CHIRAL ODD GPDS

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Work with Simonetta Liuti,

Osvaldo Gonzalez-Hernandez, et al.

Presentation for Correlations in Parton and Hadron Interactions, CERN Feb 2020

These ideas were developed in Trento ECT*, INT, Jlab, DIS2011, Frascati INF, Transversity 2011, Como 2013, PANIC, POETIC, CPHI-2020 & in consultation with many of you

New extraction of Chiral Even GPDs

Extraction of Generalized Parton Distribution Observables from Deeply Virtual Electron Proton Scattering Experiments

B. Kriesten, A. Meyer, S. Liuti, L. Calero Diaz, D. Keller,
G. Goldstein, J.O. Gonzalez-Hernandez

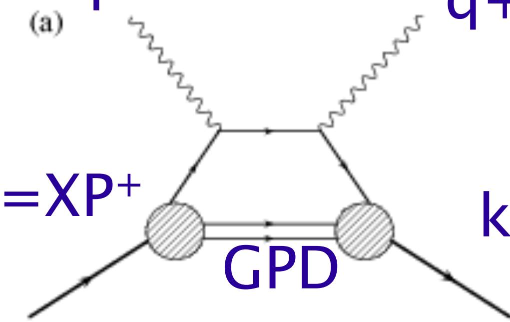
ArXiV: hep-ph 1903.05742 –> PhysRevD

- Analysis of Chiral Even GPDs from DVCS at Jlab Hall A is underway
- Include twist 3 contributions – significant for chiral even <-> odd
    - ✓ **Covariance (not just Lab frame): a desirable feature for the EIC**
    - ✓ **Transparent description of observables that ties into the TMD and other coincidence experiments picture**

- Our Flexible Model *determines Chiral Odd amplitudes from Chiral Even*

DVCS & DVMP $\gamma^*(Q^2)+P \rightarrow (\gamma \text{ or meson}) + P'$

partonic picture



q $\quad$ $q+\Delta$

$\left\{ \begin{array}{l} k_T \\ k^+ = XP^+ \end{array} \right.$ $\quad$ $\left. \begin{array}{l} k'_T = k_T - \Delta \\ k'^+ = (X-\zeta)P^+ \end{array} \right\}$

GPD

P^+ $\quad$ $\left. \begin{array}{l} P'^+ = (1-\zeta)P^+ \\ P'_T = -\Delta \end{array} \right\}$

$\zeta \rightarrow 0$
Regge

Factorized
"handbag"
picture

Quark-spectator
quark+diquark

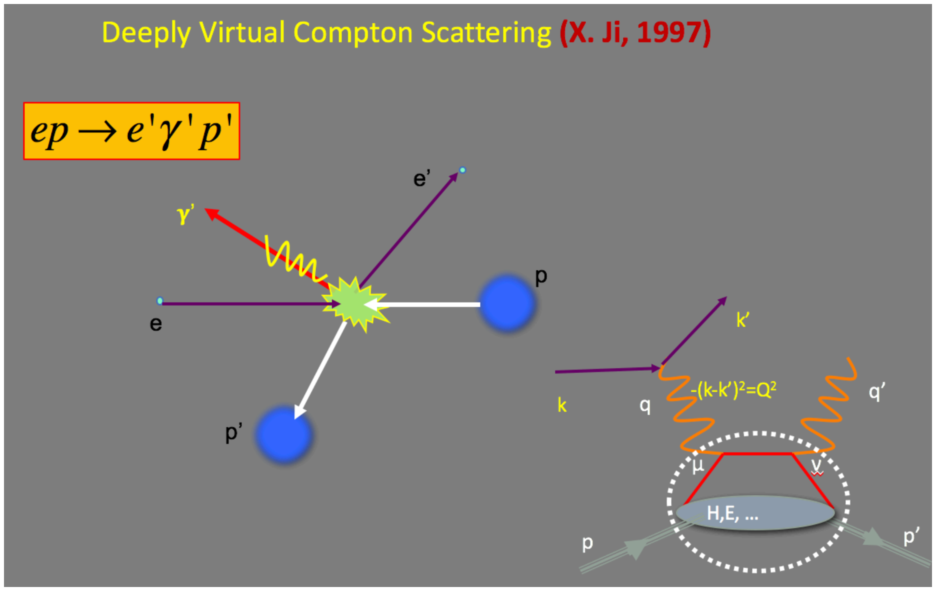
$X>\zeta$ DGLAP $\quad$ Δ_T -> b_T transverse spatial

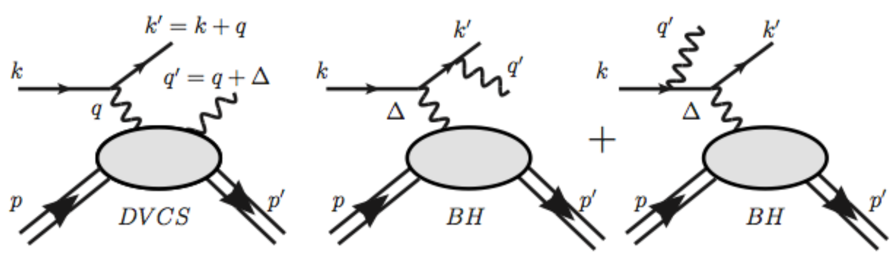
$X<\zeta$ ERBL $\quad$ $x=(X-\zeta/2)/(1-\zeta/2)$; $\xi=\zeta/(2-\zeta)$

see Ahmad, GG, Liuti, PRD79, 054014, (2009) for first chiral odd GPD parameterization Gonzalez, GG, Liuti PRD84, 034007 (2011)

Chiral even & odd processes

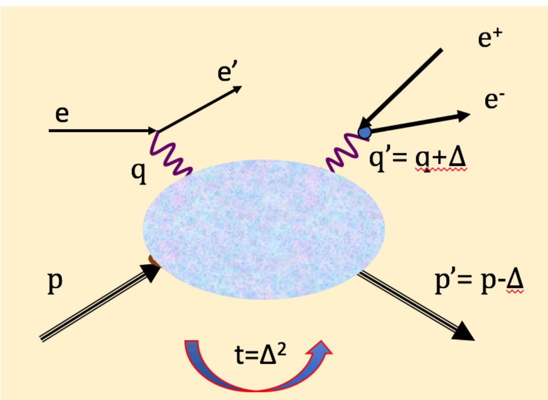
- DVCS

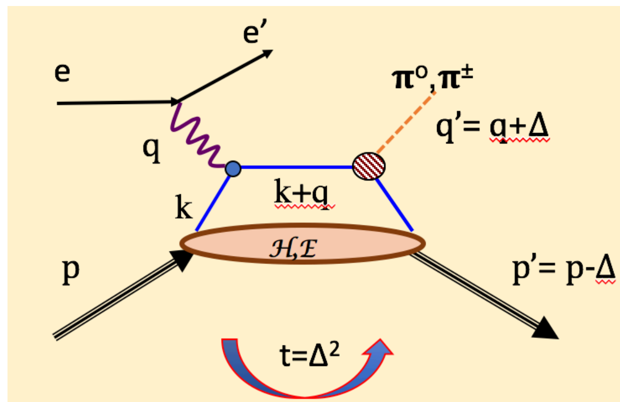


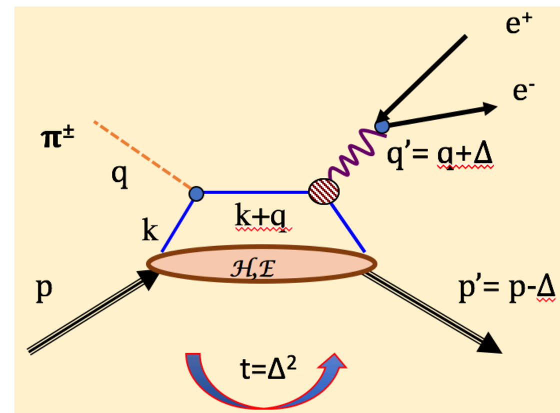


Related processes via GPDs

- TVCS DVMP Drell Yan







GPD definitions – 8 quark + 8 gluon

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[H^q\,\gamma^+ + E^q\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar u(p',\lambda')\left[\tilde H^q\,\gamma^+\gamma_5 + \tilde E^q\,\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$

Chiral even GPDs
-> Ji sum rule

$$\left\langle J_q^x \right\rangle = \tfrac{1}{2}\int dx\left[H(x,0,0)+E(x,0,0)\right]x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\,i\sigma^{+i}\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar u(p',\lambda')\Bigg[H_T^q\,i\sigma^{+i} + \tilde H_T^q\,\frac{P^+\Delta^i-\Delta^+P^i}{m^2}$$

$$+ E_T^q\,\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m} + \tilde E_T^q\,\frac{\gamma^+P^i-P^+\gamma^i}{m}\Bigg]u(p,\lambda$$

Chiral odd GPDs
-> transversity
How to measure and/or parameterize them?

Spectator inspired model of GPDs

- 2 directions –
  - 1. getting good parameterization of H, E & ~H, ~E

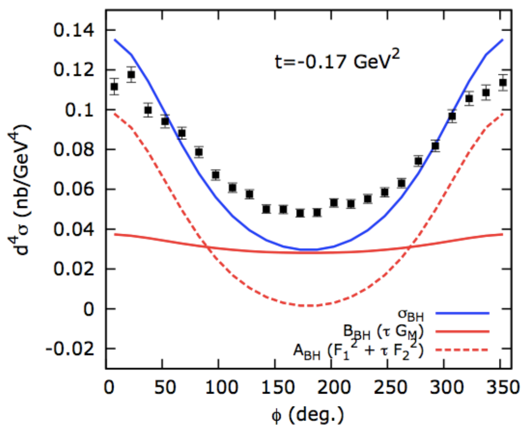
    satisfying many constraints

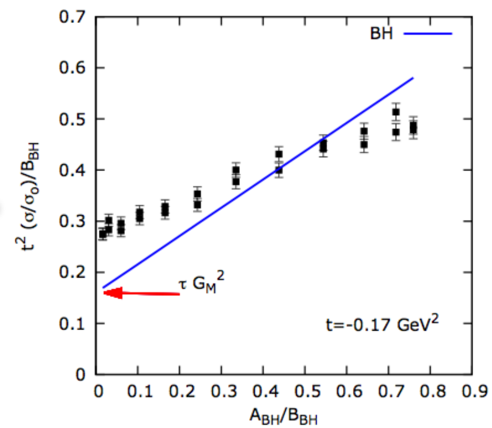
    (see O. Gonzalez-Hernandez, GG, S. Liuti - Phys.Rev. D84, 034007 (2011)

    and B. Kriesten, et al. hep-ph 1903.05742 → PRD)
  - 2. getting 8 spin dependent GPDs
    - Chiral Odd GPDs π^0 production is testing ground (Ahmad, GG, Liuti, PRD79,054014 (2009), Gonzalez, GG, Liuti, **arXiv:1201.6088 [hep-ph]** *J. Phys. G: Nucl. Part. Phys.* **39** 115001 (2012)
- => **Chiral even AND Chiral odd GPDs → normalizations**

  H, E, . . ← helicity amp relations → H_T, E_T, . .
- Small x & Regge behavior in pdf's & DVCS

Some new analyses of chiral even quantities
e.g. using Rosenbluth separation for Bethe-Heitler contribution
& separating H and E





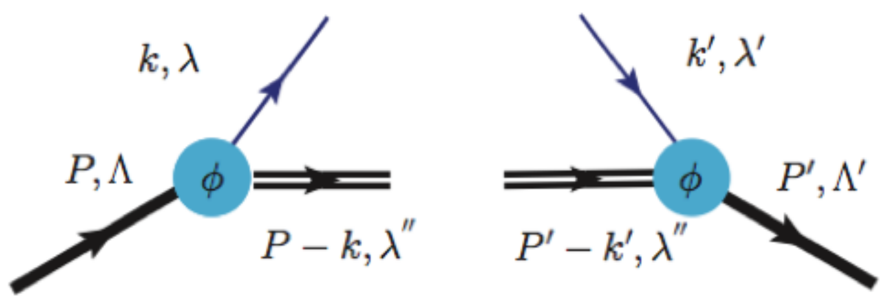
$$\frac{d^5\sigma^{BH}_{unpol}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\Gamma}{t^2}\left[A_{BH}\boxed{\left(F_1^2+\tau F_2^2\right)} + B_{BH}\boxed{\tau G_M^2(t)}\right]$$

$$\frac{d^5\sigma^{\mathcal{I}}_{unpol}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\Gamma}{Q^2(-t)}\left[A_{\mathcal{I}}\left(F_1\Re e\mathcal{H}+\tau F_2\Re e\mathcal{E}\right) + B_{\mathcal{I}}\,G_M\,\Re e(\mathcal{H}+\mathcal{E}) + C_{\mathcal{I}}\,G_M\,\Re e\widetilde{\mathcal{H}}\right]$$

Helicity Amplitudes are the Bridge

Spin(Helicity/Transversity) Basis

$$f_{\Lambda_\gamma,\Lambda;\Lambda'_\gamma,\Lambda'} = \sum_{\lambda,\lambda'} \boxed{g^{\Lambda_\gamma,\Lambda'_{\gamma}(M)}_{\lambda,\lambda'}(x,k_T,\zeta,t;Q^2)} \otimes \boxed{A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_T,\zeta,t)},$$



transversity state

final

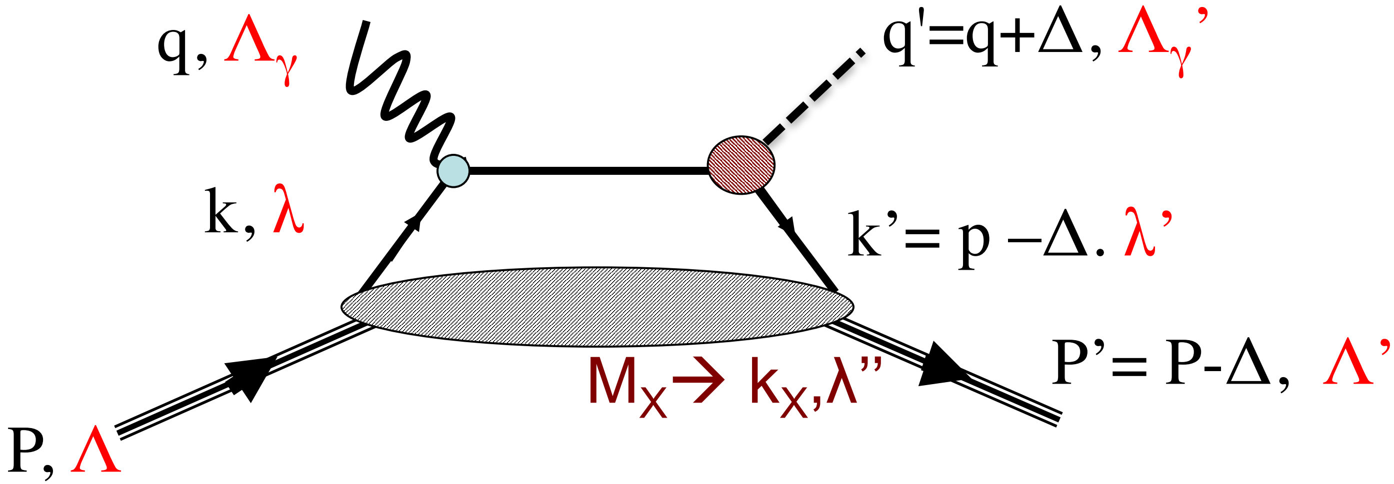
initial

$$A^{T}_{X\,\Lambda'\lambda',\,\Lambda\lambda}$$

proton

quark

Reggeization



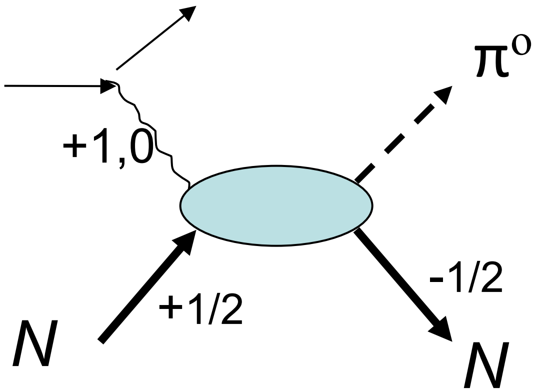
$$A = \mathcal{N} \int \frac{dk_X^2 dk^2}{(k^2 - m^2 - i\epsilon)(k'^2 - m^2 - i\epsilon)} \frac{\rho(k_X^2, k^2) \times (spin\ structure)}{(k_X^2 - M_X^2 - i\epsilon)}$$

