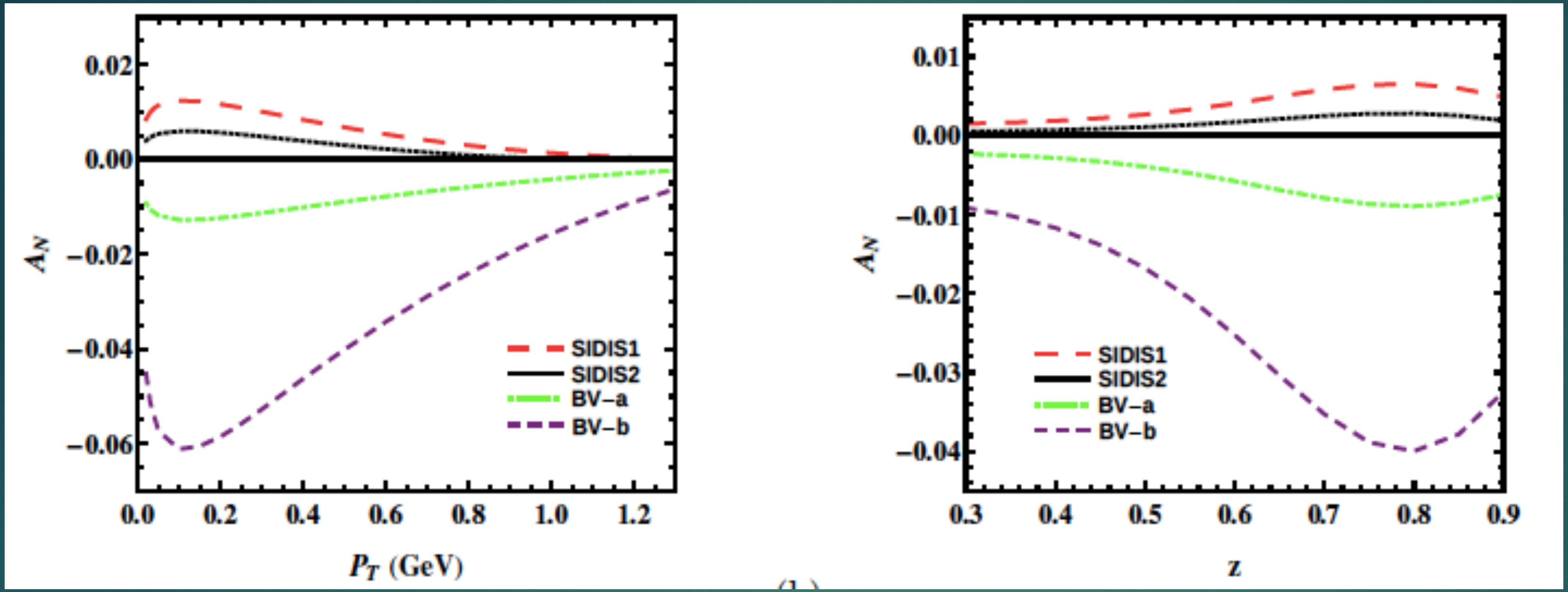
PRD 98, 014007 (2018)

Numerical Results



SSA at EIC

$\sqrt{s} = 45\ GeV$

Integration ranges :