>700

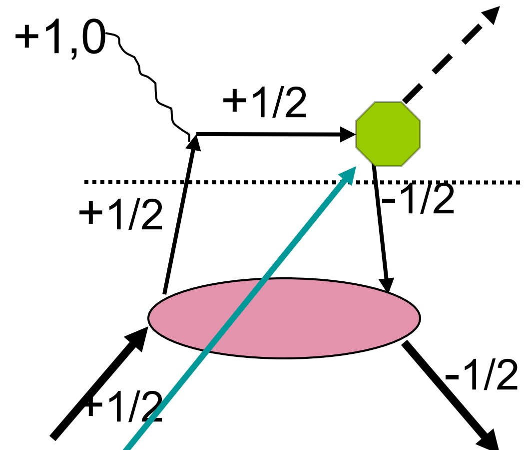
Landshoff, Polkinghorn, Short '71
Brodsky, Close, Gunion '71 Regge behavior required for Compton
AHLT '07, '09
Gorshteyn & Szczepaniak (PRD, 2010)
Brodsky, Llanes-Estrada '07
Brodsky, Llanes, Szczepaniak arXiv:0812.0395

Gonzalez, GG, Liuti, **arXiv:1201.6088 [hep-ph]** *J. Phys. G: Nucl. Part. Phys.* **39** 115001 (2012)

Exclusive Lepto-production of π^o or η, η' to measure **chiral odd GPDs & Transversity**



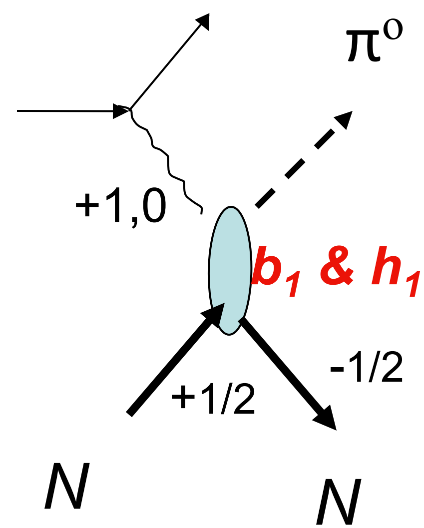
e.g. $f_{+1+,0-}(s,t,Q^2)$



$g_{+1+,0-}\ A_{++,--}$

H_T

q nos of C-odd
1^{--} exchange
ρ & ω
1^{+-} exchange
b_1 & h_1



What about coupling of π to q→q′ ? Assumed γ^5 vertex

Then for m_{quark}=0 has to flip helicity for q→π+q′ and

$$\mathcal{P} = K f_\pi \left\{ \gamma_5 \not{q}' \phi_\pi(\tau) + \gamma_5 \mu_\pi \phi_\pi^{(3)}(\tau) \right\}$$

Naïve twist 3 $\bar{\psi}\ \gamma^5\ \psi$

Rather than $\gamma^\mu\gamma^5$ – does not flip ***twist 2.*** But q' $\gamma^\mu\gamma^5$q fails to **describe transverse γ^***

Helicity amps (q'+N->q+N') are linear combinations of GPDs

$$A_{+,+;+,+} = \sqrt{1-\xi^2}\left[\frac{H+\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E+\tilde{E}}{2}\right]$$

$$A_{-,+;-,+} = \sqrt{1-\xi^2}\left[\frac{H-\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E-\tilde{E}}{2}\right]$$

$$A_{+,+;-,+} = -\frac{\sqrt{t_0-t}}{4M}(E-\xi\tilde{E})$$

$$A_{-,+;+,+} = \frac{\sqrt{t_0-t}}{4M}(E+\xi\tilde{E})$$

T-reversal at ξ =0

for chiral even GPDs and

$$A_{+-,++} = -\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1+\xi}{2}E_T - \frac{1+\xi}{2}\widetilde{E}_T\right]$$

$$A_{++,--} = \sqrt{1-\xi^2}\left[H_T + \frac{t_0-t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T + \frac{\xi}{1-\xi^2}\widetilde{E}_T\right]$$

$$A_{+-,-+} = -\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\,\widetilde{H}_T$$

$$A_{++,+-} = \frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1-\xi}{2}E_T + \frac{1-\xi}{2}\widetilde{E}_T\right],$$

for chiral odd GPDs, where for consistency with previous literature we have

In diquark spectator models $A_{++;++}$, etc. are calculated directly. Inverted -> GPDs

Invert to obtain model parameterization for GPDs

S=0 diquark

Spectator model

$A_{++,-+} = - A_{++,+-}^{*}$

$A_{-+,++} = - A_{+-,++}^{*}$

$A_{++,++} = A_{++,--}$

$$
\begin{aligned}
H(x,\xi,t) &= \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} + A_{-,+;-,+}) - \frac{2M\xi^2}{\Delta(1-\xi^2)}(A_{+,+;-,+} - A_{-,+;+,+}) \\
E(x,\xi,t) &= -\frac{2M}{\Delta}(A_{+,+,-,+} - A_{-,+;+,+}) \\
\tilde{H}(x,\xi,t) &= \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} - A_{-,+;-,+}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+,-,+} + A_{-,+;+,+}) \\
\tilde{E}(x,\xi,t) &= \frac{2M}{\Delta\xi}(A_{+,+,-,+} + A_{-,+;+,+})
\end{aligned}
$$

for chiral even GPDs and

$$
\begin{aligned}
H_T(x,\xi,t) &= \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;-,-} + A_{-,+;+,-}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+;+,-} - A_{-,+;-,-}) \\
\xi E_T(x,\xi,t) \; - \; \tilde{E}_T(x,\xi,t) &= \frac{2M}{\Delta}(A_{+,+;+,-} - A_{-,+;-,-}) \\
E_T(x,\xi,t) \; + \; \tilde{E}_T(x,\xi,t) &= \frac{\Delta}{2M(1-\xi)}\left[2A_{+,+;+,-} + \frac{4M}{\Delta\sqrt{1-\xi^2}}A_{-,+;+,-}\right]
\end{aligned}
$$

double flip

$$
\tilde{H}_T(x,\xi,t) = \frac{4M^2}{\Delta^2\sqrt{1-\xi^2}}A_{-,+;+,-}
$$

S=0 Chiral even <-> odd
Helicity Amplitudes are critical

$$
\begin{aligned}
A^{(0)}_{++,--} &= A^{(0)}_{++,++} \\
A^{(0)}_{++,+-} &= -A^{(0)}_{++,-+} \\
A^{(0)}_{+-,++} &= -A^{(0)}_{-+,++},
\end{aligned}
$$

Invert to get GPDs – same helicity amp sets

$$
\begin{aligned}
\widetilde{H}^0_T &= -(1-\zeta)^2 \frac{M(1-x)}{m+Mx'}\left[E^0 - \frac{\zeta}{2}\widetilde{E}^0\right] \\
E^0_T &= -\frac{(1-\zeta/2)^2}{1-\zeta}\left[2\widetilde{H}^0_T - E^0 + \left(\frac{\zeta/2}{1-\zeta/2}\right)^2 \widetilde{E}^0\right] \\
\widetilde{E}^0_T &= -\frac{\zeta/2(1-\zeta/2)}{1-\zeta}\left[2\widetilde{H}^0_T - E^0 + \widetilde{E}^0\right] \\
H^0_T &= \frac{H^0+\widetilde{H}^0}{2} - \frac{\zeta^2/4}{1-\zeta}\frac{E^0+\widetilde{E^0}}{2} - \frac{\zeta^2/4}{(1-\zeta/2)(1-\zeta)}E^0_T + \frac{\zeta/4(1-\zeta/2)}{1-\zeta}\widetilde{E}^0_T + \widetilde{H}^0_T,
\end{aligned}
$$

$S=0$ double helicity flip amplitude was calculated directly from Eq.(16),

$$
A^{(0)}_{+-,-+} = \frac{t_0-t}{4M}\frac{1}{\sqrt{1-\zeta}}\frac{1}{(1-\zeta/2)}\frac{\tilde{x}}{m+Mx'}\left[E^0 - \frac{\zeta}{2}\widetilde{E}^0\right].
$$

S=1 Chiral even <-> odd

$$
\begin{aligned}
A^{(1)}_{++,--} &= -\frac{x+x'}{1+xx'}\,A^{(1)}_{++,++} \\
A^{(1)}_{+-,-+} &= 0 \\
A^{(1)}_{++,+-} &= -\sqrt{\frac{\langle \tilde{k}_\perp^2 \rangle}{x'^{\,2}+\langle \tilde{k}_\perp^2 \rangle/P^{+\,2}}}\,A^{(1)}_{++,-+} \\
A^{(1)}_{+-,++} &= -\sqrt{\frac{\langle k_\perp^2 \rangle}{x^2+\langle k_\perp^2 \rangle/P^{+\,2}}}\,A^{(1)}_{-+,++},
\end{aligned}
$$

Invert to get GPDs

$$
\begin{aligned}
\widetilde{H}_T^{(1)} &= 0 \\
E_T^{(1)} &= \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)}-\frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)+a\left(E^{(1)}+\frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right] \\
\widetilde{E}_T^{(1)} &= \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)}-\frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)-a\left(E^{(1)}+\frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right] \\
H_T^{(1)} &= -\frac{x+x'}{1+xx'}\left[\frac{H^{(1)}+\widetilde{H}^{(1)}}{2}-\frac{\zeta^2/4}{1-\zeta}\,\frac{E^{(1)}+\widetilde{E}^{(1)}}{2}\right]-\frac{\zeta^2/4}{1-\zeta}E_T^{(1)}+\frac{\zeta/4}{1-\zeta}\widetilde{E}_T^{(1)}
\end{aligned}
$$

First Chiral Even: New Recursive Fitting Procedure e.g. for H and E

- Fit at $\zeta=0$, $t=0$ => $H_q(x,0,0)=q(X)$

  3 parameters per quark flavor $(M_X^q, \Lambda_q, \alpha_q)$ + initial Q_o^2

- Fit at $\zeta=0$, $t\neq0$ $\Rightarrow$

$$\int_0^1 dX\, H^q(X,t) = F_1^q(t)$$

$$\int_0^1 dX\, E^q(X,t) = F_2^q(t),$$

  - 2 parameters per quark flavor (β, p)
    - Regge factor $R \sim X^{-\alpha(t)}$

$$G_{M_X}^{\lambda}(X,t) = \mathcal{N}\frac{X}{1-X}\int d^2\mathbf{k}_\perp \frac{\phi(k^2,\lambda)}{D(X,\mathbf{k}_\perp)}\frac{\phi(k'^2,\lambda)}{D(X,\mathbf{k}_\perp+(1-X)\mathbf{\Delta}_\perp)}$$

Quark-Diquark

- Fit at $\zeta\neq0$, $t\neq0$ $\Rightarrow$ DVCS, DVMP,... data (convolutions of GPDs with Wilson coefficient functions) + lattice results (Mellin Moments of GPDs) Evolution
- Note! This is a multivariable analysis $\Rightarrow$ see e.g. Moutarde, Kumericki and D. Mueller, Guidal and Moutarde

Parametric Form

$$F(X,\zeta,t) = \mathcal{N} G^{M_\Lambda}_{M_X,m}(X,\zeta,t)\, R^{\alpha,\alpha'}_p(X,\zeta,t)$$

R ✖ Dq

Including twist 3 in DVCS helicity amps
(arXiV 1903.05742 Kriesten et al.)

$$f^{01}_{++} = W^{\gamma^1+i\gamma^2}_{++} + W^{(\gamma^1+i\gamma^2)\gamma_5}_{++} = \frac{K}{\sqrt{Q^2}}\frac{\sqrt{t_0-t}}{2M}\, e^{i\phi}\Big[2\widetilde{\mathcal{H}}_{2T} + (1-\xi)(\mathcal{E}_{2T}+\widetilde{\mathcal{E}}_{2T}) + 2\widetilde{\mathcal{H}}'_{2T} + (1-\xi)(\mathcal{E}'_{2T}+\widetilde{\mathcal{E}}'_{2T})\Big] \tag{96a}$$

$$f^{01}_{--} = W^{\gamma^1+i\gamma^2}_{--} - W^{(\gamma^1+i\gamma^2)\gamma_5}_{--} = \frac{K}{\sqrt{Q^2}}\frac{\sqrt{t_0-t}}{2M}\, e^{i\phi}\Big[2\widetilde{\mathcal{H}}_{2T} + (1+\xi)(\mathcal{E}_{2T}-\widetilde{\mathcal{E}}_{2T}) + 2\widetilde{\mathcal{H}}'_{2T} + (1+\xi)(\mathcal{E}'_{2T}-\widetilde{\mathcal{E}}'_{2T})\Big] \tag{96b}$$

$$f^{01}_{+-} = W^{\gamma^1+i\gamma^2}_{+-} + W^{(\gamma^1+i\gamma^2)\gamma_5}_{+-} = -\frac{K}{\sqrt{Q^2}}\frac{t_0-t}{4M^2}\, e^{2i\phi}\left[\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\right] \tag{96c}$$

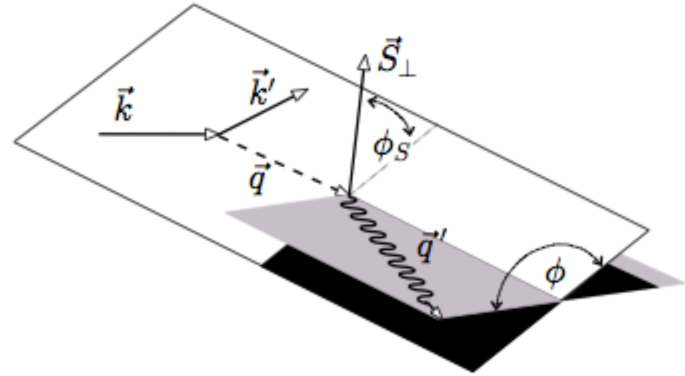
$$f^{01}_{-+} = W^{\gamma^1+i\gamma^2}_{-+} - W^{(\gamma^1+i\gamma^2)\gamma_5}_{-+} = \frac{K}{\sqrt{Q^2}}\Big[(\mathcal{H}_{2T}+\mathcal{H}'_{2T}) + \frac{t_0-t}{4M^2}\left(\widetilde{\mathcal{H}}_{2T}+\widetilde{\mathcal{H}}'_{2T}\right) + \frac{\xi}{1-\xi^2}\left(\widetilde{\mathcal{E}}_{2T}+\widetilde{\mathcal{E}}'_{2T}\right) - \frac{\xi^2}{1-\xi^2}\left(\mathcal{E}_{2T}+\mathcal{E}'_{2T}\right)\Big] \tag{96d}$$

Corresponding to these are twist 3 chiral even amps for π^0 from longitudinal photons
σ_{TL} or σ_{LT} contributions

Observables

- Cross sections
- Asymmetries

Cross sections for π^0



$$
\begin{aligned}
\frac{d^4\sigma}{dx_{Bj}dyd\phi dt} = & \ \Gamma \Big\{ \Big[F_{UU,T} + \epsilon_L F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon_L(\epsilon+1)} \cos\phi F_{UU}^{\cos\phi} + h\sqrt{2\epsilon_L(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} \Big] \\
& + S_{||} \Big[\sqrt{2\epsilon_L(\epsilon+1)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \Big(\sqrt{1-\epsilon^2} \sin\phi F_{LL} + \sqrt{2\epsilon_L(1-\epsilon)} \cos\phi F_{LL}^{\cos\phi} \Big) \Big] \\
& + S_\perp \Big[\sin(\phi-\phi_S) \Big(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \Big) + \epsilon \Big(\sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \Big) \\
& + \sqrt{2\epsilon(1+\epsilon)} \Big(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \Big) \Big] \\
& + S_\perp h \Big[\sqrt{1-\epsilon^2} \cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \Big(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \Big) \Big] \Big\}
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
F_{UU,T} &= \frac{d\sigma_T}{dt}, \quad F_{UU,L} = \frac{d\sigma_L}{dt}, \quad F_{UU}^{\cos\phi} = \frac{d\sigma_{LT}}{dt}, \\
F_{UU}^{\cos 2\phi} &= \frac{d\sigma_{TT}}{dt}, \quad F_{LU}^{\sin\phi} = \frac{d\sigma_{LT'}}{dt}
\end{aligned}
$$

Unpolarized cross section components

$$F_{UU,T} = \frac{d\sigma_T}{dt}, \quad F_{UU,L} = \frac{d\sigma_L}{dt}, \quad F_{UU}^{\cos\phi} = \frac{d\sigma_{LT}}{dt},$$
$$F_{UU}^{\cos 2\phi} = \frac{d\sigma_{TT}}{dt}, \quad F_{LU}^{\sin\phi} = \frac{d\sigma_{LT'}}{dt}$$

$$F_{UU,T} = \mathcal{N}\left[\mid f_{10}^{++} \mid^2 + \mid f_{10}^{+-} \mid^2 + \mid f_{10}^{-+} \mid^2 + \mid f_{10}^{--} \mid^2\right]$$

$$F_{UU,L} = \mathcal{N}\left[\mid f_{00}^{++} \mid^2 + \mid f_{00}^{+-} \mid^2\right]$$

$$F_{UU}^{\cos 2\phi} = -\mathcal{N}\, 2\Re e\left[(f_{10}^{++})^*(f_{10}^{--}) - (f_{10}^{+-})^*(f_{10}^{-+})\right]$$

$$F_{UU}^{\cos\phi} = -\mathcal{N}\Re e\left[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\right]$$

$$F_{LU}^{\sin\phi} = \mathcal{N}\Im m\left[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\right]$$

Asymmetries:
Longitudinal polarizations

$$F_{UL}^{\sin\phi} = \frac{1}{\sqrt{2}}\mathcal{N}\,\Im m\left[(f_{00}^{+-})^*(f_{10}^{+-} - f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} + f_{10}^{--})\right]$$

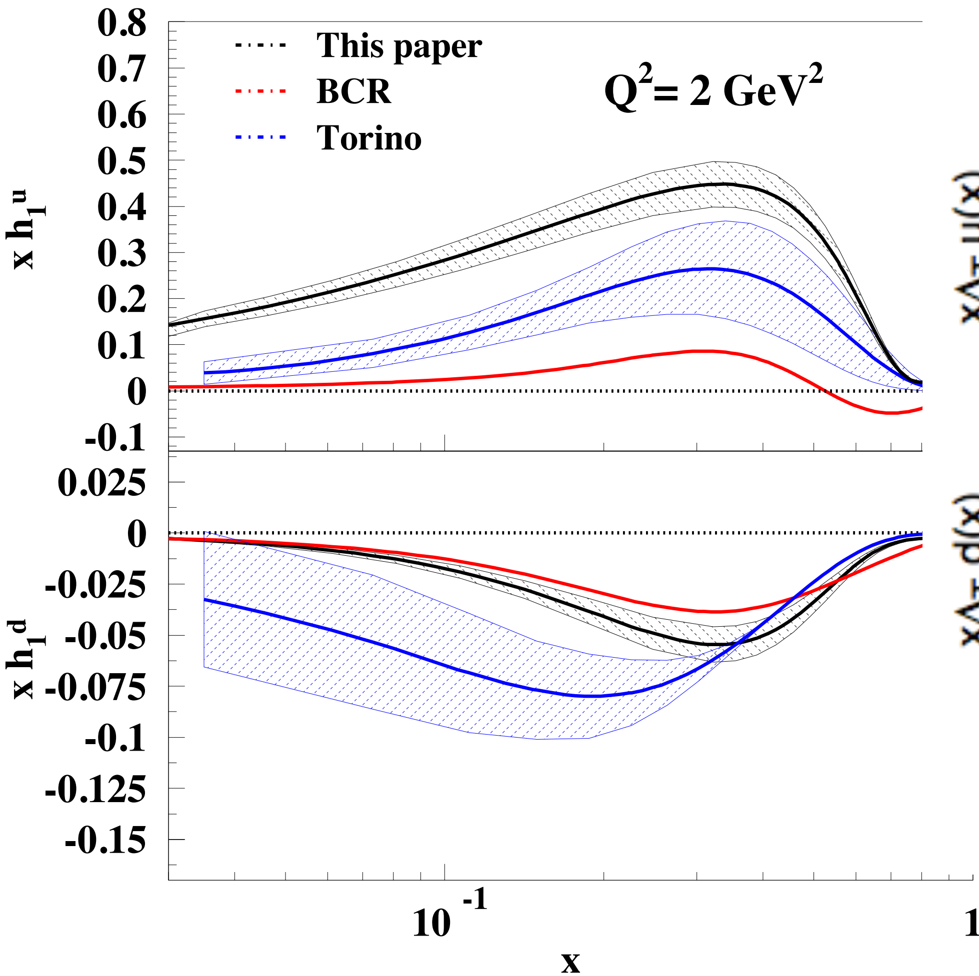
$$F_{UL}^{\sin 2\phi} = -\mathcal{N}\,\Im m\left[(f_{10}^{++})^*(f_{10}^{--}) - (f_{10}^{+-})^*(f_{10}^{-+})\right]$$

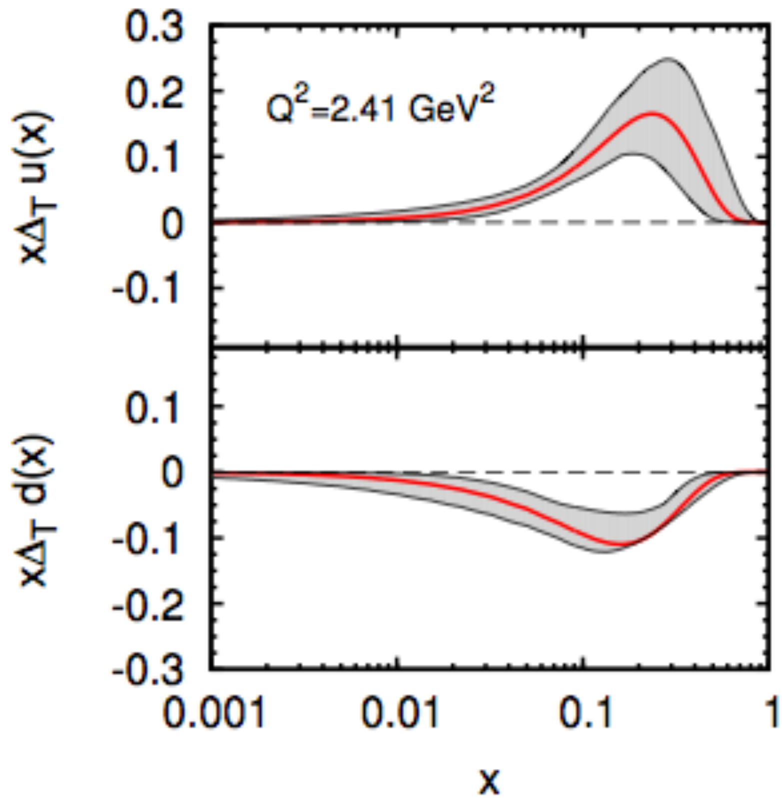
$$F_{LL}^{\cos\phi} = \frac{1}{\sqrt{2}}\mathcal{N}\,\Re e\left[(f_{00}^{+-})^*(f_{10}^{+-} - f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} + f_{10}^{--})\right]$$

$$F_{LL} = \frac{1}{2}\mathcal{N}\left[\mid f_{10}^{++}\mid^2 + \mid f_{10}^{+-}\mid^2 - \mid f_{10}^{-+}\mid^2 - \mid f_{10}^{--}\mid^2\right]$$

$$A_{LL} = \frac{\sqrt{1-\epsilon^2}\,F_{LL} + \sqrt{2\epsilon_L(\epsilon-1)}\,\cos\phi\,F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon_L F_{UU,L}}$$

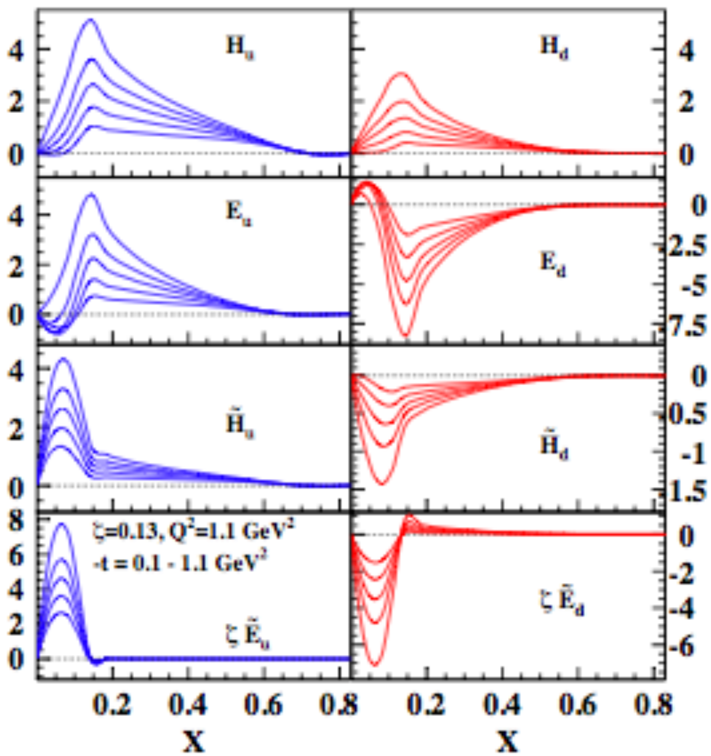
Extraction of transversity after using DVCS data via chiral even←→odd