$0 < P_T < 1\ GeV$

$0.3 < z < 0.9$

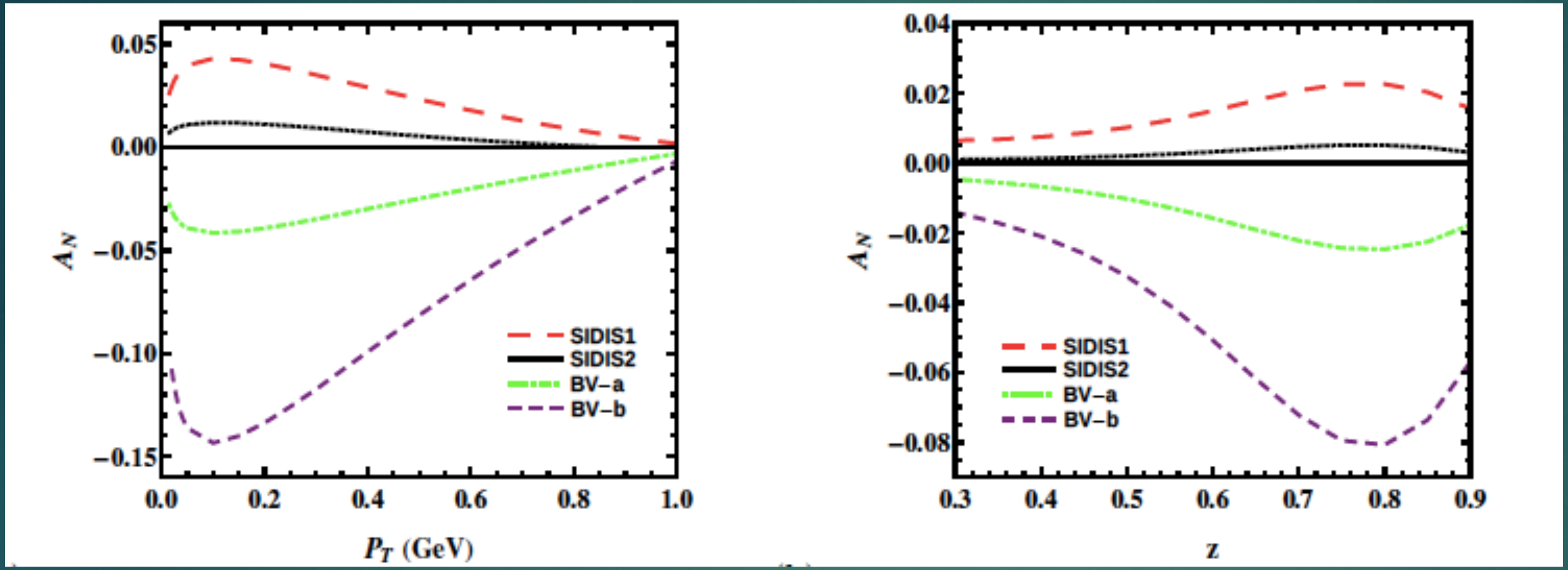
SSA increases by a maximum of 30 % if CS is not included in the denominator

SSA using LDMEs from Chao et al, PRL 108, 242004 (2012) and Zhang et al, PRL 114, 092006 (2015) are similar in magnitude

LDMEs of Butenschoen and Kniehl, PRD 84, 051501 (2011) give smaller asymmetry

S. Rajesh, Raj Kishore, AM , PRD 98, 014007 (2018)

Numerical Results



SSA at COMPASS

$$\sqrt{s} = 17.2\ GeV$$

Integration ranges :