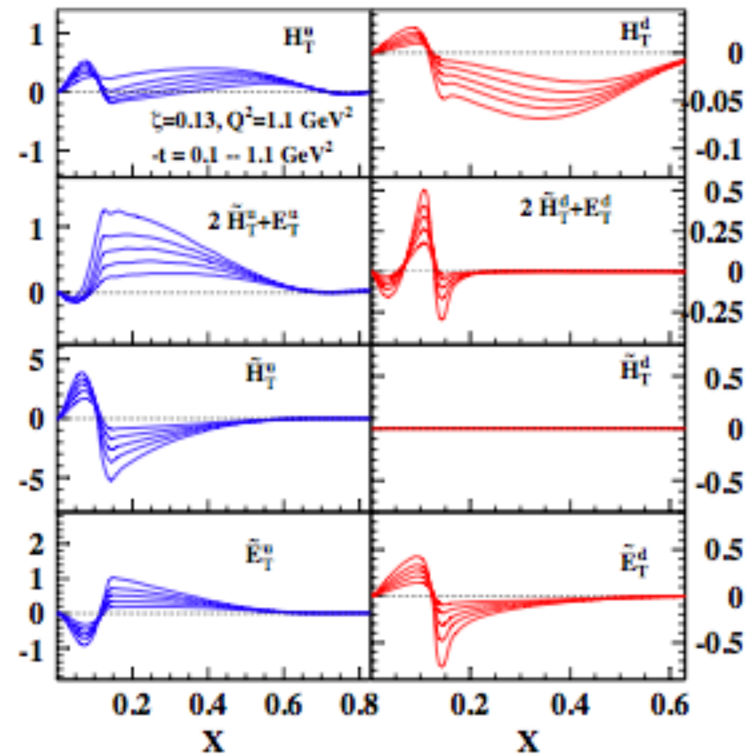




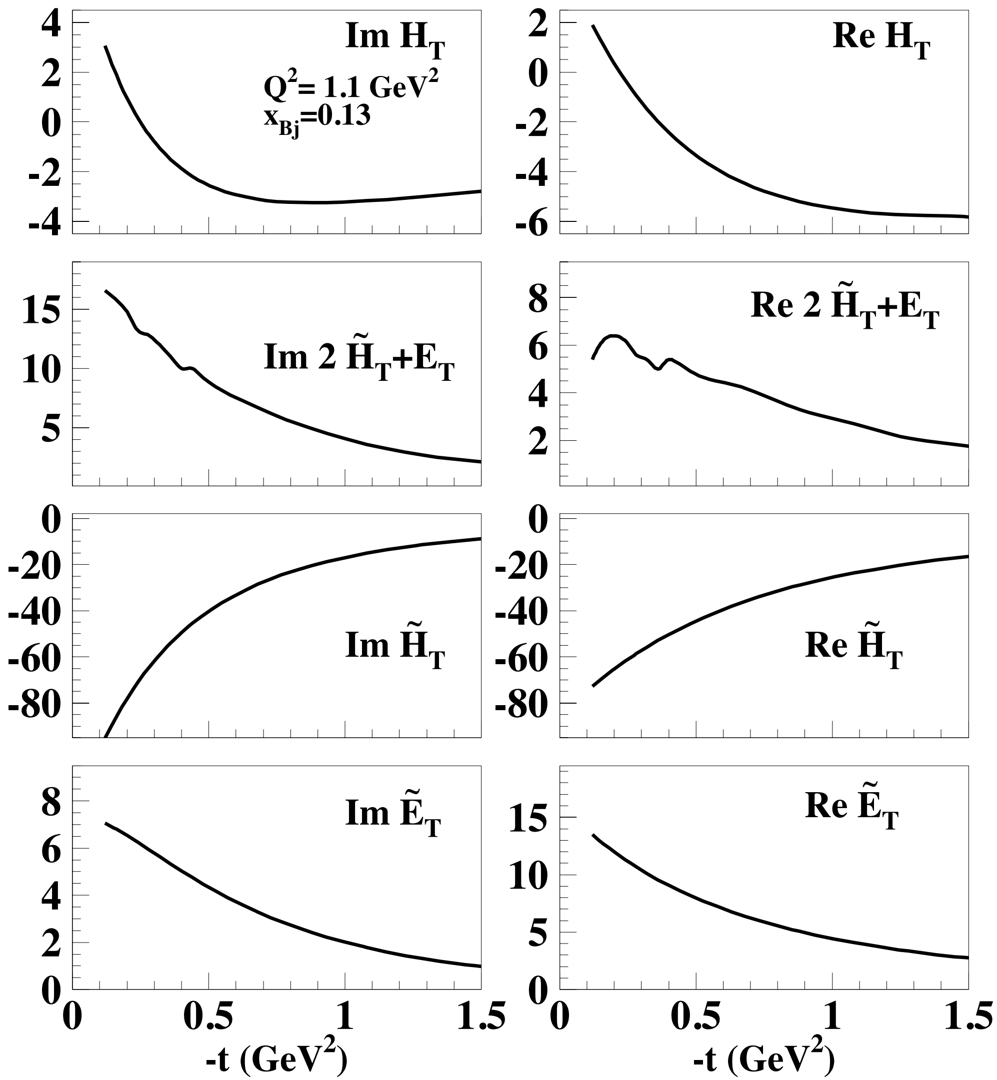
Anselmino, Boglione, et al., Phys.Rev. D87 (2013) 094019
$\delta u = 0.31_{-0.12}^{+0.16}$ $\delta d = -0.27_{-0.10}^{+0.10}$

GPDs at fixed Q^2 & ς vs. X & t

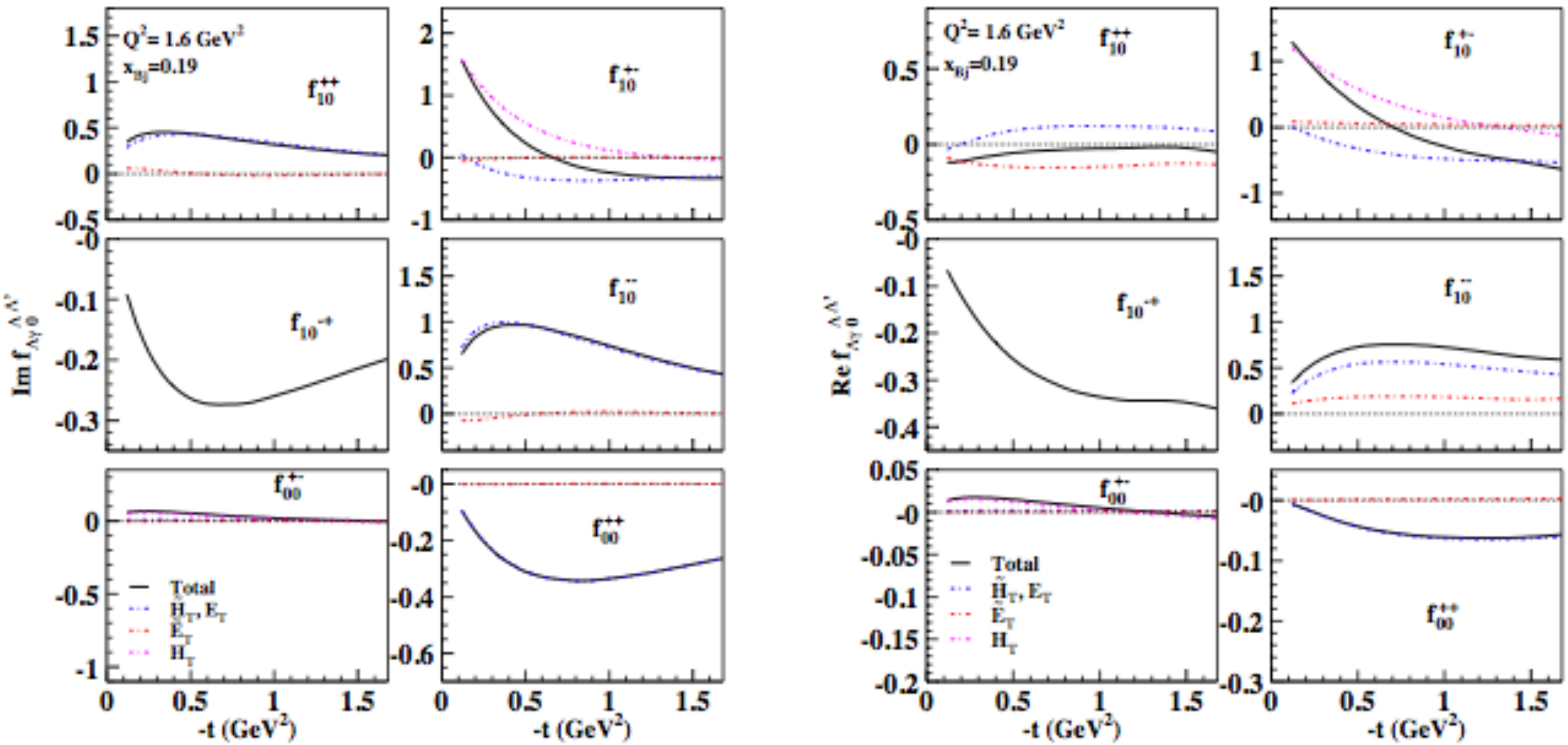




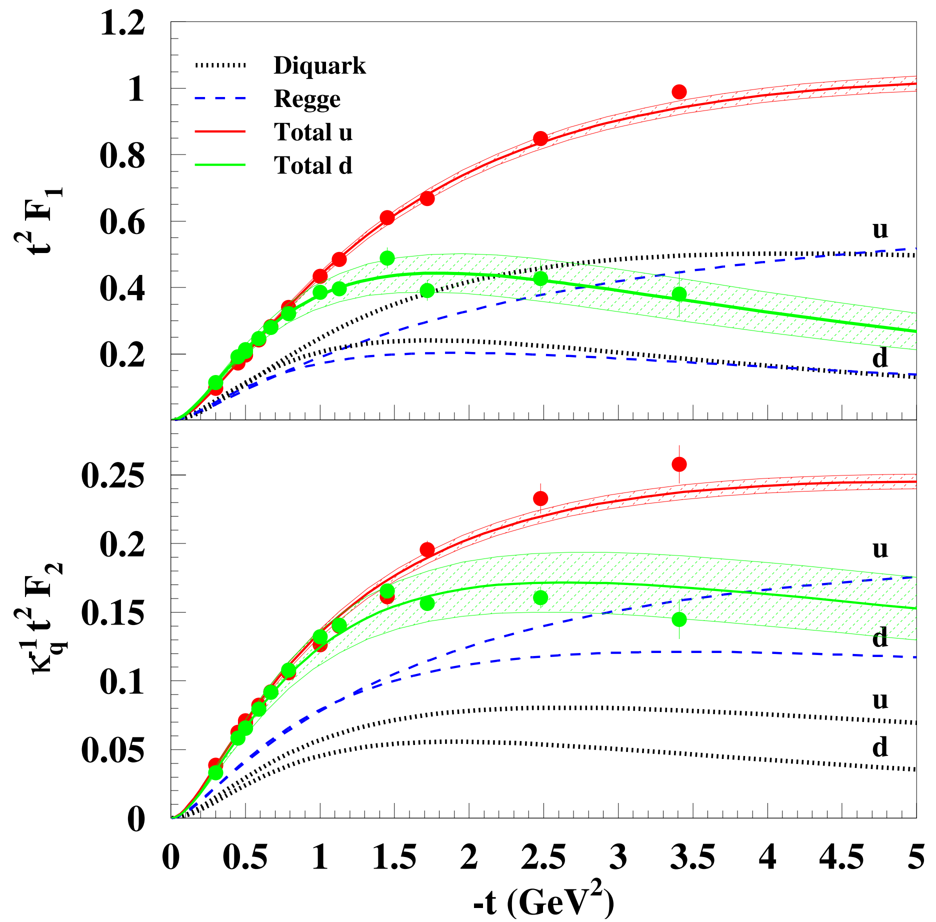
Chiral odd GPD Compton form factors



6 helicity amps for π^0 after Compton Form Factors



EM Form Factors



O.Gonzalez, GG, S.Liuti arXiv:hep/1206.1876 PRC88 065206(2013);
data: G.D. Cates, et al. PRL106,252003 (2011).

E_u & E_d
c.f. A.Bacchetta, et al.
& Ji sum rule
N.d”Hose
Disp.Rel’n B.Pasquini

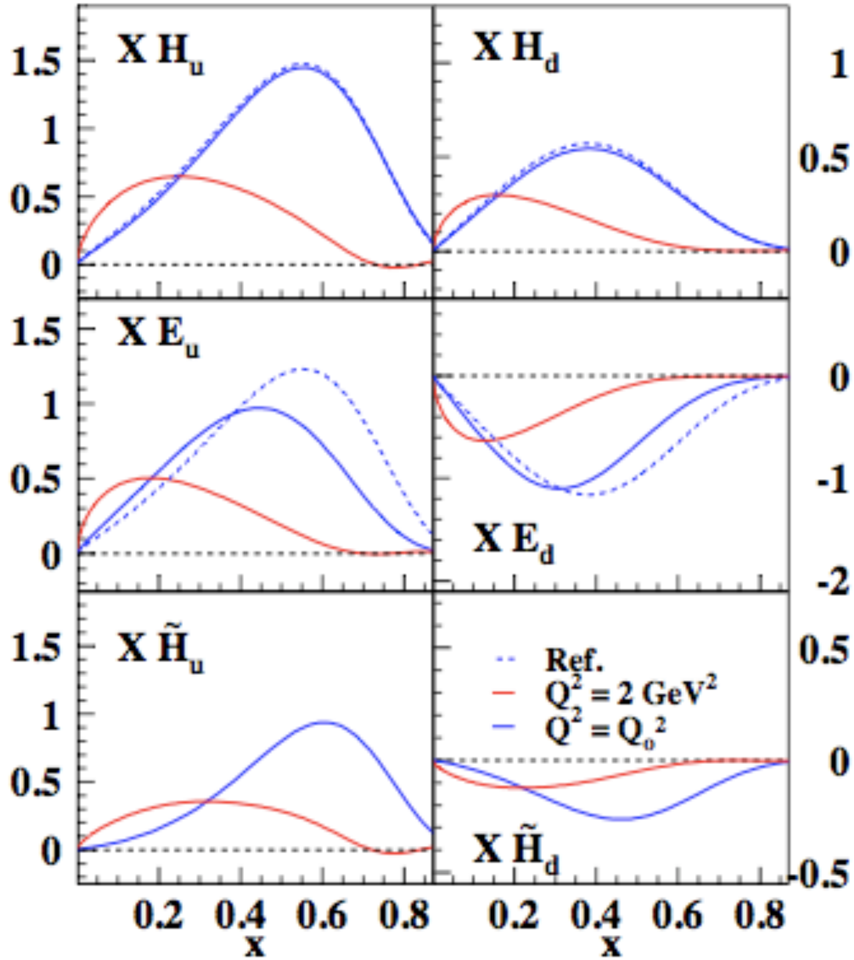


FIG. 6: (color online) GPDs $F_q(X, 0, 0) \equiv \{H_q, E_q, \tilde{H}_q\}$, for $q = u$ (left) and $q = d$ (right), evaluated at the initial scale, $Q_o^2 = 0.0936$ GeV^2, and at $Q^2 = 2$ GeV^2, respectively. The dashed lines were calculated using the model in Refs.[24, 25] at the initial scale.

Compton Form Factors → Real & Imaginary Parts

$$\mathcal{H}_q(\zeta,t,Q^2) = \int_{-1+\zeta}^{+1} dX H_q(X,\zeta,t,Q^2) \times \left(\frac{1}{X-\zeta+i\epsilon} + \frac{1}{X-i\epsilon}\right)$$

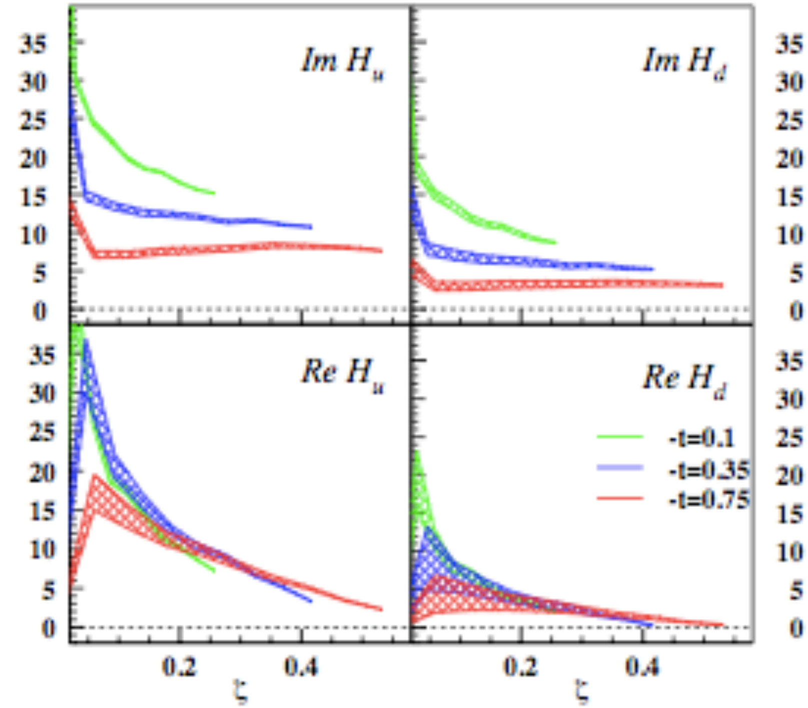


FIG. 9: (color online) Real and imaginary parts of the CFFs, $\mathcal{H}_i(\zeta,t)$, entering Eqs.(85). The CFFs are plotted vs. $x_{Bj}\equiv\zeta$, for different values of t, at $Q^2=2$ GeV^2. They are shown with the theoretical uncertainty from the parameters in TableI. Similar results are obtained for E and $\widetilde{H}$.