$$0 < P_T < 1\ GeV$$

$$0.3 < z < 0.9$$

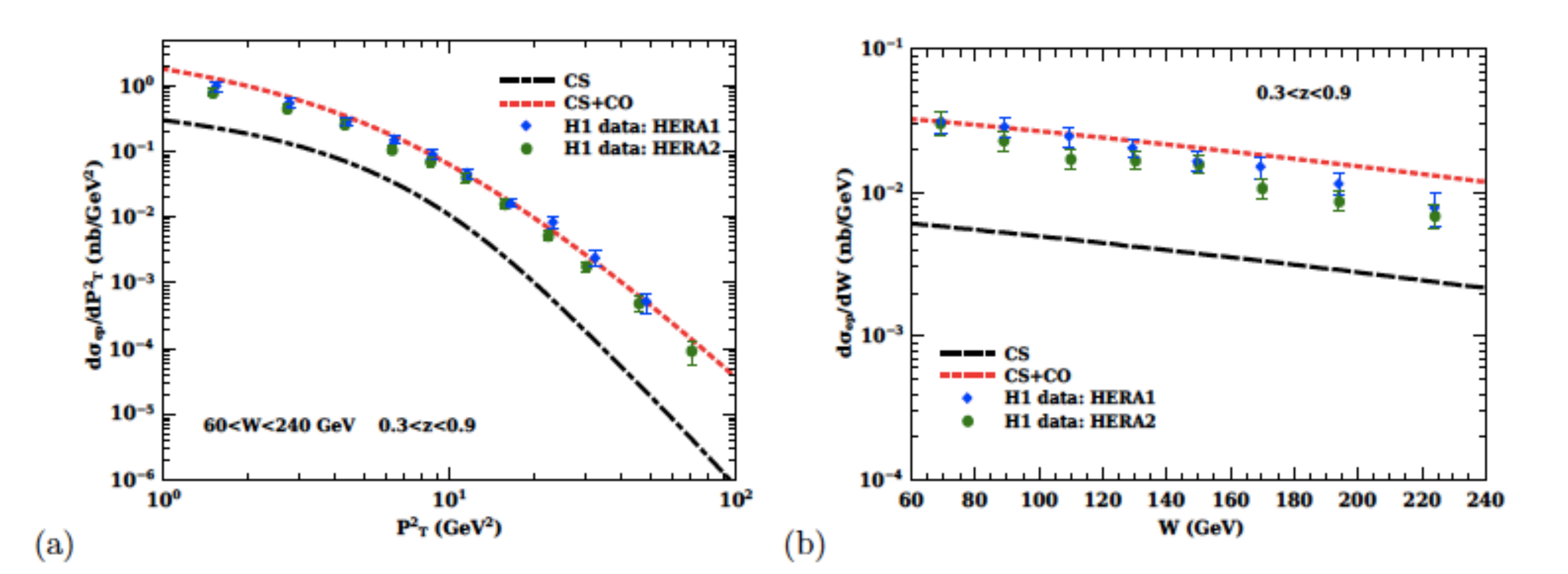
Size and sign of the asymmetry depends on the parametrization of GSF used

Asymmetry increases slightly for higher values of Gaussian widths of unpolarized TMD

SIDIS-2 gives very small asymmetry

S. Rajesh, Raj Kishore, AM
PRD 98, 014007 (2018)

Cross Section for Unpolarized Process



Cross section for the process

$$e + p \rightarrow J/\psi + X$$

$\sqrt{s} = 318 \;\; GeV$ (HERA)

$$< k_{\perp g}{}^2 >= 1 \; (GeV)^2$$

W is the invariant mass of photon-proton system

$$1 < P_T < 10 \; GeV, 60 < W < 240 \; GeV, 0.3 < z < 0.9$$

Data from H1 collaboration, EPJC 25, 25 (2002); EPJC 68, 401 (2010)

LDMEs from Zhang et al, PRL 114, 092006 (2015)

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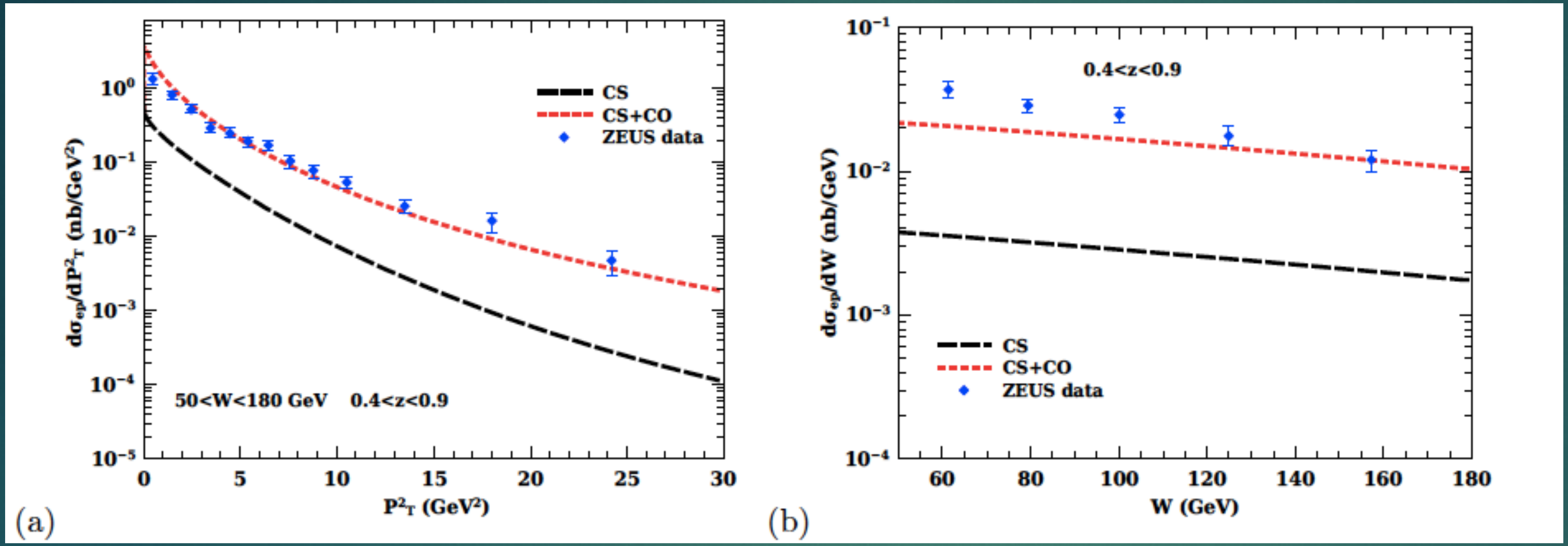
Cross Section for Unpolarized Process