Summary

- Flexible parameterization for chiral even from form factors, pdfs & DVCS **R✖Dq**
- Extended **R✖Dq** via helicity amplitudes to chiral odd sector
- DVMP – π^0 many dσ ‘s & Asymmetries
- Encorporating twist 3 GPDs

Backup Slides

6 helicity amps for π^0

f_1

$$f_{10}^{++} = g_\pi^{odd}(Q)\frac{\sqrt{t_0-t}}{2M(1+\xi)^2}\left[2\widetilde{\mathcal{H}}_T + (1-\xi)\left(\mathcal{E}_T - \widetilde{\mathcal{E}}_T\right)\right]$$

f_2

$$f_{10}^{+-} = g_\pi^{odd}(Q)\,\frac{\sqrt{1-\xi^2}}{(1+\xi)^2}\left[\mathcal{H}_T + \frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]$$

f_3

$$f_{10}^{-+} = g_\pi^{odd}(Q)\,\frac{\sqrt{1-\xi^2}}{(1+\xi)^2}\,\frac{t_0-t}{4M^2}\,\widetilde{\mathcal{H}}_T$$

f_4

$$f_{10}^{--} = g_\pi^{odd}(Q)\,\frac{\sqrt{t_0-t}}{2M(1+\xi)^2}\,\left[2\widetilde{\mathcal{H}}_T + (1+\xi)\left(\mathcal{E}_T + \widetilde{\mathcal{E}}_T\right)\right],$$

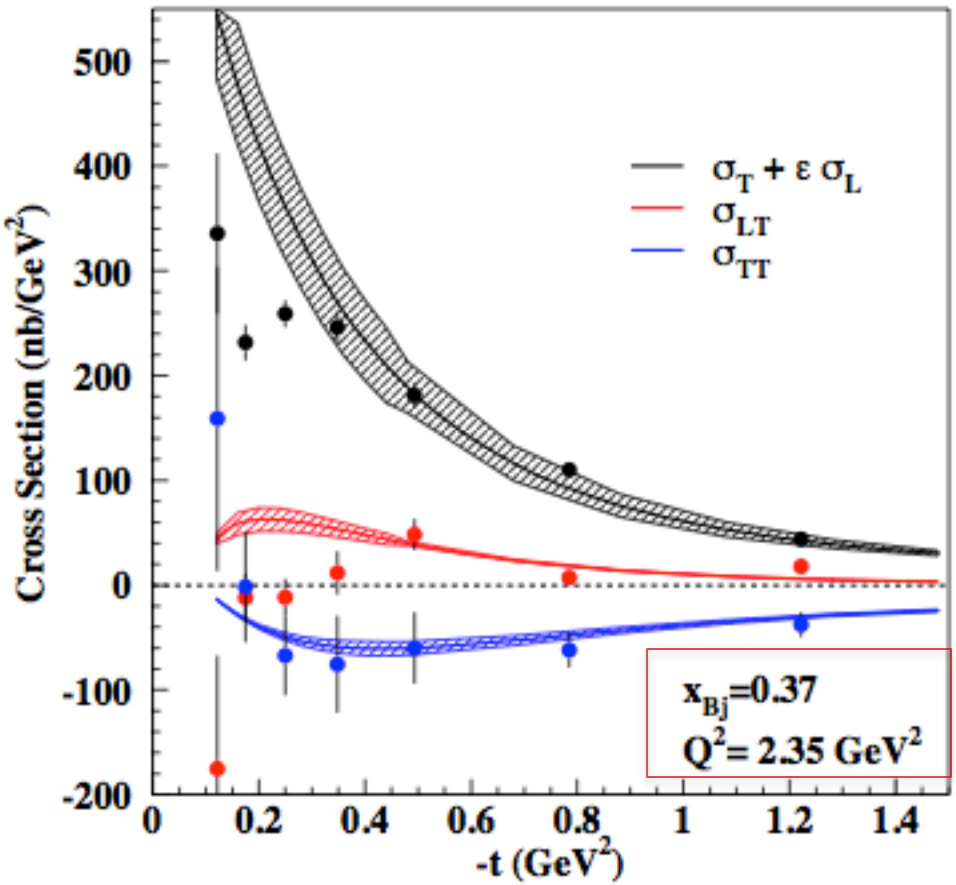
f_5

$$f_{00}^{+-} = g_\pi^{odd}(Q)\,\frac{\sqrt{1-\xi^2}}{(1+\xi)^2}\left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2}\mathcal{E}_T + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right]$$

f_6

$$f_{00}^{++} = -g_\pi^{odd}(Q)\,\frac{\sqrt{t_0-t}}{2M(1+\xi)^2}\,\left[2\widetilde{\mathcal{H}}_T + \mathcal{E}_T - \xi\widetilde{\mathcal{E}}_T\right].$$

How well do the parameters fixed with DVCS data reproduce π^0 electroproduction data?



Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010 & PRL 109, 112001 (2012)

Chiral odd integrated amplitudes

$$
\begin{aligned}
A_{+-,++} &= -A_{-+,--} = \int d^2k_\perp \phi^*_{+-}(k',P')\phi_{++}(k,P) \\
&= -\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1+\xi}{2}E_T - \frac{1+\xi}{2}\widetilde{E}_T\right] \\
A_{++,--} &= \int d^2k_\perp \phi^*_{++}(k',P')\phi_{--}(k,P) \\
&= \sqrt{1-\xi^2}\left[H_T + \frac{t_0-t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T\right] \\
A_{+-,-+} &= \int d^2k_\perp \phi^*_{+-}(k',P')\phi_{-+}(k,P) \\
&= -\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\widetilde{H}_T \\
A_{++,+-} &= \int d^2k_\perp \phi^*_{++}(k',P')\phi_{+-}(k,P) \\
&= \frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1-\xi}{2}E_T + \frac{1-\xi}{2}\widetilde{E}_T\right]
\end{aligned}
$$

The question is: how do we normalize the GPDs?

Only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x,0,0) = q_{\Uparrow}^{\uparrow}(x) - q_{\Uparrow}^{\downarrow}(x) = h_1(x)$$

Form Factors

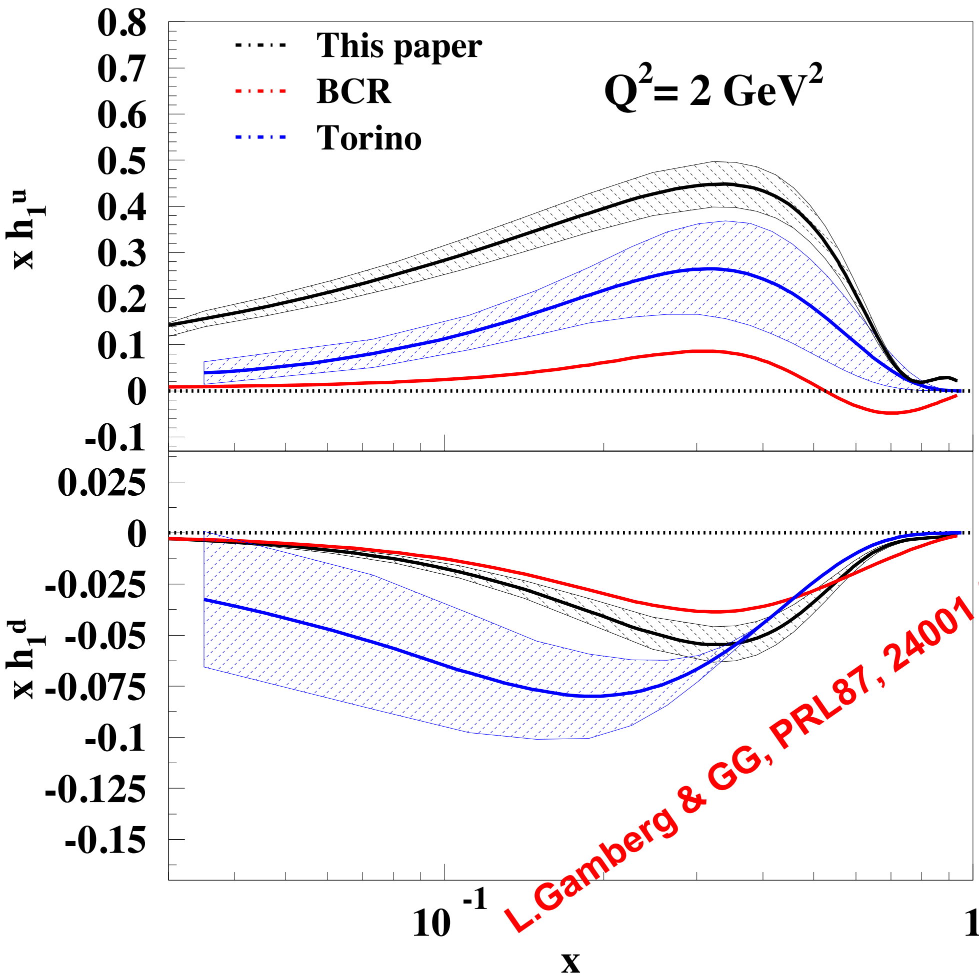
$$\int H_T(x,\xi,t)\,dx = \delta_T(t)$$

$$\int \bar{E}_T(x,\xi,t)\,dx = \int \left(2\tilde{H}_T + E_T\right) dx = \kappa_T(t)$$

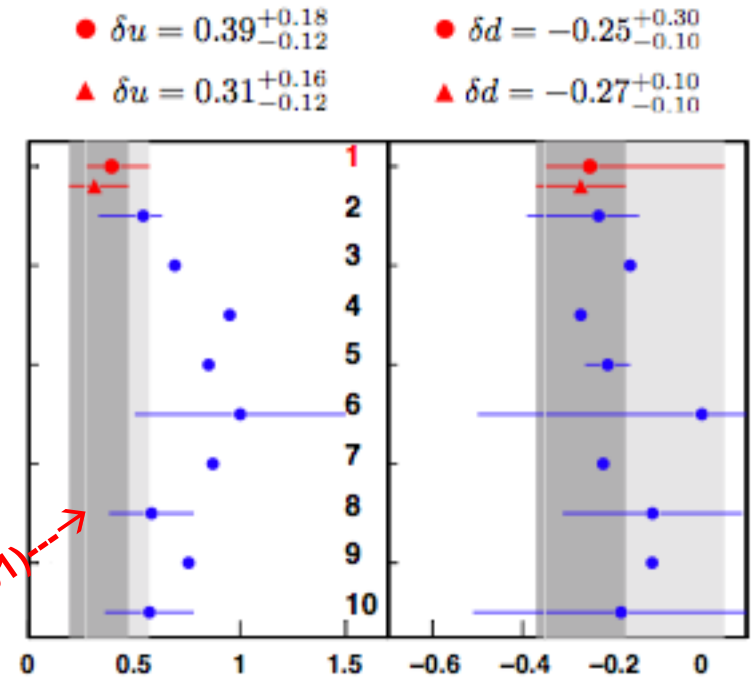
$$\int \tilde{E}_T(x,\xi,t)\,dx = 0$$

No direct interpretation of E_T

Extraction of tensor charge

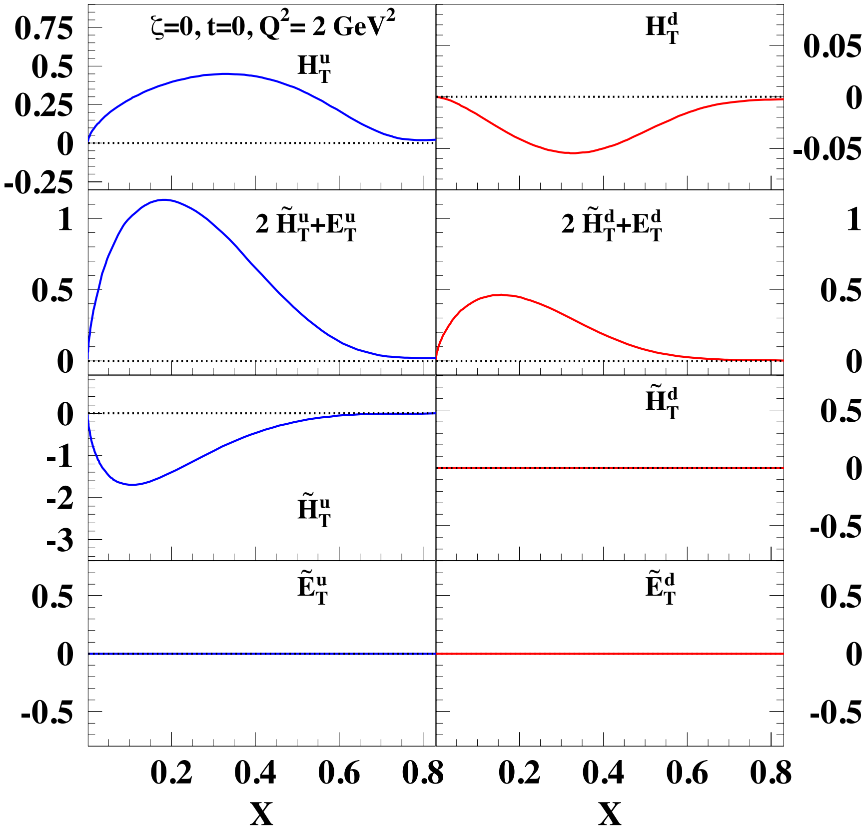


L.Gamberg & GG, PRL87, 24001 (2001)



Anselmino, Boglione, et al., Phys.Rev. D87 (2013) 094019
$\delta u = 0.31_{-0.12}^{+0.16}$ $\delta d = -0.27_{-0.10}^{+0.10}$