Cross section for the process

$$e + p \to J/\psi + X$$

$$\sqrt{s} = 300\ GeV$$

$$< {k_{\perp g}}^2 >= 1\ (GeV)^2$$

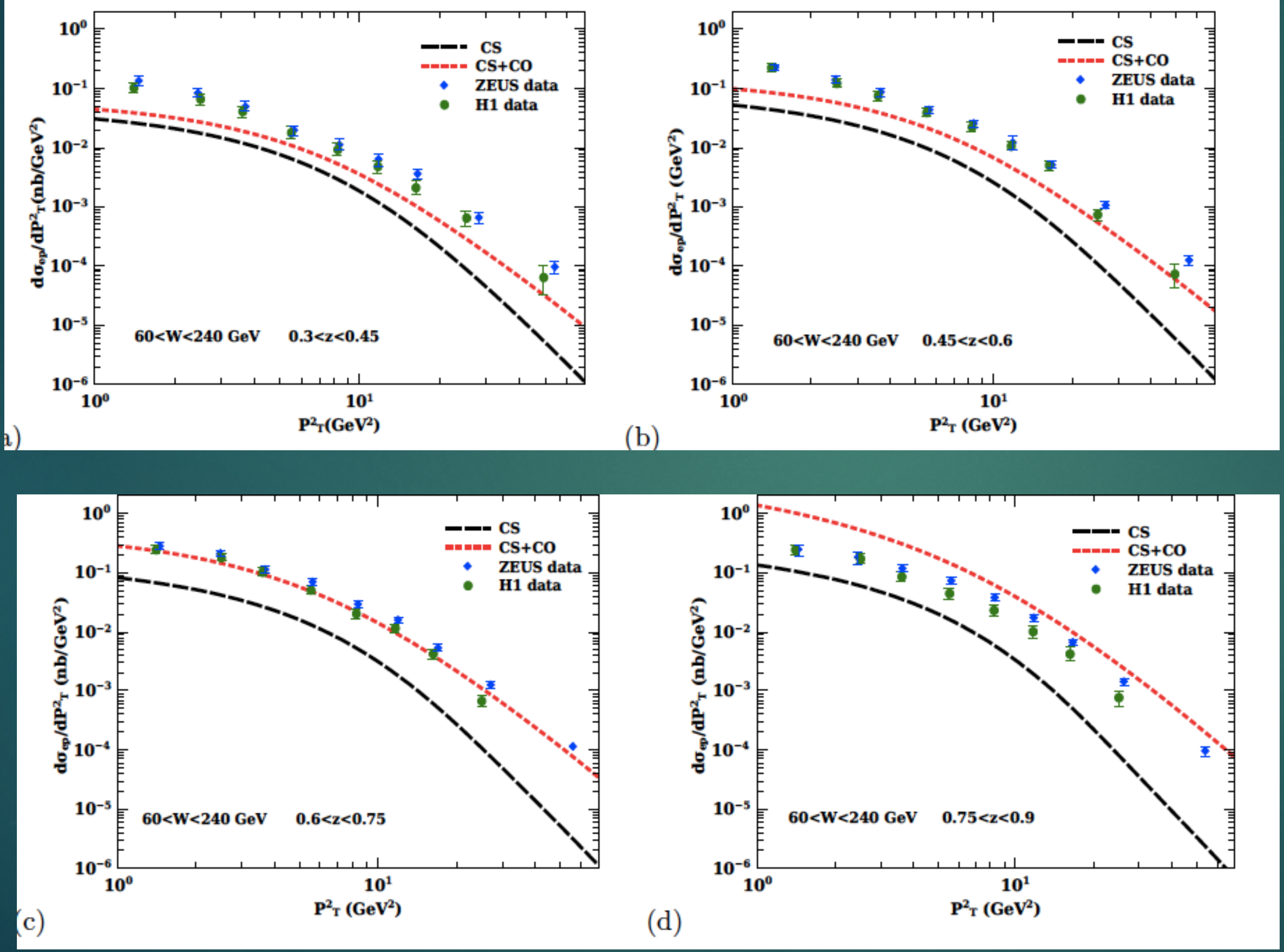
$$1 < P_T < 5\ GeV;\ 50 < W < 180\ GeV,\ 0.4 < z < 0.9$$

Data from ZEUS Collaboration, EPJC 27, 173 (2003)

LDMEs from Zhang et al, PRL 114, 092006 (2015)

S. Rajesh, Raj Kishore, AM
PRD 98, 014007 (2018)

CS vs CO contribution



$$\sqrt{s} = 318 \;\; GeV$$

$$< {k_{\perp g}}^2 >= 1 \; (GeV)^2$$

Data from

H1 collaboration, EPJC 68, 401 (2010)

ZEUS Collaboration, JHEP 02, 071 (2013)

LDMEs from

Zhang et al, PRL 114, 092006 (2015)

Back-to-back Production of J/Ψ and jet in ep Collision

$$e(l) + p^{\uparrow}(P) \rightarrow J/\psi(P_\psi) + \text{jet}(P_j) + X,$$

We consider photo production process where the virtuality of the photon is very small

$$q^2 = -Q^2 \rightarrow 0$$

Dominant subprocess

$$\gamma(q) + g(p) \rightarrow J/\psi(P_\psi) + g(P_j)$$

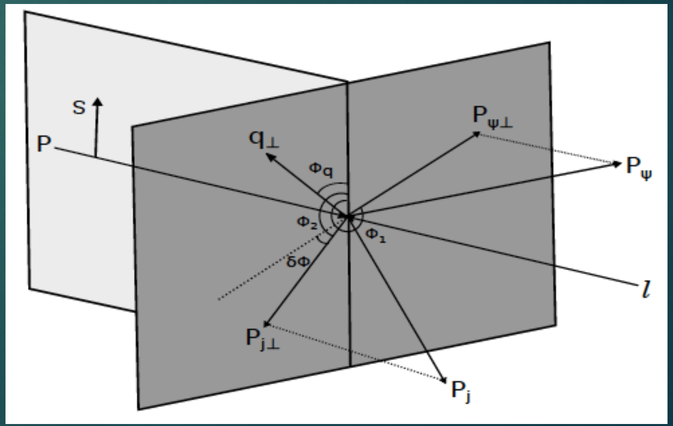
Gluon jet is observed back to back with the J/Ψ