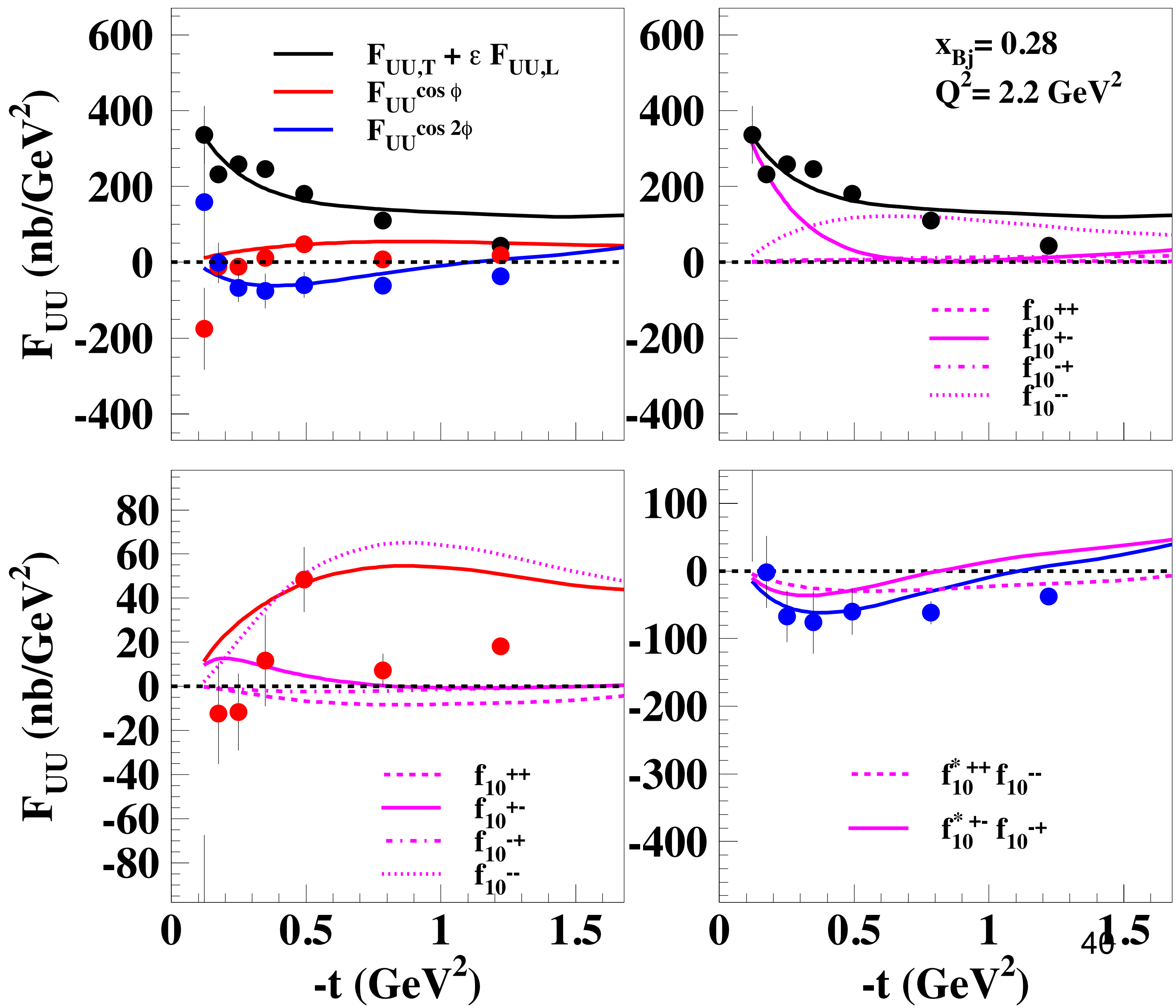
From our Reggeized form
$\delta u \approx 1.2$ $\delta d \approx -0.08$
Closer to QCD sum rule values



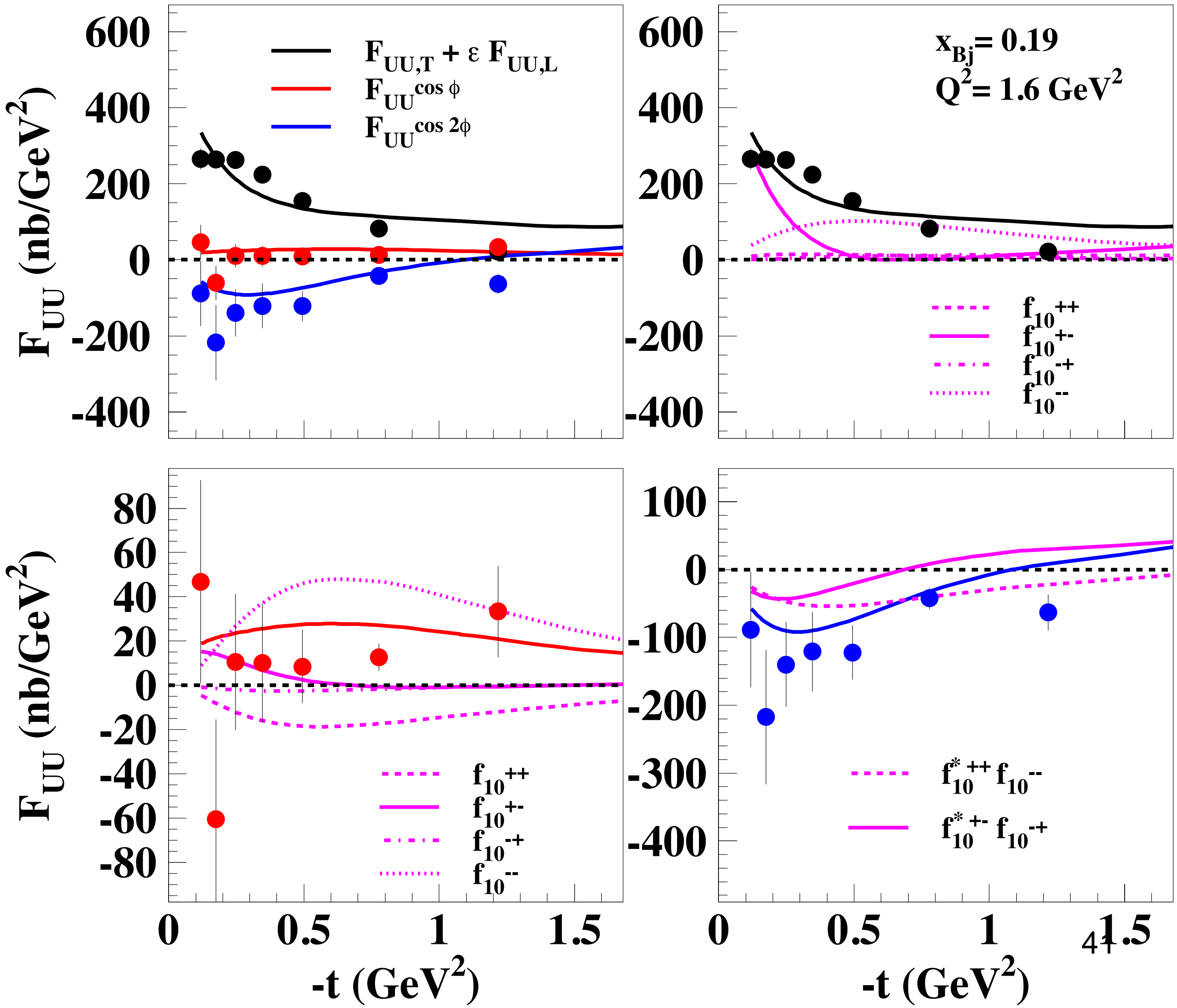
ζ=0, t=0, Q²= 2 GeV²
H_T^u
H_T^d
$2\tilde{H}_T^u+E_T^u$
$2\tilde{H}_T^d+E_T^d$
$\tilde{H}_T^u$
$\tilde{H}_T^d$
$\tilde{E}_T^u$
$\tilde{E}_T^d$
0.75
0.5
0.25
0
-0.25
0.05
0
-0.05
1
0.5
0
-1
-2
-3
-0.5
0.2
0.4
0.6
0.8
X

Comparing to other models

- The t-> 0 feature for us is that f_2 dominates & it is driven by H_T. But f_1 & f_4 also contribute as $\sim\sqrt{(t_0-t)}$, however weaker.
- f_1 & f_4 are not equal in magnitude, especially vs. ζ or ξ.
- In $A_{LL} \sim |f_1|^2 + |f_2|^2 - |f_3|^2 - |f_4|^2$
- c.f. Goloskokov & Kroll – different dominant amps.



F_UU,T + ε F_UU,L
F_UU^cos φ
F_UU^cos 2φ
x_Bj = 0.28
Q^2 = 2.2 GeV^2
f_10^++
f_10^+-
f_10^-+
f_10^--
f*_10^++ f_10^--
f*_10^+- f_10^-+
F_UU (nb/GeV^2)
-t (GeV^2)
600
400
200
0
-200
-400
80
60
40
20
-20
-40
-60
-80
100
-100
-300
0.5
1
1.5



F_UU,T + ε F_UU,L
F_UU^cos φ
F_UU^cos 2φ
x_Bj = 0.19
Q^2 = 1.6 GeV^2
f_10^++
f_10^+-
f_10^-+
f_10^--
f*_10^++ f_10^--
f*_10^+- f_10^-+
F_UU (nb/GeV^2)
-t (GeV^2)

CLAS π^0 arXiv:1206.6355v1 [hep-ex] PRL

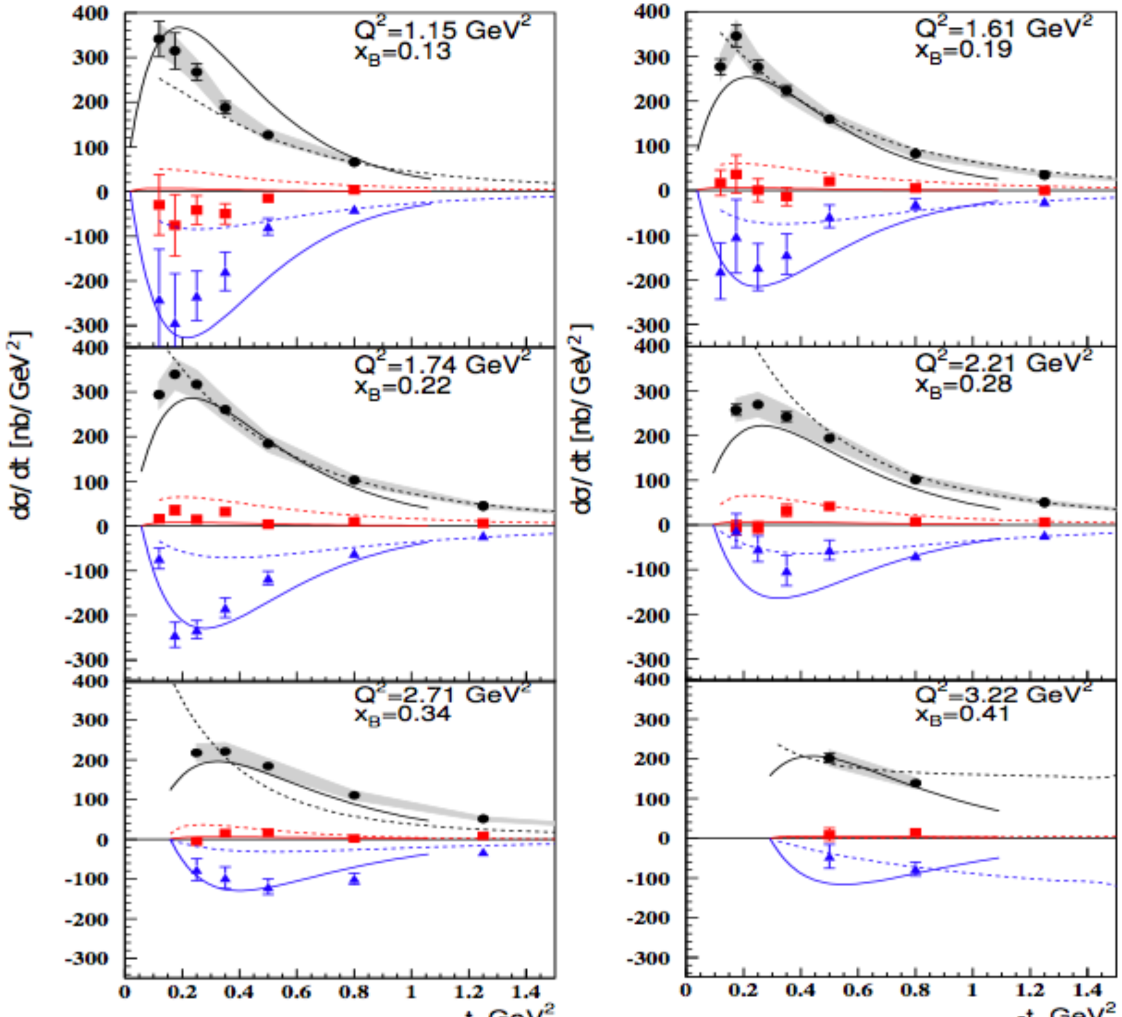
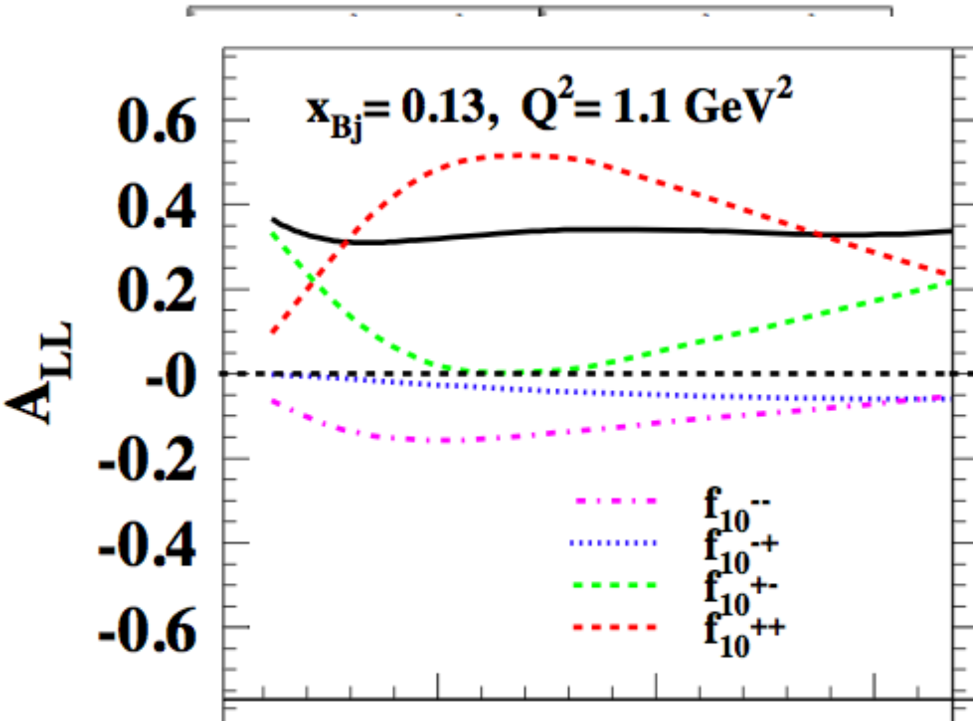
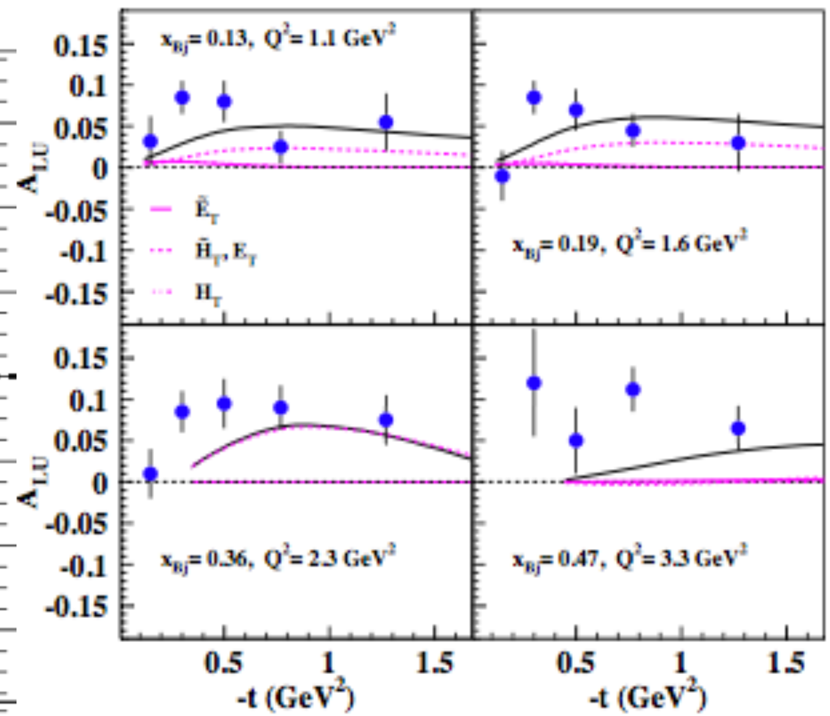