$$z = \frac{P \cdot P_h}{P \cdot q}; \quad z_1 = \frac{P \cdot P_j}{P \cdot q}$$

Energy fractions of the photon carried by J/Ψ and jet in proton rest frame

$$q_\perp = P_{\psi\perp} + P_{j\perp}, \quad K_\perp = (P_{\psi\perp} - P_{j\perp})/2$$

$$|q_\perp| << |K_\perp|$$



Numerical results for EIC

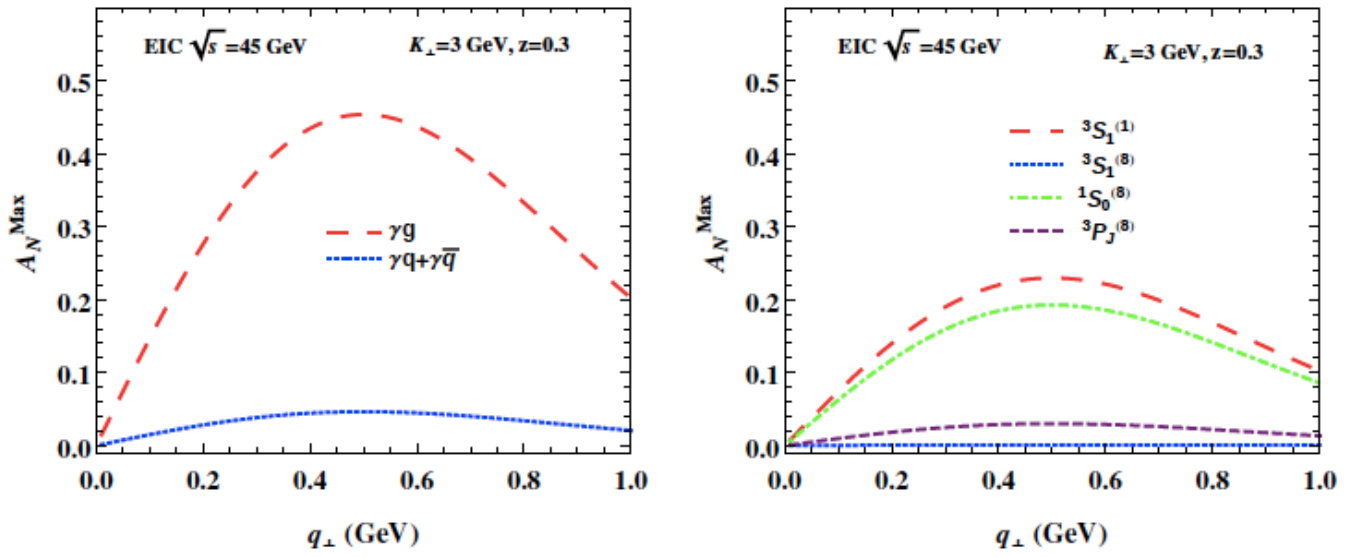


FIG. 2: (color online) Maximized Sivers asymmetry in $e + p^{\uparrow} \to J/\psi + \text{jet} + X$ process as a function of $q_\perp$ at EIC $\sqrt{s} = 45$ GeV. The Sivers function is saturated by adopting $\mathcal{N}_g(x) = 1$ and $\rho = 2/3$ for the parametrization of Sivers function given in Eq.(16). Left panel: for gluon and quark (antiquark) initiated subprocesses contribution to the asymmetry. Right panel: for different CS and CO states contribution to the maximum asymmetry.