FIG. 2: The extracted structure functions vs. t for the bins with the best kinematic coverage and for which there are theoretical calculations. The data and curves are as follows: black-$\sigma_U (= \sigma_T + \epsilon\sigma_L)$, blue-$\sigma_{TT}$, and red-σ_{LT}. The shaded bands reflect the experimental systematic uncertainties.The curves are theoretical predictions produced with the models of Refs. [14] (solid) and [15] (dashed).

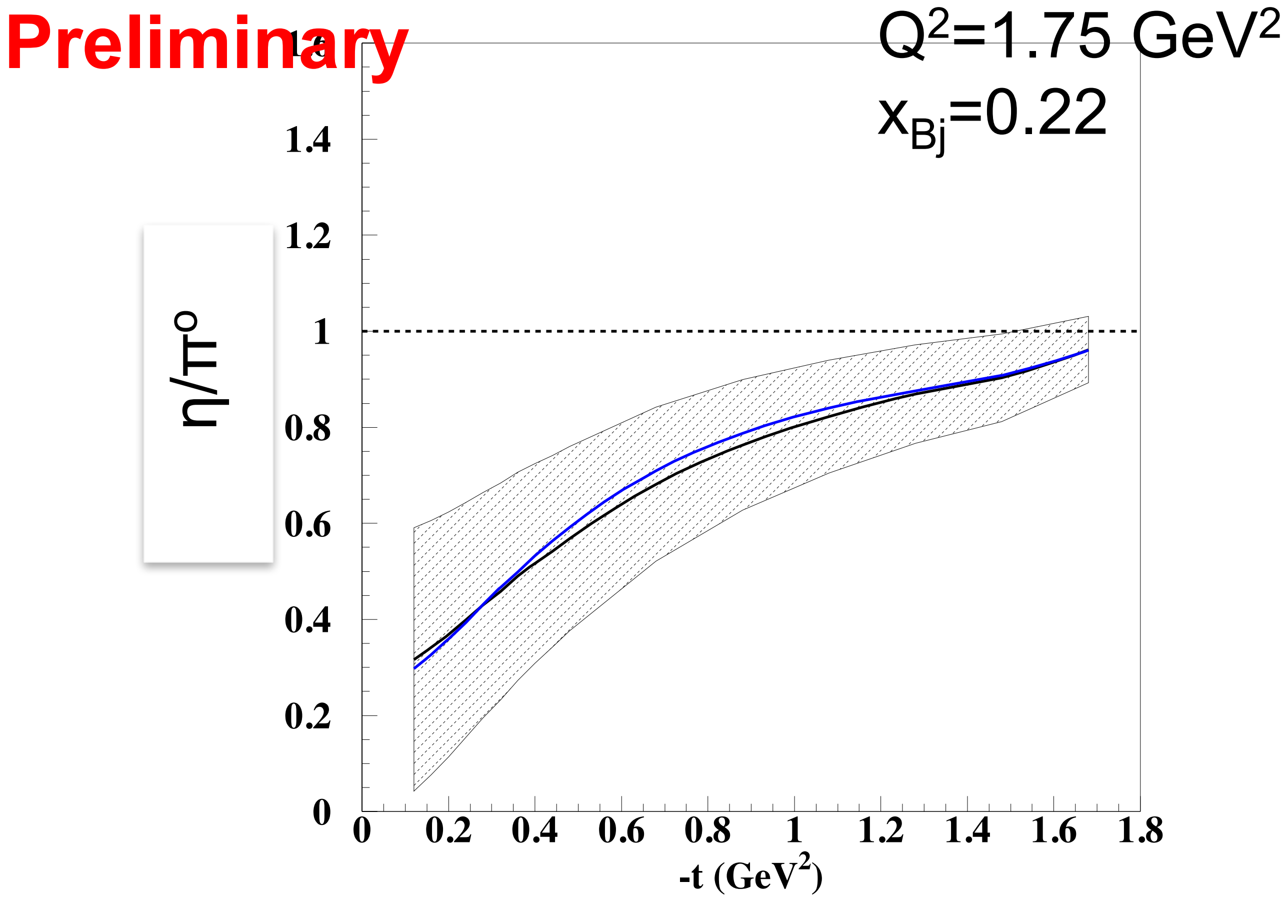
A_{LL} & A_{LU}





$$A_{LL} = \frac{\sqrt{1-\epsilon^2}\, F_{LL} + \sqrt{2\epsilon_L(\epsilon-1)}\, \cos\phi\, F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon_L F_{UU,L}}$$

$$A_{LU} = \sqrt{2\epsilon_L(\epsilon-1)}\, \frac{F_{LU}^{\sin\phi}}{F_{UU,T} + \epsilon_L F_{UU,L}}$$



Preliminary
Q2=1.75 GeV2
xBj=0.22
η/π°
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8
-t (GeV2)

Summarizing chiral odd for π^0

Based on our analysis we expect the following behaviors to approximately appear in the data.

(i) The order of magnitude of the various terms approximately follows a sequence determined by the inverse powers of Q and the powers of $\sqrt{t_o - t}$: $d\sigma_T/dt \geqslant d\sigma_{TT}/dt \geqslant d\sigma_{LT/L'T}/dt \geqslant d\sigma_L/dt$.

(ii) $d\sigma_T/dt$ is dominated by $\mathcal{H}_T$ at small t, and governed by the interplay of $\mathcal{H}_T$ and $\overline{\mathcal{E}}_T$ at larger t.

(iii) $d\sigma_L/dt$ and $d\sigma_{LT}/dt$ are directly sensitive to $\mathcal{H}_T$.

(iv) $d\sigma_{TT}/dt$ and $d\sigma_{L'T}/dt$ contain a mixture of GPDs. They will play an important role in singling out the less known terms, $\overline{E}_T$, E_T and $\widetilde{E}_T$.

The interplay of the various GPDs can already be seen by comparing to the Hall B data [4] shown in figure 2.

One can see, for instance, that the ordering predicted in (i) is followed, and that $d\sigma_T/dt$ exhibits a form factor-like fall off of $\mathcal{H}_T$ with $-t$.

Having fit other data we predicted Hermes data

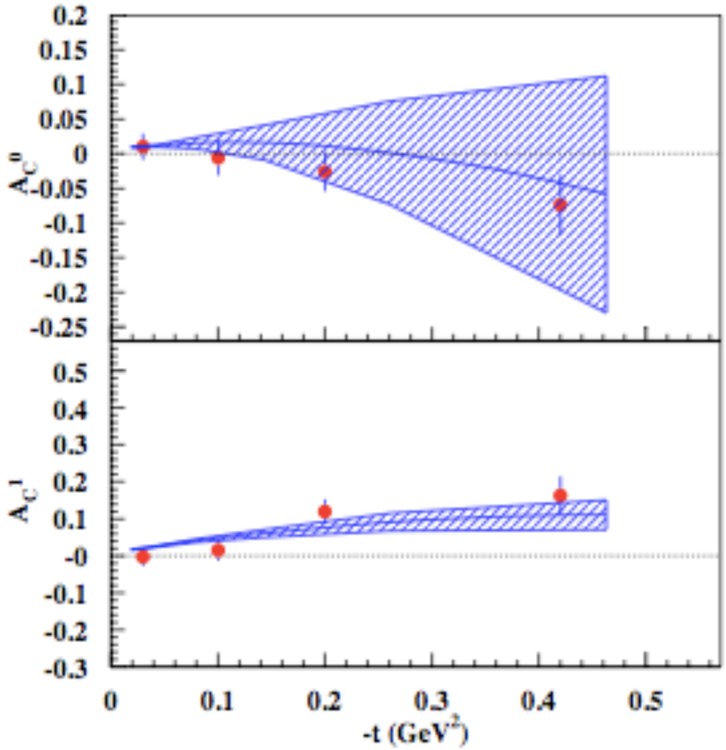


FIG. 20: Coefficients of the beam charge asymmetry, A_C, extracted from experiment [52, 53]. The lower panel is the coefficient for the $\cos\phi$ dependent term in Eq.(82), while the upper panel is the $\cos\phi$ independent term.

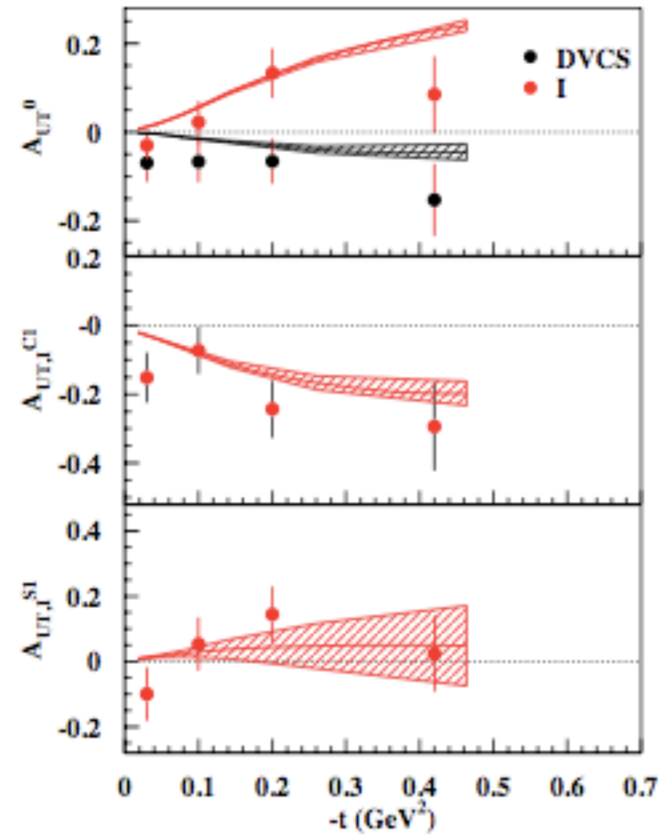


FIG. 21: Coefficients of the b[illegible]etry, A_{UT}, extracted from experiment [52, 53]. The upper panel shows the terms E and F from Eqs.(83) and (84), respectively; the middle panel shows G, and the lower panel H, both in Eq.(84). The curves are predictions obtained extending our quantitative fit of Jefferson lab data to the Hermes set of observables.

Reggeization via spectator or diquark mass formulation

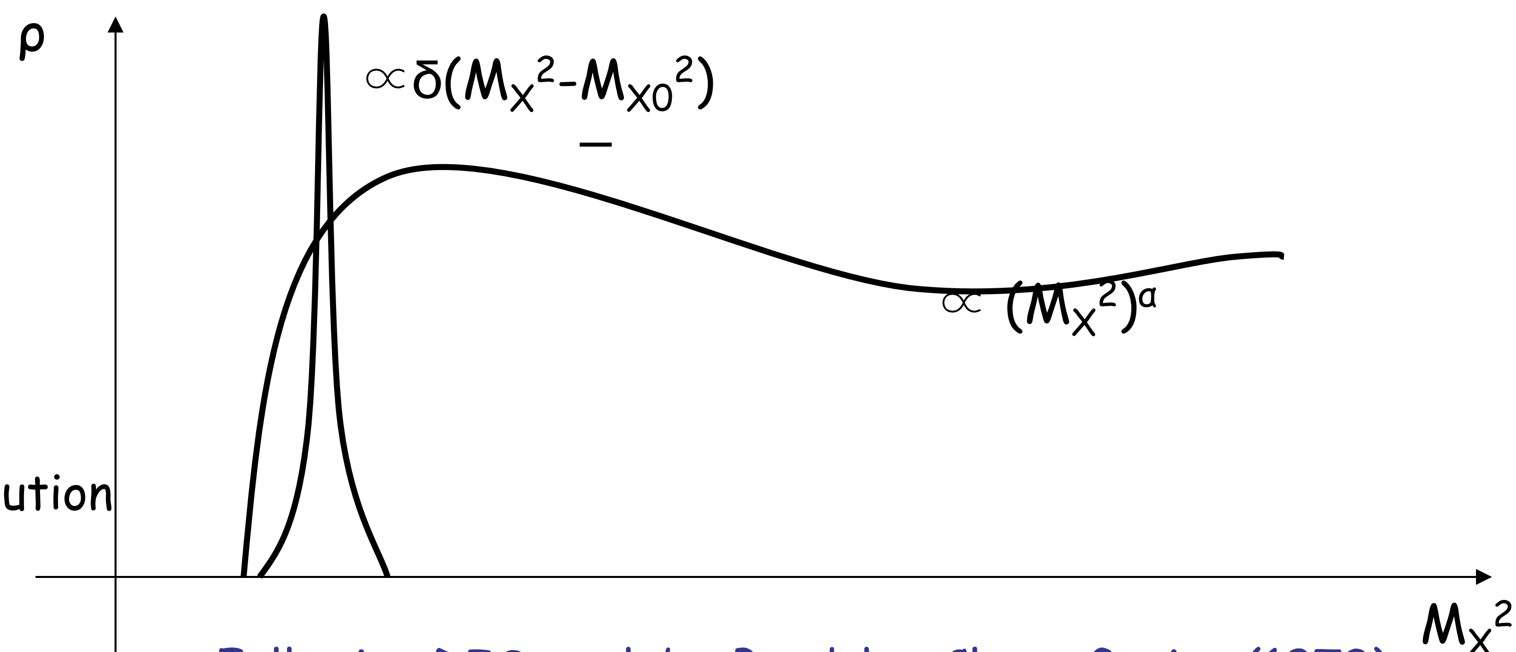
Where does the Regge behavior come from?