Contribution from both CS and CO states in NRQCD to the asymmetry

Asymmetry does not depend too much on the choice of LDMEs

Maximal asymmetry by maximizing SF saturating the positivity bound

R. Kishore, AM, S. Rajesh; arXiv: 1908.03698

Contribution to the asymmetry from individual CS and CO states

Numerical Results for EIC

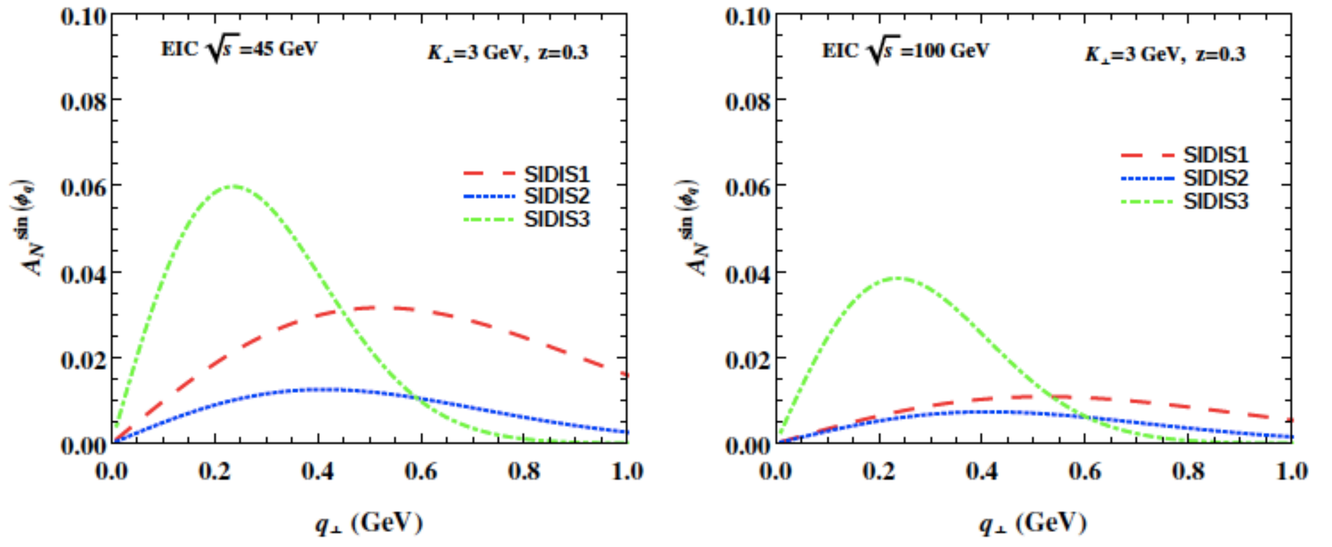


FIG. 3: (color online) The weighted Sivers asymmetry in $e+p^\uparrow \to J/\psi+\text{jet}+X$ process as a function of $q_\perp$ at EIC (a) $\sqrt{s}=45$ GeV (left panel) and (b) $\sqrt{s}=100$ GeV (right panel) using DGLAP evolution approach for SIDIS1, SIDIS2 and SIDIS3 GSF parametrization sets which are given in TABLE I.

Weighted Sivers asymmetry for EIC

TMD evolution not included

Depends on the GSF parametrization used

Both CS and CO contributions in NRQCD are included

SIDIS3 : D'Alesio et al, PRD 99, 063013 (2019)

R. Kishore, AM, S. Rajesh; arXiv: 1908.03698

Conclusion

Single spin asymmetry in J/ψ production in ep collision provides a direct way to access the GSF through the LO photon-gluon process

Sizable negative Sivers asymmetry in color octet model, agrees with COMPASS result

Inclusive photoproduction of J/ψ : wider kinematical region accessible to colliders like EIC

NRQCD based color octet model gives sizable SSA that can access GSF

Size and sign of the SSA depends strongly on the parametrization of the GSF

Theoretical calculation of the cross section using TMDs describe the HERA data well when both CS and CO contributions are taken into account

Back-to back production of J/Ψ and jet is another useful channel to probe GSF. NRQCD based calculation including both CS and CO contributions show sizable Sivers asymmetry, accessible at EIC