$$G^{\Lambda^2}_{M_X}(X,\zeta,t) = \int d^2\mathbf{k}_\perp \int dM_X^2\, \rho(M_X^2)\, \frac{\phi(k^2,\Lambda^2)}{k^2 - M_X^2}\, \frac{\phi(k'^2,\Lambda^2)}{k'^2 - m^2} \mathcal{N}_{\mathcal{H}(\mathcal{E},\tilde{\mathcal{H}},\tilde{\mathcal{E}})} \qquad \zeta \geq X$$

Diquark spectral function

$$F(X,\zeta,t) = \mathcal{N} G^{M_\Lambda}_{M_X,m}(X,\zeta,t)\, R_p^{\alpha,\alpha'}(X,\zeta,t)$$

"Regge"

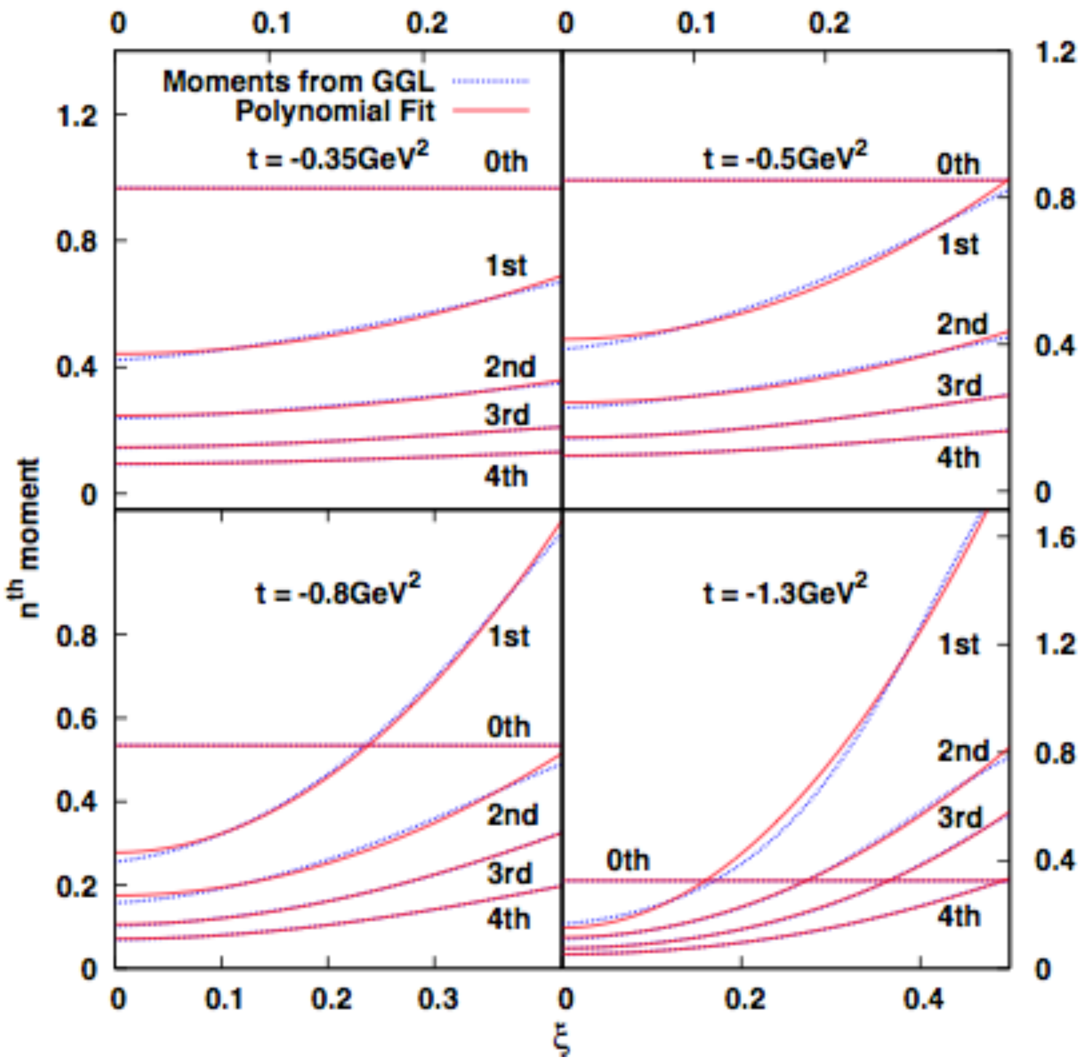


+ Q^2 Evolution

Following DIS work by Brodsky, Close, Gunion (1973)

Polynomiality!

Goldstein et al. arXiv:1012.3776



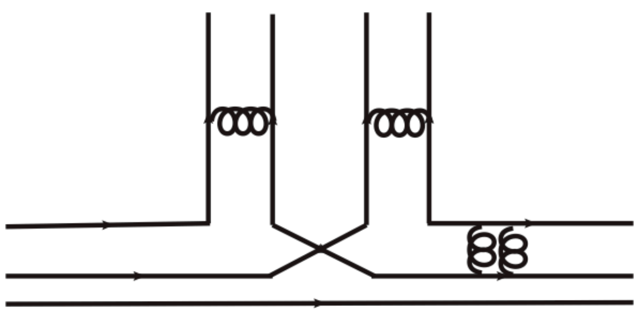
$$R_p^{\alpha,\alpha'} = X^{-[\alpha+\alpha'(X)t+\beta(\zeta)t]},$$

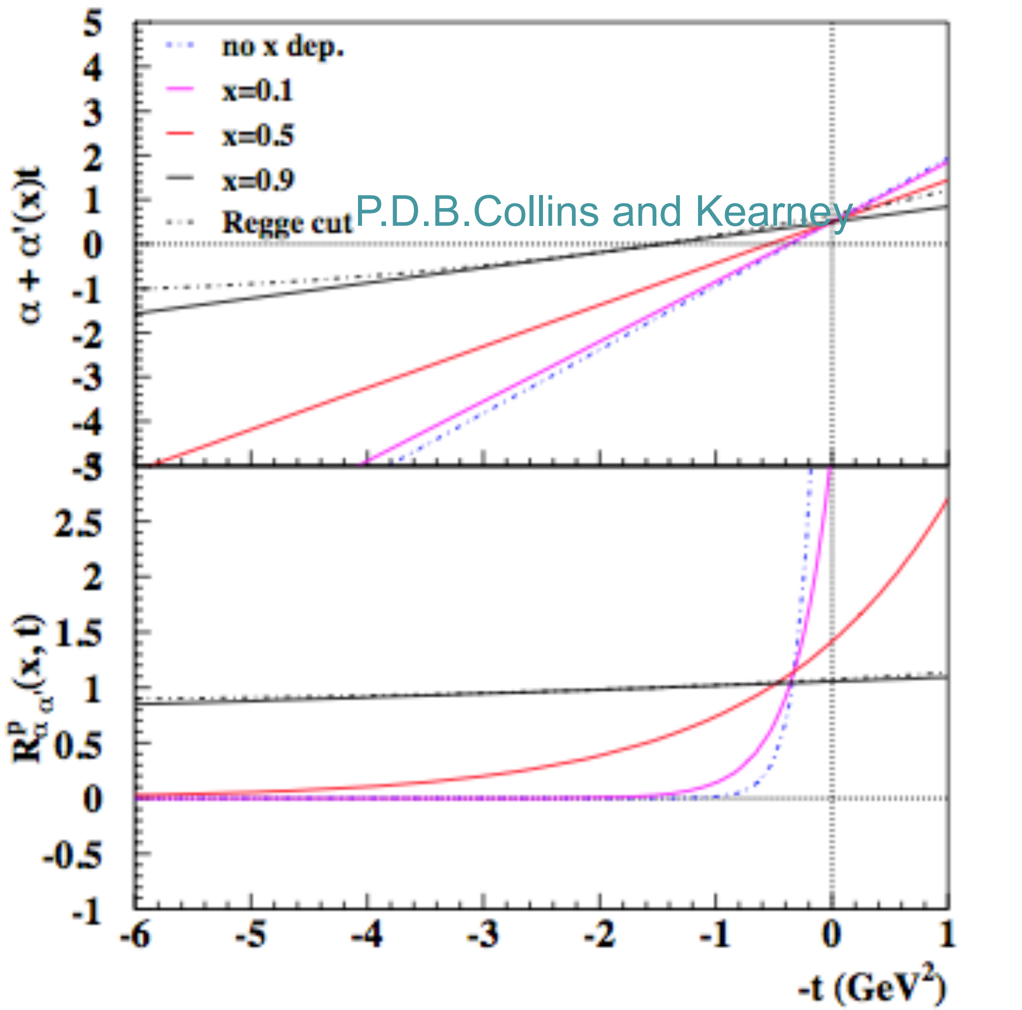
~Initially introduced by Radyushkin, Burkardt, … to account for coordinate space behavior

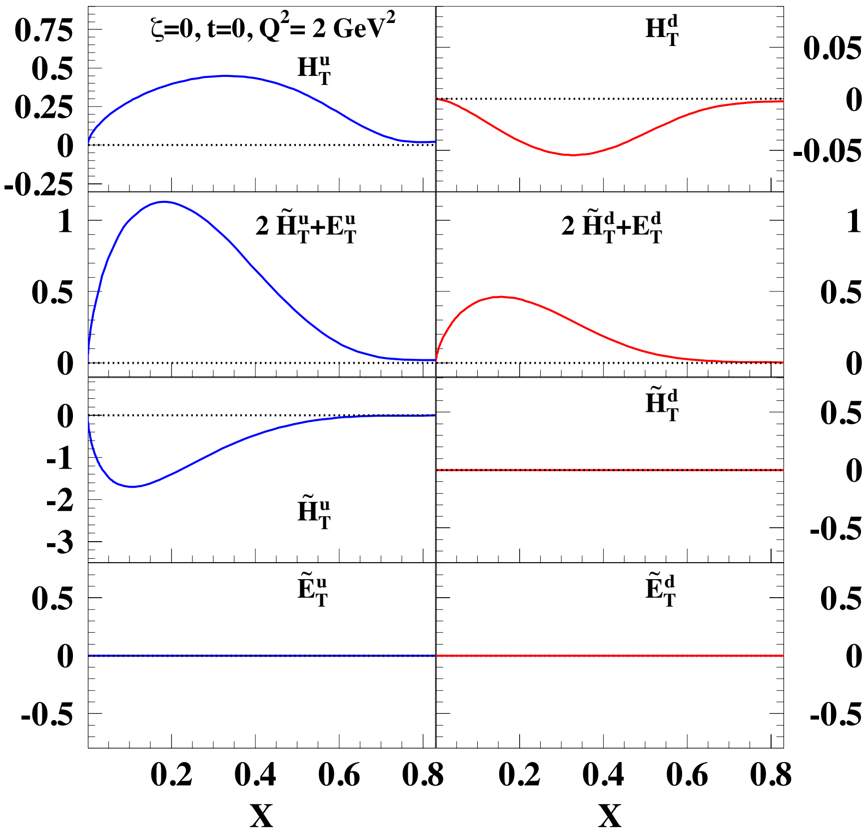
$$\alpha'(X) \equiv \alpha'(1-X)^p \qquad \beta = 0.$$

Now see as effectively taking into account Regge cuts

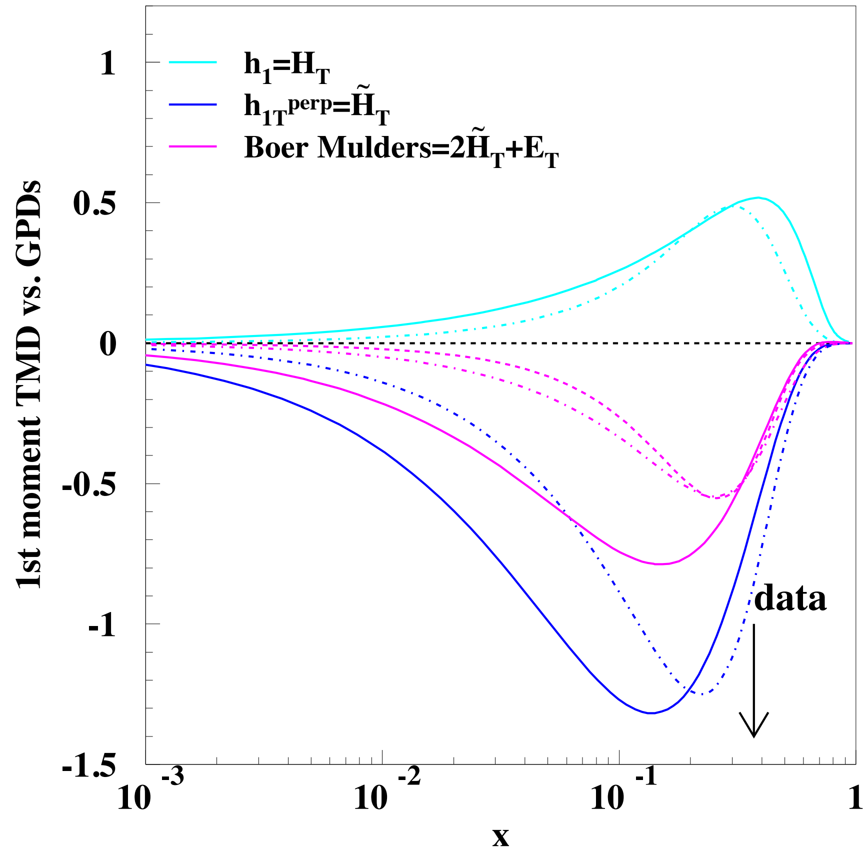
O. Gonzalez Hernandez, GG, S. Liuti arXiv 1206.1876







ζ=0, t=0, Q^2= 2 GeV^2
H_T^u
H_T^d
2 H̃_T^u+E_T^u
2 H̃_T^d+E_T^d
H̃_T^u
H̃_T^d
Ẽ_T^u
Ẽ_T^d
X
X



h_1=H_T
h_1T^perp=H̃_T
Boer Mulders=2H̃_T+E_T
1st moment TMD vs. GPDs
data
x